A Modified Regularized Adaptive Matching Pursuit Algorithm for Linear Frequency Modulated Signal Detection Based on Compressive Sensing

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Abstract—Compressive Sensing (CS) is a novel signal sampling theory under the condition that the signal is sparse or compressible. It has the ability of compressing a signal during the process of sampling. Reconstruction algorithm is one of the key parts in compressive sensing. We propose a novel iterative greedy algorithm for reconstructing sparse signals, called Modified Regularized Adaptive Matching Pursuit (MRAMP). Compared with other state-of-the-art greedy algorithms, MRAMP has the characteristics of several approaches: the speed and transparency of Orthogonal Matching Pursuit (OMP), the strong uniform guarantees of l_1-minimization and the most innovative feature is its capability of signal reconstruction without prior information of the sparsity as Sparsity Adaptive Matching Pursuit (SAMP). Recently, the idea of CS has been used in radar system, and the concept of Compressive Sensing Radar (CSR) has been proposed in which the target scene can be sparsely represented in the range domain. CS plays an important role in the detection of Linear Frequency Modulated (LFM) signal. A sparse dictionary which could match LFM signal is constructed, and with the dictionary we can get access to a more effective sparse signal, then LFM signal can be calculated according to classical least square solution. Simulation results show that by using the method this paper proposed, it outperforms many existing iterative algorithms, especially for compressible signals.

Index Terms—Compressive sensing, MRAMP, LFM

I. INTRODUCTION

Compressive sensing has attracted a great deal of attentions recently. It has been successfully applied in a multitude of scientific fields, ranging from image processing tasks to radar to coding theory, making the potential impact of advancements in theory and practice rather large. Compressive sensing methods rely on the notion of sparsity, which is primarily approximated via the l_1 norm [1], [2]. The nature and limitations of this relaxation have been well-studied [3]-[8], as well as some alternative relaxations, such as the l_p quasi norm [9], [10]. The nonconvex l_p quasi norm approaches present a tradeoff: closer approximation of sparsity for harder analysis and computation. Recent work has introduced generalized nonconvex penalties [11], [12] that have thus far demonstrated strong empirical performance [13]-[15].

The reconstruction of CS requires some non-linear algorithms to find the sparsest signal from the measurements. Finding fast reconstruction algorithm with reliable accuracy and (nearly) optimal theoretical performance is a challenging question in the CS research. So far there are about three categories of existing reconstruction algorithm; one is the famous basis pursuit with the l_1 minimization using Linear Programming (LP). It is the high computational complexity that restricts the practical applications into reality. And the convex relaxation algorithms, such as gradient projection method, have been proposed. Anther recovery algorithms based on the idea of iterative greedy pursuit are also quite popular. The Matching Pursuit (MP) and OMP are proposed early time. Their successors include the stagewise OMP (StOMP) [16] and the regularized OMP (ROMP) [17]. One notable contribution is the lower reconstruction complexity for reconstruction. However, they require more measurements for perfect reconstruction and they lack provable reconstruction quality. More recently, greedy algorithms such as the Subspace Pursuit (SP) [18] and the compressive sampling matching pursuit (CoSaMP) [19] have been proposed by incorporating the idea of backtracking. They offer comparable theoretical reconstruction quality as that of the LP methods and low reconstruction complexity. However, the common of SP and CoSAMP is that the sparsity K is known; generally K may not be available in many practical applications. And later, SAMP for blind signal recovery when K is unknown is proposed. It follows the “divide and conquer” principle through stage by stage estimation of the sparsity level and the true support set of the target signals. Both OMP and SP can be viewed as SAMP’s special cases [20].

In this paper, we propose a new greedy algorithm called modified regularized adaptive matching pursuit based on SAMP and ROMP, the specification of filtering atoms is based on regularization theory. The top-down

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methods are likely to identify the true support set more accurately, and SAMP with the most innovative feature is its capability of signal reconstruction without any prior information of the sparsity *K*. Its numerical results are even more attractive as it outperforms all of the above-mentioned algorithms in extensive simulations.

However, one popular applications of CS is in radar, LFM signal is widely used for their excellent characteristics in pulse radar to increase the range of detecting targets and get accurate resolution, which has the advantages of reducing temporal samples as well as reducing spatial samples. In the literature some papers on radar with CS can be found.

State of the art radar systems apply a large bandwidth and an increasing number of channels produce huge amount of data. Often the data handling is the most crucial matter of design. In traditional radar system, radar transmits the pulse of very short period, characterized by its very high bandwidth, to digitize a chirp signal, a very high sampling rate is required according to Shannon-Nyquist sampling theorem [21], but it is difficult to implement with a single Analog-to-Digital Converter (ADC) chip. Some parallel ADCs are developed. But it is still difficult to use the systems in practice, owing to high hardware cost (the use of multi-ADCs) and resulting unwieldy amount of sample data [22].

One approach to reduce this imbalance is that the remarkable and quite new research field is compressive sensing proposing sparse sampling for sparse scenes. Considering that LFM signal is sparse in time-frequency plane, CS technique [23], [24] is used to ease the pressure of sampling. CS provides an efficient way to sample sparse or compressible signals. The main idea of CS is that discrete-time sparse signals can be completely described and perfectly reconstructed by a number of projections over random basis. One of the main algorithms developed in this field is the conditioned minimization of l_1 norm of the vector describing the amplitude distribution of the scene under the condition that the measurements are compatible with the signal model. This theory is applicable for temporal as well as for spatial sampling. And the results of MRAMP for LFM signal recovery and echo detection are simulated.

II. Overview of Compressive Sensing

Compressive sensing seeks to represent a signal from a small number of linear measurements. Suppose $\mathbf{x}$ is an unknown *N*-dimensional signal with at most $K \ll N$ nonzero components, meaning that it has few nonzero entries

$$\mathbf{x} \in \mathbf{R}^d, \quad |\mathrm{supp}(\mathbf{x})| \le K \ll N \tag{1}$$

We call such signals *K*-sparse. According to the CS theory, such a signal can be acquired through the following linear random projections, and the linear measurements are the result of an application of the short and fat measurement matrix $\mathbf{A}$,

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{z} \tag{2}$$

In which $\mathbf{y}$ is the measurement vector with $M \ll N$ data points, $\mathbf{A}$ represents an $M \times N$ random projection matrix and $\mathbf{z}$ is the additive noise. The CS framework is attractive as it implies that $\mathbf{x}$ can be faithfully recovered from only *M* samples, suggesting the potential of significant cost reduction in digital data acquisition.

The key idea of compressive sensing is to recover a sparse signal from very few non-adaptive, linear measurements by convex optimization. Taking a different viewpoint, it concerns the exact recovery of a high-dimensional sparse vector after a dimension reduction step [25].

From a yet another standpoint, we can regard the problem as computing a sparse coefficient vector for a signal with respect to an overcomplete system. The theoretical foundation of compressive sensing has links with and also explores methodologies from various other fields such as applied harmonic analysis, frame theory, geometric functional analysis, numerical linear algebra, optimization theory, and random matrix theory [26].

However CS states that *'M'* can be far less than *'N'* provided signal is sparse (accurate reconstruction) or nearly sparse/compressible (approximate reconstruction) in original or some transform domains. Lower values for *'M'* are allowed for sensing matrices that are more incoherent within the domain (original or transform) in which signal is sparse. This explains why CS is more concerned with sensing matrices based on random functions as opposed to Dirac delta functions under conventional sensing. Although, Dirac impulses are maximally incoherent with sinusoids in all dimensions [14], however data of interest might not be sparse in sinusoids and a sparse basis (original or transform) incoherent with Dirac impulses might not exist. On the other hand, random measurements can be used for signals *K*-sparse in any basis as long as $\mathbf{A}$ obeys the following condition [17]:

$$M = K \cdot \log(N / K) \tag{3}$$

As per available literature, $\mathbf{A}$ can be a Gaussian [18], Bernoulli [19], Fourier or incoherent measurement matrix [20]. Equation (3) quantifies *'M'* with respect to incoherence between sensing matrix and sparse basis. Other important consideration for robust compressive sampling is that measurement matrix well preserves the important information pieces in signal of interest [27].

For subsequent derivations, we need two results summarized in the lemmas below.

Lemma 2.1 (Consequences of the Restricted Isometry Property (RIP))

1) (Monotonicity of δ_K) For any two integers $K \le K'$

$$\delta_K \le \delta_{K'} \tag{4}$$

(Near-orthogonality of columns) Let $I, J \subset \{1, \cdots, N\}$, be two disjoint sets. $I \cap J = \varnothing$. Suppose that $\delta_{|I|+|J|} < 1$.

For arbitrary vectors $\mathbf{a} \in \mathbf{R}^{|I|}$ and $\mathbf{b} \in \mathbf{R}^{|J|}$,

$$\left|\langle \mathbf{A}_I \mathbf{a}, \mathbf{A}_J \mathbf{b} \rangle\right| \le \delta_{|I|+|J|} \|\mathbf{a}\|_2 \|\mathbf{b}\|_2 \tag{5}$$

and

$$\|\mathbf{A}_I * \mathbf{A}_J \mathbf{b}\| \le \delta_{|I|+|J|} \|\mathbf{b}\|_2 \tag{6}$$

Both (5) and (6) represent sufficient conditions for exact reconstruction.

In order to describe the main steps of iterative greedy algorithm, we introduce next the notion of the projection of a vector and its residue.

Definition (Projection and Residue): Let $\mathbf{y} \in \mathbf{R}^m$ and $\mathbf{A}_I \in \mathbf{R}^{m \times |I|}$. Suppose that $\mathbf{A}_I^* \mathbf{A}_I$ is invertible. The projection of $\mathbf{y}$ onto span ($\mathbf{A}_I$) is defined as:

$$\mathbf{y}_p = proj(\mathbf{y}, \mathbf{A}_I) = \mathbf{A}_I \mathbf{A}_I^{\dagger} \mathbf{y} \tag{7}$$

where $\mathbf{A}_I^{\dagger} = (\mathbf{A}_I^* \mathbf{A}_I)^{-1} \mathbf{A}_I^*$ denotes the pseudo-inverse of the matrix $\mathbf{A}_I$, and $\mathbf{A}_I^*$ stands for matrix transposition.

The residue vector of the projection equals

$$\mathbf{y}_r = resid(\mathbf{y}, \mathbf{A}_I) = \mathbf{y}\text{-}\mathbf{y}_p \tag{8}$$

We find the following properties of projections and residues of vectors useful for our subsequent derivations.

Lemma 2.2 (Projection and Residue):

1) (Orthogonality of the residue) For an arbitrary vector $\mathbf{y} \in \mathbf{R}_m$, and a sampling matrix $\mathbf{A}_I \in \mathbf{R}^{m \times K}$ of full column rank, let $\mathbf{y}_r = resid(\mathbf{y}, \mathbf{\Phi}_I)$. Then

$$\mathbf{A}_I * y_r = 0 \tag{9}$$

2) (Approximation of the projection residue) Consider a matrix $\mathbf{A} \in \mathbf{R}^{m \times N}$. Let $I, J \subset \{1, \cdots, N\}$ be two disjoint sets, $I \cap J = \varnothing$, and suppose that $\delta_{|I|+|J|} < 1$. Furthermore, let $\mathbf{y} \in span(\mathbf{A}_I)$, $\mathbf{y}_p = proj(\mathbf{y}, \mathbf{A}_J)$ and $\mathbf{y}_r = resid(\mathbf{y}, \mathbf{A}_J)$. Then

$$\|\mathbf{y}_p\|_2 \le \frac{\delta_{|I|+|J|}}{1 - \delta_{\max(|I|,|J|)}} \|\mathbf{y}\|_2 \tag{10}$$

and

$$\left(1 - \frac{\delta_{|I|+|J|}}{1 - \delta_{\max(|I|,|J|)}}\right) \|\mathbf{y}\|_2 \le \|\mathbf{y}_r\|_2 \le \|\mathbf{y}\|_2 \tag{11}$$

One would like to find the sparsest vector $\mathbf{x} \in \mathbf{R}_n$ whose measurements are $\mathbf{y}$, which suggests the following optimization problem:

$$\min_{\mathbf{x}} \|\mathbf{x}\|_0 \text{ subject to } \mathbf{Ax=y} \tag{12}$$

Unfortunately, this problem is known to be NP-hard (Non-deterministic Polynomial-time hard) in general. In other words, without making further assumptions on $\mathbf{A}$ and $\mathbf{x}$, an algorithm solving this problem would be computationally intractable. For this reason, one relaxes the problem, replacing the l_0 penalty with other penalties.

III. Modified Regularized Adaptive Matching Pursuit

A. Algorithm Description

The proposed method of modified regularized adaptive matching pursuit algorithm for sparse recovery will perform more correctly than regularized adaptive matching pursuit algorithm for all measurement matrices $\mathbf{A}$ satisfying the RIP, and for all sparse or compressible signals. When we are trying to recover the signal $\mathbf{x}$ from its measurements $\mathbf{y}$=$\mathbf{Ax}$, we can use the observation vector $\boldsymbol{\theta}=\mathbf{A}^T * \mathbf{y}$ as a good local approximation to the signal $\mathbf{x}$. Namely, the observation vector $\boldsymbol{\theta}$ encodes correlations of the measurement vector $\mathbf{y}$ with the columns of $\mathbf{A}$. Note that $\mathbf{A}$ is a dictionary, and so since the signal $\mathbf{x}$ is sparse, $\mathbf{y}$ has a sparse representation with respect to the dictionary. By the Restricted Isometry Condition, every n columns form approximately an orthonormal system. Therefore, every n coordinates of the observation vector $\boldsymbol{\theta}$ look like correlations of the measurement vector $\mathbf{y}$ with the orthonormal basis and therefore are close in the Euclidean norm to the corresponding n coefficients of $\mathbf{x}$.

The local approximation property suggests to making use of the L biggest coordinates of the observation vector $\boldsymbol{\theta}$, rather than one biggest coordinate as OMP did. We thus force the selected coordinates to be more regular by selecting only the coordinates with comparable sizes. To this end, a new regularization step will be needed to ensure that each of these coordinates gets an even share of information. This leads to the following algorithm for sparse recovery.

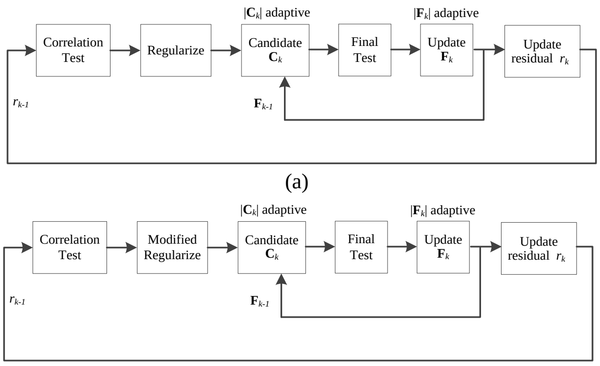


Fig. 1. Conceptual diagrams of (a) the RAMP; (b) the proposed MRAMP.

Fig. 1(a) shows the conceptual diagram of the RAMP in the k^{th} iteration. One can see that it is the combination of the ROMP algorithm and the SAMP algorithm. For the RAMP, it applies a preliminary test and a final test to build the finalist. The preliminary test is extremely similar to that of StOMP, and the final test is designed to keep the largest subset of those coordinates whose atoms' correlation differ in magnitude by at most a factor of two. This key innovation enables the RAMP to conduct blind recovery without prior information of K. And the innovation of the proposed algorithm is that we modify the process of regularization. Fig.1 (b) shows the conceptual diagram of the proposed MRAMP. It improves the accuracy of reconstruction. For simplicity, we divide the recovery process into several stages, each of which contains several iterations. $|\mathbf{F}_k|$ is kept fixed for iterations in the same stage and increased by a step size $s<K$ between two consecutive stages. Also, just as in the subspace pursuit (SP), the candidate set is chosen as $|\mathbf{C}_k| = 2|\mathbf{F}_k|$.

What's new in the proposed algorithm is that the principle of selecting group atoms is different from RAMP. In RAMP, selecting the maximal energy is the principle, and in MRAMP, we add another new screening criterion that selects the maximal mean energy to improve the performance of reconstruction.

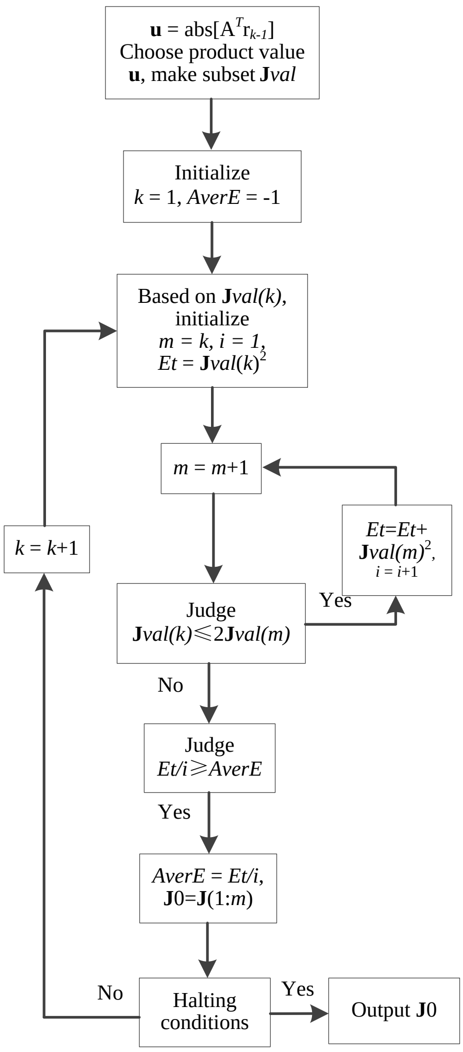


Fig. 2. Conceptual diagrams of modified regularization

B. *Procedure of Modified Regularization*

Fig. 2 shows the modified procedure of regularization based on the ROMP algorithm. Normal regularization is depicted in [17].

Let $\mathbf{u}$ be any vector in $\mathbf{R}^m$, $m>1$. Then there exists a subset $\mathbf{J}\subset\{1,\dots,m\}$ with comparable coordinates:

$$|\mathbf{u}(i)|\le 2*|\mathbf{u}(j)|\;\; i,j\in\mathbf{J} \tag{13}$$

and with big energy:

$$\|\mathbf{y}\,|_A\|_2 \ge \frac{1}{2.5\sqrt{\log m}}\|\mathbf{y}\|_2 \tag{14}$$

We will construct at most O(log$m$) subsets $\mathbf{J}_k$ with comparable coordinates and such that at least one of these sets will have large energy. It is the regularization of RAMP algorithm.

The modified RAMP algorithm is selecting maximal mean energy.

Let $\mathbf{u} = (u_1, u_2, \dots, u_m)$, and consider a partition of $\{1,\dots,m\}$ using sets with comparable coordinates:

$$\mathbf{U}_k := \{\, i : 2^{-k}\|\mathbf{u}\|_2 < |u_i| \le 2^{-k+1}\|\mathbf{u}\|_2\},\qquad k = 1,2,\dots \tag{15}$$

Let k_0 = [log m]+1, so that $|u_i| \le \frac{1}{m}\|\mathbf{u}\|_2$ for all $i\in\mathbf{J}_k$, $k>k_0$, Then the set $\mathbf{U}=\bigcup_{k\le k_0}\mathbf{J}_k$ contains most of the energy of $\mathbf{u}$:

$$\|\mathbf{u}\,|_{U^C}\|_2 \le (m(\frac{1}{m}\|\mathbf{u}\|_2)^2)^{1/2} = \frac{1}{\sqrt{m}}\|\mathbf{u}\|_2 \le \frac{1}{\sqrt{2}}\|\mathbf{u}\|_2 \tag{16}$$

Thus

$$\begin{aligned}\left(\sum_{k\le k_0}\|\mathbf{u}|\mathbf{J}_k\|_2^2\right)^{1/2} &= \|\mathbf{u}|_U\|_2\\ &= \left(\|\mathbf{u}\|_2^2 - \|\mathbf{u}|_{U^C}\|_2^2\right) \ge \frac{1}{\sqrt{2}}\|\mathbf{u}\|_2\end{aligned} \tag{17}$$

Therefore there exists $k\le k_0$ such that

$$\|\mathbf{u}\,|_{\mathbf{J}_k}\|_2 \ge \frac{1}{\sqrt{2k_0}}\|\mathbf{u}\|_2 \ge \frac{1}{2.5\sqrt{\log m}}\|\mathbf{u}\|_2 \tag{18}$$

The energy in magnitude of the selected group atoms is

$$Et = \mathbf{u}_1(j)^2 + \mathbf{u}_2(j)^2 + \cdots + \mathbf{u}_i(j)^2 \quad i<m \tag{19}$$

Sometimes selecting maximal energy group atoms will cause an error result or even when the sparsity is rising we couldn't reconstruct the original signal.

Analysis of above, we propose a new strategy for selecting the proper group, which is that select the maximal mean energy instead of the maximal energy, it can be written in formula:

$$AverE_k \ge AverE_{k+1} \tag{20}$$

here $AverE$ denotes the mean energy. It is expressed by

$$AverE = \frac{Et}{i} \tag{21}$$

here $AverE_k$ denotes the k^{th} mean value of energy, $AverE_{k+1}$ denotes next mean value of energy after the k^{th}. In the final procedure of regularization, return the sequence number of the selected group atoms in the vector $\mathbf{u}$ to subset $\mathbf{J}_0$ and take the union of $\mathbf{F}_k$ and $\mathbf{J}_0$ as candidate for next stage.

On account of their backtracking strategy, SP and ROMP of the top-down methods are likely to identify the true support set more accurately. On the other hand, the

OMP of the bottom-up approaches provided a possible solution to estimate the value of K by moving forward step by step. The SAMP with the most innovative feature is its capability of signal reconstruction without any prior information of the sparsity K. Following these observations, our MRAMP is designed to take advantages of all mentioned above: bottom-up, top-down and for blind signal recovery when K is unknown.

Algorithm I presents the procedure of the MRAMP. Here, $L=|\mathbf{F}_k|$ represents the size of finalist and for a vector $\mathbf{u}$, the function $Max(\mathbf{u},I)$ returns I indices corresponding to the largest absolute values of $\mathbf{u}$. Also, for a set $\mathbf{\Lambda} \in \{1,\ldots,N\}$, $\mathbf{\Phi}_{\Lambda}$ is the submatrix of $\mathbf{\Phi}$ with indices $i \in \mathbf{\Lambda}$. At the k^{th} iteration, $\mathbf{S}_k$,$\mathbf{C}_k$,$\mathbf{F}_k$,$\mathbf{r}_k$ stand for the short list, the candidate list, the finalist and the observation residual, respectively.

ALGORITHM I

The Proposed MRAMP Algorithm
Input: Sampling matrix $\mathbf{A}$, Sampled vector $\mathbf{y}$, step size s;
Output: A K-sparse approximation $\mathbf{\theta'}$ of the input signal;

Initialization
$\mathbf{\theta'} = \mathbf{0}$ { Trivial initialization}
$\mathbf{r}_0 = \mathbf{y}$ { Initial residue}
$\mathbf{F}_0 = \emptyset$ { Empty finalist}
$L = s$ { Size of the finalist in the first stage}
$k = 1$ { Iteration index}
$j = 1$ { Stage index}
repeat
Choose a set $\mathbf{J}$ of the L biggest coordinates in magnitude of the observation vector $u = \mathbf{A}^T * \mathbf{r}_{k-1}$, or all of its nonzero coordinates, whichever set is smaller.
Among all subsets $\mathbf{J}_0 \subset \mathbf{J}$ with comparable coordinates:
$|\mathbf{u}(i)| \le 2*|\mathbf{u}(j)|$ for all $i, j \in \mathbf{J}_0$,
$AverE_k \ge AverE_{k+1}$,
choose $\mathbf{J}_0$ with the maximal mean energy $AverE_k$
Add the set $\mathbf{J}_0$ to the index set:
$\mathbf{C}_k = \mathbf{F}_{k-1} \cup \mathbf{J}_0$ {Make Candidate List}
$\mathbf{F} = Max(|\mathbf{A}_{Ck}{}^{\dagger}\mathbf{y}|, L)$ {Final Test}
$\mathbf{r} = \mathbf{y} - \mathbf{A}_F \mathbf{A}_F{}^{\dagger} \mathbf{y}$ {Compute Residue}
if halting condition true then quit the iteration;
else if $\|\mathbf{r}_k\|_2 \ge \|\mathbf{r}_{k-1}\|_2$ then {stage switching}
$j = j + 1$ {Update the stage index}
$L = j \times s$ {Update the size of finalist}
else
$\mathbf{F}_k = \mathbf{F}$ {Update the finalist}
$\mathbf{r}_k = \mathbf{r}$ {Update the residue}
$k = k + 1$
end if
until halting condition true;
Output: The estimated signal $\mathbf{\theta'} = \mathbf{A}_F{}^{\dagger}\mathbf{y}$ { Prediction of non-zero coefficients}

The halting conditions that the residual's norm $|\mathbf{r}|_2$ is smaller than a certain threshold ε, MRAMP stops repeating, generally in which threshold ε should be set to 0. It is complex for compressible signals to stop halting. In this case, there is no known optimal way to stop the algorithm, even with convex relaxation algorithms. One common approach is to halt when a relative residue improvement between two consecutive iterations is smaller than a certain threshold. The underlying intuition is that it would not worth to take more costly iterations if the resulting improvement is too small. Based on this principle, we suggest that the MRAMP halts when the relative change of reconstructed signal's energy between two consecutive stages is smaller than a certain threshold.

And the step size s is the same as SAMP. It only requires $s \le K$. To avoid overestimation, the safest choice is certainly $s = 1$ if K is unknown. However, there is a trade-off between s and the recovery speed as smaller s requires more iterations. Also, the choice of s also depends on the magnitude distribution of the input signal. Empirical results suggest that small s is preferable for signal with exponentially decayed magnitude, while large s is advantageous for binary sparse signal. The derivation of the optimal value for s remains as an open question.

IV. Model of LFM

A. LFM Signal

Frequency modulated waveforms can be used to achieve much wider operating bandwidths. Linear Frequency Modulation is commonly used in radar system.

A typical LFM waveform can be expressed by

$$s(t) = A * Rect(\frac{t}{\tau}) e^{j2\pi\left(f_0 t + \frac{\mu}{2}t^2\right)} \tag{22}$$

in which $Rect(t/\tau)$ denotes a rectangular pulse of width τ, A is amplitude of the signal, f_0 is carrier-frequency, T_p is pulse width, $\mu = B/T_p$ is chirp rate, B is bandwidth,

$$Rect(t/T_p) = \begin{cases} 1 & -T_p/2 \le t \le T_p/2 \\ 0 & elsewise \end{cases} \tag{23}$$

The echo signal of scattering center can be expressed by:

$$\begin{aligned} s_r(t) = &\sum_{n=1}^{N} \sigma_n Rect\left(\frac{t - 2R_n/c}{T_p}\right) \\ &\times \exp\left(2\pi f_0 (t - \frac{2R_n}{c}) + \frac{1}{2}\mu (t - \frac{2R_n}{c})^2\right) \end{aligned} \tag{24}$$

where N denotes the number of the scattering points, σ_n denotes the radar cross section, c is the velocity of electromagnetic wave, R_n denotes the range from scattering point to the beginning of the signal [28].

According to CS theory, the disorganized echo signal of LFM can be decomposed under sparse basis. We are interested in constructing the sparse dictionary to complete sparse representation of the signal more effectively. For completing the detection of LFM echo signal based on CS, overcomplete atom dictionary need to be constructed first. The atom among dictionary should be constructed depends on the form of LFM signal and the purpose is we could complete sparse representation of LFM signal through atoms of dictionary. The atom dictionary $\mathbf{D}$ is a set of atoms: $\mathbf{D}=[d_1, d_2, \ldots, d_n]$. We could achieve the sparse representation of LFM signal based on the dictionary, namely: $\mathbf{D}^T\mathbf{x}=\mathbf{\theta}$. The $\mathbf{\theta}$ is sparse coefficient vector of projection which signal $\mathbf{x}$ cast on the atom database. There is only one valid sparse vector in $\boldsymbol{\theta}$ when $\mathbf{x}$ is single component signal or the same amount of valid sparse vectors as the signal components when $\mathbf{x}$ is a multicomponent signal.

*B. Construction of Dictionary **D***

The atom dictionary $\mathbf{D}$ can be constructed by the following two steps.

First step: construct the diagonal matrix, the element of matrix should satisfy:

$$\boldsymbol{\Phi}_0(m,n) = \begin{cases} s(n), & m=n \\ 0, & m \neq n \end{cases} \tag{25}$$

$$\boldsymbol{\Phi}_0 = \begin{bmatrix} s(1) & 0 & 0 & \cdots \\ 0 & s(2) & 0 & \ldots \\ \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & s(n) \end{bmatrix} \tag{26}$$

here $s(n)$ denotes the discrete sampling sequence of echo signal $s(t)$, and it can be expressed by $s(n)=\exp(j2\pi f_0 t(n)+j\pi\mu t(n)^2)$, m,n, are the indexes of cell in matrix respectively and $1\leq m,n\leq N, m,n\in N$

Second step: compute the dictionary for sparse signal in frequency domain with (Fast Fourier Transformation) FFT:

$$\boldsymbol{\Psi}_0 = \frac{1}{N}(\boldsymbol{\psi}_0, \boldsymbol{\psi}_1, \cdots, \boldsymbol{\psi}_{N\text{-}1}) \tag{27}$$

where

$$\boldsymbol{\psi}_n = (\boldsymbol{\varphi}_0^{(n)}, \boldsymbol{\varphi}_1^{(n)}, \cdots, \boldsymbol{\varphi}_{N\text{-}1}^{(n)})^T$$

$$\boldsymbol{\varphi}_m^{(n)} = \exp\left(-j2\pi nm/N\right), 1\leq m,n\leq N, m,n\in N$$

So, the dictionary is $\mathbf{D} = \boldsymbol{\Phi}_0 * \boldsymbol{\Psi}_0$

From the construction of $\boldsymbol{\Psi}_0$, it can be proved easily that $\boldsymbol{\Psi}_0\boldsymbol{\Psi}_0^H = \mathbf{E}$ is true, in which $\mathbf{E}$ is $N \times N$ identity matrix. Then, $\boldsymbol{\Psi}_0$ is invertible and $\boldsymbol{\Psi}_0^{-1} = \boldsymbol{\Psi}_0^H$ .From $\mathbf{x}=\mathbf{D}\boldsymbol{\theta}$, it can be deduced that: $x=\boldsymbol{\Phi}_0\boldsymbol{\Psi}_0\boldsymbol{\theta}$, the $\boldsymbol{\theta}$ is sparse vector.

So $\mathbf{x}$ is sparse in the matrix $\boldsymbol{\Phi}_0\boldsymbol{\Psi}_0$. This completes the proof.

The dictionary D is orthogonal

Proof: From the construction for dictionary $\boldsymbol{\Psi}_0$ by FFT, it can be proved that $\boldsymbol{\Phi}_0\boldsymbol{\Phi}_0^H = \mathbf{E}$ is true, so

$$\mathbf{D}^H\mathbf{D} = (\boldsymbol{\Phi}_0\boldsymbol{\Psi}_0)^H\boldsymbol{\Phi}_0\boldsymbol{\Psi}_0 = \boldsymbol{\Psi}_0^H(\boldsymbol{\Phi}_0^H\boldsymbol{\Phi}_0)\boldsymbol{\Psi}_0 = \boldsymbol{\Psi}_0^H\mathbf{E}\boldsymbol{\Psi}_0 = \mathbf{E}$$

Then the dictionary $\mathbf{D}$ is orthogonal. This completes the proof.

Because of the orthogonality of $\mathbf{D}$, from $\mathbf{x}=\mathbf{D}\boldsymbol{\theta}$, the calculation formula for sparse coefficient is: $\boldsymbol{\theta}=\mathbf{D}^H\mathbf{x}$. It means that it is convenient to obtain sparse coefficient and complex algorithm for sparse representation is needless. Furthermore, the orthogonality also lessens the restrict on the sensing matrix for compression when the dictionary is integrated into compressive sensing theory. As analysis mentioned above, processing compressive sensing samples based on sparse dictionary could decompose the echo signal more accurately to obtain LFM signal's sparse coefficient representation in dictionary, and further achieve better detection.

V. Simulation Results

A. Experiment 1

In this experiment, for many values of the ambient dimension N, the number of measurements M, the sparsity K, and step size s, we reconstruct random signals using MRAMP. The signals of interests are Gaussian sparse signals with length of N = 256. The partial FFT sensing operator is used with a fixed number of measurements M = 128. Our aim is to investigate the probability of exact reconstruction vs. the signal sparsity K for a given M. Different sparsitys are chosen from K= 5 to K=80 and for each K, 1000 simulations were conducted to calculate the probabilities of exact reconstruction for different algorithms. Fig.3 demonstrates the results for Gaussian sparse signals. The numerical values on x-axis denote the level of sparsity K and those on y-axis represent probability of exact recovery.

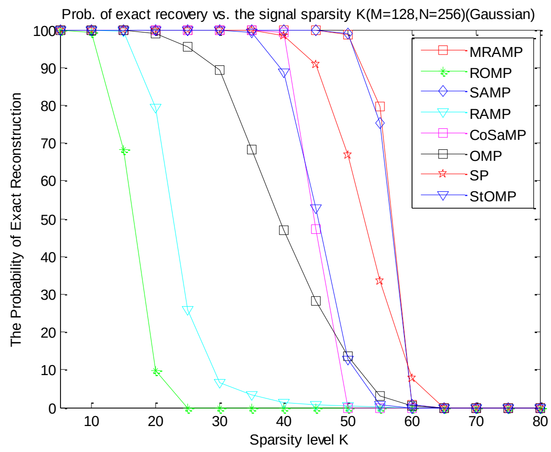


Fig. 3. Prob. of exact recovery vs. the signal sparsity K. Here, the test signal is of length N = 256 and the number of measurements is fixed as M = 128.

As can be seen, for Gaussian sparse signals, performance of the MRAMP far exceeds that of all other algorithms. While ROMP algorithm starts to fail when sparsity $K\geq 15$, and RAMP is slightly better. But MRAMP has changed a lot. MRAMP and SAMP still can afford until sparsity $K\geq 50$. When M is fixed, the probabilities of exact reconstruction of SAMP and MRAMP are quite similar with a small difference.

B. Experiment 2

This experiment investigates the probability of exact recovery vs. the number of measurements, given a fixed signal sparsity K. We use the same setups of experiment above and choose K=20, $M\in$(50, 60, 70, 80, 90, 100). For each value of M, we generate a signal $\mathbf{x}$ of sparsity K and its measurements $\mathbf{y}=\mathbf{A}\mathbf{x}$. Then we use above algorithms to recover $\mathbf{x}$. This procedure is repeated 1000 times for each value of M. We then calculate the probabilities of exact reconstruction. Fig.4 depicts these probability curves of Gaussian sparse signals. The numerical values on *x*-axis denote the number of measurements *M* and those on *y*-axis represent probability of exact recovery.

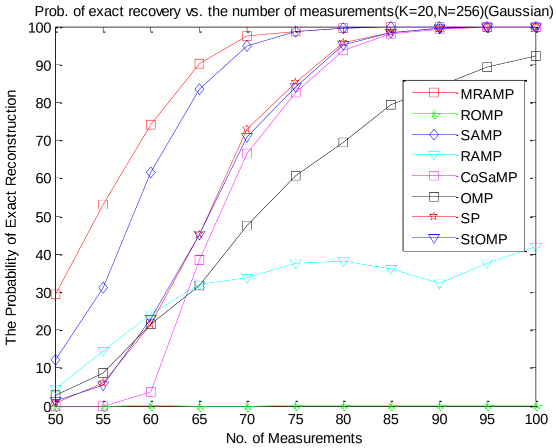


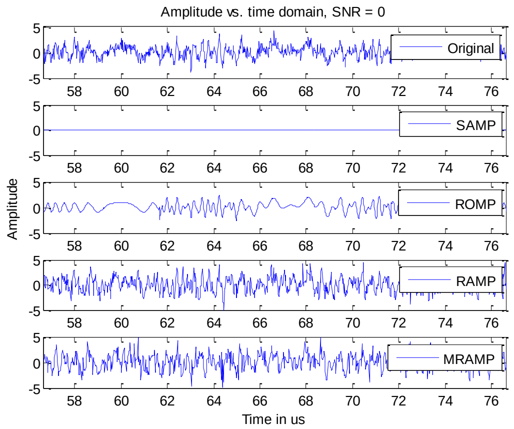
Fig. 4. The percentage of Gaussian sparse signal exactly recovered by MRAMP as a function of the number of measurements *M* in dimension *N* = 256 for various levels of sparsity *K*.

Again, we see that MRAMP is the best algorithm for recovering Gaussian sparse signals. And MRAMP will outperform better than SAMP in experiment 3. It is also interesting to observe that when the number of measurements is insufficient for guarantee of exact recovery, the probability of exact recovery of MRAMP depends on its step size and signal types. In particular, for Gaussian sparse signals, MRAMP with a smaller step size gets a higher chance of recovering signals exactly, given the same number of measurements. Although these observations could not be justified by theorems of sufficient conditions, they may be heuristically justified as follows.

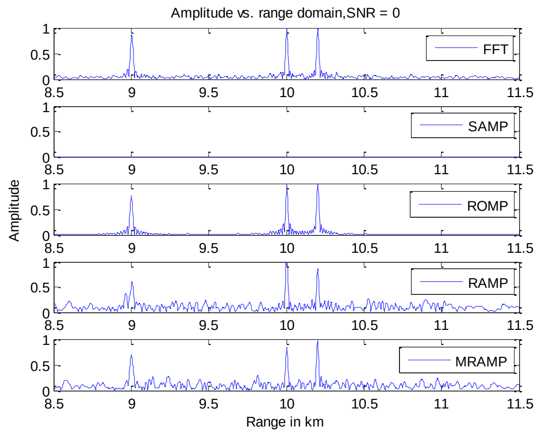
C. Experiment 3

In this experiment, the performance of MRAMP of signal reconstruction and echo detection are verified in a LFM radar system. Simulation conditions; the carrier frequency is set to 3GHz. A maximum of 1024 samples for the testing signal is considered at the receive node. Assume that this echo signal includes three targets, whose positions are at 9km, 10km and 10.2km away from the beginning of the signal. This echo signal is sparse where with respect to the waveform-matched dictionary constructed by the method. The received signal is corrupted by zero mean Gaussian noise. The Signal-to-Noise Ratio (SNR) is set to 0 dB.

From Fig. 5(a), the numerical values on *x*-axis denote the number of time in *us* and those on *y*-axis represent amplitude of the echo signals. Legend original denotes the echo signal without any processing, and successively the next four figures in Fig. 5(a) represent the recovery signal with SAMP, ROMP, RAMP and MRAMP algorithms respectively. Fig. 5(b) denotes targets detection in the range domain with algorithms above. SAMP is completely not suitable for reconstruction of LFM signal because it is not sparse strictly in frequency domain. The known sparsity *K* is the premise of ROMP and the sparsity *K* of echo is often unknown in practical applications. It is often rejected in LFM detection like OMP, SP, and CoSaMP. They are related to the sparsity. At 9km, MRAMP performs better than RAMP, and with the sparsity rising from Fig. 3, MRAMP has a good performance in reconstructing echo signals. Compared with traditional FFT, below Nyquist rate MRAMP can reconstruct original signal accurately and also save storage space. Above all the algorithms mentioned, MRAMP has the superiority for reconstructing and detecting LFM echo signals.



(a)



(b)

Fig. 5. Amplitude vs. the echo signal in time domain and range domain respectively. Here, the echo signal is of length *N* = 1024 and the number of measurements is fixed as *M* = 512. And SNR = 0.(a) time domain signal (b) range domain signal.

Furthermore, consider more practical situations that LFM signal is often corrupted by noise, and consequently the amplitude loss of reconstructed coefficients is hard to avoid. The threshold is used for echo detection in noisy environment. Although the environment noise of practical radar system is complex, in general, we describe the noise by Gaussian distribution in simulations. We choose an appropriate δ as the threshold. For noisy signal, we care about the exact position detection of target echoes.

For the case of the Gaussian distributed noise in echo signals Fig. 6 shows the detection probability for signals vs. SNR dB in this system.

From the results, we find that the tendency of detection probability varying with parameter SNR is similar for Gaussian distributed noise model.

Sparsity is chosen from SNR = -10 to K = 25 and for each K, 1000 simulations were conducted to calculate the probabilities of exact reconstruction. As can be seen, while MRAMP starts to fail when SNR≤5, and when SNR≤0, the probabilities steep drop. Compared with RAMP except ROMP, MRAMP has good performance in LFM detection with low SNR for practical applications.

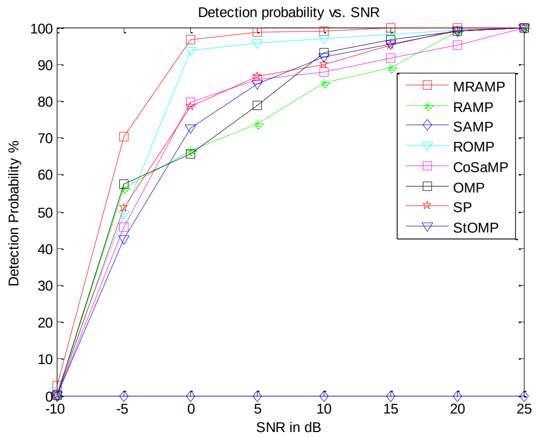


Fig. 6. Probability of echo detection of noisy signal

VI. Conclusions

In this paper, a novel iterative greedy algorithm, called modified regularization adaptive matching pursuit, is proposed and analyzed for reconstruction applications in compressive sensing. This reconstruction algorithm is most featured of not requiring information of sparsity of target signals as a prior and bottom-up, top-down approaches are also distinctive. It not only releases a common limitation of existing greedy pursuit algorithms but also keeps performance comparable with that of strongest algorithms such as ROMP, SAMP. As a result, we sampled and detected LFM echo signals which are sparse in a pre-construction matched dictionary in the framework of CS theory. This study indicates that the low-dimensional random measurement method based on the CS theory can be used to sample ultra wideband signal. The simulation results showed that the noise-free ultra wideband LFM echo signal can be exactly reconstructed and detected even in low SNR. For noisy signal, increasing the amount of samples still can guarantee overwhelming detection probability. This study suggests that in some special applications, such as radar system, construction of a basis or a dictionary to obtain very sparse representation of signal is possible. The radar signal can be sampled at a rate much lower than Nyquist rate, but still can be reconstructed with high probability.

The CS theory has remarkable advantages of reducing sampling rate and computation, which is believed to resolve the difficult that traditional methods facing in radar applications. The new MRAMP method based on CS in this paper is significant in theory and in practice. The article is still limited in theoretical analysis and simulating experiments, realizing it in engineering field should be studied further.

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References

[1] E. J. Candes, J. Romberg, and T. Tao, “Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information,” *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 489–509, Feb. 2006.

[2] D. L. Donoho, “Compressed sensing,” *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1289–1306, Apr. 2006.

[3] A. M. Bruckstein, D. L. Donoho, and M. Elad, “From sparse solutions of systems of equations to sparse modeling of signals and images,” *SIAM Rev.*, vol. 51, no. 1, pp. 34–81, 2009.

[4] T. T. Cai and A. Zhang, “Sharp RIP bound for sparse signal and low-rank matrix recovery,” *Appl. Comput. Harmon. Anal.*, vol. 35, no. 1, pp. 74–93, Jul. 2013.

[5] M. E. Davies and R. Gribonval, “Restricted isometry constants where l_p sparse recovery can fail for 0<p≤1,” *IEEE Trans. Inf. Theory*, vol. 55, no. 5, pp. 2203–2214, May 2009.

[6] D. L. Donoho and M. Elad, “Optimally sparse representation in general (nonorthogonal) dictionaries via l_1 minimization,” *Proc. Natl. Acad. Sci.*, vol. 100, no. 5, pp. 2197–2202, 2003.

[7] M. Elad and A. M. Bruckstein, “A generalized uncertainty principle and sparse representation in pairs of bases,” *IEEE Trans. Inf. Theory*, vol. 48, no. 9, pp. 2558–2567, Sep. 2002.

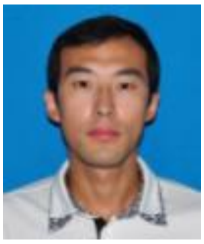
[8] S. Foucart, “A note on guaranteed sparse recovery via l_1-minimization,” *Appl. Comput. Harmon. Anal.*, vol. 29, no. 1, pp. 97–103, Jul. 2010.

[9] “Sparse recovery algorithms: Sufficient conditions in terms of restricted isometry constants,” in *Approximation Theory XIII: San Antonio 2010, ser. Springer Proceedings in Mathematics*, M. Neamtu and L. Schumaker, Eds., New York, NY: Springer, 2012, vol. 13, pp. 65–77.

[10] R. Gribonval and M. Nielsen, “Sparse representations in unions of bases,” *IEEE Trans. Inf. Theory*, vol. 49, no. 12, pp. 3320–3325, Dec. 2003.

[11] R. Gribonval, M. Nielsen, R. Gribonval, *et al.*, “Highly sparse representations from dictionaries are unique and independent of the sparseness measure,” *Appl. Comput. Harmon. Anal.*, vol. 22, no. 3, pp. 335–355, May 2007.

[12] K. Gedalyahu and Y. C. Eldar, "Time-Delay estimation from lowrate samples: A union of subspaces approach," *IEEE Trans. Signal Processing*, vol. 58, no. 6, pp. 3017–3031, Jun. 2010.
[13] R. Wu and D. R. Chen, "The improved bounds of restricted isometry constant for recovery via l_p-minimization," *IEEE Trans. Inf. Theory*, vol. 59, no. 9, pp. 6142–6147, Sep. 2013.
[14] A. Aldroubi, X. Chen, and A. Powell, "Stability and robustness of l_q minimization using null space property," in *Proc. 10th Int. Conf. Sampl. Theory Appl*., Singapore, May 2011.
[15] X. Chen, F. Xu, and Y. Ye, "Lower bound theory of nonzero entries in solutions of l_2-l_p minimization," *SIAM J. Sci. Comput*., vol. 32, no. 5, pp. 2832–2852, 2010.
[16] D. L. Donoho, Y. Tsaig, I. Drori, and J. L. Starck, "Sparse solution of underdetermined linear equations by stagewise orthogonal matching pursuit," *IEEE Transactions on Information Theory*, vol. 58, no. 2, pp. 1094-1121, 2012.
[17] D. Needell and D. Vershynin, "Uniform uncertainty principle and signal recovery via regularized orthogonal matching pursuit," *Foundations of Computational Mathematics*, vol. 9, no. 3, pp. 317-334, 2009.
[18] W. Dai and O. Milenkovic, "Subspace pursuit for compressive sensing signa J reconstruction," in *Proc. 5th International Symposium on Turbo Codes and Related Topics*, Lausanne, Switzerland, 2008, pp. 402-407.
[19] D. Needell and J. A. Tropp "CoSaMP: Iterative signal recovery from incomplete and inaccurate samples," ACM Technical Report 2008-01, California Institute of Technology, Pasadena, July 2008.
[20] T. T. Do, G. Lu, Nguyen, and Tran D. "Sparsity adaptive matching pursuit algorithm for practical compressed sensing," in *Proc. Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, California, 2008, vol. 10, pp. 581-587.
[21] D. Lao, M. W. Lenox, and G. Akabani, "The sparsity-promoted solution to the undersampling tof-pet imaging: Numerical simulations," *Progress in Electromagnetics Research*, vol. 133, 235–258, 2013.
[22] Z. Liu, X. Wei, and X. Li, "Adaptive clutter suppression for airborne random pulse repetition interval radar based on compressed sensing," *Progress in Electromagnetics Research*, vol. 128, pp. 291–311, 2012.
[23] S. J. Wei, X. L. Zhang, J. Shi, and G. Xiang, "Sparse reconstruction for SAR imaging based on compressed sensing," *Progress in Electromagnetics Research*, vol. 109, 63–81, 2010.
[24] R. Chartrand, "Fast algorithms for nonconvex compressive sensing: MRI reconstruction from very few data," in *Proc. IEEE Int. Symp. Biomed. Imaging*, Boston, Jun. 2009.
[25] M. Rossi, A. M. Haimovich, and Y. C. Eldar, "Spatial compressive sensing for MIMO radar," *IEEE Transactions on Signal Processing*, vol. 62, no. 2, pp. 419-430, 2014.
[26] Y. Gu, N. A. Goodman, and A. Ashok, "Radar target profiling and recognition based on TSI-Optimized compressive sensing kernel," *IEEE Transactions on Signal Processing*, vol. 62, no. 12, pp. 3194 -3207, 2014.
[27] S. K. Mishra, R. Gupta, A. Vaidya, and J. Mukherjee, "Printed fork shaped dual band monopole antenna for bluetooth and UWB applications with 5.5GHz WLAN band notched characteristics," *Progress in Electromagnetics Research C*., vol. 22, pp. 195-210, 2011.
[28] M. Zhang, Y. W. Zhao, H. Chen, and W. Q. Jiang, "SAR image simulation for composite model of ship on dynamic ocean scene," *Progress in Electromagnetics Research*, vol. 113, pp. 395–412, 2011.



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