Probabilistic Constraint Robust Transceiver Design for MIMO Interference Channel Networks

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Abstract -In this paper, we investigate the robust transceiver design for Multi-Input Multi-Output (MIMO) Interference Channel (IC) networks with imperfect Channel State Information (CSI). With the assumption of Gaussian CSI uncertainty, a probabilistic constraint robust transceiver design problem is formulated by maximizing the average received signal while constraining the probability of large interference plus noise, both in downlink and uplink. To solve the formulated design problem, the probabilistic constraints are first transformed as Linear Matrix Inequalities (LMIs) using Markov's inequality, and a semidefinite relaxation (SDR) technique is then applied to further recast the design problem as convex semidefinite programming (SDP) problem, which can be solved efficiently. An iterative algorithm based on alternative optimizing is proposed for the probabilistic constraint robust design. Simulation results verify that the proposed probabilistic constraint based robust transceiver design can provide robustness against Gaussian CSI errors.

Index Terms—MIMO, interference channel, imperfect CSI, probabilistic constraint, robust transceiver design, Semidefinite Programming (SDP).

I. INTRODUCTION

To accommodate the dramatically increasing demand of high data rate services, higher frequency reuse factor and ultra-dense cell coverage become the candidate solutions for the future wireless communication systems [1]. Consequently, the future wireless communication networks are typically interference limited, wherein users suffer from in-neglected co-channel interference originating from nearby cells, which severely degenerates system performance. On the other hand, Multi-Input Multi-Output (MIMO) techniques have gained a considerable amount of interest since it possesses attractive potential to improve spectral efficiency and communication reliability. Therefore, typical future wireless communication networks can be modeled as MIMO Interference Channel (IC) networks, where multiple transceiver pairs share the same frequency spectrum and the transmitted signal of a user pair constitutes interference to receivers of other user pairs. How to break the limit of interference has been an important task for MIMO IC networks.

Recent information theory advances reveal that the interference in MIMO IC networks is possible suppressed completely if the transceivers are designed cooperatively based on the idea of interference alignment under global Channel Sate Information (CSI) [2], [3]. Following these work, many efforts have been made to develop transceiver designs for MIMO IC network conditioned on accuracy CSI [4]-[12]. Among them, the representative design schemes are the alternative signal-to-interferenceand-noise ratio maximization (Max-SINR) algorithm [4] and the Minimum Mean Squared Error (MMSE) algorithm [5]. However, the CSI is inevitably imperfect due to finite-energy training and limited feedback in practical systems. Theoretical analysis in [13]-[19] indicates that transceiver design schemes neglecting the impact caused by CSI error will result in severe degradation of achievable data rate. Therefore, robust transceiver designing for MIMO IC networks that take the CSI imperfection into consideration is of great importance in practice.

Related robust transceiver design techniques for MIMO IC networks have been proposed in recent literature [20]-[24], by optimizing different metrics such as Signal-to-Interference-Plus-Noise ratio (SINR), meansquared-error (MSE), weighted sum rate and interference leakage, etc. In [20], a robust transceiver design scheme was proposed by maximizing the worst-case per-stream SINR among all the data streams in the network. MSEbased robust designs were proposed in [21] by optimizing the worst-case sum MSE or per-user MSE with respect to channel errors. The work of [22] proposed a MSE minimization robust transceiver design scheme dedicated for MIMO IC networks with limited feedback links. In [23] and [24], the weighted sum rate and interference leakage are respectively used as the optimization criteria to develop robust designs. All the above mentioned robust designs were developed for bounded CSI errors. For the case of unbounded CSI errors, several robust design schemes have been proposed in [5] and [25] assuming Gaussian distributed CSI errors. All these schemes focusing on optimizing MSE criterion, while robust designs by optimizing other criteria for unbounded CSI errors are rarely available in literature so far.

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Although these traditional robust designs can provide performance improvement compared with the non-robust design schemes, they may gain conservative performance. That is because the worst-case condition or the statistics of errors are usually considered in order to provide robustness to CSI errors, but the worst-case operation condition may rarely emerge and the statistics cannot well reflect the influence of extreme errors [26]. Recently, probabilistic constraint based robust transceiver design schemes have been investigated in [26]-[29] to achieve further performance improvement compared to traditional designs. The work in [27] and [28] proposed robust beamforming designs for Multiple-Input Single-Output (MISO) Broadcasting Channel (BC) networks by considering probabilistic SINR constraints. Reference [29] probabilistic constraints proposed based robust beamforming design for point-to-point MIMO systems with Maximum Ratio Combining (MRC) receiver. The work of [26] generalized the work of [29] into MIMO Broadcasting Channel (BC) networks. However, none of them was designed for MIMO IC networks.

In this paper, we investigate robust transceiver design schemes for MIMO IC networks undergoing Gaussian channel errors. We propose a probabilistic constraint based robust design by maximizing the average received signal while keeping low probability for large interference plus noise with imperfect CSI in both the downlink and uplink. With the help of Markov's inequality, the probabilistic constraints are transformed into Linear Matrix Inequality (LMI), and the transceiver design problems are recast as semidefinite programming (SDP) problems with rank constraints. By applying semidefinite relaxation (SDR), the non-convex rank constraint SDP problems are further relaxed as convex SDP and thus can be solved efficiently. A robust transceiver design algorithm is then proposed to alternatively optimize the transmitters and receivers. Numerical simulations show the effectiveness of the proposed probabilistic constraint robust transceiver design scheme.

The remaining sections are organized as follows. Section II discusses the system model and the channel error model. The probabilistic constraint based robust design is proposed in Section III. Simulation results are presented in Section IV. Finally, Section V concludes the paper.

Notations: C represent the complex field. Bold uppercase and lowercase letters represent matrix and column vectors, respectively. Non-bold italic letters represent scalar values. \mathbf{I}_N is an $N \times N$ identity matrix.

 \mathbf{A}^{H} , \mathbf{A}^{T} and \mathbf{A}^{-1} represent the Hermitian transpose, transpose and inverse of \mathbf{A} , respectively. $tr(\mathbf{A})$ and $rank(\mathbf{A})$ are the trace and rank of matrix \mathbf{A} , respectively. $\mathbb{E}[\cdot]$ denotes the statistical expectation. $\|\|_{2}$ denote the 2-norm. \Pr{A} denotes the probability of the event A.



Fig. 1. MIMO interference channel network. The solid arrow headed lines are signal links, and the dotted arrow headed lines are interference links.

II. SYSTEM MODEL

Consider a MIMO IC network consisting of K users as shown in Fig. 1, where each user consists of one pair of transmitter and receiver both with multiple antennas. Assume the number of transmit and receive antennas of the k th user are M_k and N_k , respectively. The channel propagation matrix from transmitter j to receiver k is denoted by $\mathbf{H}_{ki} \in \mathcal{C}^{N_k \times M_j}$, $\forall i, j \in \{1, ..., K\}$. We assume the elements of \mathbf{H}_{ki} are independent identically distributed (i.i.d.) Zero-Mean Circularly Symmetrical Complex Gaussian (ZMCSCG) random variables. We also assume a block fading model in which the channels \mathbf{H}_{ki} remain unchanged for the duration of a transmission but may change randomly between successive transmissions. The channel \mathbf{H}_{kk} describes the desired direct link between the k th user pair, and the channel $\mathbf{H}_{ki}, \forall j \neq k$ constitutes interference link from the *j* th transmitter to the k th receiver.

We assume the system operates in Time-Division-Duplex (TDD) mode, such that the reciprocity of the wireless propagation channel holds. Under such assumption, the transceivers can obtain the CSI of downlink through backward training. It is also assumed that the transmitters can exchange CSI between each other, such that the global CSI is available for all transmitters.

We assume the transmitter of user k sends d_k data streams to its paired receiver per channel use. We denote the data vector of transmitter k as $\mathbf{s}_k \in C^{d_k \times 1}$ with $\mathbb{E}[\mathbf{s}_k \mathbf{s}_k^{\mathrm{H}}] = \frac{P_k}{d_k} \mathbf{I}_{d_k}$, where P_k is the transmit power of user k. At the transmit sides, before being sent out, the data

streams are precoded by transmit precoding matrix \mathbf{V}_k , while at the receive sides, the received signals are processed by the interference suppressing matrix \mathbf{U}_k .

At the receive side, the received signals are processed accordingly by the designed interference suppressing matrices. The recovered signal vector at receiver k can be written as

$$\hat{\mathbf{s}}_{k} = \mathbf{U}_{k}^{\mathrm{H}} \mathbf{H}_{kk} \mathbf{V}_{k} \mathbf{s}_{k} + \sum_{j=1, j \neq k}^{K} \mathbf{U}_{k}^{\mathrm{H}} \mathbf{H}_{kj} \mathbf{V}_{j} \mathbf{s}_{j} + \mathbf{U}_{k}^{\mathrm{H}} \mathbf{n}_{k}$$
(1)

where $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$ denotes the noise vector at the *k* th receiver. The *l* th element of the recovered signal vector (1) can be further expressed as

$$\hat{s}_{kl} = \mathbf{u}_{kl}^{\mathrm{H}} \mathbf{H}_{kk} \mathbf{v}_{kl} s_{kl} + \sum_{\substack{m=1, m\neq l \\ \mathrm{Intra-user stream interference}}}^{d} \mathbf{u}_{kl}^{\mathrm{H}} \mathbf{H}_{kk} \mathbf{v}_{km} s_{km} + \sum_{j=1, j\neq k}^{K} \mathbf{u}_{kl}^{\mathrm{H}} \mathbf{H}_{kj} \mathbf{V}_{j} \mathbf{s}_{j} + \mathbf{u}_{kl}^{\mathrm{H}} \mathbf{n}_{k}$$

$$(2)$$

where s_{kl} denotes the *l* th element of \mathbf{s}_k , \mathbf{u}_{kl} and \mathbf{v}_{kl} denote the *l* th column vector of \mathbf{U}_k and \mathbf{V}_k , respectively. In (2), the first term on the right hand side is the desired signal for the *l* th data stream of the *k* th user, the second term represents the intra-user interference, the third term quantifies the inter-user interference and the last term represents the noise.

From (2), the downlink SINR of the l th data stream of the k th user is defined as

$$\operatorname{SINR}_{kl} = \frac{\mathbf{u}_{kl}^{\mathrm{H}} \mathbf{A}_{kl} \mathbf{u}_{kl}}{\mathbf{u}_{kl}^{\mathrm{H}} \mathbf{B}_{kl} \mathbf{u}_{kl}}$$
(3)

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where

$$\mathbf{A}_{kl} = \mathbf{H}_{kk} \mathbf{v}_{kl} \mathbf{v}_{kl} \mathbf{H}_{kk} ,$$
$$\mathbf{B}_{kl} = \sum_{j=1}^{K} \mathbf{H}_{kj} \mathbf{V}_{j} \mathbf{V}_{j}^{\mathrm{H}} \mathbf{H}_{kj}^{\mathrm{H}} - \mathbf{H}_{kk} \mathbf{v}_{kl} \mathbf{v}_{kl}^{\mathrm{H}} \mathbf{H}_{kk}^{\mathrm{H}} + \frac{1}{\rho_{k}} \mathbf{I}_{n}$$

and $\rho_k = \frac{P_k}{d_k \sigma^2}$ is defined as the per-stream SNR. The

SINR expression in (3) is a generalized Rayleigh quotient with respect to \mathbf{u}_{kl} .

In order to design transceivers, we utilize the reciprocity of interference channel [4]. The SINR along the reciprocal link of the data transmission direction can be constructed as

$$\overleftarrow{\text{SINR}}_{kl} = \frac{\mathbf{v}_{kl}^{\text{H}} \overline{\mathbf{A}}_{kl} \mathbf{v}_{kl}}{\mathbf{v}_{kl}^{\text{H}} \overline{\mathbf{B}}_{kl} \mathbf{v}_{kl}}$$
(4)

where

$$\mathbf{A}_{kl} = \mathbf{H}_{kk}^{\mathsf{H}} \mathbf{u}_{kl} \mathbf{u}_{kl}^{\mathsf{H}} \mathbf{H}_{kk},$$

$$\mathbf{\widetilde{B}}_{kl} = \sum_{j=1}^{K} \mathbf{H}_{jk}^{\mathsf{H}} \mathbf{U}_{j} \mathbf{U}_{j}^{\mathsf{H}} \mathbf{H}_{jk} - \mathbf{H}_{kk}^{\mathsf{H}} \mathbf{u}_{kl} \mathbf{u}_{kl}^{\mathsf{H}} \mathbf{H}_{kk} + \frac{1}{\hat{\rho}_{k}} \mathbf{I}_{n_{j}}$$

and $\bar{\rho} = \frac{P_k}{d_k \sigma^2}$ is defined as the per-stream SNR in the

reversed link. P is the transmit power of the reversed transmitter.

If perfect CSI is given, fixing the transmit precoders and according to the property of Rayleigh quotient, the optimal receiver to maximize SINR is given by

$$\mathbf{u}_{kl}^{\star} = \frac{\mathbf{B}_{kl}^{-1} \mathbf{H}_{kl} \mathbf{v}_{kl}}{\|\mathbf{B}_{kl}^{-1} \mathbf{H}_{kl} \mathbf{v}_{kl}\|_2}$$
(5)

Similarly, given perfect CSI and with fixed receivers, the optimal precoder is given by

$$\mathbf{v}_{kl}^{\star} = \frac{\mathbf{\overline{B}}_{kl}^{-1} \mathbf{H}_{kl}^{\mathrm{H}} \mathbf{u}_{kl}}{\|\mathbf{\overline{B}}_{kl}^{-1} \mathbf{H}_{kl}^{\mathrm{H}} \mathbf{u}_{kl}\|_{2}}$$
(6)

The classical Max-SINR algorithm proposed in [4] is then constructed by alternatively optimizing the transmit precoders according to (6) and the receive filters according to (5).

In practical communication systems, obtaining perfect CSI is always a demanding work due to either the feedback error or the channel estimation error. Our goal is to develop the robust counterparts of the transceiver design for the MIMO IC network. To model the channel uncertainty, the real CSI, \mathbf{H}_{kj} , can be expressed as a sum of the imperfect CSI and the channel uncertainty

$$\mathbf{H}_{ki} = \hat{\mathbf{H}}_{ki} + \boldsymbol{\Delta}_{ki} \tag{7}$$

where $\hat{\mathbf{H}}_{kj}$ and $\boldsymbol{\Delta}_{kj}$ represent the imperfect CSI and channel uncertainty, respectively. Specifically, we assume the elements of $\boldsymbol{\Delta}_{kj}$ are i.i.d. complex Gaussian random variables with zero mean and variance $\sigma_{\mathbf{A}}^2$.

Assume the imperfect CSI is available both at the transmitters and receivers, a straightforward way to design the transceivers is to apply the standard algorithms, e.g., the Max-SINR algorithm, with the obtained imperfect CSI. However, the system performance will definitely be degenerated if the impact of channel error is ignored in the transceiver design. To explicitly show the impact of channel error on the quality and also to facilitate transceiver optimizing, the receive SINR associated with channel error is rewritten as

$$\operatorname{SINR}_{kl} = \frac{\|\mathbf{u}_{kl}^{\mathrm{H}}(\hat{\mathbf{H}}_{kk} + \boldsymbol{\Delta}_{kk})\mathbf{v}_{kl}\|^{2}}{Z_{kl}}$$
(8)

where Z_{kl} is the received power of interference plus noise of the *l* th data stream of the *k* th user, which is written as

$$Z_{kl} = \sum_{j=1}^{k} \| \mathbf{V}_{j}^{\mathrm{H}} (\hat{\mathbf{H}}_{kj} + \mathbf{\Delta}_{kj})^{\mathrm{H}} \mathbf{u}_{kl} \|^{2}$$
$$- \| \mathbf{u}_{kl}^{\mathrm{H}} (\hat{\mathbf{H}}_{kk} + \mathbf{\Delta}_{kk}) \mathbf{v}_{kl} \|^{2} + \frac{1}{\rho_{k}} \| \mathbf{u}_{kl} \|^{2}$$
(9)

Accordingly, the SINR along the reciprocal link is denoted by

$$\overline{\mathrm{SINR}}_{kl} = \frac{\|\mathbf{u}_{kl}^{\mathrm{H}}(\hat{\mathbf{H}}_{kk} + \boldsymbol{\Delta}_{kk})\mathbf{v}_{kl}\|^{2}}{\bar{Z}_{kl}}$$
(10)

where the interference leakage from the l th data stream of the k th user is defined as

$$\begin{split} \bar{Z}_{kl} &= \sum_{j=1}^{K} \|\mathbf{U}_{j}^{\mathrm{H}}(\hat{\mathbf{H}}_{jk} + \boldsymbol{\Delta}_{jk}) \mathbf{v}_{kl}\|^{2} \\ &- \|\mathbf{u}_{kl}^{\mathrm{H}}(\hat{\mathbf{H}}_{kk} + \boldsymbol{\Delta}_{kk}) \mathbf{v}_{kl}\|^{2} + \frac{1}{\bar{\rho}_{k}} \|\mathbf{v}_{kl}\|^{2}. \end{split}$$
(11)

It is noted that a straightforward robust counterpart of the traditional Max-SINR algorithm can be derived by optimizing the average SINR with respect to the channel error. Specifically, under the channel error model (7), a method to provide robust in designing the transceivers is to maximize the average SINR with respect to channel and $\max_{\mathbf{x}_{l}} \mathbb{E}[\overline{\mathbf{SINR}}_{kl}]$. errors, i.e, $\max_{\mathbf{u}_{ll}} \mathbb{E}[\text{SINR}_{kl}]$ With the help of Jensen's inequality $\mathbb{E}[f(x)] \ge f[\mathbb{E}(x)]$ and following the method proposed in [20], the lower bounds of the average SINRs can be obtained. By further using the property of Rayleigh-Ritz quotient, the transceivers to maximize the lower bounds of average SINRs can be obtained. By alternatively optimizing the transmitters and receivers, an average SINR maximization (Max-ASINR) algorithm is then obtained. However, the performance of this Max-ASINR algorithm is conservative, because the impact of extreme errors is not well reflected in the average SINR. With the aim to further improve robust performance, in the following section, we propose an probabilistic constraint based robust design by maximizing the average signal power while keeping the probability of the worst interference low.

III. ROBUST TRANSCEIVER DESIGN BASED ON PROBABILITY CONSTRAINTS APPROACH

To optimize the SINR (8) or (10) directly is difficult, since there are no closed-form expressions for these SINRs. Instead of optimizing the SINR directly, we introduce a probabilistic constraint approach into the transceiver design in this section. Specifically, we develop a probabilistic approach to design robust transceivers for MIMO IC networks by maximizing the average received signal while keeping the probabilistic of serious interference plus noise low. The formulated transceiver design problems are non-convex, and a SDR technique is used to relax the non-convex problems as convex SDP. The optimality of the SDR problems is investigated, and it is proved that the solutions for the related SDP problems are also the optimal solutions for the original problems.

A. Receiver Design

In order to design the receivers, the basic idea of the probabilistic method is to maximize the average signal power of a user while keeping the probability of large receive interference power plus noise low. The optimization problem is then mathematically expressed as

$$\max_{\mathbf{u}_{kl}} \qquad \mathbb{E}[\| \mathbf{u}_{kl}^{\mathrm{H}}(\hat{\mathbf{H}}_{kk} + \boldsymbol{\Delta}_{kk}) \mathbf{v}_{kl} \|^{2}]$$

s.t.:
$$\Pr\{Z_{kl} \ge \gamma_{kl}\} \le p_{kl}, \qquad (12)$$
$$\| \mathbf{u}_{kl} \|^{2} \le 1,$$

where γ_{kl} is a pre-specified threshold for the power of interference plus noise and $0 \le p_{kl} \le 1$ is a given probability.

The objective in (12), i.e., the average signal power of the l th data steam for the k th user, is derived as

$$\mathbb{E}[\|\mathbf{u}_{kl}^{\mathrm{H}}(\hat{\mathbf{H}}_{kk} + \boldsymbol{\Delta}_{kk})\mathbf{v}_{kl}\|^{2}]$$

$$= \mathbf{u}_{kl}^{\mathrm{H}}\left(\hat{\mathbf{H}}_{kk}\mathbf{v}_{kl}\mathbf{v}_{kl}^{\mathrm{H}}\hat{\mathbf{H}}_{kk}^{\mathrm{H}} + \sigma_{\boldsymbol{\Delta}}^{2}\|\mathbf{v}_{kl}\|^{2}\mathbf{I}_{N_{k}}\right)\mathbf{u}_{kl} \qquad (13)$$

$$= tr\left[\left(\hat{\mathbf{H}}_{kk}\mathbf{v}_{kl}\mathbf{v}_{kl}^{\mathrm{H}}\hat{\mathbf{H}}_{kk}^{\mathrm{H}} + \sigma_{\boldsymbol{\Delta}}^{2}\|\mathbf{v}_{kl}\|^{2}\mathbf{I}_{N_{k}}\right)\mathbf{X}_{kl}\right]$$

where $\mathbf{X}_{kl} = \mathbf{u}_{kl} \mathbf{u}_{kl}^{\mathrm{H}}$, $rank(\mathbf{X}_{kl}) = 1$ and $tr(\mathbf{X}_{kl}) \leq 1$.

The probabilistic constraint in (12) is introduced to guarantee there is a low probability for the power of interference plus noise higher than a threshold. However, the probabilistic constraint has no closed-form expression, which poses challenge to solve the problem. As an alternative, we relax the probabilistic constraint to a deterministic constraint with the help of Markov's inequality, which says $\Pr\{X \ge \alpha\} \le \frac{\mathbb{E}[X]}{\alpha}$, if X is a nonnegative variable and $\alpha > 0$ [26]. The relaxed problem of (12) is then formulated as the following semidefinite programming (SDP) problem with rank constraint

$$\max_{\mathbf{X}_{kl}} tr\left[\left(\hat{\mathbf{H}}_{kk} \mathbf{v}_{kl} \mathbf{v}_{kl}^{\mathrm{H}} \hat{\mathbf{H}}_{kk}^{\mathrm{H}} + \sigma_{\mathbf{\Delta}}^{2} \| \mathbf{v}_{kl} \|^{2} \mathbf{I}_{N_{k}} \right) \mathbf{X}_{kl} \right]$$

s.t.: $\mathbb{E}[Z_{kl}] \leq \gamma_{kl} p_{kl}, \quad \mathbf{X}_{kl} \succeq \mathbf{0}$ (14)
 $rank(\mathbf{X}_{kl}) = 1, \quad tr(\mathbf{X}_{kl}) \leq 1$

where

$$\mathbb{E}[Z_{kl}] = tr\left\{\sum_{j=1}^{K} \left[\hat{\mathbf{H}}_{kj}\mathbf{V}_{j}\mathbf{V}_{j}^{H}\hat{\mathbf{H}}_{kj}^{H} + \sigma_{\Delta}^{2}tr(\mathbf{V}_{j}\mathbf{V}_{j}^{H})\mathbf{I}_{N_{k}} - \left(\hat{\mathbf{H}}_{kk}\mathbf{v}_{kl}\mathbf{v}_{kl}^{H}\hat{\mathbf{H}}_{kk}^{H} + \sigma_{\Delta}^{2}||\mathbf{v}_{kl}||^{2}\mathbf{I}_{N_{k}}\right) + \frac{1}{\rho_{k}}\mathbf{I}_{N_{k}}\left]\mathbf{X}_{kl}\right\}.$$
(15)

The problem (14) is not convex since there exists rank constraint. By dropping the rank constraint, we further relax problem (14) as the following SDP problem

$$\max_{\mathbf{X}_{kl}} tr \Big[\Big(\hat{\mathbf{H}}_{kk} \mathbf{v}_{kl} \mathbf{v}_{kl}^{\mathrm{H}} \hat{\mathbf{H}}_{kk}^{\mathrm{H}} + \sigma_{\Delta}^{2} \| \mathbf{v}_{kl} \|^{2} \mathbf{I}_{N_{k}} \Big) \mathbf{X}_{kl} \Big]$$

s.t.: $\mathbb{E}[Z_{kl}] \leq \gamma_{kl} p_{kl},$ (16)
 $\mathbf{X}_{kl} \succeq \mathbf{0},$
 $tr(\mathbf{X}_{kl}) \leq 1.$

Problem (16) is known as a semidefinite relaxation (SDR) of problem (14) [30], which can be solved by standard convex optimization tools such as CVX [31]. Note that the optimized receive vector \mathbf{u}_{kl} can be

recovered from the principal eigenvector of the optimal solution \mathbf{X}_{kl}^* .

Since the rank constraint in the original noncovex problem (14) is relaxed to formulate the convex problem (16), the rank of the solution of (16) could be not limited to one. If $rank(\mathbf{X}_{kl}^*) = 1$, the rank constraint is intrinsically satisfied, and \mathbf{X}_{kl}^* is also optimal for (14). Our simulations show that the rank of the optimal solution to the SDR problem could always be one.

B. Transmitter Design

In order to design the transmitters, we optimize the average signal power while keeping the probability of large leakage power low for the reversed link by the following problem

$$\max_{\mathbf{v}_{kl}} \quad \mathbb{E}[\| \mathbf{u}_{kl}^{\mathrm{H}}(\hat{\mathbf{H}}_{kk} + \mathbf{\Delta}_{kk})\mathbf{v}_{kl}\|^{2}]$$

s.t.:
$$\Pr\{\tilde{Z}_{kl} \ge \gamma_{kl}\} \le p_{kl}, \qquad (17)$$
$$\| \mathbf{v}_{kl}\|^{2} \le 1.$$

With the similar procedure as the receiver design, the problem (17) can be reformulated as the following SDP optimization problem with rank constraint

$$\max_{\mathbf{W}_{kl}} tr\left[\left(\hat{\mathbf{H}}_{kk}^{\mathrm{H}}\mathbf{u}_{kl}\mathbf{u}_{kl}^{\mathrm{H}}\hat{\mathbf{H}}_{kk} + \sigma_{\Delta}^{2} \| \mathbf{u}_{kl} \|^{2} \mathbf{I}_{M_{k}}\right) \mathbf{W}_{kl}\right]$$
s.t.: $\mathbb{E}[\tilde{Z}_{kl}] \leq \gamma_{kl} p_{kl},$
 $\mathbf{W}_{kl} \succeq \mathbf{0},$
 $rank(\mathbf{W}_{kl}) = 1,$
 $tr(\mathbf{W}_{kl}) \leq 1,$
(18)

where $\mathbf{W}_{kl} = \mathbf{v}_{kl} \mathbf{v}_{kl}^{\mathrm{H}}$, $rank(\mathbf{W}_{kl}) = 1$ and $tr(\mathbf{W}_{kl}) \le 1$.

By dropping the rank constraint, the non-convex problem (18) is relaxed as a convex SDP problem, which is written as

$$\max_{\mathbf{W}_{kl}} tr\left[\left(\hat{\mathbf{H}}_{kk}^{\mathrm{H}}\mathbf{u}_{kl}\mathbf{u}_{kl}^{\mathrm{H}}\hat{\mathbf{H}}_{kk} + \sigma_{\mathbf{\Delta}}^{2} \| \mathbf{u}_{kl} \|^{2} \mathbf{I}_{M_{k}}\right) \mathbf{W}_{kl}\right] \\
\text{s.t.:} \quad \mathbb{E}[\bar{Z}_{kl}] \leq \gamma_{kl} p_{kl}, \qquad (19) \\
\mathbf{W}_{kl} \succeq \mathbf{0}, \\
tr(\mathbf{W}_{kl}) \leq 1,$$

where $\mathbf{W}_{kl} = \mathbf{v}_{kl} \mathbf{v}_{kl}^{\mathrm{H}}$ and

$$\mathbb{E}[\tilde{Z}_{kl}] = tr\left\{ \left[\sum_{j=1}^{K} \left(\hat{\mathbf{H}}_{jk}^{\mathrm{H}} \mathbf{U}_{j} \mathbf{U}_{j}^{\mathrm{H}} \hat{\mathbf{H}}_{jk} + \sigma_{\Delta}^{2} tr(\mathbf{U}_{j} \mathbf{U}_{j}^{\mathrm{H}}) \mathbf{I}_{M_{k}} \right) - \left(\hat{\mathbf{H}}_{kk}^{\mathrm{H}} \mathbf{u}_{kl} \mathbf{u}_{kl}^{\mathrm{H}} \hat{\mathbf{H}}_{kk} + \sigma_{\Delta}^{2} || \mathbf{u}_{kl} ||^{2} \mathbf{I}_{M_{k}} \right) + \frac{1}{\bar{\rho}_{k}} \mathbf{I}_{M_{k}} \right] \mathbf{W}_{kl} \right\}$$

$$(20)$$

Similar with the receiver design, \mathbf{v}_{kl} can be recovered from the optimal solution of problem (19), which is always of rank one in our simulations.

C. Algorithm

Iteratively updating the transmit precoders based on (16) and the receive filters based on (19), we obtain a

probabilistic constraint approach for robust transceiver design given in Table I.

Probabilistic constraint (Prob-Cons) approach for robust design	
1:	Initialize the precoders $\mathbf{v}_{k1}, \forall k, l$.
2:	Optimize the receive filters \mathbf{u}_{kl} , $\forall k, l$ by solving the SDP
	problem (16).
3:	Optimize the transmit precoders \mathbf{v}_{k1} , $\forall k, l$ by solving the SDP
	problem (19).
4:	Repeat 2 to 3 until convergence or the maximum number of
	iterations is reached.

Similar with the Max-SINR algorithm, the convergence of the proposed Prob-Cons algorithm cannot be proved straightforwardly, because different objective functions are optimized for the transmit precoders and receive filters. However, the divergence of the algorithm never happens in our simulations. Moreover, only local CSI is required by the proposed approach.

Note that under the TDD mode, the proposed Prob-Cons algorithm can be applied in distributed manner. It can be also applied for a frequency-division-duplex (FDD) system in a centralized way.

IV. SIMULATION RESULTS

In this section, we evaluate the proposed robust transceiver design algorithms via computer simulations. The proposed algorithm is compared to the classical Max-SINR algorithm [4], the Max-ASINR robust algorithm, the robust and non-robust MMSE algorithms proposed in [5]. Without loss of generality, we consider a symmetric MIMO IC network with K = 3, $M_k = N_k = 4$, $d_k = 2$ and $P_k = P, \forall k$. For each channel realization, the elements of imperfect channel matrices $\{\hat{\mathbf{H}}_{ki}, \forall j, k\}$ are generated following $\mathcal{CN}(0,1-\sigma_{\mathbf{A}}^2)$. The elements of channel error matrices Δ_{kj} , $\forall j, k$ are generated following $\mathcal{CN}(0,\sigma_{\mathbf{A}}^2)$. For the classical Max-SINR algorithm and the non-robust MMSE algorithm, the transceivers are designed from the imperfect CSI with the impact of CSI errors neglected. We set $p_{kl} = p, \forall k, l$ and $\gamma_{kl} = \gamma, \forall k, l$ for the proposed Prob-Cons algorithm in the simulations. The SDP problems are solved using the CVX [31].

A. Convergence

In Fig. 2, we show the convergence behavior of the proposed algorithms, where the achievable sum rate is plotted against the iteration number for SNR = 30 dB.

The standard deviation of the elements of channel error matrices is set as $\sigma_{\Delta} = \{0.1, 0.2\}$. For the Prob-Cons algorithm, the probability and interference threshold are set as p = 0.05 and $\gamma = 0.01$, respectively. The sum rate is averaged over 100 channel realizations. Under the given parameter configuration, it is observed that the

performance of the Max-SINR algorithm will reach a peak in the initial several iterations and then turns down with iteration number increase, after which the performance will increase again slowly when iteration number is greater than 40. This phenomenon indicates that the conventional Max-SINR algorithm is not robust to channel errors. By contrast, the average sum rate achieved by the proposed robust Max-ASINR and Prob-Cons algorithms and the referenced robust MMSE scheme increase monotonously along with iteration number increasing at any of the investigated SNR values. It is also observed that all the robust algorithms can provide significant performance gain compared to the non-robust Max-SINR algorithm when the iterations excess 10. Moreover, simulation results show that the proposed Prob-Cons robust design scheme has similar convergence rate as the robust MMSE scheme, which has been proved to be convergent.



Fig. 2. Sum rate versus iteration number.

B. Sum Rate

Fig. 3 shows the average sum rate of the proposed Prob-Cons algorithm under CSI with different accuracies. The results are obtained by averaging simulations of 100 channel realizations. The iteration number is set as 32 for all the algorithms. It can be observed that the conventional Max-SINR algorithm is more vulnerable to CSI uncertainty compared with the non-robust MMSE algorithm, and all robust schemes achieve higher sum rate than the non-robust Max-SINR algorithms. It is observed that the robust Max-ASINR achieves conservative performance compared with other robust schemes. The Max-ASINR algorithm even performs similar as the nonrobust MMSE algorithm when the standard deviation of channel errors is extremely small (e.g., $\sigma_{\Lambda} < 0.05$). It is also shown that the Prob-Cons algorithm performs the best among all algorithms when CSI error is large (e.g., $\sigma_{\Delta} > 0.1$).

C. SINR Distribution

The distribution of per-stream SINR achieved by different algorithms is shown in Fig. 3 for $\sigma_{A} = 0.1$ at

SNR = {20,30} dB. The empirical cumulative density function (CDF) is obtained from 10000 numerical simulations by fixing the randomly generated imperfect CSI matrices { $\hat{\mathbf{H}}_{kj}$, $\forall j, k$ } and varying the channel error matrices { Δ_{kj} , $\forall j, k$ } in each simulation. It can be observed that in SINR \in [10,15] dB interval, the robust algorithms can achieve lower CDF values compared with the non-robust schemes. That is to say, for a target SINR α in [10 15] dB interval, there is higher probability for the robust schemes to achieve the target SINR, i.e., SINR $\geq \alpha$.



Fig. 3. Sum rate versus SNR at different channel accuracy.



Fig. 4. SINR distribution with $\sigma_A = 0.1$, SNR = {20, 30} dB.

It is also observed that the CDF curve of the Max-ASINR algorithm locates on the left compared to that of the robust MMSE and Prob-Cons algorithms, which further embodies that the Max-ASINR algorithm achieves conservative robust performance and the Prob-Cons algorithm outperforms the Max-ASINR algorithm with respect to robustness. Moreover, there is a cross point between the CDF curves of robust MMSE algorithm and the proposed Prob-Cons algorithm. On the left side of this cross point, the MMSE algorithm achieves smaller cumulative probability distribution value than the Prob-Cons algorithm given the SINR value, while this relation will reverse on the right side of the cross point. Therefore, the MMSE algorithm can achieve larger worst-case SINR in higher probability compared with the Prob-Cons algorithm, while the Prob-Cons algorithm achieves high SINR in high probability compared with the MMSE algorithm.

When the standard deviation of channel errors increases to $\sigma_{\rm A} = 0.2$, the simulation results are shown in Fig. 5. It is observed that the CDF curves between different algorithms behave similar as Fig. 4, but shift toward to left. This implies that the achieved SINR will deteriorate as the channel accuracy decreases for all schemes.



Fig. 5. SINR distribution with $\sigma_A = 0.2$, SNR = {20,30} dB.



Fig. 6. Impact of interference threshold on performance of the Prob-Cons algorithm

D. Impaction of Interference Threshold

The performance of Prob-Cons algorithm is obviously related with the product of probability p and interference threshold γ . Fig. 6 shows the impact of γ on the achievable rate by the Prob-Cons algorithm with fixed p = 0.05. The simulation results are obtained

through averaging 100 channel realizations. It can be observed the Prob-Cons algorithm achieves better performance when γ is taken value between 10^{-3} and 1. Moreover, the best performance of the Prob-Cons algorithm is almost invariable when γ is in that range. More simulation results at different SNR settings show that a setting of $\gamma = 0.01$ can lead to satisfied performance in general.

V. CONCLUSION

We have studied robust transceiver designs for MIMO IC networks with Gaussian channel errors. A probability constraint robust transceiver design scheme was proposed by maximizing the average received signal while keeping low probability for large interference plus noise with imperfect CSI, in both the downlink and uplink. With the help of Markov's inequality, the probabilistic constraints were recast as LMI, and the transceiver design problems were converted to SDP problems with rank constraints. The non-convex rank constraint SDP problems were further relaxed as convex SDP by relaxing the rank constraints, which can be solved efficiently. Simulation results have shown that the proposed schemes can provide robustness to CSI uncertainty significantly.

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