Frequency Estimation of 2-D Harmonics in Multiplicative and Additive Noise Based on ESPRIT

Shiyong Yang and Jinlong Jiang
School of Electronic Engineering, Jiujiang University, Jiujiang 332005, China
Email: {ysydc, jljiang8}@163.com

Abstract —This paper studies the frequency estimation of two-dimensional (2-D) harmonics in presence of multiplicative and additive noise. We construct a cyclic covariance matrix using a class of cyclic covariance of the 2-D harmonics. Exploiting the shift-invariance structure of the signal subspace, we extend ESPRIT to estimate the frequency pairs of 2-D harmonics in multiplicative and additive noise. The proposed method has high-resolution and can directly estimate the frequency pairs of 2-D harmonics without frequency pairing operation. Simulation results demonstrate the effectiveness of the proposed method.

Index Terms—Frequency estimation, multiplicative noise, ESPRIT, two-dimensional harmonic

I. Introduction

The parameter estimation of two-dimensional (2-D) harmonic has applications in many areas such as wireless communications, radio astronomy, nuclear magnetic resonance, sonar, and radar [1]-[7]. Many efficient methods have been proposed for the 2-D harmonic retrieval on constant amplitude harmonics observed only in additive noise [2]-[7]. However, the multiplicative noise often occurs in a variety of applications [8]. For example, the effects on acoustic waves due to fluctuations caused by the medium, changing orientation, and interference from scatterers of the target can be described as the multiplicative noise [8].

The authors in [9] and [10] presented the parameter estimation method based on the cyclic statistics to estimate frequency pair of 2-D harmonic in the presence of the multiplicative and additive noise. The cyclic statistics method [9], [10] developed that some statistics of that the 2-D harmonics in multiplicative and additive noise have peaks at corresponding parameters and zeros at the other, and then estimated the harmonic parameters by peak searching. However, due to affection by the pseudo peaks and the Rayleigh limit, the cyclic statistics method is low-resolution and cannot meet the requests of the high-resolution for the given observed data. This paper considers the high-resolution frequency estimation method of 2-D harmonics in multiplicative and additive noise.

It is well-known that ESPRIT is a high-resolution parameter estimation method and had been applied to many signal processing fields [11]-[14]. In this paper, we first apply ESPRIT to estimate the frequency of the 2-D harmonics in multiplicative and additive noise. We construct a cyclic covariance matrix using a class of cyclic covariance of 2-D harmonic signal, and then exploit the shift-invariance structure of the signal subspace and derive the inherent relation between the first and second dimensional frequencies. Based on the derived relations, we extend ESPRIT for the frequency estimation of 2-D harmonics in multiplicative and additive noise. The proposed method has high-resolution and can directly estimate the frequency without frequency pairing operation.

This paper is organized as follows. The signal model is given in Section II. Section III presents the ESPRIT-based frequency estimation of 2-D harmonics in multiplicative and additive noise. Section IV conducts simulations show the effectiveness of the proposed method. Finally, the conclusion is drawn in Section V.

II. Signal Model

We consider the following signal model of 2-D harmonics in presence of multiplicative and additive noise.

$$x(m,n)=\sum_{k=1}^{P}s_k(m,n)e^{j(\omega_{1k}m+\omega_{2k}n+\phi_k)}+v(m,n) \qquad (1)$$

where $m=0,1,\cdots,M-1, n=0,1,\cdots,N-1$, P is the number of sinusoidal components, $(\omega_{1k},\omega_{2k})$ and ϕ_k are the frequency pair and phase of the *k*th sinusoidal component, $s_k(m,n)$ and $v(m,n)$ are the multiplicative noise and additive noise, respectively. For the model (1), we make the following assumptions:

1) The frequency pairs $(\omega_{1k},\omega_{2k})$ are distinct in $(-\pi/2,\pi/2]\times(-\pi/2,\pi/2]-\{(0,0)\}$.

2) The phase ϕ_k are deterministic in $(-\pi,\pi]$.

3) The multiplicative noise $s_k(m,n)$ and additive noise $v(m,n)$ are mutually independent 2-D real zero-mean Gaussian white noise with variances $\sigma_{s_k}^2$ and σ_v^2 , respectively.

Manuscript received October 10, 2015; revised April 8, 2016.
This work was supported by the National Science Foundation of China under Grant No. 61362038 and the Research Foundation of Education Bureau of Jiangxi Province, China under Grant No. GJJ14738.
Corresponding author email: ysydc@163.com.
doi:10.12720/jcm.11.4.411-416

The purpose of parameter estimation in 2-D harmonic is to estimate the frequency pair $(\omega_{1k}, \omega_{2k})$ and the number of sinusoidal component P from the observed data $\{x(m,n), m=0,1,\cdots,M-1, n=0,1,\cdots,N-1\}$. In this paper, we focus on the estimation of the frequency pair $(\omega_{1k}, \omega_{2k})$ and assume that the number of the sinusoidal component P is known. The estimation of number the sinusoidal component can be found in reference [2].

III. Frequency Estimation of 2-D Harmonics in Multiplicative and Additive Noise

To obtain high-resolution frequency pair estimation method, we will apply ESPRIT to estimate the frequency pair of the 2-D harmonics in multiplicative and additive noise. We construct a cyclic covariance matrix and exploit the shift-invariance structure of the signal subspace. Then, we derive an inherent relation between the first and second dimensional frequencies. Based on the derived relations, we extend ESPRIT for the frequency pair estimation of 2-D harmonics in multiplicative and additive noise.

First, we define a cyclic covariance $c(\alpha,\beta)$ of $x(m,n)$ as

$$\begin{aligned} c(\alpha,\beta) &= E\{x^2(m,n)x^{2*}(m+\alpha,n+\beta)\} \\ &\quad - E\{x^2(m,n)\}E\{x^{2*}(m+\alpha,n+\beta)\} \end{aligned} \tag{2}$$

where $(\cdot)^*$ denotes complex conjugate, and $E\{\cdot\}$ represents the cyclic mean [7] which is defined as

$$E\{y(m,n)\} = \lim_{M,N\to\infty} \frac{1}{MN} \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} E\{y(m,n)\} \tag{3}$$

Substituting (1) into (2), we have

$$c(\alpha,\beta) = \sum_{k=1}^{P} (\sigma_{s_k}^2)^2 e^{-j(2\omega_{1k}\alpha + 2\omega_{2k}\beta)} + \eta\delta(\alpha)\delta(\beta) \tag{4}$$

where

$$\begin{aligned} \eta &= 2(\sigma_v^2)^2 + \sum_{k=1}^{P} 2(\sigma_{s_k}^2)^2 \\ &\quad + \sum_{k=1}^{P-1}\sum_{l=k+1}^{P} 4\sigma_{s_k}^2\sigma_{s_l}^2 + \sum_{k=1}^{P} 4\sigma_{s_k}^2\sigma_v^2 \end{aligned} \tag{5}$$

and $\delta(\cdot)$ denotes the Kronecker delta function

$$\delta(\alpha) = \begin{cases} 1, & \alpha = 0, \\ 0, & \text{otherwise.} \end{cases}$$

The detailed derivation of (4) is given in Appendix.

Then, we construct a cyclic covariance matrix using the cyclic covariance $c(\alpha,\beta)$ as follows

$$\mathbf{G} = \begin{bmatrix} \mathbf{D}_0 & \mathbf{D}_{-1} & \cdots & \mathbf{D}_{1-K} \\ \mathbf{D}_1 & \mathbf{D}_0 & \cdots & \mathbf{D}_{2-K} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{D}_{K-1} & \mathbf{D}_{K-2} & \cdots & \mathbf{D}_0 \end{bmatrix} \tag{6}$$

where

$$\mathbf{D}_k = \begin{bmatrix} c(k,0) & c(k,-1) & \cdots & c(k,1-L) \\ c(k,1) & c(k,0) & \cdots & c(k,2-L) \\ \vdots & \vdots & \ddots & \vdots \\ c(k,L-1) & c(k,L-2) & \cdots & c(k,0) \end{bmatrix} \tag{7}$$

K and L are positive integer greater than $(P+1)$. Substituting $c(\alpha,\beta)$ into (6), $\mathbf{G}$ can be decomposed as

$$\mathbf{G} = \mathbf{A}\mathbf{S}\mathbf{A}^H + \eta\mathbf{I}_{KL} \tag{8}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_1\mathbf{F}_1 \\ \vdots \\ \mathbf{Q}_1\mathbf{F}_1^{K-1} \end{bmatrix} \tag{9}$$

$$\mathbf{Q}_1 = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j2\omega_{21}} & e^{-j2\omega_{22}} & \cdots & e^{-j2\omega_{2P}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j2\omega_{21}(1-L)} & e^{j2\omega_{22}(1-L)} & \cdots & e^{j2\omega_{2P}(1-L)} \end{bmatrix} \tag{10}$$

$$\mathbf{F}_1 = \begin{bmatrix} e^{j2\omega_{11}} & 0 & \cdots & 0 \\ 0 & e^{j2\omega_{12}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{j2\omega_{1P}} \end{bmatrix} \tag{11}$$

$$\mathbf{S} = \begin{bmatrix} (\sigma_{s_1}^2)^2 & 0 & \cdots & 0 \\ 0 & (\sigma_{s_2}^2)^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (\sigma_{s_P}^2)^2 \end{bmatrix} \tag{12}$$

$(\cdot)^H$ represents conjugate transposition, and $\mathbf{I}_{KL}$ denotes the $KL \times KL$ identity matrix.

The rank of matrix $\mathbf{A}$ is equal to P, thus, the rank of matrix $\mathbf{A}\mathbf{S}\mathbf{A}^H$ is equal to P [15]. Computing the eigenvalue decomposition of $\mathbf{G}$ leads to a factorization of $\mathbf{G}$ such as

$$\mathbf{G} = \mathbf{U}\mathbf{D}\mathbf{U}^H = \mathbf{U}_S\mathbf{D}_S\mathbf{U}_S^H + \mathbf{U}_N\mathbf{D}_N\mathbf{U}_N^H \tag{13}$$

where $\mathbf{U}_S$ and $\mathbf{D}_S$ contain the P principal components, i.e., the eigenvalues and the eigenvectors related to the signal subspace. $\mathbf{U}_N$ and $\mathbf{D}_N$ contain the remaining components. Due to $\mathbf{A}$ and $\mathbf{U}_S$ span the same signal subspace, there must exist a $P \times P$ nonsingular matrix $\mathbf{T}$ such that

$$\mathbf{A}\mathbf{T} = \mathbf{U}_S \tag{14}$$

Throughout this paper, we will use the notation $\overline{\mathbf{M}}$ to denote the first $(K-1)L$ rows of matrix $\mathbf{M}$, and $\underline{\mathbf{M}}$ to denote the last $(K-1)L$ rows of $\mathbf{M}$, respectively.

From (9), we have

$$\underline{\mathbf{A}}\mathbf{F}_1 = \overline{\mathbf{A}} \tag{15}$$

The rank of the matrices $\underline{\mathbf{A}}$ and $\overline{\mathbf{A}}$ is equal to P since K and L are greater than $(P+1)$ [15]. Thus, the least squares solution of (15) is given by

$$\mathbf{F}_1 = \underline{\mathbf{A}}^{\dagger}\overline{\mathbf{A}} \tag{16}$$

where $(\cdot)^{\dagger}$ denotes the pseudo-inverse matrix, i.e., $\mathbf{C}^{\dagger} = (\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H$.

Substituting (14) into (16), $\mathbf{F}_1$ can be rewritten as

$$\mathbf{F}_1 = \left(\underline{\mathbf{U}_S}\mathbf{T}^{-1}\right)^{\dagger}\left(\overline{\mathbf{U}_S}\mathbf{T}^{-1}\right) = \mathbf{T}\underline{\mathbf{U}_S}^{\dagger}\overline{\mathbf{U}_S}\mathbf{T}^{-1} \tag{17}$$

Equation (16) shows that the first frequencies ω_{1k} can be estimated from $\mathbf{F}_1$, i.e., from the eigenvalues of $\mathbf{H}_1 = \underline{\mathbf{U}_S}^{\dagger}\overline{\mathbf{U}_S}$:

$$\mathbf{F}_1 = \begin{bmatrix} e^{j2\omega_{11}} & 0 & \cdots & 0 \\ 0 & e^{j2\omega_{12}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{j2\omega_{1P}} \end{bmatrix} = \mathbf{T}\mathbf{H}_1\mathbf{T}^{-1} \tag{18}$$

Now, we consider the estimation of second frequencies ω_{2k}. We define a permutation matrix $\mathbf{J}$ [4] as

$$\mathbf{J} = \sum_{k=1}^{K}\sum_{l=1}^{L}\mathbf{E}_{k,l}^{K\times L} \otimes \mathbf{E}_{l,k}^{L\times K} \tag{19}$$

where $\otimes$ denotes the Kronecker product and $\mathbf{E}_{k,l}^{K\times L}$ denotes a $K\times L$ elementary matrix with 1 for the (k,l) element and 0 elsewhere. One can easily prove that the matrix $\mathbf{J}$ has the following relations:

$$\mathbf{J}^H\mathbf{J} = \mathbf{J}\mathbf{J}^H = \mathbf{I}_{KL} \tag{20}$$

$$\mathbf{J}^H\mathbf{A} = \mathbf{B} \tag{21}$$

where

$$\mathbf{B} = \begin{bmatrix} \mathbf{Q}_2 \\ \mathbf{Q}_2\mathbf{F}_2 \\ \vdots \\ \mathbf{Q}_2\mathbf{F}_2^{K-1} \end{bmatrix} \tag{22}$$

$$\mathbf{Q}_2 = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{j2\omega_{11}} & e^{j2\omega_{12}} & \cdots & e^{j2\omega_{1P}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j2\omega_{11}(L-1)} & e^{j2\omega_{12}(L-1)} & \cdots & e^{j2\omega_{1P}(L-1)} \end{bmatrix} \tag{23}$$

$$\mathbf{F}_2 = \begin{bmatrix} e^{-j2\omega_{21}} & 0 & \cdots & 0 \\ 0 & e^{-j2\omega_{22}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-j2\omega_{2P}} \end{bmatrix} \tag{24}$$

Let $\mathbf{Y} = \mathbf{J}\mathbf{G}\mathbf{J}^H$. Combining with (8), (20), and (21), we have

$$\mathbf{Y} = \mathbf{J}\mathbf{G}\mathbf{J}^H = \mathbf{B}\mathbf{S}\mathbf{B}^H + \eta\mathbf{I}_{KL} \tag{25}$$

The rank of matrix $\mathbf{B}$ is equal to P, thus, the rank of matrix $\mathbf{B}\mathbf{S}\mathbf{B}^H$ is equal to P [15]. Computing the eigenvalue decomposition of $\mathbf{Y}$ leads to

$$\mathbf{Y} = \mathbf{V}\mathbf{Q}\mathbf{V}^H = \mathbf{V}_S\mathbf{Q}_S\mathbf{V}_S^H + \mathbf{V}_N\mathbf{Q}_N\mathbf{V}_N^H \tag{26}$$

$\mathbf{V}_S$ and $\mathbf{Q}_S$ contain the P principal components related to the signal subspace, $\mathbf{V}_N$ and $\mathbf{Q}_N$ contain the remaining components. Due to $\mathbf{B}$ and $\mathbf{V}_S$ span the same signal subspace, there must exist a $P\times P$ nonsingular matrix $\mathbf{Z}$ such that

$$\mathbf{B}\mathbf{Z} = \mathbf{V}_S \tag{27}$$

From (22), we have

$$\underline{\mathbf{B}}\mathbf{F}_2 = \overline{\mathbf{B}} \tag{28}$$

The rank of the matrices $\underline{\mathbf{B}}$ and $\overline{\mathbf{B}}$ is equal to P since K and L are greater than $(P+1)$ [15]. Then, the least squares solution of (28) is given by

$$\mathbf{F}_2 = \underline{\mathbf{B}}^{\dagger}\overline{\mathbf{B}} \tag{29}$$

Substituting (27) into (29), $\mathbf{F}_2$ can be rewritten as

$$\mathbf{F}_2 = \left(\underline{\mathbf{V}_S}\mathbf{Z}^{-1}\right)^{\dagger}\left(\overline{\mathbf{V}_S}\mathbf{Z}^{-1}\right) = \mathbf{Z}\underline{\mathbf{V}_S}^{\dagger}\overline{\mathbf{V}_S}\mathbf{Z}^{-1} \tag{30}$$

Equation (30) shows that the second frequencies ω_{2k} can be estimated from the matrix $\mathbf{F}_2$, i.e., from the eigenvalues of the matrix $\mathbf{H}_2 = \underline{\mathbf{V}_S}^{\dagger}\overline{\mathbf{V}_S}$:

$$\mathbf{F}_2 = \begin{bmatrix} e^{-j2\omega_{21}} & 0 & \cdots & 0 \\ 0 & e^{-j2\omega_{22}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-j2\omega_{2P}} \end{bmatrix} = \mathbf{Z}\mathbf{H}_2\mathbf{Z}^{-1} \tag{31}$$

Now, we derive the connection between the first and second dimensions which will be used to form the frequency pairs $(\omega_{1k}, \omega_{2k})$. Combining with (13), (15), and (26), we have

$$\mathbf{J}\mathbf{U}_S\mathbf{D}_S\mathbf{U}_S^H\mathbf{J}^H = \mathbf{V}_S\mathbf{Q}_S\mathbf{V}_S^H \tag{32}$$

Thus, we can introduce a $P\times P$ nonsingular matrix $\mathbf{R}$ such that

$$\mathbf{J}\mathbf{U}_S\mathbf{R} = \mathbf{V}_S, \tag{33}$$

$$\mathbf{R}\mathbf{Q}_S\mathbf{R}^H = \mathbf{D}_S. \tag{34}$$

The matrix $\mathbf{R}$ has two important properties. First, from the facts that $\mathbf{U}_S^H\mathbf{U}_S = \mathbf{I}_P$ and $\mathbf{V}_S^H\mathbf{V}_S = \mathbf{I}_P$ since $\mathbf{U}$ and $\mathbf{V}$ containing eigenvectors are unitary matrix, we obtain

$$\mathbf{V}_S^H\mathbf{V}_S = (\mathbf{J}\mathbf{U}_S\mathbf{R})^H\mathbf{J}\mathbf{U}_S\mathbf{R} = \mathbf{R}^H\mathbf{R} = \mathbf{I}_P \tag{35}$$

This implies $\mathbf{R}^{-1} = \mathbf{R}^H$ because $\mathbf{R}$ is nonsingular square matrix. Second, substituting (14) and (27) into (21) leads to

$$\mathbf{J}\mathbf{U}_S\mathbf{T}^{-1} = \mathbf{V}_S\mathbf{Z}^{-1} \tag{36}$$

Comparing (36) with (33), we obtain an important relation among $\mathbf{R}$, $\mathbf{T}$, and $\mathbf{Z}$

$$\mathbf{T} = \mathbf{Z}\mathbf{R}^{-1} \tag{37}$$

Substituting (33) into (30), $\mathbf{F}_2$ can be rewritten as

$$\mathbf{F}_2 = \mathbf{Z}\mathbf{R}^{-1}\left(\underline{\mathbf{J}\mathbf{U}_S}\right)^{\dagger}\overline{\mathbf{J}\mathbf{U}_S}\mathbf{R}\mathbf{Z}^{-1} = \mathbf{T}\mathbf{W}\mathbf{T}^{-1} \tag{38}$$

where $\mathbf{W} = (\underline{\mathbf{J}\mathbf{U}_S})^{\dagger}\overline{\mathbf{J}\mathbf{U}_S}$.

Combining with (18) and (38), we find an important result: the same transformation $\mathbf{T}$ diagonalizes both $\mathbf{H}_1$ and $\mathbf{W}$. However, the relations $\mathbf{F}_1 = \mathbf{T}\mathbf{H}_1\mathbf{T}^{-1}$ and $\mathbf{F}_2 = \mathbf{T}\mathbf{W}\mathbf{T}^{-1}$ cannot be directly exploited to estimate the frequency pairs. This is because that, when an eigenvalue has a multiplicity greater than 1, its eigenvectors are not uniquely defined, as it is for the transformation $\mathbf{T}$.

To avoid eigenvalues of multiplicity greater than 1, we can use the proposed method in [4] to find a unique transformation $\mathbf{N}$ to diagonalize both $\mathbf{H}_1$ and $\mathbf{W}$. The unique transformation $\mathbf{N}$ is computed from the eigenvalue decomposition of a linear combination of the matrices $\mathbf{H}_1$ and $\mathbf{W}$:

$$\gamma\mathbf{H}_1 + (1-\gamma)\mathbf{W} = \mathbf{N}^{-1}\mathbf{C}\mathbf{N} \tag{38}$$

where the parameter γ is scalar. The purpose of γ is to avoid eigenvalues of multiplicity greater than 1. Therefore, the frequency pairs $(\omega_{1k}, \omega_{2k})$ can be directly estimated from $\mathbf{N}\mathbf{H}_1\mathbf{N}^{-1}$ and $\mathbf{N}\mathbf{W}\mathbf{N}^{-1}$.

It is worth noting that the computation of $c(\alpha,\beta)$ in (2) needs $M, N \to \infty$. However, in practice, we have only the single record $x(m,n),\ m = 0,1,\cdots,M-1,$ $n = 0,1,\cdots,N-1$. Thus, it need to calculate the estimation of $c(\alpha,\beta)$. In practice, we can use the natural single record estimator $\hat{c}(\alpha,\beta)$ given in (40) at the bottom of this page to estimate cyclic covariance $c(\alpha,\beta)$.

The estimator $\hat{c}(\alpha,\beta)$ is consistent and asymptotically unbiased [16].

Finally, we summarize the key steps of the ESPRIT-based method to estimate the frequency pair of 2-D harmonics in multiplicative and additive noise as follows.

Step 1: Calculate the sample cyclic covariance $\hat{c}(\alpha,\beta)$ using (40) from the observed data $x(m,n),$ $m = 0,1,\cdots,M-1, n = 0,1,\cdots,N-1$. Construct $\mathbf{G}$ using $\hat{c}(\alpha,\beta)$ according to (6).

Step 2: Calculate the eigenvalue decomposition of $\mathbf{G}$ and obtain the eigenvector matrix $\mathbf{U}$. Construct the matrices $\overline{\mathbf{U}}$, $\underline{\mathbf{U}}$, $\overline{\mathbf{J}\mathbf{U}}$ and $\underline{\mathbf{J}\mathbf{U}}$, respectively.

Step 3: Calculate $\mathbf{H}_1 = \underline{\mathbf{U}_S}^{\dagger}\overline{\mathbf{U}_S}$ and $\mathbf{W} = (\underline{\mathbf{J}\mathbf{U}_S})^{\dagger}\overline{\mathbf{J}\mathbf{U}_S}$. Calculate the eigenvalue decomposition of matrix $\gamma\mathbf{H}_1 + (1-\gamma)\mathbf{W}$ to obtain eigenvector matrix $\mathbf{N}$.

Step 4: Calculate $\mathbf{N}\mathbf{H}_1\mathbf{N}^{-1}$ and $\mathbf{N}\mathbf{W}\mathbf{N}^{-1}$. Extract the main diagonal elements of $\mathbf{N}\mathbf{H}_1\mathbf{N}^{-1}$ and label as $\lambda_{11}, \lambda_{12}, \cdots, \lambda_{1P}$ in sequence. Extract the main diagonal elements of $\mathbf{N}\mathbf{W}\mathbf{N}^{-1}$ and label as $\lambda_{21}, \lambda_{22}, \cdots, \lambda_{2P}$ in sequence. Then, estimate frequency pairs as follows:

$$(\hat{\omega}_{1k}, \hat{\omega}_{2k}) = \left(\frac{\angle\lambda_{1k}}{2}, -\frac{\angle\lambda_{2k}}{2}\right),\ k = 1,2,\cdots,P \tag{41}$$

where $\angle$ represents calculation of phase angle.

IV. Simulation Results

To demonstrate the effectiveness of proposed ESPRIT-based method for frequency estimation of 2-D harmonics in multiplicative and additive noise, we conduct numerical Monte Carlo study in this section. In the following two examples, the 2-D harmonic signal are generated by

$$x(m,n) = \sum_{k=1}^{3} s_k(m,n) e^{j(\omega_{1k}m + \omega_{2k}n + \phi_k)} + v(m,n) \tag{42}$$

where $m = 0,1,\cdots,99; n = 0,1,\cdots,99$. The multiplicative and additive noise are 2-D real zero-mean white Gaussian noise with the variances $\sigma_{s_1}^2 = \sigma_{s_2}^2 = \sigma_{s_3}^2 = \sigma_v^2 = 1$.

Example 1: The frequency pairs are $(\omega_{11}, \omega_{21}) = (0.36, -1.45)$, $(\omega_{12}, \omega_{22}) = (-0.12, 1.23)$, and $(\omega_{13}, \omega_{23}) = (0.14, 0.85)$. The phases are $\phi_1 = 0.43$, $\phi_2 = 1.84$, and $\phi_3 = -2.45$. We choose $\gamma = 0.8$ and $K = L = 15$ in the proposed method. To compare the performance of proposed method, we also estimate the frequency pairs using the cyclic statistic method presented in [9], [10]. Table I lists the mean and standard deviation (*std*) of frequency estimations from 1000 independent realizations of two estimation methods. Table I shows that the proposed method can estimate effectively the frequency pairs.

Example 2: In this example, we test the frequency resolution of the proposed the ESPRIT-based method. The frequency pairs are very close. The frequency pairs are $(\omega_{11}, \omega_{21}) = (0.82, 0.82)$, $(\omega_{12}, \omega_{22}) = (0.82, 0.88)$, and $(\omega_{13}, \omega_{23}) = (0.88, 0.88)$. The phases are $\phi_1 = -1.39$, $\phi_2 = 0.45$, and $\phi_3 = 2.17$. We choose $\gamma = 0.8$ in the proposed method. Table II lists the mean and *std* of frequency estimations from 1000 independent realizations using the cyclic statistic method [9], [10] and the proposed

$$\begin{aligned}\hat{c}(\alpha,\beta) = & \frac{1}{(M-|\alpha|)(N-|\beta|)} \sum_{m=0}^{M-1-|\alpha|} \sum_{n=0}^{N-1-|\beta|} x^2(m,n) x^{2*}(m+\alpha, n+\beta) \\ & - \frac{1}{(M-|\alpha|)^2(N-|\beta|)^2} \left(\sum_{m=0}^{M-1-|\alpha|} \sum_{n=0}^{N-1-|\beta|} x^2(m,n) \right) \left(\sum_{m=0}^{M-1-|\alpha|} \sum_{n=0}^{N-1-|\beta|} x^{2*}(m+\alpha, n+\beta) \right)\end{aligned} \tag{40}$$

method with $K = L = 34$. Due to the frequency pairs are very closed and the distance of frequency pairs is less than the Rayleigh limit, the cyclic statistic method can only estimate one frequency pair. However, the proposed method can estimate effectively the three frequency pairs. Thus, the proposed method has high frequency resolution.

V. CONCLUSIONS

This paper investigated the frequency estimation of 2-D harmonics in multiplicative and additive noise. We extended the ESPRIT method to estimate the frequency pairs of 2-D harmonics in multiplicative and additive noise. The proposed method has high-resolution and can directly estimate the frequency pairs of 2-D harmonics without frequency pairing operation. Simulation results clearly showed the effectiveness of the proposed method.

APPENDIX: DERIVATION OF (4)

For convenience to deduce, let $(\omega_{1,P+1}, \omega_{2,P+1}) = (0,0)$, $\phi_{P+1} = 0$, and $s_{P+1}(m,n) = v(m,n)$, (1) can be rewritten as

$$x(m,n) = \sum_{k=1}^{P+1} s_k(m,n) e^{j(\omega_{1k}m + \omega_{2k}n + \phi_k)} \quad (44)$$

Therefore, we can calculate $E\{x^2(m,n)x^{2*}(m+\alpha, n+\beta)\}$ that is given in (45) at the bottom of this page.

In the derivation of (45), we have used that, for 2-D real stationary zero-mean white Gaussian noise $y(m,n)$,

$$\begin{aligned} &E\{y^2(m,n)y^{2*}(m+\alpha, n+\beta)\} \\ &= \mathrm{Cum}\{y(m,n), y(m,n), y(m+\alpha, n+\beta), y(m+\alpha, n+\beta)\} \\ &\quad + E\{y^2(m,n)\}E\{y^2(m+\alpha, n+\beta)\} \\ &\quad + 2(E\{y(m,n)y(m+\alpha, n+\beta)\})^2 \\ &= (\sigma_y^2)^2 + 2(\sigma_y^2)^2 \delta(\alpha)\delta(\beta), \end{aligned} \quad (46)$$

and all the fourth-order cumulants of 2-D Gaussian noise is equal to zero.

Then, the cyclic means are

$$\begin{aligned} &E\{x^2(m,n)\} = \lim_{M,N\to\infty} \frac{1}{MN} \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} E\{x^2(m,n)\} \\ &= \lim_{M,N\to\infty} \frac{1}{MN} \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} \left(\sum_{k=1}^{P+1} (\sigma_{s_k}^2)^2 e^{j(2\omega_{1k}m + 2\omega_{2k}n + 2\phi_k)} \right) \\ &= \sigma_{s_{P+1}}^2 . \end{aligned} \quad (47)$$

$$\begin{aligned} &E\{x^{2*}(m+\alpha, n+\beta)\} \\ &= \lim_{M,N\to\infty} \frac{1}{MN} \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} E\{x^{2*}(m+\alpha, n+\beta)\} \\ &= \lim_{M,N\to\infty} \frac{1}{MN} \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} \left(\sum_{k=1}^{P+1} (\sigma_{s_k}^2)^2 e^{j(2\omega_{1k}(m+\alpha) + 2\omega_{2k}(n+\beta) + 2\phi_k)} \right) \\ &= \sigma_{s_{P+1}}^2 . \end{aligned} \quad (48)$$

$$\begin{aligned} &E\{x^2(m,n)x^{2*}(m+\alpha, n+\beta)\} \\ &= \sum_{k=1}^{P+1} E\{s_k^2(m,n)s_k^2(m+\alpha, n+\beta)\} e^{-j(2\omega_{1k}\alpha + 2\omega_{2k}\beta)} \\ &\quad + \sum_{k=1}^{P}\sum_{l=k+1}^{P+1} 4E\{s_k(m,n)s_k(m+\alpha, n+\beta)\}E\{s_l(m,n)s_l(m+\alpha, n+\beta)\} e^{-j((\omega_{1k}+\omega_{1l})\alpha + (\omega_{2k}+\omega_{2l})\beta)} \\ &\quad + \sum_{k=1}^{P+1}\sum_{l=1, l\neq k}^{P+1} E\{s_k^2(m,n)s_l^2(m+\alpha, n+\beta)\} e^{j((2\omega_{1k}-2\omega_{1l})m + (2\omega_{2k}-2\omega_{2l})n - 2\omega_{1l}\alpha - 2\omega_{2l}\beta + 2\phi_k - 2\phi_l)} \\ &= \sum_{k=1}^{P+1} ((\sigma_{s_k}^2)^2 + 2(\sigma_{s_k}^2)^2 \delta(\alpha)\delta(\beta)) e^{-j(2\omega_{1k}\alpha + 2\omega_{2k}\beta)} + \sum_{k=1}^{P}\sum_{l=k+1}^{P+1} 4\sigma_{s_k}^2\sigma_{s_l}^2 \delta(\alpha)\delta(\beta) e^{-j((\omega_{1k}+\omega_{1l})\alpha + (\omega_{2k}+\omega_{2l})\beta)} \\ &\quad + \sum_{k=1}^{P+1}\sum_{l=1, l\neq k}^{P+1} \sigma_{s_k}^2\sigma_{s_l}^2 e^{j((2\omega_{1k}-2\omega_{1l})m + (2\omega_{2k}-2\omega_{2l})n - 2\omega_{1l}\alpha - 2\omega_{2l}\beta + 2\phi_k - 2\phi_l)} \end{aligned} \quad (45)$$

$$\begin{aligned} &E\{x^2(m,n)x^{2*}(m+\alpha, n+\beta)\} \\ &= \lim_{M,N\to\infty} \frac{1}{MN} \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} E\{x^2(m,n)x^{2*}(m+\alpha, n+\beta)\} \\ &= \sum_{k=1}^{P+1} (\sigma_{s_k}^2)^2 e^{-j(2\omega_{1k}\alpha + 2\omega_{2k}\beta)} \\ &\quad + \left(\sum_{k=1}^{P+1} 2(\sigma_{s_k}^2)^2 + \sum_{k=1}^{P}\sum_{l=k+1}^{P+1} 4\sigma_{s_k}^2\sigma_{s_l}^2 \right) \delta(\alpha)\delta(\beta) \end{aligned} \quad (49)$$

In the derivations of (47)-(49), we have used the fact that

$$\lim_{M,N\to\infty} \frac{1}{MN} \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} e^{j(\mu m + \xi n)} = \delta(\mu)\delta(\xi) \quad (50)$$

Therefore, we have

$$\begin{aligned} c(\alpha,\beta)\} &= \sum_{k=1}^{P+1} (\sigma_{s_k}^2)^2 e^{-j(2\omega_{1k}\alpha + 2\omega_{2k}\beta)} - (\sigma_{s_{P+1}}^2)^2 \\ &\quad + \left(\sum_{k=1}^{P+1} 2(\sigma_{s_k}^2)^2 + \sum_{k=1}^{P}\sum_{l=k+1}^{P+1} 4\sigma_{s_k}^2\sigma_{s_l}^2 \right) \delta(\alpha)\delta(\beta) \\ &= \sum_{k=1}^{P+1} (\sigma_{s_k}^2)^2 e^{-j(2\omega_{1k}\alpha + 2\omega_{2k}\beta)} + \eta\delta(\alpha)\delta(\beta) \end{aligned} \quad (51)$$

REFERENCES

[1] S. Yang, H. Li, and T. Jiang, "Detection of the number of two-dimensional harmonics in additive colored noise," *Wireless Communications and Mobile Computing*, vol. 14, no. 8, pp. 761-769, 2014.

[2] S. Yang, H. Li, and T. Jiang, "Detecting the number of 2-

D harmonics in multiplicative and additive noise using enhanced matrix," *Digital Signal Process.*, vol. 22, no. 2, pp. 246-252, 2012.

[3] F. J. Chen, C. C. Fung, C. W. Kok, and S. Kwong, "Estimation of two-dimensional frequencies using modified matrix pencil method," *IEEE Trans. on Signal Process.*, vol. 22, no. 2, pp. 718-724, Feb. 2007.

[4] S. Rouquette and M. Najim. "Estimation of frequencies and damping factors by two-dimensional ESPRIT type methods," *IEEE Trans. on Signal Process.*, vol. 49, no. 1, pp. 237-245, Jan. 2001.

[5] H. M. Ibrahim and R. R. Gharieb, "Estimating two-dimensional frequencies by a cumulant-based FBLP method," *IEEE Trans. on Signal Process.*, vol. 47, no. 1, pp. 262-266, Jan. 1999.

[6] J. W. Odendaal, E. Barnard, and C. W. I. Pistorius, "Two-dimensional superresolution radar imaging using the MUSIC algorithm," *IEEE Trans. Antennas Propag.*, vol. 42, no. 10, pp. 1386–1391, Oct. 1994.

[7] J. M. M. Anderson and G. B. Giannakis, "2-D harmonic retrieval using higher-order statistics," *Multidimensional Systems and Signal Process.*, vol. 6, no. 4, pp. 313-331, 1995.

[8] G. B. Giannakis and G. Zhou. "Harmonics in multiplicative and additive noise: Parameter estimation using cyclic statistics," *IEEE Trans. Signal Process.*, vol. 43, no. 9, pp. 2217-2221, Sept. 1995.

[9] F. Wang, S. Wang, and H. Dou, "Two-Dimensional parameter estimation using two-dimensional cyclic statistics," *Acta Electronic Sinica*, vol. 31, no. 10, pp. 1522-1525, 2003.

[10] S. Yang and H. Li. "Two-Dimensional harmonic parameters estimation using third-order cyclic-moments," *Acta Electronic Sinica*, vol. 33, no. 10, pp. 1808-1811, 2005.

[11] Y. Wu, L. Amir, J. R. Jensen, and G. Liao, "Joint pitch and DOA estimation using the ESPRIT method," *IEEE Trans. on Audio, Speech, and Language Process.*, vol. 23, no. 1, pp. 32-45, Jan. 2015.

[12] Y. Shi, L. Huang, C. Qian, and H. C. So, "Direction-of-arrival estimation for noncircular sources via structured least squares based ESPRIT using three-axis crossed array," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 51, no. 2, pp. 1267-1278, Feb. 2015.

[13] A. Hu, T. Lv, H. Gao, Z. Zhang, and S. Yang, "An ESPRIT-based approach for 2-D localization of incoherently distributed sources in massive MIMO systems," *IEEE Journal of Selected Topics in Signal Process.*, vol. 8, no. 5, pp. 996-1011, May 2014.

[14] J. Steinwandt, F. Roemer, M. Haardt, and G. Del Galdo, "R-Dimensional ESPRIT-type algorithms for strictly second-order non-circular sources and their performance analysis," *IEEE Trans. on Signal Process.*, vol. 62, no. 18, pp. 4824-4838, Aug. 2014.

[15] H. H. Yang and Y. Hua, "On rank of block Hankel matrix for 2-D frequency detection and estimation," *IEEE Trans. on Signal Process.*, vol. 44, no. 4, pp. 1046-1048, Apr. 1996.

[16] J. W. Brewer, "Kronecker products and matrix calculus in system theory," *IEEE Trans. on Circuits and Systems*, vol. CAS-25, no. 9, pp. 772-781, Sept. 1978.

Shiyong Yang was born in Jiangxi Province, China, in 1978. He received the Ph.D. degree in communication and information system from the Huazhong University of Science and Technology of China, Wuhan, in 2012. Currently he is an associate professor in the School of Electronic Engineering of Jiujiang University. His research interests include array signal processing, information theory, and wireless communications.

Jinlong Jiang was born in Jiangxi Province, China, in 1969. He received the M.S. degree in communication and information system from the Hohai University of China, Nanjing, in 2005. Currently he is an associate professor in the School of Electronic Engineering of Jiujiang University. His research interests include signal processing and communication theory.