Phase-Based Estimation of the Harmonic Signal Frequency

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Abstract —This paper examines estimating the frequency of the information signal based on the analysis of the probability characteristics of the phase fluctuations when a mix of signal and narrowband noise is observed. The new estimation algorithm is described while the shortcomings of the commonly applied methods are specified. The performance of the proposed approach is illustrated by a number of examples. In particular, it is demonstrated that the application of the introduced procedure allows high accuracy measurements of the carrier frequency of the non-modulated, the multilevel phase and quadrature keyed signals.

Index Terms—Single sinusoid, unknown signal frequency, phase probability density, estimated parameter, phase-shift keying, quadrature amplitude shift keying

I. INTRODUCTION

One of the tasks of radio emitting object monitoring is the operational estimation of the radio signal carrier frequency. Knowledge of the signal carrier (instantaneous) frequency may have an independent importance or be required for determining the analyzed signal modulation format [1]-[5]. As it is known, when recognizing digital keying formats, the most informative characteristics are the ones that describe the distribution laws of the instantaneous phase as well as the in-phase and quadrature components of the complex envelope. To extract these parameters from the received signal-to-noise mix, a sufficiently accurate estimate of the carrier frequency is required [6]. In particular, to determine the value of the carrier frequency when receiving multilevel phase shift keyed signals, not only the signal is raised to the power corresponding to the number of phase shift keying levels [7], but the phase-locked loop system with a digital frequency synthesizer is also used [8].

In this paper, it is examined how the signal carrier frequency can be measured using the radio signal statistical characteristics, such as the probability densities of the phase and its derivative. Non-modulated signals as well as multilevel Phase-Shift Keyed (PSK) and quadrature amplitude shift keyed (QASK) signals are considered.

Manuscript received May 4, 2020; revised March 11, 2021.
This work was supported by RFBR and CNRS under Grant No. 20-51-15001.
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doi:10.12720/jcm.16.4.126-131

II. HARMONIC SIGNAL

In [9], one obtains the expression for the conditional probability density $W(\theta)$ of the phase θ of the additive mix $\xi(t)=s(t)+n(t)$ constituted by the harmonic signal $s(t)=A_m\cos(\omega t+\varphi_0)$ and the narrowband noise $n(t)$ in the case when the phase derivative $\dot\theta(t)$ takes some fixed value $\dot\theta_0$:

$$w(\theta|\dot\theta=\dot\theta_0)=\frac{1}{\pi}\left[\left(1+\frac{q^2k^2}{2}\right)\mathrm{I}_0(g)+\frac{q^2k^2}{2}\mathrm{I}_1(g)\right]^{-1}\times$$
$$\times\exp\left[g\left(2\cos^2\theta-1\right)\right]\left\{\frac{qk\cos\theta}{\sqrt{2\pi}}\exp\left(\frac{q^2k^2\cos^2\theta}{2}\right)\times\right. \quad (1)$$
$$\left.\times\frac{q^2k^2\cos^2\theta+1}{2}\left[1+\operatorname{erf}\left(\frac{qk\cos\theta}{\sqrt{2}}\right)\right]\right\}$$

In (1), the notations are:

$$k^2=(1+\delta h_0)^2/\left(1+h_0^2\right),\qquad g=q^2\left(k^2-\delta^2\right)/4,$$
$$h_0=\delta-y_0,\quad \delta=\Delta\omega/\sqrt{-\ddot\rho(0)},\quad y_0=\dot\theta_0/\sqrt{-\ddot\rho(0)},$$

$\operatorname{erf}(x)=\left(2/\sqrt{\pi}\right)\int_0^x\exp\left(-t^2\right)\mathrm{d}t$ is the error integral, $\mathrm{I}_n(x)$ is the modified Bessel function of the n-order; $\rho(\tau)$ is the normalized correlation function (correlation coefficient) of the noise; $q=A_m/\sigma$ is the signal-to-noise ratio; A_m and σ^2 are the signal amplitude and the variance of quadrature components of noise, respectively, $\Delta\omega=\omega-\omega_0$ presents the frequency mismatch of the signal frequency ω relative to the central frequency of the noise spectrum ω_0.

In Fig. 1, the surface graph is shown calculated according to the formula (1) and characterizing the distribution of the phase θ when it is changing within the range from $-\pi$ to π for a series of the fixed values of the normalized derivative of the phase $\dot\theta_0$.

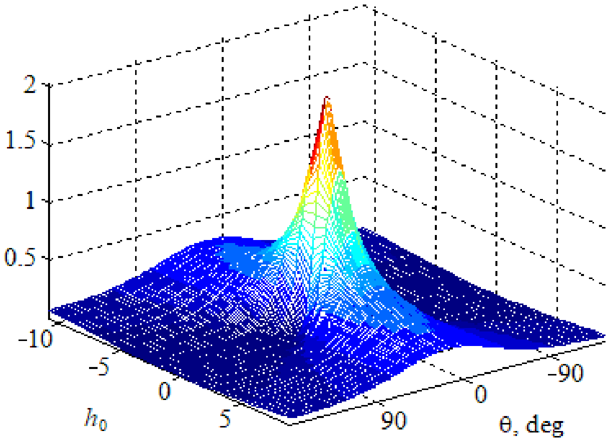


Fig. 1. The dependence of the ratio of the first two coefficients of the generalized spectrum in the Chebyshev polynomial basis under various approximations.

The detuning frequency is $\Delta\omega = 0$, while the signal-to-noise ratio value is $q = 14$ dB.

It follows from this Figure that the probability density of the phase as the function of its current value and its derivative (instantaneous frequency) is concentrated in the area of the zero values of the variables θ and h_0. This circumstance can be used to estimate the signal frequency.

For this purpose, at the output of the bandpass filter tuned to the frequency of the reference oscillator ω_0, there will be formed a sampling of N quadrature samples x_i, y_i of the signal $\xi(t)$ that has duration T for the time moments t_i, $i = \overline{1, N}$:

$$x_i = \xi(t_i)\cos(\omega_0 t_i), \qquad y_i = \xi(t_i)\sin(\omega_0 t_i). \tag{2}$$

For the range $\Delta\omega = \omega - \omega_0$ that is the possible signal frequency deviation ω from the reference oscillator frequency ω_0, the following operations are to be performed.

Let the phase of the quadrature samples x_i, y_i be changed $(2k+1)$ times by the value $\Delta\omega_1 t_i$:

$$\begin{aligned} x_{ji} &= x_i \cos(j\Delta\omega_1 t_i) + y_i \sin(j\Delta\omega_1 t_i), \\ y_{ji} &= x_i \sin(j\Delta\omega_1 t_i) - y_i \cos(j\Delta\omega_1 t_i), \end{aligned} \tag{3}$$

where $\Delta\omega_1$ is the discreteness of the frequency variation within $\Delta\omega$; $j = 0, \pm 1, \pm 2, \ldots, \pm k$. Then the phases θ_{ji} of the quadrature samples x_{ji}, y_{ji} of the signal $\xi(t)$ are calculated as:

$$\theta_{ji} = \mathrm{arctg}(y_{ji} / x_{ji}) \tag{4}$$

Further the set of $(2k+1)$ histograms – $w_j(\theta_{ji}, l)$ – is created for l phase intervals (where $l = 2\pi/\Delta\theta$, while $\Delta\theta$ is the interval width) and the histogram with the number j is selected as taking the maximum value among the histograms $w_j(\theta_{ji}, l)$:

$$w_{j\max} = \max w_j(\theta_{ji}, l) \tag{5}$$

The signal frequency estimate is determined as the frequency value $j\Delta\omega$ for which the phase histogram takes the value $w_{j\max}$. Taking into account the frequency of the reference generator, it will be:

$$\hat{\omega} = \omega_0 + j w_{j\max} \Delta\omega_1 \tag{6}$$

In Fig. 2a, the histogram $w_j(\theta_{ji}, l)$ is drawn obtained by the simulation of the signal frequency estimation according to the formulas (2)-(4) in the MATLAB.

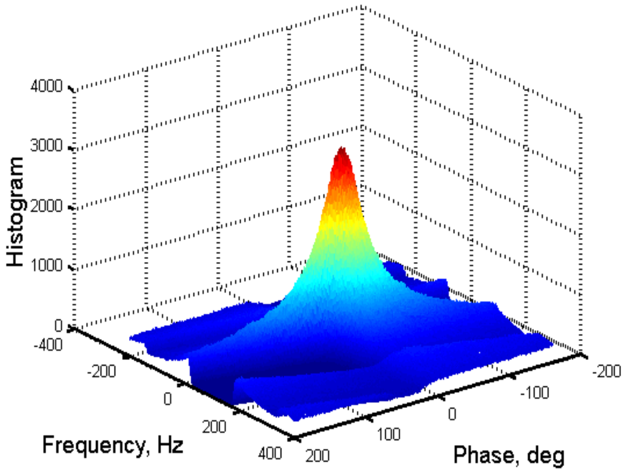
There have been examined the passing of the sum of the harmonic signal with the frequency $\omega/2\pi = 70.0001$ MHz and the noise through the Gaussian filter with the bandwidth $2\Delta f_{\mathrm{IF}} = 12$ MHz, tuned to the frequency $\omega_0/2\pi = 70$ MHz. The bandwidth of the low-pass filter in the quadrature channels is $\Delta f_{\mathrm{LPF}} = 6$ MHz. The signal-to-noise ratio is $q = 11$ dB. The range of frequency in quadrature samples is $k\Delta\omega_1/2\pi = \pm 1000$ Hz (on the chart one can see the range of $-200 \ldots +400$ Hz), the step change in frequency is $\Delta\omega_1/2\pi = 1$ Hz. The phase variation range is $\pm\pi$, the phase interval of the histogram is $\Delta\theta = \pi/180$. The duration of sampling is $T = 6.8$ ms, while the number of quadrature samples is $N = 98000$.

In Fig. 2b, the histogram $w_{j\max}$ is plotted with the number j corresponding to the formula (5). According to the results of the statistical processing of the ten measurements of the frequency $\hat{\omega}$, the confidence interval is 70000100.8±5.7 Hz, with the reliability of 0.99, while the value of the relative measurement error $(\bar{\hat{\omega}} - \omega)/\omega$ is not greater than $1.4 \cdot 10^{-8}$.

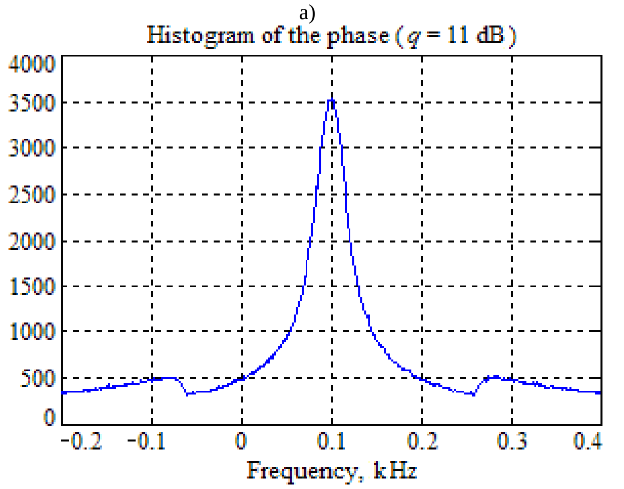
III. Phase-Shift Keyed Signal

In [10], there has been obtained an analytical expression for the probability density of the phase θ of the sum of the multilevel PSK signal and the additive narrowband Gaussian noise under $\Delta\omega = 0$:

$$\begin{aligned} W(\theta) = {} & \frac{1}{2\pi}\exp\left(-\frac{q^2}{2}\right) + \frac{q}{2M\sqrt{2\pi}} \times \\ & \times \sum_{m=1}^{m} \cos(\theta - \theta_m)\exp\left[-\frac{q^2}{2}\sin^2(\theta - \theta_m)\right] \times \\ & \times \mathrm{erfc}\left[-\frac{q}{\sqrt{2}}\cos(\theta - \theta_m)\right] \end{aligned} \tag{7}$$



a)



b)

Fig. 2. Phase histograms of the sum of the signal and the noise produced during the estimation of the harmonic oscillation frequency.

here $\theta_m = (2m-1)\pi/M$, M is the number of the phase-shift keying levels; q, $\Delta\omega$ are determined in the same way as in the formula (1); $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$.

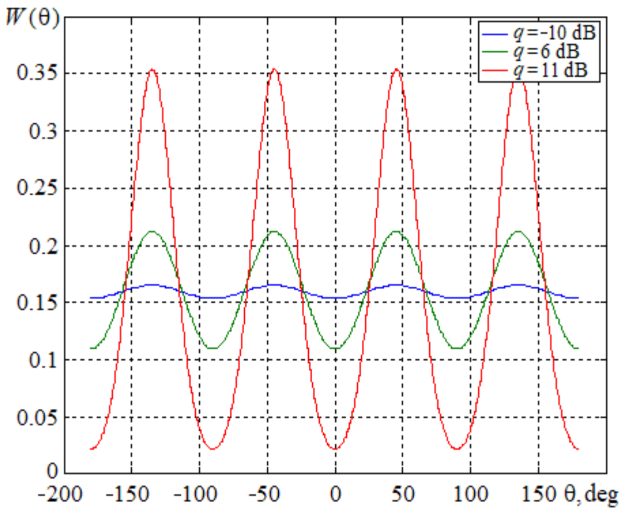


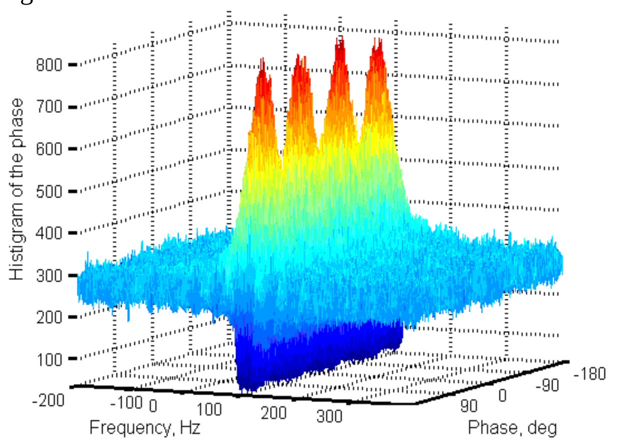
Fig. 3. The phase density of the sum of the 4-PSK signal and the noise.

In Fig. 3, the graph is shown of the dependence of the phase density of the 4-PSK signal, calculated by the formula (7), when the phase changes are within the range from $-\pi$ to π. Number of the phase-shift keying levels is $M = 4$ and signal-to-noise ratio is $q = -10$ dB (blue curve), 6 dB (green curve), 11 dB (red curve). From Fig. 3, it follows that the phase of the signal-to-noise mix is grouped around the values separated by the interval $\pi/2$. When $q \to 0$, the maximum value of $W(\theta)$ tends to $1/2\pi$.

The possibility is then examined of the application of the above introduced method of harmonic signal frequency estimation for determining the carrier frequency of the PSK signal.

In Fig. 4a and Fig. 4c, the histograms $w_j(\theta_{ji}, l)$ are presented obtained during the simulation of the process of estimation of the 4-PSK and 8-PSK signal frequency according to the formulas (2)-(4). The experimental conditions are the same as in the Section 2. The symbol rate of the PSK signals is $F_d = 2.6$ Mbit/s, the symbols are integer random numbers: $0 \ldots (M-1)$ [11].

The analysis of Fig. 4a and Fig. 4c shows that the phase of the signal-to-noise mix of the histograms $w_j(\theta_{ji}, l)$ is concentrated close to the phase values separated by the interval $\pi/2$ (for 4-PSK) or $\pi/4$ (for 8-PSK) and near the PSK signal carrier frequency values. The latter can be seen from the histograms in Fig. 4b and Fig. 4d.



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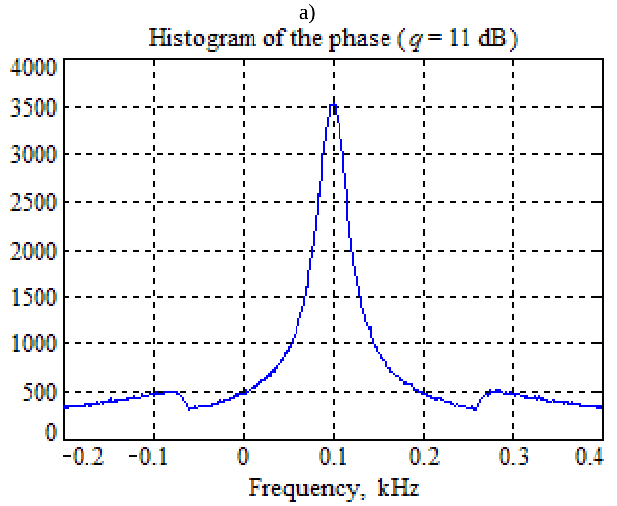

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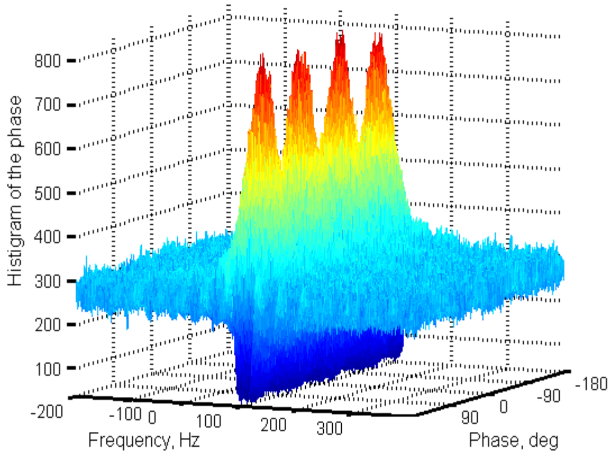

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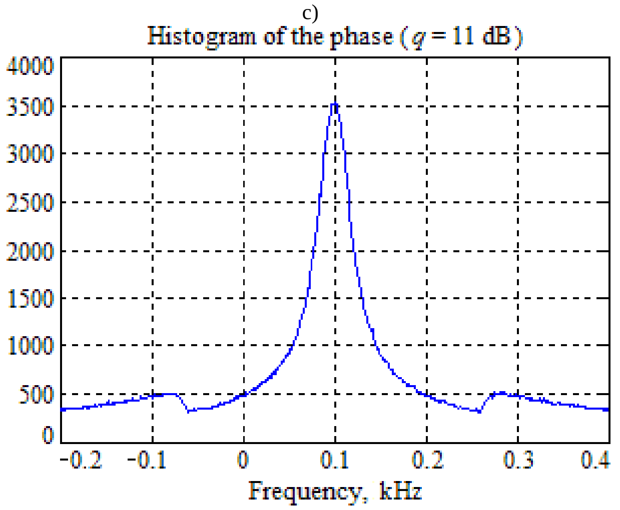


d)

Fig. 4. The histograms of the phase of the sum of the signal and the noise produced during the evaluation of the 4-PSK (a, b) and the 8-PSK (c, d) signal frequency.

Statistical processing of the ten frequency measurements gives the following results:

– For the 4-PSK signal, the confidence interval amounts to 70000099.2 ±7.1 Hz with the reliability 0.99, while the relative measurement error $\left(\bar{\hat{\omega}}-\omega\right)/\omega$ is not greater than $1.1\cdot10^{-8}$;

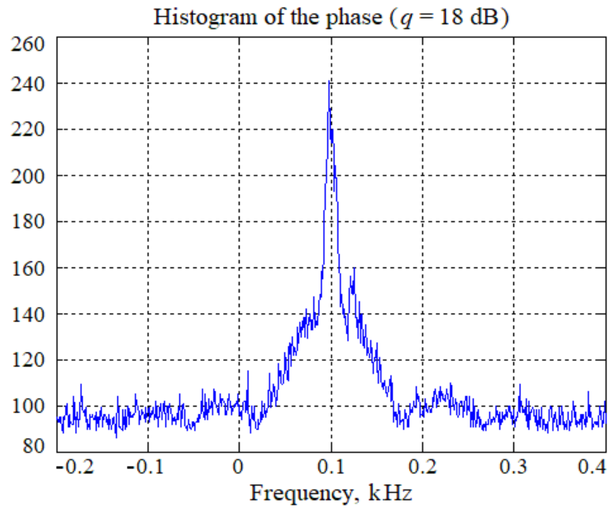
– For the 8-PSK signal, the confidence interval amounts to 70000099.6 ±6.7 Hz with the reliability 0.99, while the relative measurement error $\left(\bar{\hat{\omega}}-\omega\right)/\omega$ is not greater than $7\cdot10^{-9}$.

IV. Quadrature Amplitude Shift Keyed Keyed Signal

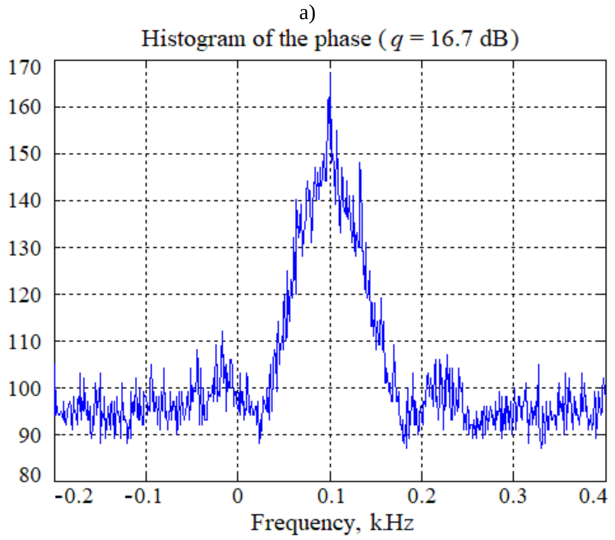
The described technique can be applied to determine the QASK signal carrier frequency. In Fig. 5, there are shown the results of calculating the histograms $w_j\left(\theta_{ji}, l\right)$ obtained by simulating the procedure for estimating the frequency of the 8-QASK (Fig. 5a) and 32-QASK (Fig. 5b) signals according to the formulas (2)-(4). The signal-to-noise ratio is $q = 18$ dB (Fig. 5a) and $q = 16.7$ dB (Fig. 5b). The other experimental conditions are the same as in Section 3.

Statistical processing of the ten frequency measurements gives the following results:

– For the 8-QASK signal, the confidence interval amounts to 70000099.8 ± 6.1 Hz with the reliability 0.99, while the relative measurement error $\left(\bar{\hat{\omega}}-\omega\right)/\omega$ is not greater than $9\cdot10^{-9}$;



a)



b)

Fig. 5. The histograms of the phase of the sum of the signal and the noise produced during the evaluation of the 8-QASK (a) and the 32-QASK (b) signal frequency.

– For the 32-QASK signal, the confidence interval amounts to 70000099.1 ± 3.3 Hz with the reliability 0.99, while the relative measurement error $\left(\bar{\hat{\omega}}-\omega\right)/\omega$ is not greater than $5\cdot10^{-9}$.

V. Implementation of the Signal Frequency Estimation Algorithm

Implementation of the developed algorithm for estimating the signal frequency requires the use of high-performance computing facilities to support the time rate.

In order to solve this problem, a graphics accelerator GPU (Graphics Processor Unit) can be used supporting CUDA (Compute Unified Device Architecture) technology introduced by the corporation NVIDIA, facilitating the GPU-based development of applications [12].

A GPU includes several texture processing units. Each unit consists of an enlarged block of texture samples and two or three data-flow multiprocessors, each of which includes eight computing devices and two super-functional blocks. All instructions are executed according to the SIMD (Single Instruction, Multiple Data – one instruction, multiple data flow) principle, when one instruction is applied to all the threads in the Warp platform (in technology CUDA, it is a group of 32 threads that is the minimum amount of data processed by multiprocessors). This is a way of executing SIMT (single instruction, multiple threads – one instruction, several threads) instructions. CUDA programming model assumes the thread grouping.

The most labor-intensive computational procedure in the frequency estimation algorithm is the determination of the probability density of the signal phase samples by forming histograms of these samples, the number of histograms is equal to the number of frequency steps within the range of frequency values that are searched close to the expected frequency value.

Since in the CUDA execution model the threads are united in blocks and have the shared memory, its use increases the efficiency of the histogram building, with one block building one phase histogram of the signal with a frequency shift corresponding to the block number in the shared memory. After the building of one histogram is complete, each block searches for the maximum in its own histogram and saves the result to the array in the global memory. From this array, the maximum and its index are chosen and its index is the desired frequency.

The histogram building is implemented in such a way that the initial data in the global memory does not change. That is, the main calculations for the frequency shift for the phases occur in registers. The obtained result is rounded and increases the corresponding element of the histogram. The search for the maximum in the histogram is carried out in parallel.

It has been experimentally determined that, when the sampling consists of $N = 98000$ quadrature samples (that corresponds to the sampling duration of $T = 6.8$ ms) and the search boundaries are ±10 kHz, the introduced algorithm execution time needed for estimating the signal frequency is 0.47 s, if one uses GTX 560ti, a GPGPU device supporting CUDA technology and equipped with 384 CUDA-cores.

The execution of the same algorithm under the same conditions but applying a general-purpose central processing unit Core i7 4700MQ with a clock frequency of 3.4 GHz takes 67 seconds, because in this case the execution time depends linearly upon the signal sample length and the number of steps in the frequency estimation.

The use of more modern GPGPU devices allows reducing the time for obtaining the estimate of the signal frequency to the values that make real-time measurements possible.

VI. Conclusion

The proposed approach to estimating the signal carrier frequency, based on the analysis of the phase density of the mix of the signal and the noise, allows performing a high-accuracy measurement of the carrier frequency of both the harmonic signal and the PSK and QASK signals with the unknown parameters of the phase shift keying. Frequency estimation is associated with significant computational costs, but the current level of technology can ensure the implementation of the introduced algorithm in real time. To reduce the calculation time, it is advisable to start with a rough estimate of the frequency that may be based, for example, on the signal spectrum, in accordance with the existing implementation. The obtained results can be used for rapid estimation of the carrier frequency by the short signal realization during the radio object monitoring, including the automatic recognition of the radio signal modulation format.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Our project research team jointly conducted the presented study, carried out numerical calculations and analyzed the results obtained both individually and cooperatively. All authors had approved the final version of the paper.

References

[1] O. V. Chernoyarov, M. I. Maksimov, A. V. Salnikova, and A. V. Zakharov, “Detection of the fluctuating pulse with unknown time of arrival and intensity,” *International Journal of Applied Engineering Research*, vol. 12, no. 18, pp. 9367-9376, Oct. 2017.

[2] O. V. Chernoyarov, M. I. Maksimov, A. V. Salnikova, and M. M. Shahmoradian, “The estimate of the dispersion of the fast-fluctuating stationary Gaussian random process,” in *Proc. 3rd Int. Conf. Frontiers of Signal Processing*, Paris, 2017, pp. 25-29.

[3] O. A. Dobre, A. Abdi, Y. Bar-Ness, and W. Su, “Survey of automatic modulation classification techniques: Classical approaches and new trends,” *IET Communications*, vol. 1, no. 2, pp. 137-156, Apr. 2007.

[4] S. S. Adzhemov, A. A. Stogov, M. V. Tereshonok, D. S. Chirov, and M. V. Ivankovich, “The algorithm for recognizing the digital signal modulation format by the constellation shape using the Kohonen self-organizing

maps," *T-Comm: Telecommunications and Transport*, no. 11, pp. 4-6, Nov. 2011. (in Russian)

[5] A. B. Sergienko and A. V. Osipov, "Digital modulation recognition using circular harmonic approximation of likelihood function," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, Florence, 2014, pp. 3460-3463.

[6] S. N. Kirillov and P. S. Pokrovsky, "Algorithm of modulation type classification for unknown radio signal of space radio communication systems," *Vestnik of Ryazan State Radio Engineering University*, no. 44, pp. 3-26, Oct. 2013. (in Russian)

[7] A. V. Stepanov and S. A. Matveev, *The Methods of Computer Signal Processing in Radio Communication Systems*, Moscow: SOLON-press, 2007. (in Russian)

[8] A. V. Shakhmatov, "Algorithm of digital demodulation of multi-position phase-shift keying," in *Proc. Tomsk State University of Control Systems and Radioelectronics*, no. 2-1, pp. 74-77, Dec. 2011. (in Russian)

[9] E. V. Volkhonskaya, "The conditional probability densities of envelope and phase of the sum of a harmonic signal and narrow-band random process," *Computer Modelling and New Technologies*, vol. 6, no. 1, pp. 121-127, Jun. 2002.

[10] L. Friederichs, *Probability Density Functions of Envelope and Phase of the Sum of a PSK Modulated Carrier and Narrowband Gaussian Noise*; Starnberg: AUDENS Telecommunications, 2000.

[11] A. B. Sergienko, *Digital Signal Processing*, St.-Petersburg: Peter, 2002. (in Russian)

[12] J. Cheng, M. Grossman, and T. McKercher, *Professional CUDA C Programming*, Hoboken: Wiley, 2014.

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