Resource Allocation for AF-OFDMA System Using Combinatorial Auction

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Abstract —We propose a combinatorial auction-based subcarrier assignment algorithm for single-relay Amplify-and-Forward (AF) Orthogonal-Frequency-Division-Multiple-Access (OFDMA) relaying systems. The proposed algorithm is based on a one-shot multiple-item auction, where each user submits bundles of subcarriers and their corresponding bids. Bundles are generated based on the Shapley and the pair-wise Synergy-Shapley values computed for the user's data rate. After receiving all bids, the Winner-Determination-Problem (WDP) is solved using the structured search algorithm to allocate the subcarriers, then the power is allocated optimally at the source and relay nodes to maximize the sum rate. The effect of the number of submitted bundles/bids on the throughput and fairness indices is investigated. The proposed combinatorial auction outperforms in the throughput and fairness indices an auction algorithm without bundling strategies even though for the case where users are allowed to bid for few bundles in addition to the singleton bid. Numerical results are used to show the advantages of the proposed algorithm.

Index Terms—AF, OFDMA, resource allocation, one-shot auction, Shapley value, bundle auction

I. Introduction

Cooperative and relaying wireless communication systems can significantly improve the throughput and reliability of wireless communications. Amplify-and-Forward (AF) is the simplest relaying scheme [1]. Orthogonal – Frequency – Division – Multiple – Access (OFDMA) allows multiple users to transmit simultaneously on different subcarriers during the same symbol period. Resource allocation is an essential consideration to enhance the overall system performance while satisfying certain constraints at the same time. Different algorithms are used to allocate resources in wireless communication systems to achieve different objectives based on optimization and game theory formulations.

Recently, auction theory has been considered for resource allocation in wireless communication systems to handle the problem of resource competition among selfish users as in [2]-[7]. In [2], auction algorithms for wireless communication systems were comprehensively surveyed. In [3], the authors proposed a channel allocation algorithm based on the second-price-auction to allow users to compete for a wireless fading channel. In [4], an auction algorithm for sub-channel allocation was proposed using the difference of throughput among sub-channels to allow users to compete through bidding. In [5], the authors proposed auction-based scheduling algorithm for OFDMA systems to achieve proportional fair resource allocation. In [6], an auction algorithm was proposed for subcarrier allocation aiming to balance efficiency and fairness with service differentiation in OFDMA relay networks. In [7], different bidding strategies for AF-OFDMA relay networks with optimal power allocation at the source and relay nodes were proposed aiming to achieve different objectives.

The bundling (combinatorial) auction is the auction that allows users to place bids on combinations of items; the value of the bundle is not the sum of the values of its individual items. Bundling auction compared to other auction mechanisms often increases the efficiency of the auction, while keeping risks for the bidders low [8]. In [9], an energy efficient combinatorial auction algorithm was proposed to allocate the resources between device-to-device equipments and cellular users. Generally, each user in OFDMA systems can be allocated several subcarriers. In this sense, combinatorial auction can be used to allocate the subcarriers in OFDMA systems. However, for the best of the authors knowledge, resource allocation for OFDMA systems based on combinatorial auction is not considered in literature.

The main contribution of this paper is to develop a combinatorial auction for subcarrier allocation in AF-OFDMA relaying systems. The bundles are generated based on the synergy model. A bundle value comprises two parts, the values of individual subcarriers and the pair-wise synergy among pairs of subcarriers as in [10]. The values of individual subcarries are evaluated using the Shapley value (i.e., cooperative game solution concept) as in [7]. Based on the Shapley value, we propose the concept of pair-wise Synergy-Shapley value and use it to evaluate the worth of each pair of subcarriers. Using the user's data rate with optimal power allocation at the source and relay nodes, the Shapley and pair-wise Synergy-Shapley values are computed. The computational complexity of calculating the Shapley value and pair-wise Synergy-Shapley value is avoided by using a sampling method to approximate both values within a reasonable accuracy. Bundles are generated in a hierarchical fashion based on the average value of the bundle; start by two-element bundles then move to three-element bundles and so on. The bundle size is increased until the average bundle value starts to decrease. Then the

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subcarriers are allocated by solving the Winner-Determination-Problem (WDP).

The remaining of this paper is organized as follows. In Section II, the system model is presented. The resource allocation algorithm is presented in Section III. Numerical results are presented and discussed in Section IV. Finally, conclusions are drawn in Section V.

II. AF-OFDMA System Model

A single relay two-hop AF-OFDMA system is considered. The available bandwidth W is divided into N subcarriers in which the channel coefficients of each subcarrier is assumed to be frequency flat. Let $\mathcal{J}=\{1,2,\cdots,N\}$ be the set of subcarriers, and $\mathcal{I}=\{1,2,\cdots,I\}$ be the set of active users. Sender (source) nodes S_i for $i=1,2,\cdots,I$ are communicating with the destination node D. The first-hop channel coefficients of the i th user between the source and destination and the source and relay nodes at the j th subcarrier are denoted as $h_{SD}^{(i)}(j)$ and $h_{SR}^{(i)}(j)$, respectively. The second-hop channel coefficient between the relay and destination at the j th subcarrier is denoted by $h_{RD}(j)$. The data rate $R_{AF}^{(i)}(j)$ of the j th subcarrier in AF relaying systems is computed as [1]:

$$R_{AF}^{(i)}(j)=\frac{W}{2N}\log(1+\frac{\Gamma_{SD}^{(i)}(j)+\Gamma_{AF}^{(i)}(j)}{\Gamma}) \tag{1}$$

where Γ is a constant representing the capacity gap, $\Gamma_{SD}^{(i)}(j)$ is the signal-to-noise-ratio (SNR) for direct transmission of the i th user using the j th subcarrier and $\Gamma_{AF}^{(i)}(j)$ is the end-to-end SNR of the i th user for subcarrier j using AF relaying. The SNR $\Gamma_{SD}^{(i)}(j)$ is computed as: $\Gamma_{SD}^{(i)}(j)=\hat{P}_S^{(i)}(j)\gamma_{SD}^{(i)}(j)$, where $\hat{P}_S^{(i)}(j)$ is the i th user transmit power on the j th subcarrier, $\gamma_{SD}^{(i)}(j)=\frac{|h_{SD}^{(i)}(j)|^2}{\sigma^2}$ and σ^2 is the variance of the additive-white-Gaussian-noises (AWGN). The end-to-end SNR of AF relaying $\Gamma_{AF}^{(i)}(j)$ is computed as [11]:

$$\Gamma_{AF}^{(i)}(j)=\frac{\gamma_{SR}^{(i)}(j)\gamma_{RD}(j)\hat{P}_S^{(i)}(j)\hat{P}_R^{(i)}(j)}{1+\gamma_{SR}^{(i)}(j)\hat{P}_S^{(i)}(j)+\gamma_{RD}(j)\hat{P}_R^{(i)}(j)} \tag{2}$$

where $\gamma_{SR}^{(i)}(j)=\frac{|h_{SR}^{(i)}(j)|^2}{\sigma^2}$, $\gamma_{RD}(j)=\frac{|h_{RD}(j)|^2}{\sigma^2}$ and $\hat{P}_R^{(i)}(j)$ is the relay transmitted power for the i th user at the j th subcarrier.

III. Resource Allocation Algorithm

The resource allocation problem of AF-OFDMA communication systems aiming to achieve different objectives (e.g. maximize sum rate, maximize fairness, etc.) comprises two parts: the power allocation at the source and relay nodes and the subcarrier assignment profile. Splitting the resource allocation problem into two resource allocation problems (i.e., power allocation problem and subcarrier assignment problem) is interesting for the following reasons: first, it can be used to reduce the computational complexity and may lead to distributed algorithms, since the joint resource allocation problem is a mixed integer non-linear programming problem, which is computationally complex to solve. However, the obtained solution may be sub-optimal. Second, it can be used to obtain a solution that compromises between two objectives, e.g., compromising between the throughput and fairness; the subcarriers can be allocated to achieve fairness and the relay and source power can be allocated to maximize the throughput. Third, it opens the door to use different frameworks (i.e. game theory, market mechanism, etc.) to formulate the resource allocation problem and obtain solutions with some desirable properties, e.g., competitive based, truth revealing, etc. In this sense, we propose a combinatorial auction mechanism for the subcarrier assignment problem and convex optimization techniques are used to solve the power allocation problem. In the followings, the power allocation problem is formulated and solved for a given subcarrier assignment profile and then the combinatorial auction framework is presented for the subcarrier assignment problem. Finally, the WDP is formulated and solved.

A. Power Allocation

Let $\mathbf{Y}$ be the subcarrier assignment profile, i.e., $[\mathbf{Y}]_{ij}=Y_j^{(i)}\in\{0,1\}$ with $Y_j^{(i)}=1$ indicates that subcarrier j is assigned to user i and $\sum_{i\in\mathcal{I}}Y_j^{(i)}=1,\forall j\in\mathcal{J}$ indicates that subcarrier j is assigned to only one user.

The power allocation problem is aiming to maximize the sum rate formulated as:

$$\max_{\mathbf{P}}\frac{W}{2N}\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}Y_j^{(i)}\log(1+\frac{\Gamma_{SD}^{(i)}(j)+\Gamma_{AF}^{(i)}(j)}{\Gamma Y_j^{(i)}}), \tag{3a}$$

$$s.t.\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}P_R^{(i)}(j)\le P_R^{\max}, \tag{3b}$$

$$\sum_{j\in\mathcal{J}}P_S^{(i)}(j)\le P_i^{\max},\forall i\in\mathcal{I}, \tag{3c}$$

$$P_S^{(i)}(j)\ge 0, P_R^{(i)}(j)\ge 0, \tag{3d}$$

where $\mathbf{P}$, is the vector of the sources and the relay power profiles, which contains $P_S^{(i)}(j)=\hat{P}_S^{(i)}(j)Y_j^{(i)}$ and

$P_R^{(i)}(j) = \hat{P}_R^{(i)}(j)Y_j^{(i)} \quad \forall i \in \mathcal{I}$ and $\forall j \in \mathcal{J}$. Constraint (3b) means that the total power allocated to forward the data from all users assisted by the relay is limited to the maximum relay power $P_R^{\max}$, whereas constraint (3c) indicates that the source power allocated for user i is limited to the maximum power $P_i^{\max}$. For high SNR, $\Gamma_{AF}^{(i)}(j)$ can be approximated by its upper bound as in [11]:

$$\Gamma_{AF}^{(i)}(j) \approx \frac{\gamma_{SR}^{(i)}(j)\gamma_{RD}(j)P_S^{(i)}(j)P_R^{(i)}(j)}{\gamma_{SR}^{(i)}(j)P_S^{(i)}(j) + \gamma_{RD}(j)P_R^{(i)}(j)} \tag{4}$$

which will be used from now on to derive the forthcoming results. With this approximation, the data rate $R_{AF}^{(i)}(j)$ becomes a jointly concave function with respect to $P_R^{(i)}(j)$ and $P_S^{(i)}(j)$ as can be proved by the second order derivative test. Hence, the optimal power profile $\mathbf{P}$ can be obtained using convex optimization techniques. An analytical solution can be obtained by differentiating the Lagrangian function with respect to $P_S^{(i)}(j)$ and $P_R^{(i)}(j)$ and then equating by zero as [7]:

$$P_S^{(i)}(j) = \begin{cases} \dfrac{\left(\dfrac{W/2N}{\lambda_i}(\gamma_{SD}^{(i)}(j) + A_j^{(i)}) - \Gamma\right)^+}{\gamma_{SD}^{(i)}(j) + B_j^{(i)}} & if \ \ P_R^{(i)}(j) > 0, \\ \left(\dfrac{W/2N}{\lambda_i} - \dfrac{\Gamma}{\gamma_{SD}^{(i)}(j)}\right)^+ & if \ \ P_R^{(i)}(j) = 0, \end{cases} \tag{5}$$

where $(x)^+ = \max\{0, x\}$, $A_j^{(i)} = \dfrac{\gamma_{SR}^{(i)}(j)\gamma_{RD}^2(j)C_j^{2(i)}}{(\gamma_{SR}^{(i)}(j) + \gamma_{RD}(j)C_j^{(i)})^2}$,

and $B_j^{(i)} = \dfrac{\gamma_{SR}^{(i)}(j)\gamma_{RD}(j)C_j^{(i)}}{\gamma_{SR}^{(i)}(j) + \gamma_{RD}(j)C_j^{(i)}}$.

The relay power profile is obtained as:

$$P_R^{(i)}(j) = P_S^{(i)}(j)D_j^{(i)} \tag{6}$$

where $D_j^{(i)}$ is computed as:

$$D_j^{(i)} = \frac{\gamma_{SR}^{(i)}(j)\left(-1 + \sqrt{1 + (1 + \dfrac{\gamma_{SR}^{(i)}(j)}{\gamma_{SD}^{(i)}(j)})(\dfrac{\lambda_i}{\lambda_R}\dfrac{\gamma_{RD}(j)}{\gamma_{SD}^{(i)}(j)} - 1)}\right)^+}{\gamma_{RD}(j)(1 + \dfrac{\gamma_{SR}^{(i)}(j)}{\gamma_{SD}^{(i)}(j)})}.$$

The Lagrange multiplier λ_i is selected to satisfy the total source power constraint $\sum_{j \in \mathcal{J}_i} P_S^{(i)}(j) = P_i^{\max}$ for the set of subcarriers $\mathcal{J}_i$ allocated to user i with $\mathcal{J}_i \neq \varnothing$, and the Lagrange multiplier λ_R is selected to satisfy the total relay power constraint (3b).

B. Combinatorial Auction for Subcarrier Assignment

The combinatorial auction framework is proposed to decide on the subcarrier assignment profile $\mathbf{Y}$ because of its efficiency compared to a single bid auction. Each user submits bids for bundles of subcarriers, besides it submits singleton bids for all subcarriers. The bundle value reflects the worth of the bundle to the user. Since it is impossible to compute all possible bundle values. The simple synergy model is adopted to generate the bundles as in [10]. In this sense, the subcarriers are combined into bundles based on the individual subcarrier's value and the pair-wise synergy values. The normalized Shapley value is used as the individual subcarrier's value as in [7]. For the pair-wise synergy value we extend the concept of Shapley value and introduce the Synergy-Shapley value as detailed next.

In the followings; we will defined the Shapley and Synergy-Shapley values, then we will present a sampling approach to approximate their values and then we will present the proposed bundling algorithm based on both the Shapley and Synergy-Shapley values. Finally, we will show how the bundles (subcarriers) are assigned to the users by solving the WDP.

The AF-OFDMA cooperative game is defined as [7]:

Definition: An AF-OFDMA cooperative game with transferable utility is defined as the pair ($\mathcal{J}, v_i$), where v_i is a real valued mapping defined over all possible subsets of $\mathcal{J}$. The mapping v_i is called the characteristic function or the value function of the i th user and it is computed based on the achievable data rate of AF-OFDMA as explained next.

The characteristic function $v_i(\mathcal{J}_i)$ of the i th user using the set of subcarriers $\mathcal{J}_i$, where $\mathcal{J}_i$ is one of the 2^N possible subsets of $\mathcal{J}$ is defined as [7]:

$$v_i(\mathcal{J}_i) = \frac{W}{2N}\sum_{j \in \mathcal{J}_i}\log(1 + \frac{\Gamma_{SD}^{*(i)}(j) + \Gamma_{AF}^{*(i)}(j)}{\Gamma}) \tag{7}$$

where $\Gamma_{SD}^{*(i)}(j)$ and $\Gamma_{AF}^{*(i)}(j)$ are obtained by using optimal power profiles at the source and relay nodes as in (5) and (6), respectively. Computing the relay optimal power profile for $j \in \mathcal{J}_i$, requires the optimal Lagrange multiplier λ_R, which is dependent on the subcarrier assignment of all other users. To reduced the computational complexity and the overhead between the users and destination node of computing λ_R, we will assume that the destination node broadcasts λ_R and $h_{RD}(j)$ for all users to generate the bundles.

The Shapley value from cooperative games is used to evaluate the worth of the subcarrier in AF-OFDMA systems, as in [7]. The Shapley value allows the user to quantify accurately the contribution of each subcarrier towards the data rate.

Let $\mathbf{\Phi}^{(i)} = (\Phi_1^{(i)}, \Phi_2^{(i)}, \cdots, \Phi_N^{(i)})$ be the Shapley value vector of the i th user. The Shapley value of subcarrier j for user i is given as [12]:

$$\Phi_j^{(i)} = \frac{1}{N!}\sum_{\pi\in\Omega} v_i(C_j(\pi)\cup\{j\}) - v_i(C_j(\pi)) \quad (8)$$

where Ω is the set of all possible $N!$ permutations on $\mathcal{J}$ assuming all orderings are equally likely, π is a permutation in Ω, $C_j(\pi)$ is the set of all subcarriers appearing before subcarrier j in the permutation π, and $v_i(C_j(\pi)\cup\{j\}) - v_i(C_j(\pi)), \forall C_j \subseteq \mathcal{J}\setminus\{j\}$ is the marginal contribution of subcarrier j to the coalition of subcarriers $C_j(\pi)\cup\{j\}$. The Synergy-Shapley value denoted as $\Phi_{j,k}^{(i)}$ is defined as:

$$\Phi_{j,k}^{(i)} = \frac{1}{N!}\sum_{\pi\in\Omega} v_i(C_{j,k}(\pi)\cup\{j,k\}) - v_i(C_{j,k}(\pi)) \quad (9)$$

where $C_{j,k}(\pi)$ is the set of all subcarriers appearing before the pair of subcarriers $\{j,k\}$ in the permutation π, and $v_i(C_{j,k}(\pi)\cup\{j,k\}) - v_i(C_{j,k}(\pi))$, $\forall C_{j,k} \subseteq \mathcal{J}\setminus\{j,k\}$ is the marginal contribution of the pair of subcarriers $\{j,k\}$ to the coalition of subcarriers $C_{j,k}(\pi)\cup\{j,k\}$. The Synergy-Shapley value vector of the i th user is $\mathbf{\Phi}_{Syn}^{(i)} = (\Phi_{1,2}^{(i)}, \cdots, \Phi_{1,N}^{(i)}, \Phi_{2,3}^{(i)} \cdots, \Phi_{N-1,N}^{(i)})$.

It is clear that expressions (8) and (9) require to work with $N!$ permutations on $\mathcal{J}$ to compute the marginal contributions of the subcarriers which is computationally complex especially for large N. Hence, the direct approach for computing the Shapley and Synergy-Shapley values are not tractable. Therefore, the Shapley and the Synergy-Shapley values are computed approximately in this work using a sampling-based approach that works in polynomial time, as in [12]. The estimation of the Shapley and Synergy-Shapley values are presented in Algorithms 1 & 2, respectively. The estimation of the Shapley value/ pair-wise Synergy-Shapley value, scales linearly with the number of subcarriers N and the number of samples M.

Algorithm 1: Estimation of the Shapley value

Require: M, $\gamma_{SD}^{(i)}(j)$, $\gamma_{SR}^{(i)}(j)$, $\gamma_{RD}(j)$, $\forall j \in \mathcal{J}$, λ_R, $P_i^{\max}$.

for $j \in \mathcal{J}$ **do** $\hat{\Phi}_j^{(i)} \leftarrow 0$, **end for**
for $m = 1{:}M$ **do**
Generate a sample $\pi_m \in \Omega$ with probability $\frac{1}{N!}$.
for $j \in \mathcal{J}$ **do**
Find the set $C_j(\pi_m)$.
Find $v_i(C_j(\pi_m)\cup\{j\})$ and $v_i(C_j(\pi_m))$ using (7).
Calculate $M_c^{(i)}(j) = v_i(C_j(\pi_m)\cup\{j\}) - v_i(C_j(\pi_m))$
Calculate $\hat{\Phi}_j^{(i)} = \hat{\Phi}_j^{(i)} + M_c^{(i)}(j)$.
end for
end for
$\hat{\Phi}_j^{(i)} = \frac{\hat{\Phi}_j^{(i)}}{M}$, $\forall j \in \mathcal{J}$
return $\mathbf{\Phi}^{(i)}$

Algorithm 2: Estimation of the Synergy-Shapley value.

Required: M, $\gamma_{SD}^{(i)}(j)$, $\gamma_{SR}^{(i)}(j)$, $\gamma_{RD}(j)$, $\forall j \in \mathcal{J}$, $P_i^{\max}$, λ_R

for $j \in \mathcal{J}$, $k \in \mathcal{J}$, $j \neq k$ **do** $\hat{\Phi}_{j,k}^{(i)} \leftarrow 0$, **end for**
for $m = 1{:}M$ **do**
Generate a sample $\pi_m \in \Omega$ with probability $\frac{1}{N!}$.
for $j \in \mathcal{J}$ **do**
for $k \in \mathcal{J}$ $j \neq k$ **do**
Find the set $C_{j,k}(\pi_m)$
Calculate $v_i(C_{j,k}(\pi_m)\cup\{j,k\})$ and $v_i(C_{j,k}(\pi_m))$
Calculate $M_c^{(i)}(j,k) = v_i(C_{j,k}(\pi_m)\cup\{j,k\}) - v_i(C_{j,k}(\pi_m))$
Calculate $\hat{\Phi}_{j,k}^{(i)} = \hat{\Phi}_{j,k}^{(i)} + M_c^{(i)}(j,k)$.
end for
end for
end for
$\hat{\Phi}_{j,k}^{(i)} = \frac{\hat{\Phi}_{j,k}^{(i)}}{M}$, $\forall j,k \in \mathcal{J}$
return $\mathbf{\Phi}_{Syn}^{(i)}$

To generate the bundles and their corresponding bids, the simple synergy model is used, as in [10]. According to this model, the bundle value includes two parts; the value of the individual items in the bundle and the synergy values among the items in the bundle. We assume that users bid with their true valuations. The bidding strategy of the i th user for a single subcarrier (i.e., singleton bid) defined as $\mathbf{b}_1^{(i)} = [b_1^{(i)}, b_2^{(i)}, \cdots, b_N^{(i)}]^T$, where $b_j^{(i)} = b_{Sh}^{(i)}(j)$ is obtained by the normalized Shapley value as: $b_{Sh}^{(i)}(j) = \frac{\hat{\Phi}_j^{(i)}}{\sum_{j \in \mathcal{J}} \hat{\Phi}_j^{(i)}}$, $\forall j \in \mathcal{J}$. The normalized pair-wise Synergy-Shapley value is defined as: $b_{Syn}^{(i)}(j,k) = \frac{\hat{\Phi}_{j,k}^{(i)}}{\sum_{j,k \in \mathcal{J}} \hat{\Phi}_{j,k}^{(i)}}$, $\forall j,k \in \mathcal{J}$.

For a doubleton bid, the bundle value is the sum of the two subcarriers values (normalized Shapley values) plus the normalized pair-wise Synergy-Shapley value between the two subcarriers. The value of the i th user for bundle $\mathcal{B}^{(i)}$, with size $|\mathcal{B}^{(i)}| \geq 2$ is computed as [10]:

$$b_{\mathcal{B}}^{(i)} = \sum_{j=1}^{|\mathcal{B}^{(i)}|} b_{Sh}^{(i)}(j) + \frac{2}{(|\mathcal{B}^{(i)}|-1)} \sum_{j=1}^{|\mathcal{B}^{(i)}|} \sum_{k>j} b_{Syn}^{(i)}(j,k) \quad (10)$$

The average unit contribution (AC) of bundle $\mathcal{B}^{(i)}$ for user i denoted as $AC_{\mathcal{B}}^{(i)}$ is computed as:

$$AC_{\mathcal{B}}^{(i)} = \frac{b_{\mathcal{B}}^{(i)}}{|\mathcal{B}^{(i)}|} \quad (11)$$

The bundle creation algorithm starts from each individual subcarrier and searches for subcarriers to add to the current bundle to increase the AC, if such subcarrier can be found, then the subcarrier which increases the AC the most is added. The process proceeds until the AC cannot be increased further. All generated bundles until the algorithm stops are considered as desirable bundles. Once the bundles are generated, the user may submit some or all of the generated bundles depending on whether or not the destination puts a limit on the maximum number of bundle bids allowed per the user or the number of times the subcarrier appears in all the bundles.

Algorithm 3 describes the process of generating the bundles of subcarriers for user i using the estimated normalized Shapley and Synergy-Shapley values. In this algorithm:

1. $\mathcal{B}_{n_j}^{(i)}(j)$ represents the i user bundle of size n_j and contains subcarrier j.
2. $b_{n_j}^{(i)}(j)$ is the bid of bundle $\mathcal{B}_{n_j}^{(i)}(j)$.
3. $\mathcal{B}^{(i)}$ represents the set of the generated bundles for user i.
4. $\mathbf{b}^{(i)}$ represents the set of bids corresponding to the set of bundles $\mathcal{B}^{(i)}$.
5. $\mathcal{B}_{n>1}^{(i)} =$ denotes the i th user set of bundles of size n greater than 1, i.e., $\mathcal{B}_{n>1}^{(i)} = \mathcal{B}^{(i)} - \{\mathcal{B}_1^{(i)}(1), \cdots, \mathcal{B}_1^{(i)}(j), \cdots, \mathcal{B}_1^{(i)}(N)\}$.

Algorithm 3: Bundles generating algorithm

Require: $b_{Sh}^{(i)}(j)$, $b_{Syn}^{(i)}(j,k)$, $\forall j \in \mathcal{J}$, $\forall k \in \mathcal{J}$.

1: Set $\mathcal{B}^{(i)} = \varnothing$, $\mathbf{b}^{(i)} = \varnothing$ and $n_j = 1, \forall j = 1, \cdots, N$.
2: Create a single subcarrier bundle $\mathcal{B}_{n_j}^{(i)}(j) = \{j\}$, $\forall j \in \mathcal{J}$.
3: Assign it the bidding strategy $b_{n_j}^{(i)}(j) = b_{Sh}^{(i)}(j)$, $\forall j \in \mathcal{J}$.
4: Update $\mathcal{B}^{(i)} = \{\mathcal{B}^{(i)}, \mathcal{B}_{n_j}^{(i)}(j)\}$, and $\mathbf{b}^{(i)} = \{\mathbf{b}^{(i)}, b_{n_j}^{(i)}(j)\}$.
5: **for** $\forall j \in \mathcal{J}$ **do**
6: Set $\mathcal{B} = \mathcal{B}_{n_j}^{(i)}(j)$.
7: Calculate $AC_{\mathcal{B} \cup \{k\}}^{(i)}$, $\forall k \notin \mathcal{B}$ and $\mathcal{B} \cup \{k\} \notin \mathcal{B}^{(i)}$ using (10) and (11).
8: Find $\hat{k} = \arg\max_k AC_{\mathcal{B} \cup \{k\}}^{(i)}$
9: **If** $AC_{\mathcal{B} \cup \{\hat{k}\}}^{(i)} > AC_{\mathcal{B}}^{(i)}$ **then**
10: $n_j = n_j + 1$
11: Set $\mathcal{B}_{n_j}^{(i)}(j) = \mathcal{B} \cup \{\hat{k}\}$ and $b_{n_j}^{(i)}(j) = |\mathcal{B}_{n_j}^{(i)}(j)| AC_{\mathcal{B} \cup \{\hat{k}\}}^{(i)}$
12: Update $\mathcal{B}^{(i)} = \{\mathcal{B}^{(i)}, \mathcal{B}_{n_j}^{(i)}(j)\}$ and $\mathbf{b}^{(i)} = \{\mathbf{b}^{(i)}, b_{n_j}^{(i)}(j)\}$.
13: **Go to** 6.
14: **end if**
15: **end for**
16: **return** $\mathcal{B}^{(i)}$ and $\mathbf{b}^{(i)}$

After all users submit their bids $\mathbf{b}^{(i)}, \forall i \in \mathcal{I}$ and the corresponding bundles $\mathcal{B}^{(i)}, \forall i \in \mathcal{I}$, the destination node assigns the subcarriers to the users aiming to maximize its own benefit by solving the WDP formulated as [8]:

$$\max_{x_n^{(i)}; \forall n \in n_i, \forall i \in \mathcal{I}} \sum_{i \in \mathcal{I}} \sum_{n \in n_i} x_n^{(i)} [\mathbf{b}^{(i)}]_n \quad (12a)$$

$$s.t. \sum_{i \in \mathcal{I}} \sum_{n \in n_i} x_n^{(i)} I_n^{(i)}(j) \leq 1, \forall j \in \mathcal{J}, \quad (12b)$$

$$x_n^{(i)} \in \{0,1\}, \forall n \in n_i, \forall i \in \mathcal{I}, \quad (12c)$$

where n_i is the set of bundles submitted by user i, $[\mathbf{b}^{(i)}]_n$ is the n th bundle submitted by user i and $[\mathbf{b}^{(i)}]_n$ is its corresponding bid. $x_n^{(i)}$ is a binary variable representing whether bid $[\mathbf{b}^{(i)}]_n$ is selected or not, and $I_n^{(i)}(j)$ is an indicator function such that $I_n^{(i)}(j)=1$ if subcarrier j is included in bundle $[\mathcal{B}^{(i)}]_n$ and $I_n^{(i)}(j)=0$, otherwise. Constraints (12b) means that the subcarrier can be assigned to only one bundle.

Even though problem (12) is an NP-hard problem, many effective search algorithms were proposed to find the optimal allocation based on cleaver structuring of the search space, preprocessing, heuristic ordering methods and pruning techniques as in [13]. Stochastic local search algorithm and population-based swarm particle optimization can also be used to solve the WDP as in [14], [15], respectively. In this paper, the structured search algorithm is applied to solve the WDP as in [13].

Finally, the source and relay power profiles are determined using (5) and (6) for the subcarrier assignment profile obtained from the structured search algorithm.

Two performance measures are used to compare the performance of the proposed algorithm: Jain's fairness index F_I computed as:

$$F_I = \frac{(\sum_{i\in\mathcal{I}} v_i(\mathcal{J}_i))^2}{|\mathcal{I}|\sum_{i\in\mathcal{I}} v_i^2(\mathcal{J}_i)} \tag{13}$$

where a value of fairness index closer to 1 means a relative better fairness, and the throughput index T_I computed as:

$$T_I = \frac{\sum_{i\in\mathcal{I}} v_i(\mathcal{J}_i)}{\sum_{i\in\mathcal{I}} \hat{v}_i(\hat{\mathcal{J}}_i)} \tag{14}$$

where $\hat{\mathcal{J}}_i$ is the allocation that maximizes the sum rate $\sum_{i\in\mathcal{I}} \hat{v}_i(\hat{\mathcal{J}}_i)$. In order to find $\sum_{i\in\mathcal{I}} \hat{v}_i(\hat{\mathcal{J}}_i)$, the joint resource allocation problem is solved using the dual approach based on the assumption of zero-duality gap, since OFDMA systems satisfy the time sharing property for large number of subcarriers as shown in [16]. The proposed combinatorial auction for resource allocation in AF-OFDMA is of a distributed nature; the bundles are generated at the users side and the WDP is solved at the base-station side. The computational complexity of the proposed combinatorial auction for AF-OFDMA is summarized in Table I. Limiting the number of submitted bids reduces the computational complexity of the WDP as shown in [13].

TABLE I: THE COMPUTATIONAL COMPLEXITY OF THE PROPOSED COMBINATORIAL AUCTION

Step	Complexity Order	Comments
Shapley value	$\mathcal{O}(NM)$	Sampling approach
Synergy Shapley value	$\mathcal{O}(NM)$	Sampling approach
Bundle generation	$\mathcal{O}(N^2)$	Generalized synergy model
WDP	$\mathcal{O}(N^3)$	Structured search algorithm

IV. SIMULATION RESULTS AND DISCUSSION

We model the subcarrier channel coefficients between any two nodes with a separating distance d as $h(j)\sim\mathcal{CN}(0,\frac{1}{L(1+d)^n})$, where $\mathcal{CN}$ is the complex normal distribution, $n=4$ is the propagation loss factor, and $L=4$ is the number of channel taps as in [17]. The scenario under consideration is shown in Fig. 1 consists of I users ($S_1, S_2, \cdots, S_I$), one relay and a common destination D. The sources are uniformly distributed in the shaded area.

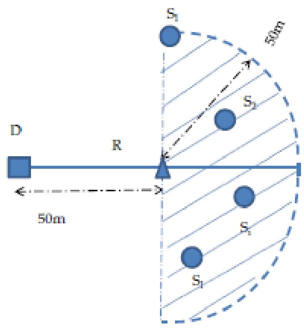

Fig. 1. Sources, relay and destination nodes positions.

The subcarrier noise power σ^2 is set as 4×10^{-11} Watt. The source maximum transmit power is $P_i^{\max}=1$ Watt, and the relay maximum transmit power is $P_R^{\max}=10$ Watt unless otherwise specified. The number of subcarriers $N=32$, the subcarrier bandwidth $W/N=4$ KHz and the capacity gap $\Gamma=1$. The sample size M is set to 50 as in [7].

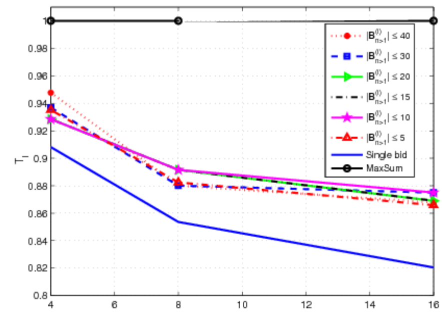

Fig. 2. The throughput index T_I as a function of the number of users.

Fig. 2 shows the throughput index T_I of the proposed bundle auction as a function of the number of users. The maximum data rate $\sum_{i\in\mathcal{I}}\hat{v}_i$ is obtained as in [16]. The throughput index T_I of the proposed bundle auction is shown for different number of bids. The maximum number of permitted bids is used to label the curves. Clearly, the throughput index T_I of the bundle auction outperforms the throughput index T_I of a single bid auction (singleton bid) as expected. It is noticed that the optimal number of submitted bundles is dependent on the number of users, e.g., for the scenario of 4 users, the highest T_I is obtained when $|\mathcal{B}_{n>1}^{(i)}| \leq 40$, whereas for the scenario of 16 users, the highest T_I is obtained with $|\mathcal{B}_{n>1}^{(i)}| \leq 10$.

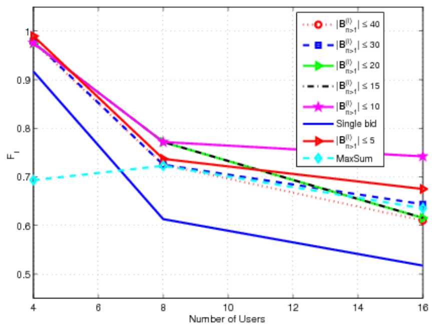

Fig. 3. The fairness index F_I as a function of the number of users.

Fig. 3 shows the fairness index F_I as a function of the number of users for the proposed bundle auction as a function of the number of users for different number of bids. The fairness index of the maximum sum rate is also shown for comparison purposes. The fairness index F_I of the proposed auction algorithm outperforms the fairness index of a single bid auction. This is due to the fact that each subacrrier is allocated to the user who submits the highest bid in a single bid auction, whereas in the bundle auction and the solution of the WDP is more efficient than the solution of a single bid auction. It is noticed that limiting the number of submitted bundles, increases the fairness index F_I, however, the optimal number of submitted bundles is dependent on the number of users, e.g., for the scenario of 4 users, the highest F_I is obtained when $|\mathcal{B}_{n>1}^{(i)}| \leq 5$, whereas for the scenario of 16 users, the highest F_I is obtained with $|\mathcal{B}_{n>1}^{(i)}| \leq 10$.

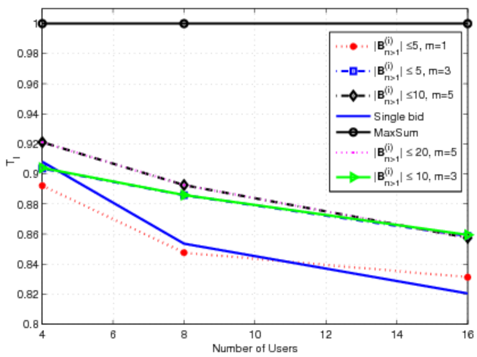

Fig. 4. The throughput index T_I as a function of the number of users.

Figs. 4 & 5 show the throughput T_I and fairness F_I indices of the proposed bundle auction as a function of the number of users, respectively, where the destination puts a limit on the number of times m the subcarrier appears in all bundles submitted by the user.

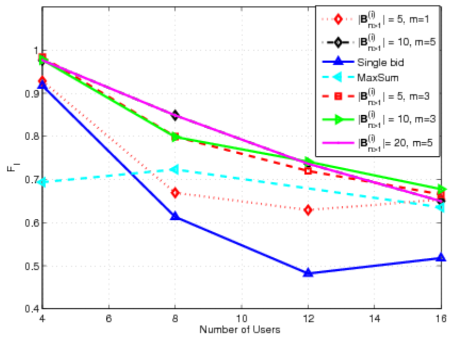

Fig. 5. The fairness index F_I as a function of the number of users.

Clearly, limiting the time of appearance of the subcarrier affects the throughput and fairness indices: i.e., increasing n may improve the performance of the fairness and throughput indices as shown in Figs. 4 & 5. For $m = 1$, it is noticed that the throughput index T_I may decrease beyond the throughput index T_I of the single bid, but the fairness index F_I is greater than the fairness index of the single bid auction.

V. Conclusion

In this paper, a competitive approach is proposed for resource allocation in up-link AF-OFDMA relaying systems. The subcarriers are assigned based on the combinatorial auction framework. The bidding strategy of the auction is based on the estimation of the Shapley and the Synergy-Shapley values.

Each user generates bundles of subcarriers based on the simple synergy model and submits the corresponding bids to the destination. The structured search algorithm is used to solve the WDP and assign the subcarriers to the users. The throughput and fairness indices are used to evaluate the performance of the proposed algorithm. Numerical simulation results show that the proposed algorithms achieve high-performance measures compared to the one-shot single bid auction.

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