Dynamic Rate Allocation for Multipath Routing under Path Stability and Prioritized Traffic Session Constraints for Cognitive Radio Ad Hoc Networks with Selfish Secondary Users

Long Zhang[1], Chunhong Bai[1], Fan Zhuo[1], Haitao Xu[2], and Wei Huang[3]
[1] School of Information and Electrical Engineering, Hebei University of Engineering, Handan 056038, China
[2] School of Computer and Communication Engineering, University of Science and Technology Beijing, Beijing 100083, China
[3] Energy Efficiency Evaluation Center, China Electric Power Research Institute, Beijing 100192, China
Email: zhanglong@hebeu.edu.cn; {chhbai, fnzhuo}@outlook.com; xuhaitao@ustb.edu.cn; huangwei2@epri.sgcc.com.cn

Abstract —In this paper, we investigate the dynamic rate allocation for multipath routing with the node-disjoint paths in cognitive radio ad hoc networks composed of selfish secondary users (SUs) which are unwilling to forward the packets. We model the path stability factor of each path under a certain traffic session by distributing available channel in view of the carrier frequency of the available uplink channel on each path. Then the traffic sensing factor is also devised to characterize the priorities of different traffic sessions along the node-disjoint paths. In addition, we propose the differential game model of dynamic rate allocation for multi-path routing by taking into account both the mobility and random nature of the selfish SUs. A set of the non-cooperative optimal solutions to the dynamic rate allocation model is further obtained by deriving an optimization problem. Moreover, the distributed optimal rate update rule is developed to dynamically regulate the rate of traffic session over a certain path. Numerical results confirm that the effectiveness of the proposed dynamic rate allocation model.

Index Terms—Cognitive radio, ad hoc networks, rate allocation, path stability, prioritized traffic session

I. Introduction

Cognitive radio (CR) or dynamic spectrum access in [1] has newly emerged as a promising solution to improve the spectrum utilization by allowing unlicensed secondary users (SUs) to access idle licensed spectrum. In a CR network, SUs can periodically sense the licensed spectrum and opportunistically access the spectrum holes or spectrum opportunities (SOPs) unoccupied by primary users (PUs). Most of the existing research efforts in CR networks mainly focus on the issues of the physical and MAC layers under an infrastructure based single-hop scenario, including cooperative spectrum sensing, spectrum access and sharing techniques [2]-[7]. In addition, SUs can further form a multi-hop ad hoc network without the support of infrastructure. In a cognitive radio ad hoc network (CRANET) [8], SUs can only access the SOPs by seeking to underlay, overlay, or interweave their signals with those of existing PUs without significantly impacting their communications.

In comparison with the lower layer solutions explored by the existing research efforts as reported before, recent work indicates that there are many new challenges towards the routing problem upon the network layer in multi-hop CRANETs, such as the coupling between routing and spectrum-awareness, path stability and spectrum availability, and route maintenance, etc [9]-[11]. In this context, routing design in CRANETs differs sufficiently from the ad hoc networks, and must be carefully considered to deal with the creation and the maintenance of multi-hop paths among SUs by selecting both the next hop SUs and the spectrum availability on each link of the path due to the presence of PUs. To facilitate the distribution of the traffic load and also to avoid the route failure, there is a need to exploit multi-path routing from the perspective of path diversity to improve the multi-hop transmission efficiency and stability in CR networks, notably [12]-[16]. Also, rate allocation for multi-path routing in CR networks is a key technology to ensure the effective packet delivery in different paths at the same time as well as the traffic load balance. Furthermore, effective rate allocation will also help to minimize end-to-end delay and control message overhead. Various rate allocation methods have been developed for CR networks from different perspectives, including end-to-end delay optimization [17], QoS-constrained bi-objective optimization [18], joint rate and power allocation [19], etc.

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Corresponding author email: zhanglong@hebeu.edu.cn.
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As an alternative framework for modeling, game-theoretic approach has gained more attention as an economic tool to study the rate allocation problem in wireless networks [20], [21]. In [20], a non-cooperative game-theoretic framework for distributed asynchronous power and rate allocation in ad hoc networks was proposed. The rate allocation within this framework deals with its convergence to network-wide acceptable equilibrium states under stochastic communication channels. In [21], a problem of joint channel and rate assignment was formulated as a static non-cooperative game with charging scheme, in which the different individual demand constraints in ad hoc networks are taken into account. The existing rate allocation models [20], [21] using the static game theoretic approach cannot take into account the underlying constraint that CR networks in practice were subject to the dynamic time-varying network topology wherein the assignment of rate should be continuous in time. Another body of work formulates the dynamic rate allocation problem for multi-path routing in CR networks as differential game model [22], [23]. In [22], an efficiency-awareness rate assignment problem of multi-path routing was devised as a non-cooperative differential game model with the hop number and bandwidth constraints, and the equilibrium of the game is also obtained. In [23], a multi-person non-cooperative differential game model for stable-aware traffic load sharing over multi-path routing was presented, and the feedback Nash equilibrium of the game is derived. However, although [22] and [23] used differential game theoretic approach to study the dynamic rate allocation problem, they did not consider some practical factors such as path stability and prioritized traffic session for multi-path routing in dynamic rate allocation model.

Our work in this paper mainly focuses on the underlay CRANET scenario due to the high spectrum utilization and the simplicity of implementation. Both mobility and sometimes random nature of SUs result in the dynamics of the CRANET topology with respect to time dependency. In general, given the dynamic time-varying network topology, it is not realistic to keep the traffic rate unchangeably for multi-path routing. Hence, it is crucial to dynamically allocate the rate for multi-path routing according to the dynamic nature of time dependency. Meanwhile, in order to avoid the reroute operations due to the break of the end-to-end paths, an available and stable path should be selected as one of the node-disjoint multi-path routes, aiming to achieve the effective assignment of the traffic load along the node-disjoint multi-path routing. Taking into consideration the impact of the differentiated types of traffic sessions on the demand of transmission quality, this observation motivates us to characterize the priorities of the different traffic sessions to deal with the rate allocation problem. With this respect, the dynamic rate allocation framework for the node-disjoint multi-path routing is formulated bearing in mind the constraints of path stability and prioritized traffic session.

In this work, we study the dynamic rate allocation for multipath routing with node-disjoint paths by taking into account the constraints of the path stability and the prioritized traffic session for CRANET. We define the path stability factor of each path under a certain traffic session in multipath routing scenario. Specifically, the path stability factor is calculated by distributing the available channel according to the carrier frequency of the available uplink channel on each path with different distance. To characterize the priorities of different traffic sessions along the node-disjoint paths, we also devise the traffic sensing factor to evaluate the impact of differentiated types of traffic sessions on the rate allocation problem. By analyzing the characterized factors, the payoff function of a certain path from the upstream source SU is further presented based on two cost functions including the path stability cost and the traffic session cost. Owing to both the mobility and sometimes random nature of the selfish SUs, we propose the differential game model of dynamic rate allocation for multi-path routing. In this model, the transmission rate of traffic session over a certain path from the upstream source SU is dynamically regulated according to the dynamic nature of time dependency. Moreover, we devise an optimization problem to obtain a set of the non-cooperative optimal solutions to the dynamic rate allocation model. The distributed optimal rate update rule is also employed to dynamically regulate the rate of traffic session over a certain path.

The remainder of this paper is organized as follows. In Section II, we introduce the preliminaries including network model, path stability model, and prioritized traffic session model. The dynamic rate allocation model is formulated in Section III. Then the non-cooperative optimal solution is described in Section IV. Section V presents the numerical results. Finally, we conclude the whole paper in Section VI.

II. Preliminaries

A. Network Model

We consider a distributed underlay CRANET scenario, where PUs send their packets to the primary base station through the cellular primary networks with a finite set of cells. We employ the independent and identically distributed alternating ON-OFF process to model the occupation time length of PUs in uplink channels. Specifically, the OFF state indicates the idle state where the unoccupied uplink channels or the SOPs can be freely occupied by SUs. Let t_0 and T denote the starting time and the terminal time of the dynamic rate allocation for multi-path routing in this scenario, respectively. By the means of collaborative spectrum sensing, SUs can only leverage the OFF state to access the SOPs over the authorized uplink channels to further form a distributed multi-hop underlay CRANET with a set $\mathcal{M}$ of stationary SUs within time interval $\left[t_0, T\right]$. Let $\mathcal{T}$ be the set of traffic sessions. We assume that there is a traffic session $\ell \in \mathcal{T}$ from a single upstream source SU $\mathcal{S}_\ell \in \mathcal{M}$ to a downstream destination SU $\mathcal{D}_\ell \in \mathcal{M}$ along n node-disjoint paths. Let $\mathcal{N} = \{1, 2, \cdots n\}$ be the set of the node-disjoint paths. Note that a path $P(i) \subset \mathcal{M}$, $i \in \mathcal{N}$ under the traffic session $\ell \in \mathcal{T}$ is a subset of SUs.

B. Path Stability Model

We assume that there exist k links with different lengths on path $P(i)$, $i \in \mathcal{N}$ over m available uplink channels. Let $l_{\varpi}^{P(i)}$ be the ϖ-th link of path $P(i)$, for $\varpi = 1, 2, \cdots, k$. The length of the ϖ-th link of path $P(i)$ is assumed to be $d_{\varpi}^{P(i)}$. We then rank the length of k links in descending order and denote by $L_{\xi}^{P(i)}$ the ξ-th sorted link, for $\xi = 1, 2, \cdots, k$, where ξ is the label in descending order. We remark that the greater value of ξ will result in the shorter length of the corresponding link. Analogously, we denote by $f_{\upsilon}^{P(i)}$ the carrier frequency of the available uplink channel on path $P(i)$, for $\upsilon = 1, 2, \cdots, m$. The m available uplink channels will also be ranked in ascending order based on the values of the carrier frequency. The ascending order is denoted by τ, for $\tau = 1, 2, \cdots, m$. Notice that the greater value of τ yields the greater value of the carrier frequency. Therefore, the stability coupling factor θ_ξ of the ξ-th sorted link $L_{\xi}^{P(i)}$ on path $P(i)$, $i \in \mathcal{N}$ can be formally defined as

$$\theta_\xi = \begin{cases} \dfrac{1}{k}, & \text{if } \xi \geq \tau \\ \dfrac{1}{k\lambda |\xi - \tau|}, & \text{if } \xi < \tau \end{cases} \tag{1}$$

where λ is a constant, for $\lambda > 1$. Note that Eq. (1) indicates that the ξ-th sorted link $L_{\xi}^{P(i)}$ can find its most appropriate channels in view of the length of links and the carrier frequency of uplink channels. Obviously, $1/(k\lambda|\xi - \tau|)$ is smaller than $1/k$. We assume that there have A sorted links with stability coupling factor $1/k$ and $(k - A)$ sorted links with stability coupling factor $1/(k\lambda|\xi - \tau|)$. For the convenience of calculation, we assume that the value of $|\xi - \tau|$ in $(k - A)$ sorted links are represented by $G_1, G_2, \cdots, G_{k-A}$, respectively. Hence, we define the path stability factor of path $P(i)$, $i \in \mathcal{N}$ as θ_i, which is the sum of the stability coupling factor θ_ξ of the ξ-th sorted link $L_{\xi}^{P(i)}$ on path $P(i)$. Specifically, the path stability factor θ_i of path $P(i)$ can be written as

$$\theta_i = A \cdot \frac{1}{k} + \sum_{\iota=1}^{k-A} \frac{1}{k\lambda G_\iota} \tag{2}$$

C. Prioritized Traffic Session Model

Appealing to the Enhanced Distributed Channel Access (EDCA) mechanism for the support of applications with QoS requirements in IEEE 802.11e standard [24], we devise the traffic sensing factor under the EDCA mechanism to quantify the priorities of the different traffic sessions. Our objective is to characterize the priorities of different traffic sessions along the node-disjoint paths bearing in mind the differentiated types of traffic sessions. From the point of view of the priority, we classify the traffic session types for multi-path routing into four access categories (ACs) [24]. Let q be an integer index of the associated traffic type to an AC, for $q \in [1, 4]$. As shown in Table I, four ACs correspond to four types of traffic sessions, including Voice Traffic (VO), Video Traffic (VI), Best Effort Traffic (BE), and Background Traffic (BK). We denote by $F^{\ell \in \mathcal{T}}(q)$ the traffic sensing factor for traffic session $\ell \in \mathcal{T}$ under traffic type q. According to the EDCA mechanism [24], the traffic sensing factor $F^{\ell \in \mathcal{T}}(q)$, for $\ell \in \mathcal{T}$ can be given as

$$F^{\ell \in \mathcal{T}}(q) = Random(q) \times aSlotTime \times AIFSN(q) \tag{3}$$

where $Random(q)$ is a pseudo-random integer drawn from a uniform distribution over the interval $[0, CW(q)]$, $aSlotTime$ is a constant time number in microseconds based on different physical layer techniques in the EDCA mechanism, and $AIFSN(q)$ is the arbitration interframe space number of the EDCA mechanism, which is further formulated in Table I. It should be noted that $CW(q) \in [CW\min(q), CW\max(q)]$ is an integer within the range of values of the contention window limits $CW\min(q)$ and $CW\max(q)$.

TABLE I. TRAFFIC SESSION TYPES AND CONTENTION PARAMETERS

Traffic Session Type	Priority	*CW*min(*q*)	*CW*max(*q*)	*AIFSN*(*q*)
VO	*Pri*(1)	(*aCWmin*+1)/4-1	(*aCWmin*+1)/2-1	2
VI	*Pri*(2)	(*aCWmin*+1)/2-1	*aCWmin*	2
BE	*Pri*(3)	*aCWmin*	*aCWmax*	3
BK	*Pri*(4)	*aCWmin*	*aCWmax*	7

III. DYNAMIC RATE ALLOCATION MODEL

Practically, during the multi-path transmission with the traffic session $\ell \in \mathcal{T}$ from source SU $\mathcal{S}_\ell \in \mathcal{M}$ to destination SU $\mathcal{D}_\ell \in \mathcal{M}$, SUs in charge of forwarding the packets appear to the selfishness. This is a natural idea due to the fact that the SUs are unwilling to forward the packets on behalf of other SUs since these operations will lead to the consumption of their own resources, such as energy and available bandwidth. Because of the selfish SUs in forwarding actions, the upstream source SU $\mathcal{S}_\ell \in \mathcal{M}$ as the rate controller of the traffic load under the traffic session $\ell \in \mathcal{T}$ should pay for each downstream SUs along path $P(i)$ to accommodate the consumption of resources. Based on both the mobility and sometimes random nature of the selfish SUs, differential game is adopted to construct the dynamic rate allocation model for multi-path routing by taking into account the time-continuous rate change of the traffic session $\ell \in \mathcal{T}$. The objective of the upstream source SU $\mathcal{S}_\ell \in \mathcal{M}$ is to minimize the cost or the so called payoff function of path $P(i)$ under the constraint of the path stability and prioritized traffic session.

Let $r_i(s)$ denote the instant transmission rate of traffic session $\ell \in \mathcal{T}$ over path $P(i)$, $i \in \mathcal{N}$ from the upstream source SU $\mathcal{S}_\ell \in \mathcal{M}$ at time $s \in [t_0, T]$. According to Shannon's capacity theorem, the bandwidth of path $P(i)$ can be approximately calculated as $r_i(s)/\log_2(1+S/N)$, where S/N is the signal-to-noise ratio over path $P(i)$. Under this description, the coupling constraint between the traffic sensing factor $F^{\ell\in\mathcal{T}}(q)$ and the bandwidth of path $P(i)$, $i \in \mathcal{N}$, denoted by δ_i, is further represented as

$$\delta_i = \frac{r_i(s)}{\log_2(1+S/N)} \cdot \frac{1}{F^{\ell\in\mathcal{T}}(q)} \tag{4}$$

To formulate the payoff function of path $P(i)$ that the upstream source SU $\mathcal{S}_\ell \in \mathcal{M}$ should pay, we then proceed to propose the path stability cost function and the traffic session cost function. Let $h(s)$ be the current price that upstream source SU $\mathcal{S}_\ell \in \mathcal{M}$ is willing to pay at time $s \in [t_0, T]$. Recall that the sum of the stability coupling factor θ_ξ of the ξ-th sorted link $L_\xi^{\mathcal{P}(i)}$ on path $P(i)$ is characterized by the path stability factor θ_i, Hence, the path stability cost of path $P(i)$, $i \in \mathcal{N}$ at time $s \in [t_0, T]$, denoted by $PSC_i(s)$, is modeled as

$$PSC_i(s) = \theta_i \cdot h(s) \tag{5}$$

To quantify the impact of prioritized traffic session on the dynamic rate allocation, we devise the traffic session cost of path $P(i)$, $i \in \mathcal{N}$ at time $s \in [t_0, T]$, denoted by $TSC_i(s)$, which can be calculated as

$$TSC_i(s) = \delta_i \cdot \frac{u_i}{h(s)} \cdot r_i(s) \tag{6}$$

where u_i is a price coefficient of traffic session $\ell \in \mathcal{T}$ over path $P(i)$. Therefore, based on the cost functions as discussed above, the total cost of path $P(i)$, $i \in \mathcal{N}$ at time $s \in [t_0, T]$ is given by

$$\theta_i \cdot h(s) + \frac{u_i}{F^{\ell\in\mathcal{T}}(q) \cdot \log_2(1+S/N)} \cdot \frac{r_i^2(s)}{h(s)} \tag{7}$$

Appealing to differential game theory by [25], $h(s)$ can be regarded as the state variable, and $r_i(s)$ can be considered as the strategy. The strategy means the choice of action or behavior by player in differential game. We emphasize that the path $P(i)$ refers to the player i of the game. For convenience, we view the starting time of the game as $t_0 = 0$ throughout the paper. From Eq. (7), the aim of path $P(i)$ at time $s \in [0, T]$ is to minimize the total cost function, which can be formally expressed as

$$\begin{aligned}\text{Minimize}: \int_0^T \bigg\{ & \frac{u_i}{F^{\ell\in\mathcal{T}}(q) \cdot \log_2(1+S/N)} \cdot \frac{r_i^2(s)}{h(s)} \\ & + \theta_i \cdot h(s) \bigg\} \cdot e^{-\sigma s} ds\end{aligned} \tag{8}$$

where σ is a constant discount factor, for $0 < \sigma < 1$. According to differential game theory [25], the state variable $h(s)$ in Eq. (8) is assumed to satisfy the differential equation as follows

$$\begin{cases} \dot{h}(s) = h(s) - \sum_{i\in\mathcal{N}} r_i(s) \\ h(0) = h_0 \end{cases} \tag{9}$$

Therefore, Eqs. (8) and (9) constitute the differential game model of dynamic rate allocation for multi-path routing.

IV. Non-Cooperative Optimal Solution

In this section, we apply the differential game theory [25] to derive the non-cooperative optimal rate of traffic session $\ell \in \mathcal{T}$ over path $P(i)$ under our dynamic rate allocation model in Eqs. (8) and (9). Specifically, we devise an optimization problem to obtain a set of the non-cooperative optimal solutions to the dynamic rate allocation model as follows

$$\begin{aligned}\text{Minimize}: \int_0^T \bigg\{ & \frac{u_i}{F^{\ell\in\mathcal{T}}(q) \cdot \log_2(1+S/N)} \cdot \frac{r_i^2(s)}{h(s)} \\ & + \theta_i \cdot h(s) \bigg\} \cdot e^{-\sigma s} ds \\ \text{Subject to}: \ & \dot{h}(s) = h(s) - \sum_{i\in\mathcal{N}} r_i(s) \\ & h(0) = h_0\end{aligned} \tag{10}$$

Let $\{\phi_1^*(t,h), \cdots, \phi_i^*(t,h), \cdots, \phi_n^*(t,h)\}$ be the set of the non-cooperative optimal solutions to the optimization problem in Eq. (10). Moreover, we assume that there exists continuously differentiable function $V^i(t,h)$, satisfying the following partial differential equations

$$-\frac{\partial V^i(t,h)}{\partial t}$$
$$= \text{minimize}:$$
$$\left\{\left(\frac{u_i}{F^{\ell\in\mathcal{T}}(q)\cdot\log_2(1+S/N)}\cdot\frac{r_i^2(s)}{h(s)}+\theta_i\cdot h(s)\right)\cdot e^{-\sigma t}\right. \quad (11)$$
$$\left.+\frac{\partial V^i(t,h)}{\partial h}\cdot\left[h(t)-\sum_{j\in\mathcal{N}\setminus i}\phi_j^*(t,h)-r_i(t)\right]\right\}$$

$$V^i(T,h)=0 \quad (12)$$

Theorem 1: Under the condition of $V^i\ t,h\ =\ A_i\ t\ b\ t\ +B_i\ t\ \ e^{-\sigma t}$, the set of the non-cooperative optimal solutions to the optimization problem in Eq. (10) is formulated as follows

$$\left\{\phi_i^*(t,h)\middle|\phi_i^*(t,h)=\frac{F^{\ell\in\mathcal{T}}(q)\log_2(1+S/N)}{2u_i}\cdot A_i(t)\cdot h(t)\right\} \quad (13)$$

where $A_i(t)$ and $B_i\ t$ satisfy following equations

$$\dot{A}_i(t)=\frac{F^{\ell\in\mathcal{T}}(q)\log_2(1+S/N)}{4u_i}A_i^2(t)+(\sigma-1)A_i(t)$$
$$+\frac{F^{\ell\in\mathcal{T}}(q)\log_2(1+S/N)}{2}\sum_{j\in\mathcal{N}\setminus i}\frac{A_j(t)}{u_j}A_i(t)-\theta_i \quad (14)$$
$$\dot{B}_i(t)=\sigma B_i(t)$$

Proof: Performing the minimization operation of Eq. (11), we obtain

$$\phi_i^*(t,h)=\frac{F^{\ell\in\mathcal{T}}(q)\log_2(1+S/N)}{2u_i}\cdot\frac{\partial V^i(t,h)}{\partial h}\cdot h(t)\cdot e^{\sigma t} \quad (15)$$

Substituting $\phi_i^*(t,h)$ in Eq. (15) into Eqs. (10) and (11), we have

$$V^i\ \ t,h\ =\ A_i\ \ t\ b\ t\ +B_i\ \ t\ \ e^{-\sigma t} \quad (16)$$

The non-cooperative optimal solution to the optimization problem in Eq. (10) is formulated as

$$\phi_i^*(t,h)=\frac{F^{\ell\in\mathcal{T}}(q)\log_2(1+S/N)}{2u_i}\cdot A_i(t)\cdot h(t) \quad (17)$$

Note that $A_i(t)$ and $B_i\ t$ are two extra introduced auxiliary variables to better represent $V^i\ t,h$, where $A_i(t)$ and $B_i\ t$ can be given in Eq. (14).

Hence, this completes the proof.

For notational simplicity, we set $F^{\ell\in\mathcal{T}}(q)\log_2(1+S/N)=\psi$, and we also omit the time t in the auxiliary variables $A_i(t)$ and $B_i\ t$ throughout the following paper.

Theorem 2: Let c denote a constant number. A_i in the non-cooperative optimal solution to the optimization problem in Eq. (10) can be further formulated as

$$A_i=\frac{((\sigma-1)+H)e^{(t-c)H}-(\sigma-1)+H}{2\eta\left(1-e^{(t-c)H}\right)} \quad (18)$$

$$\eta=\frac{\psi}{4u_i}+\frac{\psi}{2}\sum_j\frac{1}{u_j} \quad (19)$$

$$H=\sqrt{(\sigma-1)^2+4\eta\cdot\theta_i} \quad (20)$$

if and only if the following condition is satisfied

$$\frac{2\eta\cdot A_i+(\sigma-1)-H}{2\eta\cdot A_i+(\sigma-1)+H}=e^{(t-c)\mathrm{H}} \quad (21)$$

Proof: Considering the symmetric form of A_i and B_i in Eq. (14), we can immediately denote Eq. (14) by Riccati equation

$$\frac{dA_i}{dt}=\left(\frac{\psi}{4u_i}+\frac{\psi}{2}\sum_j\frac{1}{u_j}\right)A_i^2+(\sigma-1)A_i-\theta_i \quad (22)$$

Without loss of generality, the form of Eq. (22) can be rearranged by the following differential equation

$$X(A_i,t)dt+Y(A_i,t)dA_i=0 \quad (23)$$

where $X(A_i,t)=\left(\frac{\psi}{4u_i}+\frac{\psi}{2}\sum_j\frac{1}{u_j}\right)A_i^2+(\sigma-1)A_i-\theta_i$ and $Y(A_i,t)=-1$.

In view of $\frac{\partial X(A_i,t)}{\partial A_i}\neq\frac{\partial Y(A_i,t)}{\partial t}$, it is not difficult to recognize that Eq. (22) is not exact. Hence, we introduce a non-zero integrating factor $U(A_i,t)$ which can make the equation an exact form by multiplying it on both sides of Eq. (23). Here, for $(\sigma-1)^2-4\eta(-\theta_i)>0$, we can easily obtain

$$U(A_i,t)=\frac{1}{\left|\eta A_i^2+(\sigma-1)A_i-\theta_i\right|} \quad (24)$$

where $\eta=\frac{\psi}{4u_i}+\frac{\psi}{2}\sum_j\frac{1}{u_j}$. So Eq. (23) multiplied by $U(A_i,t)$ is exact, and then we obtain

$$U(A_i,t)\cdot X(A_i,t)dt+U(A_i,t)\cdot Y(A_i,t)dA_i$$
$$=dU(A_i,t) \quad (25)$$

Hence, we have

$$\frac{\partial U(A_i,t)}{\partial A_i}=-\frac{1}{\left|\eta A_i^2+(\sigma-1)A_i-\theta_i\right|}$$
$$\frac{\partial U(A_i,t)}{\partial t}=1 \quad (26)$$

Let $H=\sqrt{(\sigma-1)^2+4\eta\theta_i}$. If and only if $\eta A_i^2+(\sigma-1)A_i-\theta_i>0$, by integrating Eq. (26) with respect to A_i , we have

$$\begin{aligned} U(A_i,t) &= -\frac{1}{H}\ln\left|\frac{2\eta A_i+(\sigma-1)-H}{2\eta A_i+(\sigma-1)+H}\right|+b(t) \\ \frac{\partial U(A_i,t)}{\partial t} &= b'(t) \end{aligned} \tag{27}$$

We can easily obtain $b(t)=t$. Let $U(A_i,t)=c$. Upon solving Eq. (28) as follows

$$-\frac{1}{H}\ln\left|\frac{2\eta A_i+(\sigma-1)-H}{2\eta A_i+(\sigma-1)+H}\right|+t=c \tag{28}$$

so we have Eq. (18).

Hence, this completes the proof.

Algorithm 1: Distributed Optimal Rate Update Rule

Identify the set $\mathcal{N}=\{1,2,\cdots n\}$;
Initialize $F^{\ell\in\mathcal{T}}(q)$ for $\ell\in\mathcal{T}$ under traffic type q , and θ_i for $P(i)$;
Initialize $r_i(0)$ of $\ell\in\mathcal{T}$ over $P(i)$;
$t_0\leftarrow 0$;
loop
  for each $i\in\mathcal{N}$ **do**
    $\eta\leftarrow\frac{\psi}{4u_i}+\frac{\psi}{2}\sum_j\frac{1}{u_j}$, $H\leftarrow\sqrt{(\sigma-1)^2+4\eta\cdot\theta_i}$;
    Compute $A_i(t)$ given by Eq. (18);
    $r_i(t)\leftarrow\phi_i^*(t,h)$ within time interval $[0,T]$;
  end for
  if $q\leftarrow q'$ or $t>T$ **then**
    Terminate the loop;
  end if
end loop

In summary, the non-cooperative optimal rate of traffic session $\ell\in\mathcal{T}$ over path $P(i)$ in our dynamic rate allocation model is strictly derived as follows

$$\phi_i^*(t,h)=\frac{F^{\ell\in\mathcal{T}}(q)\log_2(1+S/N)}{2u_i}\times \frac{\left((\sigma-1)+H\right)e^{(t-c)H}-(\sigma-1)+H}{2\eta\left(1-e^{(t-c)H}\right)}h(t) \tag{29}$$

Accordingly, based on Eq. (29), we devise Algorithm 1 to achieve the optimal rate update rule in a distributed manner.

V. Numerical Results

Consider a distributed CRANET scenario depicted in Fig. 1, involving $|\mathcal{M}|=150$ SUs located randomly in the range of 100m×100m square area. We assume that there is a traffic session $\ell\in\mathcal{T}$ from a single upstream source SU $\mathcal{S}_\ell\in\mathcal{M}$ to a downstream destination SU $\mathcal{D}_\ell\in\mathcal{M}$ along $n=4$ node-disjoint paths, denoted by $P(1)$, $P(2)$, $P(3)$ and $P(4)$. The distance $d_\varpi^{\mathcal{P}(i)}$ of each link is clear in this scenario. We assume that the number of the available uplink channel on $n=4$ node-disjoint paths is set to $m=7$. The choice of the channel for links are given in Fig. 2, where $l_\varpi^{P(i)}$, ξ , and τ means that the distance of the ϖ -th link of path $P(i)$ is ranked the ξ -th and the value of the carrier frequency of selected channel is ranked the τ -th. The stability factor θ_i of $n=4$ node-disjoint paths can be obtained by means of combination between Fig. 2 and Eq. (2). In Eq. (2), we set the constant λ in Eq. (1) to 1.2. In the differential game model of dynamic rate allocation for multi-path routing, we assume that the price coefficient for $n=4$ node-disjoint paths has the same value, which is defined as $u_i=120$. The signal-to-noise ratio S/N is set to 70dB, and the constant c in Eq. (18) is assumed to be -1000. The contention window limits of the EDCA mechanism in IEEE 802.11e standard are given by $aCWmin=31$ and $aCWmax=1023$.

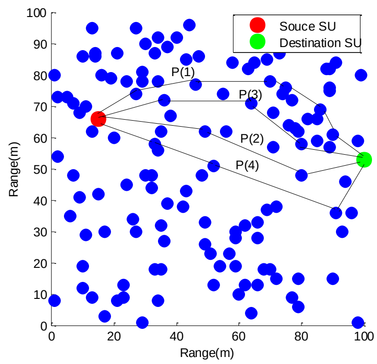

Fig. 1. The simulation scenario

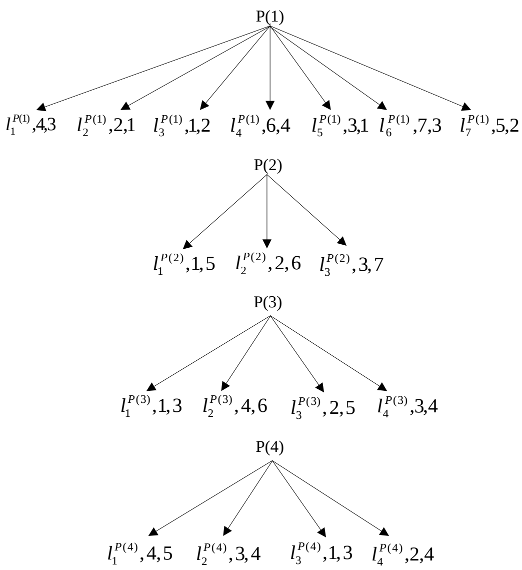

Fig. 2. Links and the corresponding channels

Firstly, we show the non-cooperative optimal rate comparison of $n=4$ node-disjoint paths under different discount factor σ. We assume that the BK as the type of data traffic is employed by the traffic session $\ell \in \mathcal{T}$ from a single upstream source SU $\mathcal{S}_\ell \in \mathcal{M}$ to a downstream destination SU $\mathcal{D}_\ell \in \mathcal{M}$ along $n=4$ node-disjoint paths. Thus, we choose the arbitration interframe space number $AIFSN(0)=7$. As for the traffic sensing factor $F^{1\in T}(0)$, we choose the pseudo-random integer $Random(0)=49$ and the constant $aSlotTime = 20us$. Fig. 3 compares the non-cooperative optimal rate of $n=4$ node-disjoint paths versus game time $t\in[0,4]$ s under different discount factor σ. From Fig. 3, we can see that the non-cooperative optimal rate will increase when the discount factor σ increases. This can be explained by the fact that σ is the discount factor which is a ratio of expected earnings in the future and present value. The higher discount factor will result in the higher rate for data transmission. Meanwhile, Fig. 3 also shows non-cooperative optimal rate changes of four paths under different stability factor with the growth of game time $t\in[0,4]$ s. It is apparent that the non-cooperative optimal rate of four paths under the same type of traffic session depends on the stability factor θ_i according to Eq. (13). It can be observed the non-cooperative optimal rate of $P(1)$ is obviously larger than that of $P(2)$, $P(3)$ and $P(4)$. This can be explained by the fact that the higher stability factor can reduce the ratio of the break of the end-to-end paths.

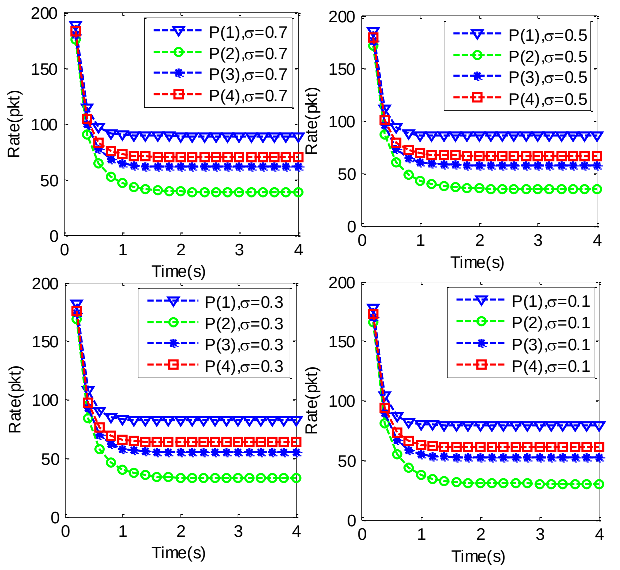

Fig. 3. The non-cooperative optimal rate of four node-disjoint paths under different discount factor σ

Fig. 4 compares the payoff of $n=4$ node-disjoint paths versus game time $t\in[0,4]$ s under different types of traffic sessions. From Fig. 4, it is apparent that the increase of the path stability factor θ_i of path $P(i)$ will result in an enhancement of the payoff of path $P(i)$. In general, high path stability factor θ_i needs high average payoff. This can be explained by the fact that higher stability factor will result in more payoff that path $P(i)$ needs to pay according to Eq. (7). Moreover, it is clear that the payoff of path $P(i)$ with traffic sessions BE and BK could be less than that of VO and VI. The reason for this is that the priority of traffic session VO and VI is higher than that of BE and BK. Then we find that the lower traffic sensing factor $F^{\ell\in\mathcal{T}}(q)$ will lead to the rising of the factor δ_i.

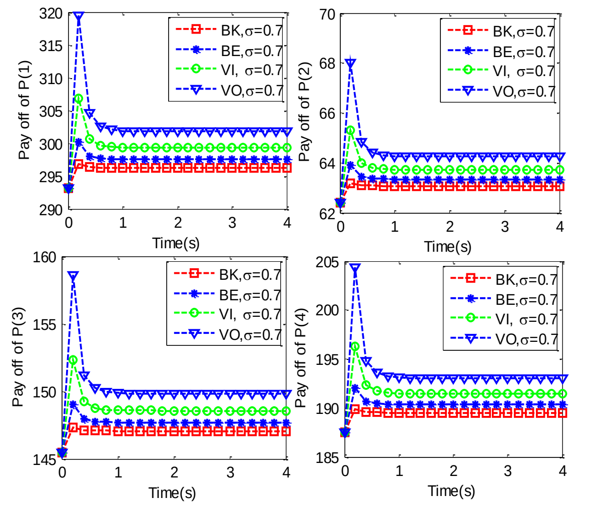

Fig. 4. The payoff of $n=4$ node-disjoint paths under different types of traffic sessions

Fig. 5 compares the non-cooperative optimal rate of path $P(2)$ versus game time $t\in[0,4]$ s under different types of traffic sessions. From Fig. 5, we can see that the non-cooperative optimal rate of traffic session VO and VI is significantly higher than that of BE and BK. This can be explained by the fact that the priority of VO and VI is higher than that of BE and BK.

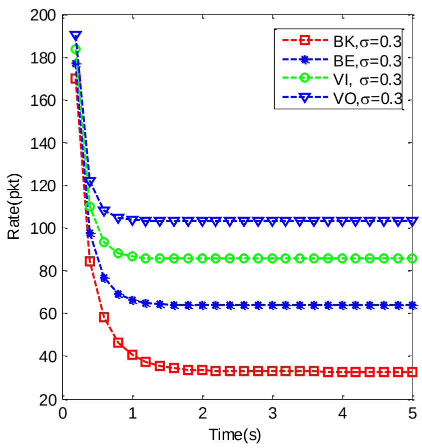

Fig. 5. The non-cooperative optimal rate comparison of $P(2)$ under different types of traffic sessions

VI. Conclusion

In this paper, we have developed the differential game model of dynamic rate allocation for multi-path routing in CRANETs by taking into account the mobility and random nature of the selfish SUs. We devise the payoff function of a certain path from the upstream source SU based on two cost functions including the path stability cost and the traffic session cost. By devising an optimization problem, we also derive the non-cooperative optimal rate of traffic session over a certain path under our dynamic rate allocation model. Moreover, we propose a distributed optimal rate update rule with the purpose of dynamically regulating the rate of traffic session over a certain path.

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Long Zhang received the B.S. degree in communication engineering from China University of Geosciences, Wuhan, China, in June 2006 and the Ph.D. degree in communication and information systems from University of Science and Technology Beijing (USTB), Beijing, China, in January 2012. Since February 2012, he has been with the School of Information and Electrical Engineering from Hebei University of Engineering, Handan, China. His current research interests include opportunistic Internet of Vehicles, cognitive radio ad hoc networks, satellite and space communications, delay and disruption tolerant networks, resource allocation in wireless networks, network modeling and optimization, and cross-layer design.

Chunhong Bai received the B.S degree in electronics and information engineering from Hebei University of Engineering, Handan, China in 2014. She is currently working toward the M.S. degree in computer technology at the Department of Communication Engineering, Hebei University of Engineering, Handan, China. Her current research interests include cognitive radio, game theory, and wireless resource optimization.

Fan Zhuo received the B.S degree in communications engineering from Hebei University of Engineering, Handan, China in 2014. He is currently working toward the M.S. degree in computer technology at the Department of Communication Engineering, Hebei University of Engineering, Handan, China. His current research interests include cognitive radio, routing algorithm, and cross-layer design.

Haitao Xu received his B.S. degree in Department of Electronics Communication & Software Engineer from Sun Yat-Sen University, China, in 2007, and his M.S. degree in Faculty of Engineering from University of Bristol in 2009. He obtained the Ph.D. degree in Department of Communication Engineering, School of Computer & Communication Engineering, from University of Science and Technology Beijing. Now he is a lecturer in University of Science and Technology Beijing. His research interests include cognitive radio networks, self-organized networks, mathematical model, game theory and next-generation networks.

Wei Huang received the B.S. degree in automation from the School of Electronic and Information Engineering, University of Science and Technology Liaoning, Liaoning, China, in June 2004, and the M.S. degree in control science and engineering from the School of Automation and Electrical Engineering, University of Science and Technology Beijing (USTB), Beijing, China, in January 2010. Currently, he is an Engineer of Energy Efficiency Evaluation Center of Electric Power Research Institute, Beijing, China. His current research interests include energy efficiency evaluation, power demand management, and communication technology.