A Pilot Assignment Scheme for Single-Cell Massive MIMO Circumstances

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Abstract —In Time-Division Duplex (TDD) massive Multiple-Input Multiple-Output (MIMO) systems, pilot contamination becomes the performance bottleneck when the number of terminals is larger than that of available pilot sequences. In this paper, we first deduce the general formulas for uplink Signal-to-Interference Ratio (SIR) and channel capacity in single-cell massive MIMO circumstances with insufficient pilot sequences. Then, in order to enhance the Quality of Service (QoS) for the terminals who suffer from severe pilot contamination, an optimization problem is formulated to maximize the minimum uplink capacity of all terminals. Next, because the complexity of finding the optimal pilot assignment is too high, a suboptimal two-step assignment approach with low complexity is proposed to solve the optimization problem. Compared with the random pilot assignment scheme in single-cell massive MIMO systems, simulation results prove the effectiveness of this two-step scheme in obtaining better minimum uplink SIR and minimum channel capacity when the number of BS antennas is great but finite. Meanwhile, the average capacity of all terminals can also be improved.

Index Terms—Pilot assignment scheme, single-cell massive MIMO, pilot contamination, performance analysis

I. Introduction

By utilizing a large number of antennas at the Base Station (BS) to serve a relatively small amount of terminals simultaneously, massive MIMO technology is possible not only to improve the spectral and energy efficiency, but also to enhance the robustness and reliability of systems. After recent years' development, it has become a key role in the beyond fourth generation (B4G) cellular systems [1].

Channel State Information (CSI) at the BS is essential for achieving high-performance communications. For TDD protocol, the BS can greatly reduce the overhead of CSI acquisition by exploiting channel reciprocity [2]. Therefore, the typical massive MIMO systems rely on TDD style. However, because of the limited pilot resources, some terminals have to share the same pilot sequence during the uplink transmission, resulting in pilot contamination. Pilot contamination is one of the most important shortcomings of TDD operation. The study shows that it becomes the only remaining impairment with unlimited number of BS antennas [3].

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To vanish pilot contamination, many methods on pilot design and precoding are studied. In [4], a modified frame structure is proposed by shifting the location of pilot so that the pilot transmissions in different cells are done at non-overlapping times. In [5], the fundamental problem of pilot contamination in multi-cell systems is analyzed and a corresponding precoding method is presented based on the Minimum Mean Squared Error (MMSE) principle. Moreover, in [6], an advanced-fractional frequency reuse scheme for multi-cell massive MIMO circumstances is presented to mitigate pilot contamination. In [7], a pilot allocation scheme which allows the terminals in each cell close to their BSs to reuse the same pilot sequence is proposed. However, all these papers assign the available pilot sequences to different terminals randomly, and ignore the fact that the channel quality varies among different terminals. In addition, some researches recognize this fact. And in [8], a Smart Pilot Assignment (SPA) scheme considering the different channel qualities is described in multi-cell massive MIMO systems. In [9], a Soft Pilot Reuse (SPR) combined with a Multi-Cell Block Diagonalization (MBD) precoding is proposed to enhance the QoS for terminals who suffer from severe pilot contamination. However, in [8] and [9], the number of terminals in each cell is assumed to be equal to that of available pilot sequences. In most cases, this assumption is invalid.

In this paper, we first present the general formulas, including the uplink SIR and channel capacity, for a target terminal in single-cell massive MIMO systems when the number of terminals is larger than that of available pilot sequences. Then, in order to improve the performance for the terminals suffering from severe pilot contamination, an optimization problem is formulated to maximize the minimum uplink capacity of all terminals. Because of the high complexity of finding the optimal pilot assignment, we propose a suboptimal two-step assignment scheme with low complexity to solve the optimization problem. Unlike the conventional pilot assignment scheme which allocates all available pilot sequences to the terminals randomly in single-cell massive MIMO circumstances [10], [11], this two-step approach considers the different channel qualities and assigns the insufficient pilot sequences to the terminals according to their large-fading factors. When the number of BS antennas is great but finite, simulation results prove the effectiveness of this assignment scheme in obtaining

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better minimum uplink SIR, minimum channel capacity, as well as the average capacity of all terminals, compared with the random assignment scheme.

The rest of the paper is organized as follows: Section II presents a canonical single-cell TDD massive MIMO system and analyzes the uplink SIR and channel capacity for a target terminal. Section III describes the optimization problem and the suboptimal two-step pilot assignment scheme in detail. Section IV shows the simulation results. And section V is the conclusions.

Notion: In this paper, boldface symbols represent vectors or matrices. CN denotes the field of complex numbers. $(\cdot)!$ denotes the factorial operator. $(\cdot)^T$ and $(\cdot)^*$ denote the transpose operator and Hermitian transpose operator, respectively. $\{\cdot\}$ denotes a collection of parameters and $\{\cdot\}_i$ denotes the i-th element of it. The additive noises mentioned below are independent and identically distributed (i.i.d.) zero-mean circularly-symmetric complex Gaussian random variables.

II. System Specification

In this section, we first present a typical single-cell TDD massive MIMO system. Then, we analyze the uplink SIR and channel capacity for a target terminal when the number of terminals is larger than that of available pilot sequences.

A. System Model

As shown in Fig. 1, we consider a single-cell TDD massive MIMO system. The radius of the hexangular cell is R. The only BS with M antennas, located at the central position, serves K $(K << M)$ single-antenna terminals which are uniformly random distributed in the cell (with the exclusion of a disk of radius r_c centered on the BS). In addition, $\mathbf{g}_k = [g_{1k}, g_{2k}, \cdots, g_{Mk}]^T \in CN^{M\times 1}$ denotes the propagation factor between the BS and the k-th terminal. It can be modeled as

$$\mathbf{g}_k = \mathbf{h}_k \sqrt{\beta_k} \tag{1}$$

where $\mathbf{h}_k \in CN^{M\times 1}$ denotes the small-scale fading matrix between the BS and the k-th terminal. Each element of $\mathbf{h}_k$ is assumed to be i.i.d. Gaussian random variable with zero mean and unit variance. β_k denotes the large-scale fading factor between the BS and the k-th terminal. Because β_k changes slowly over space, it is assumed to be a non-negative constant and known to the BS.

For TDD protocol, we assume $\mathbf{g}_k$ remains constant during the coherence interval with the length of T symbols. In addition, for uplink transmission, the coherence interval contains of two phases. At the first phase, all terminals transmit their pilot sequences simultaneously, after that the BS estimates the channels based on these uplink pilots. At the second phase, all terminals transmit data symbols, afterwards the BS decodes these data by exploiting the channel estimates implemented at the first phase.

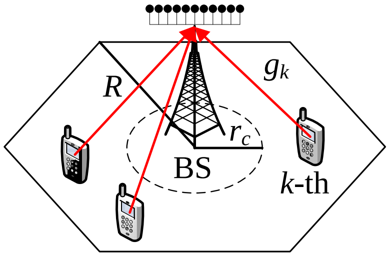

Fig. 1. The single-cell TDD massive MIMO system.

B. Performance Analysis

Assuming the pilot sequences used in this cell $\{\mathbf{\Phi}^1, \mathbf{\Phi}^2, \cdots, \mathbf{\Phi}^\tau\}$ are mutually orthogonal, i.e.

$$\begin{cases} \mathbf{\Phi}^{j_1} (\mathbf{\Phi}^{j_2})^* = 1 & j_1 = j_2 \\ \mathbf{\Phi}^{j_1} (\mathbf{\Phi}^{j_2})^* = 0 & j_1 \neq j_2 \end{cases} \tag{2}$$

where $\mathbf{\Phi}^j \in CN^{M\times\tau}$, and τ $(\tau < K)$ denotes the number of available pilot sequences.

In this paper, we consider the i-th terminal as the target terminal. At the first phase of uplink transmission, the received pilot sequences at the BS $\mathbf{r}_i \in CN^{M\times\tau}$ can be represented as

$$\mathbf{r}_i = \sqrt{\rho_p} \sum_{k=1}^{K} \mathbf{g}_k \mathbf{\Phi}_k + \mathbf{n}_i \tag{3}$$

where ρ_p denotes the average transmission power for uplink pilots. $\mathbf{\Phi}_k \in CN^{1\times\tau}$ denotes the pilot sequence assigned to the k-th terminal, and $\mathbf{\Phi}_k \in \{\mathbf{\Phi}^1, \mathbf{\Phi}^2, \cdots, \mathbf{\Phi}^\tau\}$. $\mathbf{n}_i \in CN^{M\times\tau}$ denotes the additive noise. Then, the BS estimates the channels by correlating $\mathbf{r}_i$ with the pilot sequence $\mathbf{\Phi}_i^*$

$$\begin{aligned} \hat{\mathbf{g}}_i &= \mathbf{r}_i \mathbf{\Phi}_i^* \\ &= (\sqrt{\rho_p} \sum_{k=1}^{K} \mathbf{g}_k \mathbf{\Phi}_k + \mathbf{n}_i) \mathbf{\Phi}_i^* \\ &= \sqrt{\rho_p} \sum_{k=1}^{K} \mathbf{g}_k \xi(k) + \mathbf{n}_i \mathbf{\Phi}_i^* \end{aligned} \tag{4}$$

where $\hat{\mathbf{g}}_i = [\hat{g}_{1i}, \hat{g}_{2i}, \cdots, \hat{g}_{Mi}]^T \in CN^{M\times 1}$ denotes the estimate for $\mathbf{g}_i$. $\xi(k)$ denotes an indicator function and the definition of it can be represented as

$$\xi(k) = \begin{cases} 1 & \mathbf{\Phi}_k \mathbf{\Phi}_i^* = 1 \\ 0 & \mathbf{\Phi}_k \mathbf{\Phi}_i^* = 0 \end{cases} \tag{5}$$

$\mathbf{n}_i \mathbf{\Phi}_i^*$ denotes the equivalent noise. In (5), $\xi(k) = 1$ denotes that the k-th terminal and i-th terminal share the same pilot sequence.

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At the second phase, the received data signals at the BS $\mathbf{y}_i \in CN^{M\times(T-\tau)}$ can be expressed as

$$\mathbf{y}_i = \sqrt{\rho_d}\sum_{k=1}^{K}\mathbf{g}_k\mathbf{x}_k + \mathbf{n}_i \tag{6}$$

where ρ_d denotes the average transmission power for data. $\mathbf{x}_k \in CN^{1\times(T-\tau)}$ denotes the uplink data from the k-th terminal. Then the BS decodes the received data with maximum ratio combining (MRC)

$$\begin{aligned}\frac{\hat{\mathbf{x}}_i}{M\sqrt{\rho_p\rho_d}} &= \frac{\hat{\mathbf{g}}_i^*\mathbf{y}_i}{M\sqrt{\rho_p\rho_d}}\\ &= \frac{1}{M\sqrt{\rho_p\rho_d}}(\sqrt{\rho_p}\sum_{k_1=1}^{K}\xi(k_1)\mathbf{g}_{k_1} + \mathbf{n}_i\mathbf{\Phi}_i^*)^*\\ &\quad\times(\sqrt{\rho_d}\sum_{k_2=1}^{K}\mathbf{g}_{k_2}\mathbf{x}_{k_2} + \mathbf{n}_i)\\ &= \sum_{k_1=1}^{K}\sum_{k_2=1}^{K}\frac{\xi(k_1)\mathbf{g}_{k_1}^*\mathbf{g}_{k_2}}{M}\mathbf{x}_{k_2} + \frac{\vartheta}{M\sqrt{\rho_p\rho_d}}\end{aligned} \tag{7}$$

where ϑ denotes the uncorrelated noise and

$$\begin{aligned}\vartheta = &\sqrt{\rho_p}\sum_{k_1=1}^{K}\xi(k_1)\mathbf{g}_{k_1}^*\mathbf{n}_i + \mathbf{\Phi}_i\mathbf{n}_i^*\mathbf{n}_i\\ &+\sqrt{\rho_d}\mathbf{\Phi}_i\mathbf{n}_i^*\sum_{k_2=1}^{K}\mathbf{g}_{k_2}\mathbf{x}_{k_2}\end{aligned} \tag{8}$$

For an infinite M, $\frac{\vartheta}{M\sqrt{\rho_p\rho_d}}$ can be significantly reduced and ignored [3]. In addition, according to the law of large numbers, we can get

$$\frac{\xi(k_1)\mathbf{g}_{k_1}^*\mathbf{g}_{k_2}}{M} \overset{M\to\infty}{=} \begin{cases}\xi(k_1)\beta_{k_1} & \mathbf{\Phi}_{k_1}\mathbf{\Phi}_{k_2}^* = 1\\ 0 & \mathbf{\Phi}_{k_1}\mathbf{\Phi}_{k_2}^* = 0\end{cases} \tag{9}$$

In (9), $\mathbf{\Phi}_{k_1}\mathbf{\Phi}_{k_2}^* = 1$ denotes that the k_1-th terminal and k_2-th terminal share the same pilot sequence. Therefore, with an infinite number of BS antennas, the processed signal after MRC (7) can be simplified as

$$\frac{\hat{\mathbf{x}}_i}{M\sqrt{\rho_p\rho_d}} \overset{M\to\infty}{=} \beta_i\mathbf{x}_i + \sum_{k=1,\ k\neq i}^{K}\xi(k)\beta_k\mathbf{x}_k \tag{10}$$

Based on (10), the uncorrelated noise and small-scale fading factor are eliminated completely when M is increased without bound. The only factor influencing the system performance is the pilot contamination from terminals who use the same pilot sequence as that assigned to the target terminal.

Further, the effective uplink SIR for the target terminal can be expressed as

$$\mathrm{SIR}_i = \frac{\beta_i^2}{\sum_{k=1,\ k\neq i}^{K}\xi(k)\beta_k^2} \tag{11}$$

According to (11), the values of ρ_p and ρ_d do not impact the uplink SIR. It is proportional to a ratio of the squares of β. The denominator of (11) denotes the sum of β^2 from all impact terminals which share the same pilot sequence as that assigned to the i-th terminal. With insufficient pilot sequences, some terminals must reuse the same pilot. Therefore, how to assign the available pilot sequences to the terminals has an important influence on uplink SIR.

The available capacity for the target terminal can be expressed as

$$\mathrm{C}_i = W\frac{(T-\tau)}{T}\log_2(1+\mathrm{SIR}_i) \tag{12}$$

where W denotes the total bandwidth. In addition, the average uplink capacity of all terminals can be expressed as

$$\mathrm{C}_{\mathrm{ave}} = \frac{1}{K}\sum_{i=1}^{K}C_i \tag{13}$$

III. Proposed Pilot Assignment Scheme

In this section, we first formulate an optimization problem to enhance the QoS for the terminals with poor performance, and analyze the complexity of the complete search and random assignment scheme. Then a two-step pilot assignment scheme with low complexity is presented to obtain the suboptimal solution.

Specially, to simplify the pilot assignment scheme, we assume that the number of terminals is an integral multiple of that of available pilot sequences in this section. By utilizing an iterative algorithm, the two-step scheme is still valid if we relax this assumption.

A. Optimization Problem Formulation

For the conventional pilot assignment scheme in single-cell massive MIMO systems [10], [11], all available pilot sequences are assigned to the terminals randomly. Similarly, these two papers assume that the number of terminals equals to that of available pilot sequences $(K=\tau)$. However, K is an integral multiple of τ there. With the precondition of randomness and effectiveness, we extend the random pilot assignment scheme to this configuration as:

1). Randomly select τ from all terminals and assign available pilot sequences to them.

2). Randomly select another τ terminals and assign pilot sequences.

3). Repeat this process until all terminals are operated.

According to (11), the random pilot assignment scheme will result in the terminals with small large-scale fading factors suffering from severe pilot contamination, and the terminals with large large-scale fading factors suffering from modest pilot contamination. Therefore, the performance for the terminals with severe pilot contamination cannot be guaranteed. In this subsection,

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we present an optimization problem to improve the QoS of these terminals. Specifically, the optimization problem Ψ is formulated to maximize the minimum uplink capacity of all terminals. It can be expressed as

$$\Psi: \max_{\{\Gamma\}} \min_{\forall i} \mathrm{C}_i \tag{14}$$

with identical W, T and τ, (14) can be simplified as

$$\bar{\Psi}: \max_{\{\Gamma\}} \min_{\forall i} \log_2\left(1+\frac{\beta_i^2}{\sum_{k=1,\ k\neq i}^{K} \xi(k)\beta_k^2}\right) \tag{15}$$

In (14) and (15), $\{\Gamma\}$ denotes all possible kinds of pilot assignments. And $\forall i = 1,2,\cdots,K$ denotes any of K terminals.

The most straightforward way to find the optimal pilot assignment scheme of $\bar{\Psi}$ is complete search. It means that once terminal scheduling is done, each terminal can be given one pilot randomly. The complexity of this optimal scheme can be expressed as

$$\tau^K \tag{16}$$

In addition, when the number of terminals is an integral multiple of that of pilot sequences ($K = N\tau$), the complexity of the extended random assignment scheme can be expressed as

$$C_K^N C_{K-N}^N \cdots C_N^N = \frac{K!}{(N!)^\tau} \tag{17}$$

In (17), C_K^N denotes that we randomly select N from K terminals and assign an available pilot sequence to them. In short, (17) denotes the number of all possible kinds of pilot assignments.

B. Suboptimal Pilot Assignment Scheme

As mentioned in section II-A, β_k is assumed to be known to the BS. Based on this, the suboptimal pilot assignment scheme contains of two steps. First, the BS sorts the large-scale fading factors of all terminals in descending order by using quicksort function. Second, the available pilot sequences are allocated to the terminals according to the order offered by the first step.

For the first step, we define a collection that includes the large-scale fading factors of all terminals as

$$\{\beta_{\text{before}}\} = \{\beta_1, \beta_2, \cdots, \beta_K\} \tag{18}$$

After the quicksort function, we obtain another collection as

$$\{\beta_{\text{after}}\} = \{\beta^1, \beta^2, \cdots, \beta^K\} \tag{19}$$

For the elements of $\{\beta_{\text{after}}\}$, we have

$$\beta^1 \geq \beta^2 \geq \beta^3 \geq \cdots \geq \beta^K \tag{20}$$

Similarly, we sort the K terminals according to (19) as

$$\{U_{\text{after}}\} = \{U^1, U^2, \cdots, U^K\} \tag{21}$$

Specially, there is a one-to-one correspondence between U^i in (21) and β^i in (19) for $\forall i = 1,2,\cdots,K$. The simple pseudocode of quicksort function is shown as follows. And to sort the entire $\{\beta_{\text{before}}\}$, we set $a=1$, $b=K$ in function 1 and function 2.

Function 1 quicksort **(** $\{\beta_{\text{before}}\}$, a, b **) is**

if $a < b$ then m = partition **(** $\{\beta_{\text{before}}\}$, a, b **)**;
quicksort **(** $\{\beta_{\text{before}}\}$, a, $m-1$ **)**;
quicksort **(** $\{\beta_{\text{before}}\}$, $m+1$, b **)**;

Function 2 partition **(** $\{\beta_{\text{before}}\}$, a, b **)** is

$p = \{\beta_{\text{before}}\}_K$; $l = a$;
for $j = 1$ to b do
if $\{\beta_{\text{before}}\}_j \leq p$ then
swap $\{\beta_{\text{before}}\}_l$ with $\{\beta_{\text{before}}\}_j$; $l = l+1$;
swap $\{\beta_{\text{before}}\}_l$ with $\{\beta_{\text{before}}\}_K$; return l;

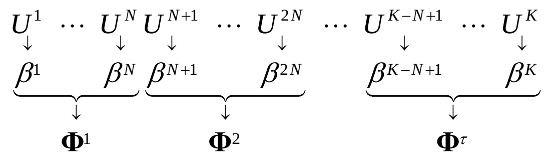

Fig. 2. The suboptimal pilot sequence assignment.

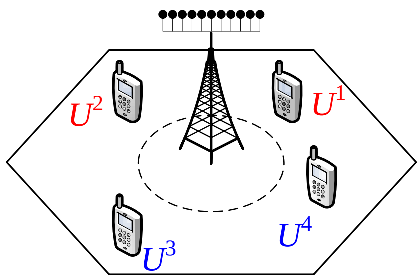

Fig. 3. An example of suboptimal pilot assignment scheme.

For the second step, all orthogonal pilot sequences are allocated to the terminals according to (19). In order to enhance the QoS for the terminals who suffer from severe pilot contamination, we try to avoid the situation that the terminals with small large-scale fading factors share the same pilot sequence as that assigned to the terminals with large large-scale fading factors. Based on this principle, as shown in Fig. 2, we assign one of the orthogonal pilot sequences to the first N elements in $\{U_{\text{after}}\}$, i.e., $\{U_{\text{after}}\}_1 \sim \{U_{\text{after}}\}_N$. Then another pilot sequence is assigned to the next N elements, i.e., $\{U_{\text{after}}\}_{N+1} \sim \{U_{\text{after}}\}_{2N}$. This process will be repeated τ times, for all terminals are evenly divided into τ groups there. Fig. 3 shows an example of the suboptimal pilot assignment scheme for 4 terminals and 2 pilot sequences. In Fig. 3, we assume $\beta_1 \geq \beta_2 \geq \beta_3 \geq \beta_4$; therefore, U^1

and U^2 share the same pilot; U^3 and U^4 share the same pilot.

The total complexity for the proposed pilot assignment scheme can be expressed as

$$K\log_2 K + \tau! \tag{22}$$

The first item in (22) originates from the sorting action in the first step. The second item in (22) originates from the pilot assignment which assigns τ pilot sequences to τ groups in the second step.

Table I shows the numerical results for (16), (17) and (22) under different values of (K,τ). In Table I, the red letters denote the least complexity among three schemes. It is obvious that the complexity for optimal scheme is always the highest. When the value of (K,τ) is small (for example, (4,2)), the complexity for random scheme is lower than that of proposed scheme. This because there is few kinds of pilot sequence assignments for the random scheme if the value of (K,τ) is small. However, at the same time, the quicksort action in the first step of proposed scheme has heavy computation. With the increase of (K,τ), the low complexity advantage of proposed scheme emerges. Specially, in the case of a larger number of terminals and pilot sequences (for example, (9,3) and (8,4)), the complexity for proposed scheme can be neglected.

TABLE I: COMPLEXITY

(K,τ)	optimal scheme	random scheme	proposed scheme
(4,2)	16	6	$4\log_2 4+2!$
(6,2)	64	20	$6\log_2 6+2!$
(6,3)	729	90	$6\log_2 6+3!$
(9,3)	19683	1680	$9\log_2 9+3!$
(8,4)	65536	2520	$8\log_2 8+4!$

IV. SIMULATION RESULTS

TABLE II: SIMULATION PARAMETERS

Evaluation for independent trials	10^5
Number of BS antennas M	128
Coherence interval T	20 symbols
Cell radius R	250 m
Radius of the disk r_c	25 m
Average transmission power ρ_p and ρ_d	0 dBm
Additive noise spectrum density	-174 dBm/HZ
Total bandwidth W	20 MHZ
Decay exponent γ	3.8
Log normal shadowing fading σ	8dB

In this section, we evaluate the performances of different pilot assignment schemes through Monte-Carlo simulations. The system parameters are summarized in Table II. Specifically, as described in [3], the large-scale fading factor for the k-th terminal β_k can be modeled as

$$\beta_k = \frac{z_k}{r_k^\gamma} \tag{23}$$

where r_k denotes the distance between the BS and the k-th terminal. γ denotes the decay exponent. The quantity $10\log(z_k)$ is distributed zero-mean Gaussian with a standard deviation of σ.

A. Minimun Uplink SIR

In this subsection, we analyze the minimum uplink SIR for three assignment schemes when the number of terminals $K=4$ and the number of pilot sequences $\tau=2$.

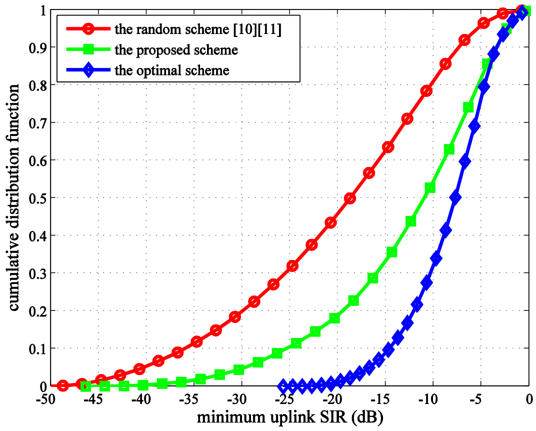

Fig. 4. Cumulative distribution function for the minimum uplink SIR.

Fig. 4 plots the Cumulative Distribution Function (CDF) for the minimum uplink SIR of all terminals. It shows that the random pilot assignment scheme has the worst performance. This because for the random scheme, the situation that the terminals with small large-scale fading factors share the same pilot sequence as that assigned to the terminals with large large-scale fading factors often happens. The proposed assignment scheme can reduce the probability of occurrence of this situation, and it outperforms the random scheme. Not surprisingly, the optimal scheme is the best at the cost of highest complexity. When the minimum uplink SIR is already high (about -4dB), the performance of the proposed assignment is almost the same as that of the optimal assignment. Table III summarizes the SIR gains compared to the random approach for Fig. 4.

TABLE III: SIR GAINS

CDF	the proposed scheme	the optimal scheme
20%	11dB	18dB
50%	7dB	10dB
80%	5dB	6dB

In addition, by combining Table I with Fig. 4, we have: for complexity, optimal scheme > proposed scheme > random scheme; for minimum uplink SIR, optimal scheme > proposed scheme > random scheme. This result conforms to the common sense that the high-performance and low-complexity always cannot be satisfied simultaneously.

B. Minimun Uplink Capacity

In this subsection, we analyze the minimum uplink capacity for three assignment schemes when the number of terminals $K=8$ and the number of pilot sequences $\tau=4$.

Fig. 5 plots the CDF for the minimum uplink capacity of all terminals. Similarly, the random scheme has the worst performance, and the optimal scheme is the best. The minimum capacity of the proposed scheme falls in between the two schemes listed above. The performance of the proposed assignment approaches that of the optimal assignment when the minimum uplink capacity is already high (about 9Mbps). Table IV summarizes the capacity gains compared to the random approach for Fig. 5. And these gains originate from the improved minimum uplink SIR.

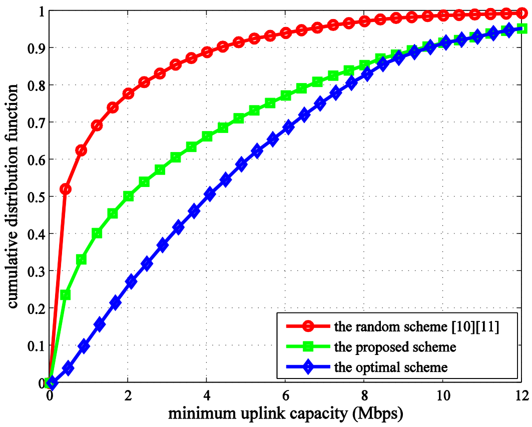

Fig. 5. Cumulative distribution function for the minimum uplink capacity.

It is worth noting that, by combining Table I with Fig. 5, we have: for complexity, optimal scheme > random scheme > proposed scheme; however, for minimum uplink capacity, optimal scheme > proposed scheme > random scheme. This result shows that with the increase of (K,τ), the proposed scheme can improve the performance for the terminals suffering from severe pilot contamination, meanwhile, decrease the computational complexity.

TABLE IV: CAPACITY GAINS

CDF	the proposed scheme	the optimal scheme
20%	0.2Mbps	1.3Mbps
50%	1.6Mbps	3.6Mbps
80%	4.3Mbps	5.3Mbps

C. Average Uplink Capacity

In this subsection, we analyze the average uplink capacity for three assignment schemes when the number of terminals $K=8$ and the number of pilot sequences $\tau=4$.

Fig. 6 depicts the CDF for the average uplink capacity (13). In this figure, when the average uplink capacity is about 34Mbps, the curve of the proposed assignment and that of the random assignment intersect each other. On the left of this intersection, the curve of the proposed scheme is lower than that of the random scheme. This proves the effectiveness of the proposed scheme in obtaining better minimum uplink capacity. On the right of the intersection, the curve of the proposed scheme is higher than that of the random scheme. This means the minimum uplink capacity improvement happens at the expense of capacity degradation of the terminals with modest pilot contamination.

In addition, in Fig. 6, the vertical bar denotes the average uplink capacity for 10^5 independent trials. And the average capacity for random scheme, proposed scheme and optimal scheme are about 30.6Mbps, 31.7Mbps and 100.1Mbps, respectively. This result shows that compared with the random scheme, the proposed scheme can also improve the average uplink capacity for 1.1Mbps with the related parameter settings.

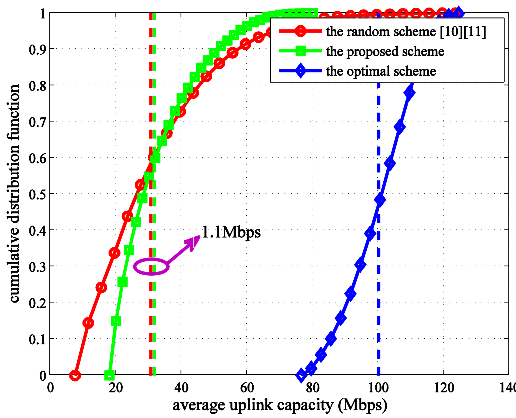

Fig. 6. Cumulative distribution function for the average uplink capacity.

V. CONCLUSION

This paper proposes a two-step pilot assignment scheme in single-cell TDD massive MIMO systems to enhance the performance of terminals suffering from severe pilot contamination when the number of terminals is greater than that of available pilot sequences. The general formulas including the uplink SIR and channel capacity for a target terminal are deduced in Section II. By exploiting the large-scale fading factors, the two-step pilot assignment scheme with low complexity is presented to obtain the suboptimal solution of the optimization problem which is formulated to maximize the minimum uplink capacity of all terminals. With the typical parameter of 128 BS antennas, simulation results in Section IV prove that the proposed scheme can improve the minimum uplink SIR and minimum channel capacity. Meanwhile, the average capacity of all terminals can also be improved. In addition, one of our further works is to discuss the extensibility of this two-step approach when the pilot sequences are not strictly orthogonal.

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