Weak Signal Detection Based on Lifting Wavelet Threshold Denoising and Multi-Layer Autocorrelation Method

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Abstract —To solve the problem that the weak signal is difficult to detect under a strong background noise, a detection method based on lifting wavelet threshold denoising and multi-layer autocorrelation method is proposed. Firstly, the original signal is denoised by lifting wavelet threshold to improve the signal-to-noise ratio. Secondly, the multi-layer autocorrelation function of the noise-reconstructed signal is calculated, and its time-frequency signature are analyzed. Finally, the combined algorithm is used on weak signals with low signal-to-noise ratio to extract weak signal features. Simulation and experimental results demonstrate that the proposed method can detect weak signal features buried in the heavy noise effectively. The proposed method is compared with the traditional noise reduction method, which reflects its effectiveness and superiority.

Index Terms—Weak signal detection, lifting wavelet, threshold function, multi-layer autocorrelation

I. Introduction

The detection of weak signals in the strong background noise is a research hotspot in the field of signal processing, as well as an important problem in fault diagnosis, sonar detection, communication transmission, biomedicine, and other engineering areas. The weak signal has a low signal-to-noise ratio (SNR), which is due to the low amplitude of the feature signal itself, in addition, strong noise interference will also make the SNR lower. The core of weak signal detection is to apply various signal processing methods to enhance the SNR of the target signal.

Wavelet transform has received much attention from scholars by virtue of its variable scale and adaptive matching features [1]. For the shortcomings of the traditional wavelet algorithm, which is computationally complex and time-consuming, Sweldens *et al.* [2] proposed lifting wavelet threshold denoising based on wavelet transform. The lifting wavelet is detached from the dependence of the classical wavelet on the Fourier transform, and the noise reduction performance is further optimized. It also can be completely reconstructed, which is more rapid and effective. Many scholars have done a series of improvement studies on lifting wavelets, and successfully applied them to weak signal detection, fault diagnosis, and other fields [3]-[5]. However, the denoising effect of both classical wavelet and lifting wavelet depends heavily on the characteristics of the signal itself and the parameters of the denoising algorithm. For signals with low SNR, wavelet threshold method or lifting wavelet threshold method often needs to be combined with other methods to achieve the optimal denoising effect. In signal processing, the autocorrelation function can describe the dependence between the values of the signal at different moments and is an effective mathematical tool to find repetitive patterns or to identify vanishing fundamental frequencies implicit in the harmonic frequencies of the signal. The frequency components of the signals in engineering are often extremely complex. When the noise interference is weak, a single autocorrelation process can obtain a high SNR, while when the noise interference is strong, the degree of improvement is often very limited, so it is necessary to carry out multi-layer autocorrelation processes. Multi-layer autocorrelation can remove random nonperiodic Gaussian white noise while preserving the effective periodic signal in the mixed signal. Theoretically, the more times the autocorrelation is done, the higher the SNR obtained, so that the weak signals that are buried in the noise can be detected. Therefore, this method is also widely utilized in the denoising step of weak signal detection [6].

Based on the study of traditional wavelet threshold denoising, this paper proposes a novel method for detecting weak signals under strong noise background by introducing lifting wavelet threshold denoising method into the process of weak signal noise reduction and combining it with multi-layer autocorrelation, and verifies the effectiveness of the method by using two engineering application examples.

The rest of the paper is organized as follows. Section II introduces the basic theory of lifting wavelet thresholding denoising and multi-layer autocorrelation, and proposes an improved wavelet threshold function, which provides theoretical support for the subsequent noise reduction process of the signal. Section III describes the procedure of the whole detection algorithm. Section IV and Section

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V use one simulation example and two experimental examples to verify the effectiveness of the proposed method, respectively. Finally, conclusions are drawn in Section VI.

II. Lifting Wavelet Threshold Denoising

A. The Principle of Lifting Wavelet Transform

Wavelet analysis has a strong de-correlation, resulting in the energy of the effective signal being concentrated in a small number of larger wavelet coefficients in the wavelet domain, while the corresponding wavelet coefficients of the noise are small, and the corresponding coefficients of the noise still satisfy the Gaussian white noise distribution. Based on this characteristic, the time domain signal or spatial domain signal can be converted into the wavelet domain by lifting wavelet decomposition. The noise reduction process can be achieved by setting an appropriate threshold value to filter out the noisy wavelet coefficients and subsequently reconstructing the threshold-filtered wavelet coefficients. The lifting scheme process includes three stages of decomposition, prediction, and update [7].

B. Parameter Selection

1) The choice of wavelet basis functions

In the wavelet transform process, the choice of wavelet basis function directly determines the noise reduction effect. At present, dbN and symN series wavelets are more widely used, these two series wavelets have superior regularity and better sensitivity for the detection of signal mutation points. To compare the performance of various wavelets, a simulated sinusoidal signal y=sin(2π 20*t*) with a SNR of -10dB is constructed. The number of decomposition layers is set to 3. The threshold function selects soft threshold. The SNR of reconstructed signal, root mean square error (RMSE) and correlation coefficient of original signal are used as evaluation indexes. The SNR, RMSE and correlation coefficient ρ are calculated as follows [8]:

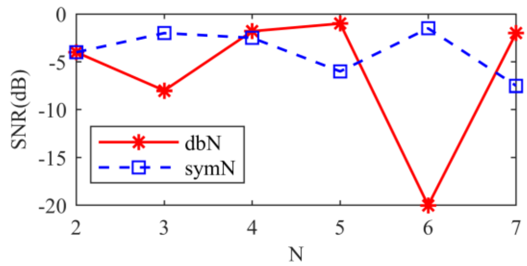
$$\mathrm{SNR} = 10\lg\frac{\sum_{n=1}^{N} S^2(n)}{\sum_{t=1}^{N}[S(n)-\bar{S}(n)]^2} \tag{1}$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}[S(n)-\bar{S}(n)]^2} \tag{2}$$

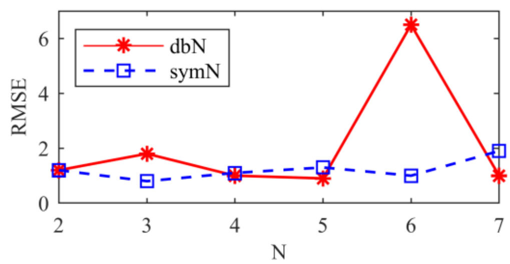
$$\rho = \frac{\sum_{n=1}^{2N-1}\bar{S}(n)S(n)}{\sqrt{\sum_{n=1}^{2N-1}\bar{S}^2(n)\sum_{n=1}^{2N-1}S^2(n)}} \tag{3}$$

where N is the number of sampling points of the signal, $S(n)$ is the original signal sequence, $\bar{S}(n)$ is the reconstructed signal sequence after lifting denoising.

The SNR, RMSE and correlation coefficient of the wavelet reconstruction signal is shown in Fig. 1. The horizontal coordinates indicate the type of wavelet, including both dbN and symN series. The results show that when db5 wavelet basis function is selected, the reconstructed signal has the highest output SNR, the highest correlation coefficient with the source signal, and the lowest RMSE. This demonstrates that the db5 wavelet has the best noise reduction and the lowest distortion. Therefore, without special instructions, db5 wavelets are used in this paper.



(a)



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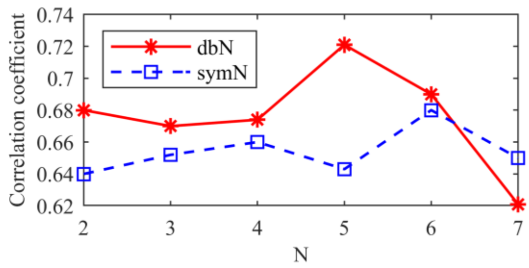


(c)

Fig. 1. Denoising effects of various types of wavelets. (a) SNR; (b) RMSE; (c) Correlation coefficient.

2) Selection of the threshold value

According to the reference [9], among the generic threshold estimation, extreme value threshold estimation, unbiased likelihood estimation, and heuristic estimation, the generic threshold estimation has a better denoising effect. The expressions are as follows:

$$\lambda = \sigma\sqrt{2lnN} \tag{4}$$

where N is the signal length and σ is the standard deviation of Gaussian white noise, which is estimated as shown in (5).

$$\sigma = med(|d_1(k)|)/0.6745 \tag{5}$$

where, $d_1(k)$ is the first wavelet coefficient sequence after lifting wavelet decomposition of the original signal, and *med* means taking the median calculation.

3) Selection of the threshold function

The basic principle of threshold function is to filter the wavelet coefficients that contain noise coefficients, and retain or shrink the large wavelet coefficients. Both the smoothness and distortion of the reconstructed signal depend on the threshold function. Therefore, the selection of the threshold function is one of the key procedures to lifting wavelet threshold denoising. Traditional thresholding functions include hard and soft thresholding functions. In this paper, based on the characteristics of these two traditional threshold functions, an improved threshold function is proposed with the following expressions:

$$\hat{w}_{j,k} = \begin{cases} w_{j,k}\text{-}sign(w_{j,k})\bullet\lambda(1-\mu)^2, |w_{j,k}| \ge \lambda \\ 0, |w_{j,k}| < \lambda \end{cases} \tag{6}$$

where μ is the adjustment factor of the improved threshold function, which takes values in the range [0, 1]. When μ tends to 0, the improved threshold function is close to the soft threshold function, and when μ tends to 1, it approximates the hard threshold function. Taking the threshold values λ=1, μ=0.5, the function schematic of the three methods is shown in Fig. 2.

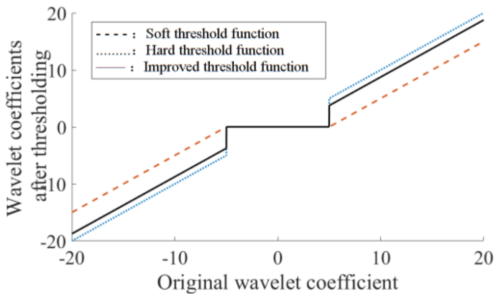


Fig. 2. Schematic diagram of the threshold function.

It can be obtained that in the hard thresholding function, the reconstructed signal may produce oscillations. Although the wavelet coefficients estimated by the soft-threshold method have better continuity, when the absolute value of the wavelet coefficients is higher than the threshold value, it leads to deviations in λ and μ, which seriously affects the approximation of the reconstructed signal to the real signal [10]. Compared with the two traditional ones, along with the change of adjustment factor μ, the improved thresholding function considers the advantages of both soft and hard thresholding functions, and overcomes the defects of both, so that the reconstructed signal is smoother and the useful information of the signal can be better retained.

Still using the sinusoidal signal y=sin(2π 20t) with a SNR of -10dB as an example. The number of decomposition layers is still set to 3, and the wavelet basis function is selected as db5. The SNR and the RMSE are used to judge the noise reduction performance of different threshold functions. Table I shows the denoising results of different threshold functions. By using the improved threshold function, the highest SNR and correlation coefficient are obtained, and the RMSE is the lowest. This demonstrates that the proposed improved threshold function has the most superior denoising effect.

TABLE I: THE COMPARISON OF DENOISING EFFECT OF DIFFERENT THRESHOLD FUNCTIONS

threshold function	SNR (dB)	RMSE	Correlation Coefficient
Hard threshold	-2.89	0.986	0.645
Soft threshold	-1.19	0.812	0.718
Improved threshold	-0.91	0.784	0.736

C. Multi-layer Autocorrelation

The denoising effect and distortion degree of wavelet threshold denoising is influenced by many factors such as the number of decomposition layers, wavelet basis function, and threshold function. It is impossible to combine the denoising effect and the distortion degree of the reconstructed signal when the SNR is low, which leads to the limitation of its application in the field of weak signal detection. In contrast, time-delayed multi-layer autocorrelation does not lose useful signal components [11], and combining the lifting wavelet threshold denoising method with multi-layer autocorrelation can improve the accuracy of autocorrelation calculation while reducing the distortion of reconstructed signals, thus expanding the SNR detection range.

By the characteristics of the autocorrelation function, the autocorrelation function of a periodic signal still behaves as a periodic signal, and the period remains the same as the original signal. The traditional autocorrelation detection method is based on the uncorrelated characteristics between noise and signal and noise and noise, and the autocorrelation operation is performed between the original signal and its signal after delay τ. Multi-layer autocorrelation treats the autocorrelation function of the original signal as a new periodic signal and repeats the autocorrelation operation, and the higher the number of autocorrelations, the better the improvement of SNR [12].

Assume that the signal obtained after the lifting wavelet threshold denoising process is:

$$Y(t) = s(t) + q(t) \tag{7}$$

where $s(t)$ is the reconstructed target signal and $q(t)$ is the reconstructed noise signal. An autocorrelation is done for the reconstructed signal and is expressed as follows:

$$R_{YY}(t) = E[Y(t)\bullet Y(t+\tau)] = R_{ss}(\tau) + R_{sq}(\tau) + R_{qq}(\tau) \tag{8}$$

where $R_{ss}(\tau)$ is the autocorrelation function of the signal; $R_{sq}(\tau)$ means the cross-correlation function of signal and noise and $R_{qq}(\tau)$ refers to the autocorrelation function of the noise, respectively, as follows:

$$R_{ss}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_0^T s(t)s(t+\tau)dt \quad (9)$$

$$R_{sq}(\tau) = E[s(t)\bullet q(t+\tau)] + E[s(t+\tau)\bullet q(t)] \quad (10)$$

$$R_{qq}(\tau) = \lim_{T\to\infty}\frac{1}{T}\int_0^T q(t)q(t+\tau)dt \quad (11)$$

It is worth noting that if $q(t)$ is standard Gaussian white noise, then $R_{sq}(\tau)=0$, and its autocorrelation function is 0 for all delayed quantities except at $\tau=0$.

To further analyze the effect of multi-layer autocorrelation, proceed to construct the sinusoidal signal $y=\sin(2\pi\cdot 20t)$ by varying its SNR linearly from -30dB to 10dB. Firstly, the signal is pre-processed by using the previously mentioned lifting wavelet threshold denoising. Then, the processed signal is subjected to different times of autocorrelation operations. The correlation coefficient of the final processing result with respect to the source signal can be calculated. The results are shown in Table II, and the data plot are shown in Fig. 3. The simulation results show that the denoising performance of the autocorrelation method improves with the increase of the number of autocorrelation calculations. Especially under the condition of low SNR. the advantage of autocorrelation with more layer is more obvious.

In summary, when the effective signal of mixed Gaussian white noise is calculated by autocorrelation, the noise reduction of the effective signal and the extraction of periodic signal features can be achieved. Theoretically, when the signal period T is infinite, both $R_{sq}(\tau)$ and $R_{qq}(\tau)$ converge to 0. However, in practical engineering, T cannot be infinite, so both will always exist and the denoising situation is not ideal. If the autocorrelation function a of the signal is treated as a new periodic signal and multiple autocorrelation operations are performed, although it will bring about changes in signal amplitude and phase, it will not change the frequency, and the improvement in SNR will be improved more and more with the number of autocorrelation operations, so multi-layer autocorrelation can be used to detect the weak signal drowned in noise.

TABLE II: THE TABLE OF CORRELATION COEFFICIENT WITH SOURCE SIGNAL UNDER AUTOCORRELATION WITH DIFFERENT LAYERS

SNR (dB)	-30	-25	-20	-15	-10	-5	0	5	10
One-time	0.002	0.119	0.384	0.648	0.779	0.842	0.865	0.904	0.947
Two-time	0.139	0.217	0.533	0.661	0.748	0.903	0.924	0.962	0.972
Three-time	0.212	0.325	0.661	0.703	0.861	0.955	0.972	0.972	0.973
Four-time	0.383	0.519	0.704	0.862	0.915	0.957	0.972	0.973	0.968

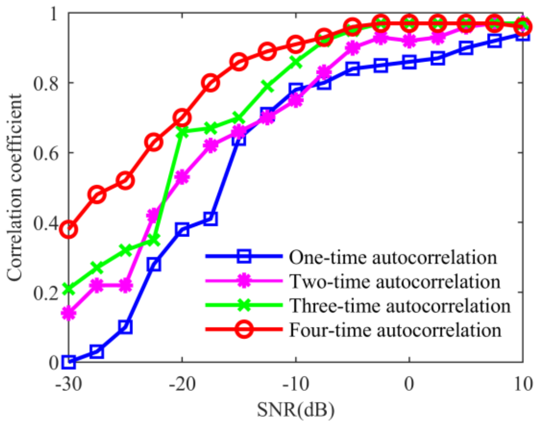


Fig. 3. Correlation coefficient with source signal under autocorrelation with different layers.

III. ALGORITHM PROCESS

In this paper, we propose a weak signal detection method based on improved lifting wavelet threshold denoising and multi-layer autocorrelation, the specific process of the method is as follows:

- Select the appropriate wavelet basis function, threshold value λ and decomposition layers to perform the lifting wavelet transform on the original signal and obtain the wavelet coefficients $w_{j,k}$.
- Apply the improved threshold function to find the optimal adjustment factor and threshold quantization to process the wavelet coefficients of each layer to obtain the new wavelet coefficients.
- The new wavelet coefficients are inverse transformed to obtain the reconstructed signal and complete the noise reduction of the original signal.
- Inverse lifting wavelet transform is done for the new wavelet coefficients to obtain the reconstructed signal and complete the noise reduction of the original signal.
- Perform envelope multi-layer autocorrelation operation on wavelet reconstructed signal, combine with spectrum analysis to complete the detection of weak single frequency signal under strong noise background.

IV. SIMULATION ANALYSIS

To verify the effectiveness of the proposed method, the target signal $s(t) = 0.1\sin(2\pi\times 100t)$ is selected, the sampling frequency is set to 12k Hz, the sampling time is 1s, and the Gaussian white noise is mixed to an initial signal-to-noise ratio of -30dB. After constructing the simulated signal, the time domain waveform is shown in Fig. 4(a), the frequency spectrum is shown in Fig. 4(b), and the STFT time-frequency diagram is shown in Fig. 4(c). Obviously, the noise interference is very serious, and the SNR is low, so the feature signal is drowned in the severe background noise.

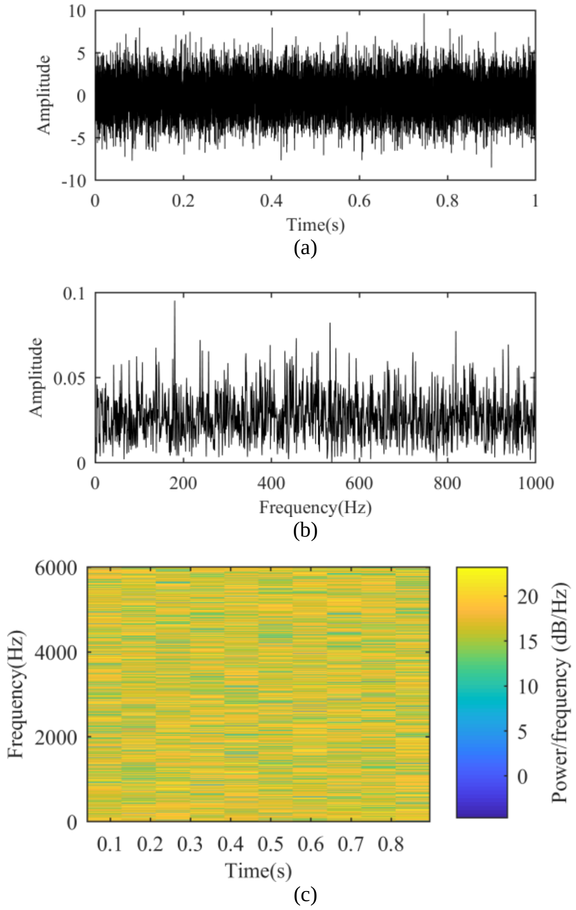


Fig. 4. Time-frequency diagram of simulated signal. (a) Time-domain waveform; (b) Frequency spectrum; (c) STFT time-frequency diagram.

Different values of μ in the threshold function are taken to compare the RMSE of the reconstructed signal with that of the original signal. The parameter μ corresponding to the smallest RMSE value is the best. Fig. 5 shows the trend of RMSE with μ, and μ=0.05 is the best value. The original signal is decomposed and reconstructed by 3-layer wavelet decomposition, and the time-frequency diagram of the reconstructed signal using lifting wavelet threshold denoising are shown in Fig. 6. After lifting wavelet denoising, the frequency of the characteristic signal appears in the spectrum. Four distinct energy bands appear in the STFT time-frequency diagram. This illustrates that the proposed method preserves the information of multiple frequency bands, and the distortion level of the signal is suppressed. However, the noise interference component still exists. No useful information is observed at the characteristic frequency in the STFT time-frequency diagram. The precise detection of weak signals has not yet been achieved.

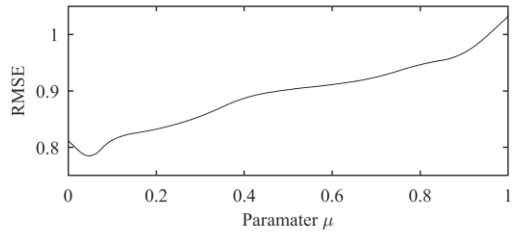


Fig. 5. The trend of RMSE with parameter μ.

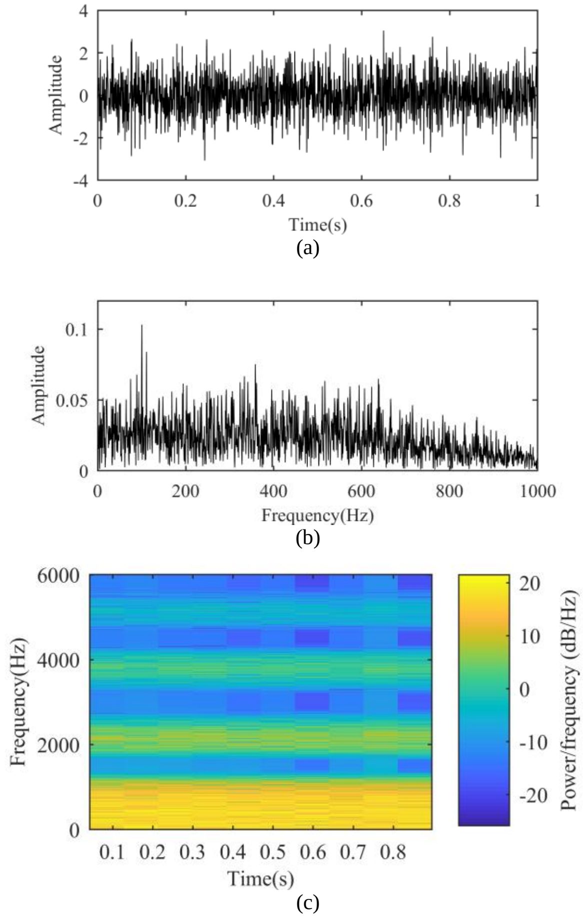


Fig. 6. Time-frequency diagram of reconstructed signal. (a) Time-domain waveform; (b) Frequency spectrum. ;(c) STFT time-frequency diagram.

When the signal is under a strong noise background, Fig. 6 reveals that if only the proposed lifting wavelet threshold noise reduction method is used, it is not enough to accomplish the precise detection of weak signal. Therefore, the autocorrelation process is performed on the reconstructed signal obtained after the lifting wavelet threshold denoising process, and the obtained one-time autocorrelation and four-time autocorrelation frequency spectrums are shown in Fig. 7 and Fig. 8, respectively. Fig. 9 shows the STFT time-frequency diagram of four-time autocorrelation of reconstructed simulation signal. From the comparison effect, the higher the number of autocorrelations, the more obvious the detection effect is. In addition, comparing with Fig. 6(c), Fig. 9 shows a clear concentration of energy at the characteristic frequency of 100 Hz, and there is no interference around.

From above simulation analysis, the results show that the proposed method can improve the SNR of single-frequency weak signals and obtain obvious detection effects.

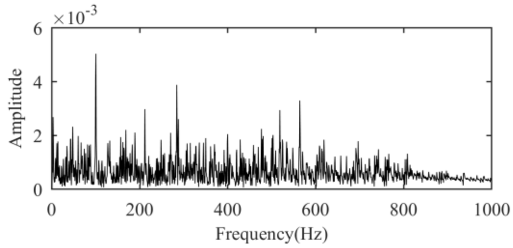


Fig. 7. One-time autocorrelation frequency spectrum of reconstructed simulation signal.

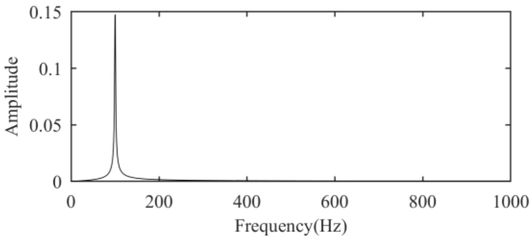


Fig. 8. Four-time autocorrelation frequency spectrum of reconstructed simulation signal.

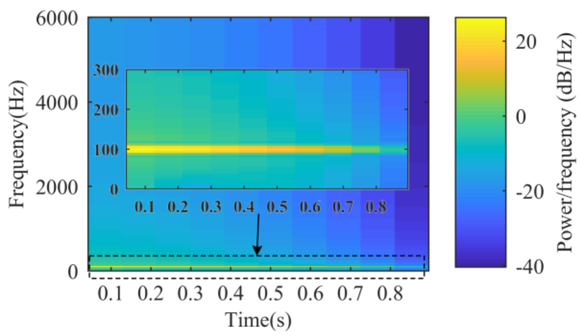


Fig. 9. STFT time-frequency diagram of four-time autocorrelation of reconstructed simulation signal.

V. Experiment Verification

A. Rolling Bearing Fault Feature Extraction

Bearing vibration fault signals are susceptible to contamination by various noises, especially at the early fault occurrence when the characteristic information is weak [13]. To verify the practicality of the proposed method, the fault signals from the rolling bearing database of Case Western Reserve University in the United States are utilized. The data of the outer race faults numbered 130 are chosen for analysis. The sampling frequency is 12 kHz, and take the first 8192 points of the experimental data for analysis and add -10 dB of Gaussian white noise to the original signal. The rotate speed is 1797 rpm and the fault frequency of the bearing is 107.36 Hz. The time-domain waveform and envelope spectrum of the signal are shown in Fig. 10.

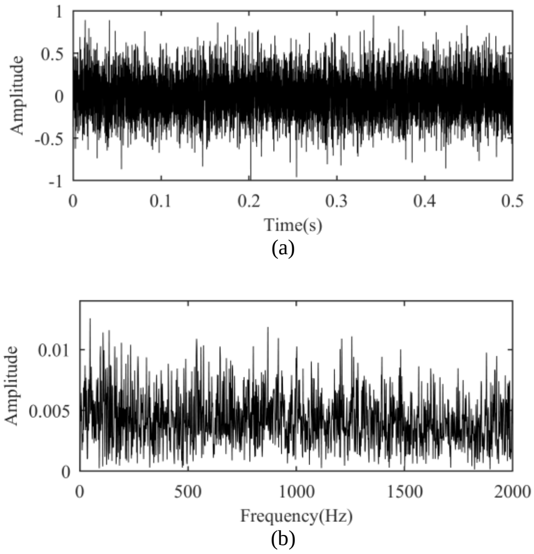


Fig. 10. Time domain waveform and envelope spectrum of bearing vibration signal. (a) Time-domain waveform; (b) Envelope spectrum.

As can be observed from Fig. 10, the time domain waveform of the rolling bearing outer ring vibration signal after noise addition is complex. There is severe noise interference in the envelope spectrum, and the fault characteristic frequencies are completely submerged. The signal is denoised by using the lifting wavelet to obtain the reconstructed signal. By choosing different parameters of the improved threshold function, the RMSE of the reconstructed signal and the original signal are compared, and the best value of the parameter μ is obtained when the RMSE value is the smallest. As can be seen from Fig. 11, the best value is 1. The time-domain waveform and envelope spectrum of the reconstructed signal are shown in Fig. 12.

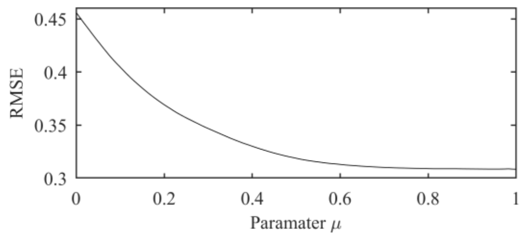
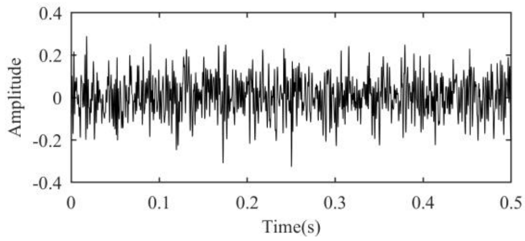


Fig. 11. The trend of RMSE with parameter μ.

Comparing Fig. 10(b) and Fig. 12(b), it can be found that the reconstructed signal after lifting wavelet achieves a fine noise reduction effect, less obvious characteristic frequency in the envelope spectrum can be observed, but the spectrum is still very complicated. Using the multi-layer autocorrelation method for subsequent processing, the envelope autocorrelation function of the reconstructed signal is solved. After the reconstructed signal is autocorrelated once and autocorrelated four times, their frequency spectrums are shown in Fig. 13 and Fig. 14, respectively. Besides, Fig. 15 shows the STFT time-frequency diagram of four-time autocorrelation of reconstructed simulation signal.



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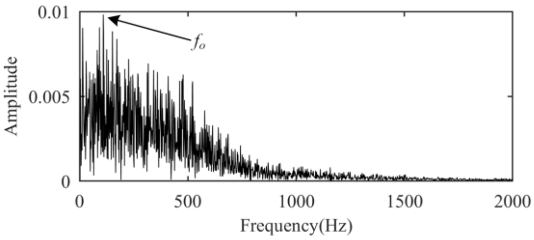

(b)

Fig. 12. Time domain waveform and envelope spectrum of reconstructed vibration signal. (a) Time-domain waveform; (b) Envelope spectrum.

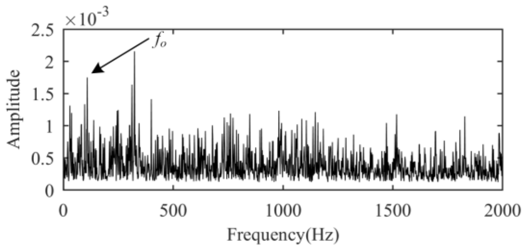

Fig. 13. One-time autocorrelation frequency spectrum of reconstructed vibration signal.

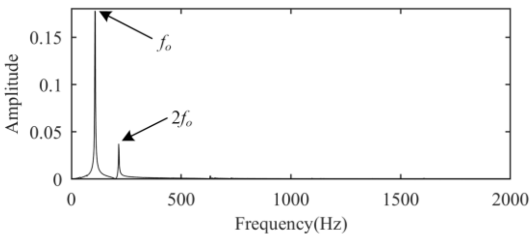

Fig. 14. Four-time autocorrelation frequency spectrum of reconstructed vibration signal.

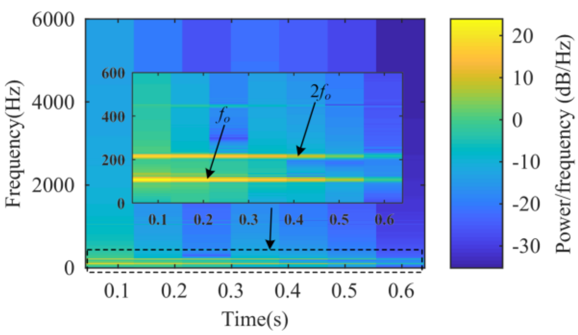

Fig. 15. STFT time-frequency diagram of four-time autocorrelation of reconstructed vibration signal.

As can be seen from Fig. 13, after one-time autocorrelation process, the frequency spectrum of the outer ring reconstructed signal can be observed at the fault feature frequency f_o, but its multiples cannot be obtained, and the noise interference is still numerous. After four-time autocorrelation processes, Fig. 14 and Fig. 15 clearly shows the fault frequency and its double frequency, with obvious components, which significantly reduce the influence of noise. Therefore, the proposed method in this article can accurately extract the fault characteristic frequency in the rolling bearing fault vibration signal under a strong noise background.

B. Weak Electrical Signal Detection

To further verify the generality of the method in this paper, a weak electrical signal provided by the research group is used for detection. The test system modulates and outputs the weak sinusoidal signal and converts it into a digital signal for analysis [14]. Fig. 16 shows a test system consisting of a signal generator (generating a sine wave), a weak signal amplification analog circuit board (amplifying the signal by modulation), a signal acquisition instrument (whose type is the Donghua DH5908N, with a maximum sampling frequency of 128 kHz per channel and a minimum voltage resolution of 5 μV), a shielded box and a PC. The sampling frequency is set to 128 kHz, the number of sampling points is 1274860, and the target signal frequency is 93 Hz.

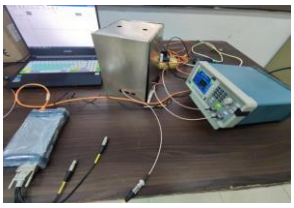

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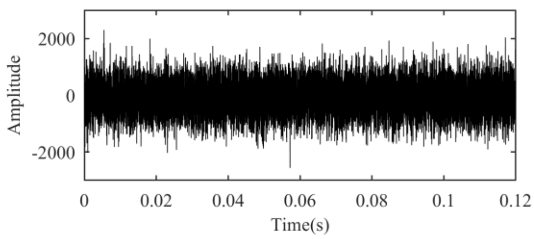
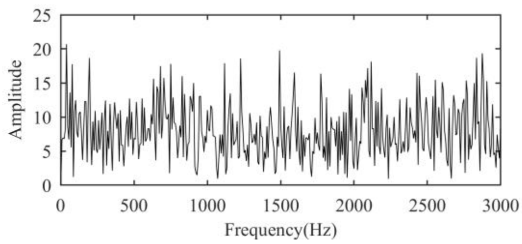
Fig. 16. The test system. (a) The overall test system; (b)The analog amplifier circuit board.



(a)



(b)

Fig. 17. Time domain waveform and envelope spectrum of the original weak signal. (a) Time-domain waveform; (b) Envelope spectrum.

A signal with a length of 16384 is randomly selected from the modulation signal for analysis. Fig. 17 shows the time-domain waveform and envelope spectrum of the modulating signal. The traditional envelope demodulation method cannot extract the frequency of the target signal due to the noise interference and the weakness of the original signal itself.

Like the previous section, firstly, by choosing different threshold function parameters μ, the RMSE variation curve of the reconstructed signal and the original signal with μ is obtained, and Fig. 18 shows the optimal μ is 0.6. Then, the original signal is processed using the lifting wavelet threshold denoising method with the optimal parameter, and the time domain waveform and envelope spectrum are shown in Fig. 19.

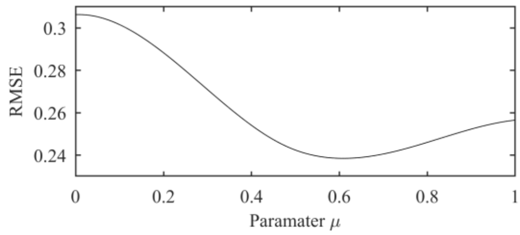


Fig. 18. The trend of RMSE with parameter μ.

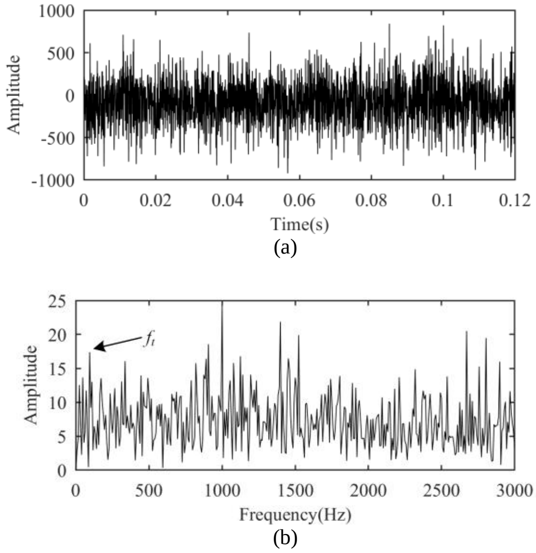


Fig. 19. Time domain waveform and envelope spectrum of reconstructed weak signal (a) Time-domain waveform; (b) Envelope spectrum.

From Fig 19(b), after the original modulated signal is processed by lifting wavelet threshold denoising, the frequency of the target signal can be extracted by using traditional envelope demodulation. However, the amplitude corresponding to the target frequency is not prominent and there is still serious noise interference. Therefore, relying only on lifting wavelet threshold denoising cannot achieve precise detection of the target signal frequency. At this point, the reconstructed signal is processed in combination with the multi-layer autocorrelation algorithm. The envelope autocorrelation function of the reconstructed signal is calculated, and its frequency spectrum is obtained. Comparing the frequency spectrum with only one-time autocorrelation process and four-time autocorrelation processes, the results are shown in Fig. 20 and Fig. 21, respectively. In addition, Fig. 22 shows the STFT time-frequency diagram of four-time autocorrelation of reconstructed weak signal.

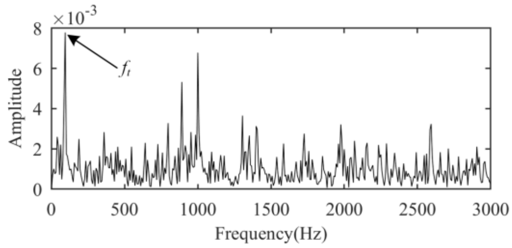


Fig. 20. One-time autocorrelation frequency spectrum of reconstructed weak signal.

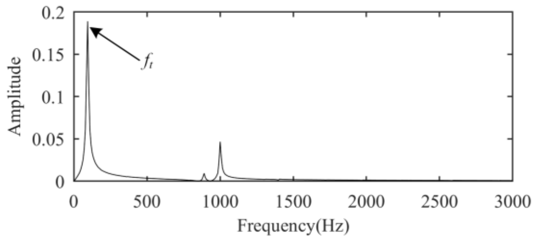


Fig. 21. Four-time autocorrelation frequency spectrum of reconstructed weak signal.

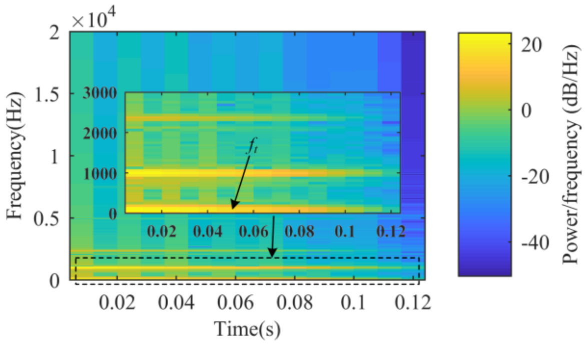


Fig. 22. STFT time-frequency diagram of four-time autocorrelation of reconstructed weak signal.

Fig. 20 illustrates that the frequency of the target signal becomes more prominent after one-time autocorrelation process. However, there is interference from this high frequency modulation component in the frequency spectrum, and the noise in the frequency band range is not completely removed. Fig. 21 and Fig. 22 displays the results of four-time autocorrelation processes, the high-frequency modulation components are effectively suppressed, the target detection frequency is extremely prominent, and there is almost no extra noise interference. In summary, the proposed method in this paper can achieve the precise detection of weak signals, and it is applicable to different engineering fields.

VI. Comparative Study

In order to further demonstrate the superiority of the proposed method, it is compared with the traditional singular value decomposition (SVD) denoising and narrow-band filtering method. The correlation coefficient with respect to the source signal was used as the criterion for quantitative evaluation. For the simulation test, Case 1 and Case 2, the correlation coefficients were obtained by the above three denoising algorithms. It should be noted that for Case 1, the source signal is the bearing vibration signal without any noise added. For Case 2, the source signal is the original sine wave before high-frequency modulation. The results obtained are shown in Table III, and the bar graph is shown in Fig. 23.

TABLE III: THE TABLE OF CORRELATION COEFFICIENT WITH SOURCE SIGNAL UNDER DIFFERENT METHODS

Method	Simulation	Case 1	Case2
Proposed	0.647	0.527	0.486
SVD	0.426	0.419	0.478
Narrow-band filter	0.366	0.278	0.285

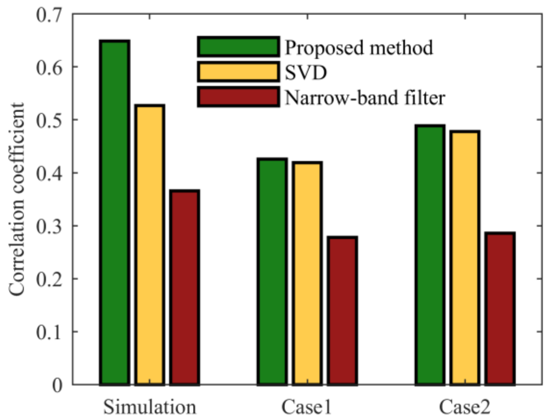


Fig. 23. Comparison of denoising effects of different methods

As can be seen from Table I and Fig. 5, in simulation case, the noise reduction performance of the proposed method is superior to the other two methods. This demonstrates that the proposed method is suitable for weak sinusoidal signal detection under low SNR conditions. In two experimental cases, the proposed method achieves a similar effect to SVD decomposition, but is still optimal overall. The performance of the algorithm is affected by the coupling of rotating parts and the interference thermal noise in the circuit board. However, combined with the above analysis, the detection accuracy of the proposed method can meet the requirements of related engineering applications. Compared with traditional methods, it has certain advantages.

VII. CONCLUSION

In this article, the detection of weak signals based on lifting wavelet threshold denoising and multi-layer autocorrelation is investigated and some conclusions can be obtained as follows.

- The proposed method combines lifting wavelet threshold denoising with multi-layer autocorrelation. It can effectively reduce the distortion of the reconstructed signal while improving the SNR and expanding the detection range for the precision detection of weak signals.
- This article proposes a new improved threshold function, which overcomes the defects of traditional soft and hard threshold function. The improved threshold function has favorable adaptability, and facilitates the retention of target signal wavelet coefficients. Thus, the filtering effect has been improved.
- Simulation and experimental analysis verify that the proposed method can effectively suppress Gaussian white noise, improve the SNR of the target signal, reduce the RMSE, as well as successfully detect the single-frequency weak electrical signal under a strong noise background. In addition, the proposed method can also accurately extract the rolling bearing fault signal frequency and its multiplier. The detection results are more prominent, indicating that the method has strong engineering practicality.
- Compared with the traditional noise reduction methods, the method proposed in this paper has advantages, and has a certain wide range of applications. For multi engineering background, the anti-interference performance of this method still has room to improve. Moreover, the rounding error caused by multi-layer autocorrelation can also be used as the focus of future research. The solution of the rounding error can overcome the problem of unstable accuracy caused by the increase of autocorrelation times, and further improve the effect of this method.

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