

# Low-Complexity Fine Frequency Synchronization for MB-OFDM Based UWB Systems

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**Abstract**—In this paper, a fine frequency synchronization method with low complexity is proposed for Multi-Band Orthogonal Frequency Division Multiplexing (MB-OFDM) based Ultra-Wideband (UWB) systems. Low complexity is still one of the major difficulties in the design of fine frequency synchronizer for MB-OFDM UWB systems due to arctangent operations and Numerically Controlled Oscillator (NCO), both of which have high computational complexity, are usually needed in fine frequency synchronizer. In this paper, the proposed fine frequency synchronization scheme directly estimates the sine/cosine values of the residual phases of all sub-carriers by simplifying them as values of a linear function. Then, the estimated sine/cosine values are directly used for frequency offset compensation. In this way, the highly complex arctangent operation and NCO are not needed any more in the proposed scheme. Simulation results show the proposed scheme has high performance.

**Index Terms**—MB-OFDM, UWB, fine frequency synchronization

## I. INTRODUCTION

Due to advantages of wide bandwidth and low Radiation Spectral Density (RSD), Ultra-Wideband (UWB) technology is an ideal approach for the high-speed Wireless Personal Area Network (WPAN), which requires both high data rate and low power. Among approaches for UWB, Multi-Band Orthogonal Frequency Division Multiplexing (MB-OFDM) technique [1]-[2] can offer high spectrum efficiency and robustness in multipath diversity, is most promising for high speed transmission.

However, as normal OFDM techniques, MB-OFDM is also sensitive to frequency offset. Both Carrier Frequency Offset (CFO) and Sampling Frequency Offset (SFO) can destroy the orthogonality among sub-carriers, which will introduce Inter-Carrier Interference (ICI) and greatly degrade the system performance. Though, CFO in MB-OFDM systems have been compensated based on preamble symbols in time domain, the accumulated phase offset based on Residual Carrier Frequency Offset (RCFO) and SFO still can greatly degrade the system performance if they are not corrected.

Based on pilots inside OFDM symbols, many fine frequency synchronization methods have been proposed for OFDM based systems [3]-[20]. Most of these algorithms [3], [4], [6]-[8], [10], [11], [13]-[16], [18]-[20] need the assistance of arctangent operation and Numerically Controlled Oscillator (NCO) to calculate phase offset and compensate the signal with distorted phase, respectively. Both the arctangent operation and the NCO have very high computational complexity. To reduce the computational complexity of the fine frequency synchronization in OFDM based systems, some simplified methods [5], [9] are proposed by transforming phase calculation into a series of trigonometric computation. Of them, scheme in [9] is specially designed for MB-OFDM UWB systems. However, these methods sacrifice certain accuracy performance because getting approximation operations are introduced during trigonometric computation.

In this paper, a highly simplified fine frequency synchronizer is proposed for MB-OFDM UWB systems. Instead of computing phase with arctangent operations and mapping phase to sine/cosine through NCO, the sin/cosine values of the residual phases of all subcarriers are directly computed in the proposed scheme by simplifying the sin/cosine values as results of a linear function. The simulation results show the proposed fine frequency synchronizer has high accuracy.

The rest of the paper is organized as follows. Section II gives a brief description of the system model. The proposed fine frequency synchronizer is proposed in Section III. Evaluation results are shown in Section IV. Finally, conclusions are given in Section V.

## II. SYSTEM MODEL DESCRIPTION

In MB-OFDM UWB systems, a transmitted OFDM symbol is constructed with  $N = 128$  Inverse Fast Fourier Transform (IFFT) outputs followed by 32 null samples, called zero-padding, and 5 null guard samples. Therefore, a transmitted OFDM symbol in MB-OFDM UWB systems is composed of  $N_s = N + N_g = 165$  samples. The transmitted baseband signal of symbol  $l$  can be described as [21]

$$s_l(t) = \sum_{k=0}^{N-1} X_{l,k} e^{j2\pi(k/T_d)(t-lT_s)} u(t-lT_s) \quad (1)$$

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$$u(t) = \begin{cases} 1 & 0 \leq t \leq T_d \\ 0 & \text{else} \end{cases} \quad (2)$$

where  $X_{l,k}$  is the data on subcarrier  $k$  of symbol  $l$ ,  $T_s$  defines the total length of an OFDM symbol,  $T_d$  is the duration of the data part of a symbol.

The equivalent frequency fading selective channel, which is composed of the actual UWB channel impulse response (CIR) and the effect of filters in transceiver, can be simplified as [22]

$$h(\tau) = \sum_i h_i \delta(\tau - \tau_i) \quad (3)$$

where  $h_i$  denotes the path gain of path  $i$  and  $\tau_i$  is the path delay of path  $i$ .

In MB-OFDM UWB systems, it can be assumed that the length of ZP is longer than the maximum path delay. And according to specification [1],  $h(\tau)$  can be seen as time-invariant within the duration of one frame. By assuming that sampling signals at  $t_n = nT_s$ , the discrete baseband signal at receiver is given by

$$r(t_n) = \sum_i h_i s(nT_s - \tau_i) + w(nT_s) \quad (4)$$

where  $w(nT_s)$  is a zero-mean Gaussian noise with variance of  $\sigma^2$ .

Fine frequency synchronization in MB-OFDM UWB systems is carried out after the fast Fourier transform (FFT), before which both coarse CFO and timing offset have been compensated. By only taking RCFO and SFO into consideration, the received frequency-domain data on subcarrier  $k$  of symbol  $l$  can be simplified as

$$Y_{l,k} = \sum_{k=0}^{N-1} X_{l,k} H_{l,k} e^{j\theta_{l,k}} + W_{l,k} \quad (5)$$

where  $H_{l,k}$  is the channel transfer function on subcarrier  $k$  of symbol  $l$ ,  $\theta_{l,k}$  is the phase distortion and  $W_{l,k}$  is a Gaussian noise with the same characteristic as  $w(nT_s)$ .

In specification [1], twelve of the  $N=128$  subcarriers in each data symbol are allocated as pilots for channel estimation, which also can be used for fine frequency synchronization. The pilots in a data symbol are shown in Fig. 1. The twelve pilots are allocated at subcarriers  $\{1, 5, 15, 25, 35, 45, 55, 73, 83, 93, 103, 113, 123\}$  and are marked with pilot indexes  $\{5, 15, 25, 35, 45, 55, -55, -45, -35, -25, -15, -5\}$ .

### III. PROPOSED FINE FREQUENCY SYNCHRONIZER

In traditional fine frequency synchronizers [3], [4], [15], [16], they simplify the phase distortion of the  $k$ th subcarrier of symbol  $l$  as

$$\theta_{l,k} = a + k\zeta \quad (6)$$

where  $a$  is the initial phase offset, which is caused by RCFO, and  $\zeta$  is caused by SFO. Then, the estimator in

[3] estimate  $a$  and  $\zeta$  as

$$a = \frac{1}{2\pi\rho} \times \frac{\phi_{l,1} + \phi_{l,2}}{2}, \quad \zeta = \frac{1}{2\pi\rho} \frac{\phi_{l,1} - \phi_{l,2}}{K/2} \quad (7)$$

$$\phi_{l,(1|2)} = \arg \left[ \sum_{k \in C_{(1|2)}} Y_{l,k} Y_{l-1,k}^* \right] \quad (8)$$

where  $K$  is the number of subcarriers in the system, and  $C_1 = [1, K/2]$  and  $C_2 = [K/2+1, K]$  are the first and second half of the subcarriers, respectively. From (7) and (8), it can be seen that two arctangent operations are consumed to achieve  $\theta_{l,k}$ . Furthermore,  $\theta_{l,k}$  cannot be directly used for frequency offset compensation. In these methods, they need to take an additional function, in which at least one highly complex NCO is included, to achieve  $\sin(\theta_{l,k})/\cos(\theta_{l,k})$  from the  $\theta_{l,k}$  for compensating the signal with phase offset.

Different to coarse CFO synchronization, which only be activated to detect several preamble symbols, fine frequency synchronization need to be executed when every data symbol received. Furthermore, most MB-OFDM devices is very sensitive to power consumption. Therefore, these traditional methods are not ideal solution for fine frequency synchronization in MB-OFDM UWB systems. In this section, we propose a highly simplified fine frequency synchronizer.

In MB-OFDM UWB systems, fine frequency synchronization is executed in frequency domain after channel equalization. By assuming that the CIR is correctly compensated, the received signal of subcarrier  $k$  is given by

$$\begin{aligned} Y_{l,k} &= H_{l,k} X_{l,k} e^{j\theta_{l,k}} H_{l,k}^* + W_{l,k} \\ &= |X_{l,k} + V_{l,k}| e^{j(\alpha_{l,k} + \theta_{l,k} + I_{l,k})} \end{aligned} \quad (9)$$

where  $V_{l,k}$  and  $I_{l,k}$  can be seen as the effect of noise on the amplitude and phase of the received complex signal, respectively, and  $\alpha_{l,k}$  is the initial phase of the signal. Therefore,  $\theta_{l,k}$  can be estimated correctly based on  $Y_{l,k}$  if the influence of  $V_{l,k}$  and  $I_{l,k}$  can be eliminated.

Firstly, we reduce the effect of  $V_{l,k}$ . In MB-OFDM UWB systems, pilot subcarriers can be used for fine frequency synchronization. The pilot signals in MB-OFDM UWB systems is given by

$$P_m = \frac{\pm 1 \pm j}{\sqrt{2}} \quad (m = \pm 5, \pm 15, \pm 25, \pm 35, \pm 45, \pm 55) \quad (10)$$

where  $m$  are pilot indexes.

To distinguish definitions  $\theta_{l,k}$ ,  $I_{l,k}$ ,  $Y_{l,k}$  and  $V_{l,k}$ , which are indexed by the subcarrier indexes, from their corresponding presentation on pilot subcarriers, which are indexed by pilot indexes, their corresponding presentation under pilot indexes are defined as  $\tilde{\theta}_{l,m}$ ,  $\tilde{I}_{l,m}$ ,  $\tilde{Y}_{l,m}$  and  $\tilde{V}_{l,m}$ .

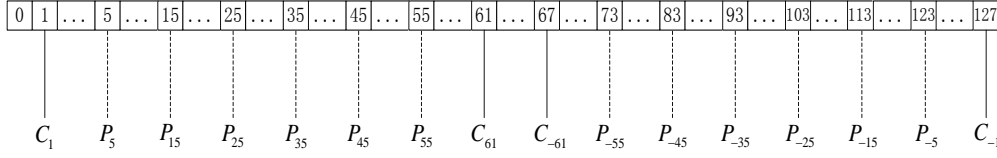


Fig. 1. The subcarrier indexes and pilots in a data symbol

To make reading easier, some important notations defined in this section are listed in appendix A.

From (10), it can be seen that  $|P_m| = 1$ , ( $m = \pm 5, \pm 15, \pm 25, \pm 35, \pm 45, \pm 55$ ). Therefore, the effect of  $\tilde{Y}_{l,m}$  can be reduced by normalizing  $\tilde{Y}_{l,m}$  as

$$\tilde{Y}_{l,m}' = \tilde{Y}_{l,m} / |\tilde{Y}_{l,m}| \quad (11)$$

Without considering the effect of noise,  $e^{j\hat{\theta}_{l,m}}$  can be estimated based on  $\tilde{Y}_{l,m}'$ . Let  $T_{l,m} = e^{j\hat{\theta}_{l,m}}$ , ( $m = \pm 5, \pm 15, \pm 25, \pm 35, \pm 45, \pm 55$ ). It can be estimated as

$$\hat{T}_{l,m} = \tilde{Y}_{l,m}' P_m^* \quad (m = \pm 5, \pm 15, \pm 25, \pm 35, \pm 45, \pm 55) \quad (12)$$

As  $\hat{T}_{l,m}$  is achieved by ignoring the effect of  $\tilde{I}_{l,m}$ ,  $\hat{T}_{l,m}$  is only a coarse estimation.

In MB-OFDM UWB systems, channel noise follows a complex Gaussian distribution with zero-mean. Therefore  $\tilde{I}_{l,m}$  approximately follows a uniform distribution with a probability distribution function (PDF) of  $P(\tilde{I}_{l,m}) = 1/2\pi$ ,  $-\pi \leq \tilde{I}_{l,m} \leq \pi$ , and  $P(\tilde{I}_{l,m}) = 0$  otherwise. Therefore, phase noise on other pilot subcarriers can be used to smooth the effect of  $\tilde{I}_{l,m}$ .

As RCFO and SFO is very small in MB-OFDM systems, the maximum phase distortion difference  $\theta_{l,127} - \theta_{l,0}$  between two subcarriers meets  $\theta_{l,127} - \theta_{l,0} < \pi/16$ . Therefore,  $\cos(\theta_{l,k}) / \sin(\theta_{l,k})$  of all subcarriers can be simplified with the linear form of  $\cos(\theta_{l,k}) / \sin(\theta_{l,k}) = \alpha + k\beta$ .

In the MB-OFDM UWB system, the pilots in a symbol are symmetrical to subcarrier 64. Therefore, the phase distortion at the 64th subcarrier of symbol  $l$  can be defined as the initial phase distortion  $\delta_l$  for symbol  $l$ .

$$\cos(\delta_l) = \text{Re} \left( \frac{1}{12} \sum_{m \in C} \hat{T}_{l,m} \right), \quad (13)$$

$$(C = \pm 55, \pm 45, \pm 35, \pm 25, \pm 15, \pm 5)$$

$$\sin(\delta_l) = \text{Im} \left( \frac{1}{12} \sum_{m \in C} \hat{T}_{l,m} \right), \quad (14)$$

$$(C = \pm 55, \pm 45, \pm 35, \pm 25, \pm 15, \pm 5)$$

Based on  $\cos(\delta_l) / \sin(\delta_l)$ , the  $\cos(\theta_{l,k}) / \sin(\theta_{l,k})$  of all subcarriers can be simplified as

$$\cos(\theta_{l,k}) = \cos(\delta_l) + (k - 64)\beta_1 \quad (k = 0, 1, \dots, N-1) \quad (15)$$

$$\sin(\theta_{l,k}) = \sin(\delta_l) + (k - 64)\beta_2, \quad (k = 0, 1, \dots, N-1) \quad (16)$$

where  $k$  is the subcarrier index and  $\beta_1$  and  $\beta_2$  are slopes of the two linear functions, respectively.

By dividing the pilots into two parts,  $C_1 = \{m = -5, -15, -25, -35, -45, -55\}$  and  $C_2 = \{m = 5, 15, 25, 35, 45, 55\}$ ,  $\beta_1$  and  $\beta_2$  are given by

$$\beta_1 = \text{Re} \left[ \frac{1}{60} \left( \sum_{m \in C_2} \hat{T}_{l,m} - \sum_{m \in C_1} \hat{T}_{l,m} \right) \right] \quad (17)$$

$$\beta_2 = \text{Im} \left[ \frac{1}{60} \left( \sum_{m \in C_2} \hat{T}_{l,m} - \sum_{m \in C_1} \hat{T}_{l,m} \right) \right] \quad (18)$$

The achieved  $\cos(\theta_{l,k}) / \sin(\theta_{l,k})$  ( $k = 0, 1, \dots, N-1$ ) can be directly used to compensate the distorted signal. In this way, the fine frequency synchronization can be implemented with low computational complexity.

#### IV. EVALUATION

In this section, we evaluate the proposed fine frequency synchronizer. The complexity of it is evaluated firstly. Then, the MSE performance is analyzed, and the BER performance is simulated. Simulations are carried out by assuming that signals are transmitted under UWB channel model CM1 with a data rate of 200Mbps. The parameters of the MB-OFDM system in simulations follow the ones in specification [1]:  $N = 128$ ,  $N_s = 165$ , carrier frequencies are  $\{3432, 3960, 4488\}$  MHz, and the sub-carrier spacing is 4.125 MHz. According to specification [1], the maximum absolute CFO between the receiver and transmitter is  $\pm 40$  ppm.

The complexity, which includes hardware complexity and computational complexity, of the proposed fine frequency synchronization method is shown in Table I. The complexity of the traditional method in [3] and the trigonometric function based method in [9], which has low complexity, are also shown in the table for comparison. As the circuit level designs are not shown in the three algorithms, the hardware complexity of these algorithms can be defined as the hardware cost on one pilot and one data subcarrier (because operations on other pilots and sub-carriers can reuse the hardware resources used on the first pilot and the first data subcarrier, respectively). From (11), it can be seen that  $\tilde{Y}_{l,m}'$  can be achieved through 2 real divisions and 1 operation of getting amplitude of a complex value. As  $P_m = (\pm 1 \pm j) / \sqrt{2}$ , the computation of  $T_{l,m}$  also can be

simplified, which can be implemented with 2 real multiplications and 2 real additions based on  $Y'_{l,m}$ . The remainder operations for the first pilot and the first data subcarrier in the fine synchronization process can be implemented with about 6 real multiplications and 6 real additions. To simplify comparison, the hardware complexity of one real division is shown as 4/3 real multiplication in Table I. Therefore, the proposed fine synchronization algorithm has a hardware cost of about 10+2/3 real multipliers, 8 real adders and 1 calculator for getting the amplitude of a complex value, as shown in Table I.

In Table I, ARC is the operation of getting phase from a complex value, ABS represents the operation of getting the amplitude of a complex value. To simplify comparison, one complex divider is transformed into 4/3 complex multiplier, and one complex multiplier and one complex adder are presented in the form of real multiplier and real adder, respectively, according to that one complex multiplier can be implemented with 3 real multipliers and 5 real adders, and that one complex adder can be implemented by using 2 real additions. In the table, the hardware cost of a NCO calculator is far higher than that of a complex multiplication, and one ARC calculator also has much higher hardware complexity than a complex multiplier. As ABS can be calculated based on linear approximation, it can be implemented with a same hardware complexity level as a real adder by choosing appropriate parameters for the linear approximation operation. It can be seen from the table that the method in [3] needs one NCO calculator and one ARC calculator under the situation that the real multipliers used in the proposed method, the method in [3] and the method [9] are 10+2/3, 6 and 10, respectively. Therefore, the method in [3] has much higher hardware cost than the proposed method and the method in [9]. Compared with the method in [9], the proposed method needs 2/3 more real adder and 1 more ABS calculator but 4+2/3 fewer real adders. Therefore, the hardware complexity of proposed method and the method in [9] is equivalent.

There are 12 pilot subcarriers and 112 data subcarriers in MB-OFDM UWB systems. Based on the aforementioned analysis on hardware cost, it can be obtained the proposed method needs about 284 real multiplications, 272 real additions and 12 ABSs to complete the whole fine synchronization process. Compared with the method in [9], which has a computational cost of about 296 real multiplications and 352 real additions, the proposed algorithm has lower computational complexity. The method in [3] needs 134, 76 and 12 fewer real multiplications, real additions and ABSs, respectively, than the proposed method, but it needs 124 more NCOs and 2 more ARCs. As one NCO has far higher computational complexity than a complex multiplication, the method in [3] has far higher computational complexity than the proposed method.

According to (29) in appendix B, the mean square error (MSE) of the estimated  $\sin(\delta_l)/\cos(\delta_l)$  is given by

TABLE I. THE COMPARISON ON COMPUTATIONAL COMPLEXITY AMONG FINE FREQUENCY SYNCHRONIZATION METHODS

	Operation	Proposed	Method in [3]	Method in [9]
Hardware cost	NCO calculator	0	1	0
	ARC calculator	0	1	0
	Real multiplier	10+2/3	6	10
	Real adder	8	10	12+2/3
	ABS calculator	1	0	0
	NCO	0	124	0
Computational cost	Arctangent	0	2	0
	Real multiplication	284	150	296
	Real addition	272	196	352
	ABS	12	0	0

$$MSE[\sin(\delta_l)] \approx \frac{1}{2} \times \left( \frac{1}{12} \right)^2 \sigma^2 \quad (19)$$

From (19), it can be seen that the MSE of the estimated  $\sin(\delta_l)/\cos(\delta_l)$  is much smaller than the variance of  $\sin(\tilde{I}_{l,m})/\cos(\tilde{I}_{l,m})$ , which implies that the proposed fine frequency synchronization method based on  $\sin(\delta_l)/\cos(\delta_l)$  has much better noise immunity than based on getting phase from individual pilot subcarrier directly.

For the MB-OFDM system with approximately 4000MHZ carrier frequency and 4.125MHZ sub-carrier spacing, the normalized value for the 40ppm CFO is approximately 0.04. The simulated bit error rate (BER) results of the proposed fine CFO synchronizer are shown in Fig. 2, Fig. 3 and Fig. 4. The BER results of the method in [3] and the scheme in [9] are included as comparison.

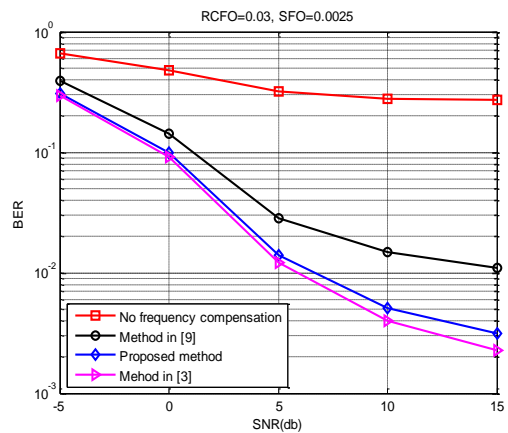


Fig. 2. BER comparison among fine frequency synchronization methods under RCFO=0.03 and SFO=0.0025

Fig. 2 shows the simulated results achieved by assuming the normalized RCFO and SFO are 0.03 and 0.0025 respectively. The results shown in Fig. 3 are achieved by setting 0.015 and 0.002 for the normalized RCFO and SFO, respectively. The results in these two figures correspond to the MB-OFDM systems with high RCFO and SFO. As shown in the figures, the BER performance of the proposed scheme is much higher than the method in [9] because some trigonometric operations in method [9] are simplified as 0 and 1 by assuming that the SFO is very small. It also can be seen that the BER performance of the proposed method is close to the BER performance of the method in [3], which needs arctangent operations and NCO operation.

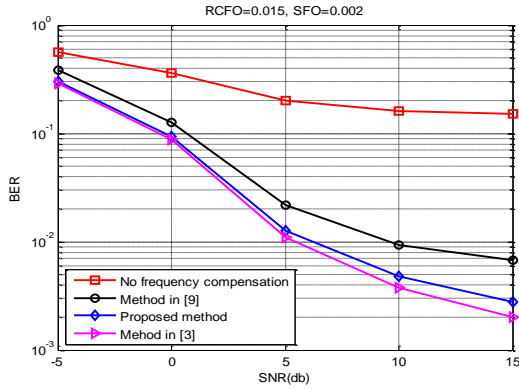


Fig. 3. BER comparison among fine frequency synchronization methods under RCFO=0.015 and SFO=0.002

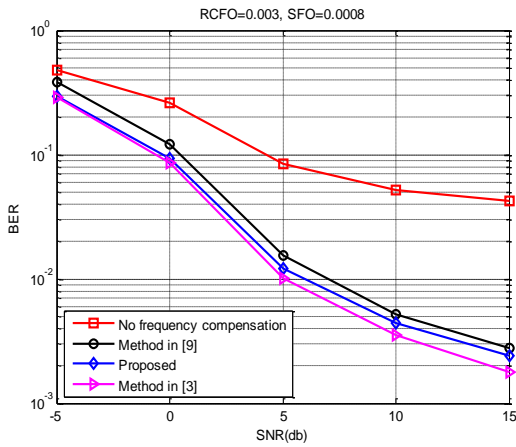


Fig. 4. BER comparison among fine frequency synchronization methods under RCFO=0.003 and SFO=0.0008

After CFO compensation, the normalized RCFO of MB-OFDM UWB systems are usually lower than 0.005. By setting 0.003 and 0.0008 for RCFO and SFO respectively, the simulated BER performance is shown in Fig. 4. From this figure, it also can be seen that the BER performance of the proposed method is also close to the BER performance of the method in [3]. In this case, the BER performance of the method in [9] is much closer to the proposed method than the cases in Fig. 2 and Fig. 3 under high SNRs (SNR>5). However, the proposed method still has much better BER performance than the method in [9]

under low SNRs (SNR<0) due to it is quite robust to noise.

## V. CONCLUSION

In this paper, a new fine frequency synchronization method is proposed for MB-OFDM based UWB systems. In this method, the sine/cosine values of the residual phases of all sub-carriers are estimated directly based on simplifying them as values of a linear function. As arctangent operation and NCO operation are not used, the proposed scheme has low complexity. Evaluation results show the proposed scheme has very low computational complexity with high performance.

## APPENDIX A IMPORTANT NOTATIONS IN SECTION III

As there are many notations defined in section III, descriptions for some important notations are listed in Table II to make reading easier.

## APPENDIX B DERIVATION FOR (19)

To simplify analysis, QPSK modulation is assumed in the analysis process. The estimated  $T_{l,m}$  in (12) can be given by

$$\hat{T}_{l,m} = e^{j(\tilde{\theta}_{l,m} + \tilde{I}_{l,m})} \quad (20)$$

where  $\tilde{I}_{l,m}$  is the phase distortion caused by noise.

The error of the estimated  $\sin(\delta_l)$  can be given by

$$\begin{aligned} & \sin(\delta_l) - \sin(\theta_{l,64}) \\ &= \frac{1}{12} \sum_{m \in C} \sin(\tilde{\theta}_{l,m} + \tilde{I}_{l,m}) - \sin(\theta_{l,64}) \end{aligned} \quad (21)$$

where  $C$  is the set of all pilot indexes .

TABLE II. IMPORTANT NOTATIONS IN SECTION III

Notations	Descriptions
$\theta_{l,k}$	Phase distortion on the subcarrier indexed by subcarrier index $k$
$\tilde{\theta}_{l,m}$	Phase distortion on the pilot indexed by pilot index $m$
$V_{l,k}$	The effect of noise on the amplitude of the received subcarrier signal indexed by subcarrier index $k$
$\tilde{V}_{l,m}$	The effect of noise on the amplitude of the received pilot signal indexed by pilot index $m$
$I_{l,k}$	The effect of noise on the phase of the received subcarrier signal indexed by subcarrier index $k$
$\tilde{I}_{l,m}$	The effect of noise on the phase of the received pilot signal indexed by subcarrier index $m$
$T_{l,m}$	$e^{j\tilde{\theta}_{l,m}}$
$\delta_l$	Initial phase distortion of symbol $l$
$\beta_1$	Slop of the linear form of $\cos(\theta_{l,k}) = \alpha + k\beta$
$\beta_2$	Slop of the linear form of $\sin(\theta_{l,k}) = \alpha + k\beta$

As  $\theta_{l,127} - \theta_{l,0}$  is very small in MB-OFDM UWB systems,  $\sin(\theta_{l,64})$  can be linearized as

$$\sin(\theta_{l,64}) \approx \frac{1}{12} \sum_{m \in C} \sin(\tilde{\theta}_{l,m}) \quad (22)$$

Therefore,

$$\begin{aligned} \sin(\delta_l) - \sin(\theta_{l,64}) &\approx \frac{1}{12} \sum_{m \in C} \sin(\tilde{\theta}_{l,m} + \tilde{I}_{l,m}) - \frac{1}{12} \sum_{m \in C} \sin(\tilde{\theta}_{l,m}) \\ &= \frac{1}{12} \sum_{m \in C} [\sin(\tilde{\theta}_{l,m} + \tilde{I}_{l,m}) - \sin(\tilde{\theta}_{l,m})] \end{aligned} \quad (23)$$

The MSE of the estimated  $\sin(\delta_l)$  is defined as

$$MSE[\sin(\delta_l)] = E \left\{ [\sin(\delta_l) - \sin(\theta_{l,64})]^2 \right\} \quad (24)$$

By substituting (23) into (24),

$$\begin{aligned} MSE[\sin(\delta_l)] &= E \left\{ \left[ \frac{1}{6} \sum_{m \in C} \cos[(2\tilde{\theta}_{l,m} + \tilde{I}_{l,m})/2] \sin(\tilde{I}_{l,m}/2) \right]^2 \right\} \\ &= E \left\{ \left[ \frac{1}{6} \sum_{m \in C} [\cos(\tilde{\theta}_{l,m}) \cos(\tilde{I}_{l,m}/2) - \right. \right. \\ &\quad \left. \left. \sin(\tilde{\theta}_{l,m}) \sin(\tilde{I}_{l,m}/2) \right] \sin(\tilde{I}_{l,m}/2) \right]^2 \right\} \end{aligned} \quad (25)$$

As residual phase is very small in MB-OFDM UWB systems,  $\sin(\tilde{\theta}_{l,m}) \approx 0$  and  $\cos(\tilde{\theta}_{l,m}) \approx 1$  can be assumed.  $MSE[\sin(\delta_l)]$  can be simplified as

$$MSE[\sin(\delta_l)] \approx E \left\{ \left[ \frac{1}{12} \sum_{m \in C} \sin(\tilde{I}_{l,m}) \right]^2 \right\} \quad (26)$$

Let  $D_l = \sum_{m \in C} P_m P_m^*$ , and let  $A_w = \sum_{m \in C} \tilde{W}_{l,m} P_m^*$ , where  $\tilde{W}_{l,m}$  is the Gaussian noise on the pilot subcarrier  $m$  of symbol  $l$ . As  $\tilde{W}_{l,m} (m \in C)$  are zero-mean Gaussian noises,  $\tilde{W}_{l,m} P_m^* (m \in C)$  will neutralize each other, which means that  $|A_w|$  is much smaller than  $|D_l|$  with high probability. Thus  $\sum_{m \in C} \sin(\tilde{I}_{l,m})$  can be approximately given by

$$\sum_{m \in C} \sin(\tilde{I}_{l,m}) \approx \frac{|A_w| \sin(\alpha_w)}{|D_l| + |A_w| \cos(\alpha_w)} \approx \frac{|A_w| \sin(\alpha_w)}{|D_l|} \quad (27)$$

where  $\alpha_w$  is the phase of  $A_w$ .  $|A_w| \sin(\alpha_w)$  is approximately a Gaussian random variable with zero mean and variance  $\frac{1}{2} \sum_{m \in C} |P_m|^2 \sigma^2$ . As  $|P_m|$  is equal to  $P_m P_m^*$ , (26) can be given by

$$MSE[\sin(\delta_l)] \approx \frac{1}{2} \times \left( \frac{1}{12} \right)^2 \sigma^2 \quad (28)$$

As  $\cos(\delta_l)$  has the same characteristic as  $\sin(\delta_l)$ ,

$\cos(\delta_l)$  has the same MSE as  $\sin(\delta_l)$ . Therefore, the MSE of  $\sin(\delta_l)/\cos(\delta_l)$  can be given by

$$MSE[\sin(\delta_l)] \approx \frac{1}{2} \times \left( \frac{1}{12} \right)^2 \sigma^2 \quad (29)$$

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