The Research of Frequency Domain Least Squares Channel Estimation in OFDM Systems

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Abstract**—**This paper focuses on comb-type or scattered pilot arrangement in OFDM systems with a frequency guard band under the time-variant channel. If the maximum channel delay is large, there is a nearly singular matrix problem when a receiver adopts the Frequency Domain LS (FDLS) method to estimate the data subcarrier channel frequency response. This problem renders FDLS channel estimation more sensitive to noise. M. Yu and P. Sadeghi proposed a simply method to avoid the nearly singular matrix problem; however, they did not describe in detail how to select the important regularization parameter. Therefore, this study will discuss and analyze the regularization parameter, and propose a flow to select a suitable regularization parameter. From BER simulations, the proposed flow shows good performance.

Index Terms—OFDM, channel estimation, frequency domain LS channel estimation

I. Introduction

In the environment of wireless transmission, the transmitted signals are often influenced by channels, resulting in signal distortion. Generally, an equalizer is used at the receiver of a communication system to compensate the channel effect, and the equalizer of the traditional time domain has complexity problems. However, under OFDM technology, the concept of parallel transmission is adopted, which divides a section of a broadband signal into several mutually orthogonal narrowband sub-channels (subcarriers) to transmit, thus, when facing frequency selective fading channels, each sub-channel has relatively smooth channel response; therefore, in OFDM systems, an equalizer with one coefficient can be used in frequency domain to compensate the channel influence of each subcarrier. Compared with an ordinary system using a time-domain equalizer, one-tap frequency domain equalizer has simple hardware. The key point in OFDM systems is how to realize accurate channel estimation to determine the equalizer coefficients and compensate the distorted signals.

Currently, in communication specifications adopting OFDM technology, it is common to insert pilots (such as DVB-T) or training sequence (such as IEEE 802.11a) in the transmitted signals, which, on the one hand, can make the receiver realize time synchronization, and on the other hand, can estimate the channel frequency response, in order to determine the coefficients of the frequency domain equalizer. In the time-variant channel environment, as channel frequency response will change over time, that is, the channel frequency responses will be different on different OFDM symbols, at this time, the channel frequency response, as estimated by the headmost training sequence in the signals, will be unable to adapt to the latter OFDM symbols. Therefore, in the time-variant channel, the method of inserting pilots on some specific subcarriers of OFDM symbols is adopted. Generally, pilot arrangement is classified into block-type and comb-type [1], in which, block-type is relatively insensitive to frequency selectivity, but it is not suitable for fast-fading channels; the comb-type is generally adopted when the channel is fast-fading, but it is sensitive to frequency selectivity. The corresponding channel estimation has been discussed in [1]-[3]. In addition, there is another pilot arrangement: scattered pilot, which is beyond the above two, while related channel estimation is discussed in [4], [5].

In the time-variant channel, if it is supposed that the channel in one OFDM symbol is approximately constant, the estimation of the channel frequency response using the pilot generally requires two steps. First, work out the channel estimation of the pilot sub-carriers: generally, the LS method is adopted in calculation, that is, divide the received signal on pilot by known transmitted content, and the advantage of such method is that it is relatively simple. In addition, there is the MMSE method. Although it has better effect than LS method, it requires calculation of some statistical values. Thus, it is complicated and hard to realize through hardware. Second, the channel frequency responses on data sub-carriers are estimated by the channel frequency responses on pilot sub-carriers. In this step, the common methods are shown, as follows: one-dimensional or two-dimensional linear interpolation [4], [6], polynomial interpolation [7], FFT interpolation [8], [9], LS method [10]-[13] and other methods [14], [15]. The simplest method is one-dimensional linear interpolation, which can be used no matter whether block-type or comb-type pilot arrangement; however, its efficiency will be affected due to the limitation of its environment.

When the LS estimation method is adopted to estimate the channel frequency responses on data sub-carriers, although it is more complicated than linear interpolation, it is free of problem, such as linear interpolation, which is subject to some environmental limitations, and can be used even when the statistical properties of the channel are unknown. However, the LS method has the problem

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that the matrix would be close to singular, and this problem may amplify the noise and result in difficult realization of hardware. Reference [11], [13] attempted to put forward a method to solve this problem; Reference [13] used the theory of Tikhonov regularization; and [11] used a similar method. However, there was no detailed mathematics discussion in the two papers, and the selection of a key parameter was not clearly specified. In addition, in [11], the matrix size is directly affected by the number of pilots, which affects the actual application, so there will be a heavy computation and flexibility is lacking. Therefore, under an OFDM system with a guard band, this paper takes the LS estimation method [11] to solve the problems in [11] in order that the LS method can be applied in practice. In addition, we will use an example based on a simple OFDM system, and conduct simulations in a time-invariant channel and a time-variant channel, in order to verify the effect and feasibility of the method put forward.

Section II will introduce the FDLS estimation method, Section III will discuss how to overcome the problem of a matrix close to singular in the FDLS estimation method, Section IV presents simulations and discussions, and Section V offers the conclusions.

II. Frequency Domain LS (FDLS) Estimation Method

First, define related parameters and environment: the OFDM system is based on a comb-type pilot arrangement with a guard band, where pilot spacing is N_p, P is the number of pilot sub-carriers, the size of FFT is N, and N/N_p is an integer ($N/N_p \neq P$). Suppose that the time-variant channel is approximately constant in one OFDM system. Let $h_l[n]$ be the digital channel impulse response in the lth OFDM symbol, and suppose M is the maximum channel delay in samples, then $h_l[n]=0$ when $n>(M-1)$ or $n<0$ (causal system in real world). The transmitted frequency domain data in the lth OFDM symbol is $X_l[k]$, where k is the subcarrier index, and the N-point IDFT of $X_l[k]$ is $x_l[n]$ without CP (cyclic prefix) in the lth OFDM symbol, where n is the time domain sample index without CP in one OFDM symbol. Let $y_l[n]$ be the time domain received signal after removal of CP in lth OFDM symbol, when CP is long enough that there is no ISI between two adjacent OFDM symbols. Therefore, $y_l[n]$ can be expressed as:

$$y_l[n]=x_l[n]\otimes h_l[n]+w_l[n] \tag{1}$$

where, $w_l[n]$ is white Gaussian noise, and $\otimes$ is the circular convolution operation. Let the channel frequency response $H_l[k]$ be the N-point DFT of $h_l[n]$, and the mathematical expression shown, as follows:

$$H_l[k]=\sum_{n=0}^{N-1} h_l[n]e^{-j2\pi kn/N}=\sum_{n=0}^{M-1} h_l[n]e^{-j2\pi kn/N} \tag{2}$$

Let $Y_l[n]$ be the N-point DFT of $y_l[n]$, then

$$Y_l[k]=X_l[k]H_l[k]+W_l[n] \tag{3}$$

where, $W_l[n]$ is the N-point DFT of $w_l[n]$, as well as white Gaussian noise. Next, the LS method is adopted to estimate the channel frequency response on the pilot sub-carriers, and the channel frequency response estimated on the k_pth sub-carrier is shown, as follows:

$$\begin{aligned}\hat{H}_l[k_p]&=Y_l[k_p]/X_l[k_p]\\ &=H_l[k_p]+\frac{W_l[k_p]}{X_l[k_p]}\\ &=H_l[k_p]+V_l[k_p]\ ,\ k_p\in\text{pilot sub-carrier}\end{aligned} \tag{4}$$

where, $|X_l[k_p]|=1$, $V_l[k_p]=W_l[k_p]/X_l[k_p]$, as well as white Gaussian noise. Then, the FDLS method is used to estimate the channel frequency response on data sub-carrier, and the error function E_l is shown, as follows:

$$E_l=\sum_{k_p}\left|\sum_{n=0}^{M-1} h_l[n]e^{-j2\pi k_p n/N}-\hat{H}_l[k_p]\right|^2 \tag{5a}$$

Equation (5a) can also be expressed in matrix form

$$E_l=(\mathbf{F}_p\mathbf{h}_l-\hat{\mathbf{H}}_l)^H(\mathbf{F}_p\mathbf{h}_l-\hat{\mathbf{H}}_l) \tag{5b}$$

where $M\leq P$, $\mathbf{h}_l$ and $\hat{\mathbf{H}}_l$ are the column vector of $h_l[n]$ and $\hat{H}_l[k_P]$, respectively, forming $M\times1$and $P\times1$, $\mathbf{F}_p$ is a $P\times M$ partial Fourier transformation matrix, and the element thereof is $e^{-j2\pi k_p n/N}$. The LS method is adopted to minimize E_l to obtain the estimated channel impulse response $\hat{\mathbf{h}}_l$:

$$\hat{\mathbf{h}}_l=(\mathbf{F}_p^H\mathbf{F}_p)^{-1}\mathbf{F}_p^H\hat{\mathbf{H}}_l \tag{6}$$

In actual hardware realization, $(\mathbf{F}_p^H\mathbf{F}_p)^{-1}\mathbf{F}_p^H$ is unrelated to the noise energy or statistical properties of the channel, and it is a fixed matrix when P and M are determined by the system, which can be calculated in advance and stored in the memory. When in time-variant channel, the channel impulse response can be estimated by (6) on each OFDM symbol. Equation (6) is deduced, as follows:

$$\hat{\mathbf{h}}_l=\mathbf{R}^{-1}\mathbf{F}_p^H\mathbf{H}_l+\mathbf{R}^{-1}\mathbf{F}_p^H\mathbf{V}_l \tag{7}$$

where, $\mathbf{R}=\mathbf{F}_p^H\mathbf{F}_p$ is the square matrix of size $M\times M$, $\mathbf{H}_l$ is the column vector of $P\times1$, as composed by $H_l[k]$, and the ideal channel impulse response is $\mathbf{h}_l=\mathbf{R}^{-1}\mathbf{F}_P^H\mathbf{H}_l$. As $\mathbf{F}_p$ is a $P\times M$ partial Fourier transformation matrix, $\mathbf{R}$ is not a diagonal matrix. When M is bigger, $\mathbf{R}$ will become a matrix close to singular; the last item in (7) is an error term caused by noise. The closer $\mathbf{R}$ is to singular, the bigger the error energy is. In [11], it is ordered that $M=P$; however, in some actual applications, the maximum channel delay will be not too large. The smaller M is, the less the computation will be, and the slighter the nearly singular problem will be.

The average error energy E_v, as caused by the last term in (7), is shown as follows:

$$\begin{aligned}E_v&=E[(\mathbf{R}^{-1}\mathbf{F}_P^H\mathbf{V}_l)^H\mathbf{R}^{-1}\mathbf{F}_P^H\mathbf{V}_l]\\ &=E[\mathbf{V}_P^H\mathbf{F}_P(\mathbf{R}^{-1})^H\mathbf{R}^{-1}\mathbf{F}_P^H\mathbf{V}_l]\end{aligned} \tag{8}$$

E_v is a positive real value, thus, (8) can be deduced, as follows:

$$\begin{aligned} E_v &= \mathrm{tr}[E_v] \\ &= \mathrm{tr}\{E[\mathbf{V}_l^H \mathbf{F}_P (\mathbf{R}^{-1})^H \mathbf{R}^{-1} \mathbf{F}_P^H \mathbf{V}_l]\} \\ &= E[\mathrm{tr}\{(\mathbf{R}^{-1})^H \mathbf{R}^{-1} \mathbf{F}_P^H \mathbf{V}_l \mathbf{V}_l^H \mathbf{F}_P\}] \\ &= \mathrm{tr}\{(\mathbf{R}^{-1})^H \mathbf{R}^{-1} \mathbf{F}_P^H E[\mathbf{V}_l \mathbf{V}_l^H] \mathbf{F}_P\} \\ &= \sigma_v^2 \mathrm{tr}\{(\mathbf{R}^{-1})^H\} \end{aligned} \tag{9}$$

where, $\sigma_v^2 = E[|V_l[k_P]|^2]$, $\mathbf{R}$ is an $M\times M$ Toeplitz matrix. In the FDLS estimation method, when the M value is larger, $\mathbf{R}$ is very close to singular; thus, the value of $\mathrm{tr}\{(\mathbf{R}^{-1})^H\}$ will be too large. It can be discovered from (9) that, the nearly singular problem will cause the LS estimation method to be very sensitive to noise; therefore, the estimation effect becomes poor. The following is a simple example to illustrate this problem.

Example 1: The related simulation parameters are set, as follows: FFT size N=2048, signal is QPSK modulation, consider comb-type pilot arrangement, pilot interval N_p=16, guard band is the region from the 849th to 1199th sub-carriers, CP length is 128, the total number of pilots is P=107, thus, theoretically, M can reach 107 at maximum. Aiming at different M values, we can obtain the following data: when M=30, $\mathrm{tr}\{(\mathbf{R}^{-1})^H\}$=3.5732e+003; when M=60, $\mathrm{tr}\{(\mathbf{R}^{-1})^H\}$=2.6780e+011; when M=70, $\mathrm{tr}\{(\mathbf{R}^{-1})^H\}$= 2.4336e+014, basically, $\mathbf{R}$ is closer to singular with the increased M, and sensitivity to noise of the LS estimation method increases with M.

III. Method to Overcome the Nearly Singular Problem of Matrix in FDLS Estimation Method

In [11], the method to overcome $\mathbf{R}$ closeness to singular is given based on the characteristics in the following formula:

$$(\alpha\mathbf{I}+\mathbf{R})^{-1}\mathbf{R}\approx\mathbf{I} \tag{10}$$

where, $0<\alpha<<\mathbf{R}(0,0)=M$, with a proper α, $(\alpha\mathbf{I}+\mathbf{R})$ will be free of the nearly singular problem; thus, reference [4] offers the following formula:

$$\hat{\mathbf{h}}_l = (\alpha\mathbf{I}+\mathbf{R})^{-1}\mathbf{F}_P^H \mathbf{F}\hat{\mathbf{H}}_l \tag{11}$$

Equation (11) is used to replace Equation (6) to estimate the channel impulse response. In [4], although there is an equation for α selection, there is no detailed instruction.

Let the average error energy of the estimated channel impulse be E_l, which is calculated, as follows:

$$E_l = E[(\hat{\mathbf{h}}_l - \mathbf{h}_{li})^H (\hat{\mathbf{h}}_l - \mathbf{h}_{li})] \tag{12}$$

where, $\mathbf{h}_{li}$ is the ideal channel impulse response on the lth OFDM symbol. Substitute (4) and (11) into (12) to obtain (13):

$$\begin{aligned} E_l = E[&(\mathbf{R}_n^{-1}\mathbf{F}_P^H \mathbf{H}_l + \mathbf{R}_n{}^{-1}\mathbf{F}_P^H \mathbf{V}_l - \mathbf{h}_{li})^H \\ &\times (\mathbf{R}_n^{-1}\mathbf{F}_P^H \mathbf{H}_l + \mathbf{R}_n{}^{-1}\mathbf{F}_P^H \mathbf{V}_l - \mathbf{h}_{li})] \end{aligned} \tag{13}$$

where, $\mathbf{R}_n = (\alpha\mathbf{I}+\mathbf{R})$, substitute $\mathbf{F}_P^H\, \mathbf{H}_l = \mathbf{R}\mathbf{h}_{li}$ into (13) to further simplify, shown as follows:

$$\begin{aligned} E_l &= E[(\mathbf{R}_n^{-1}\mathbf{R}\mathbf{h}_{li} + \mathbf{R}_n^{-1}\mathbf{F}_P^H \mathbf{V}_l - \mathbf{h}_{li})^H \\ &\qquad \times (\mathbf{R}_n^{-1}\mathbf{R}\mathbf{h}_{li} + \mathbf{R}_n^{-1}\mathbf{F}_P^H \mathbf{V}_l - \mathbf{h}_{li})] \\ &= E[((\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})\mathbf{h}_{li})^H (\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})\mathbf{h}_{li}] \\ &\quad + E[(\mathbf{R}_n^{-1}\mathbf{F}_P^H \mathbf{V}_l)^H \mathbf{R}_n^{-1}\mathbf{F}_P^H \mathbf{V}_l] \\ &= E[\mathbf{h}_{li}^H (\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})^H (\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})\mathbf{h}_{li}] \\ &\quad + E[\mathbf{V}_l^H \mathbf{F}_P (\mathbf{R}_n^{-1})^H \mathbf{R}_n^{-1}\mathbf{F}_P^H \mathbf{V}_l] \end{aligned} \tag{14}$$

By using the characteristics of tr(**AB**)=tr(**BA**), the above equation is simplified, as follows:

$$\begin{aligned} E_l &= \mathrm{tr}(E[\mathbf{h}_{li}^H (\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})^H (\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})\mathbf{h}_{li}]) \\ &\quad + \mathrm{tr}(E[\mathbf{V}_l^H \mathbf{F}_P (\mathbf{R}_n^{-1})^H \mathbf{R}_n^{-1}\mathbf{F}_P^H \mathbf{V}_l]) \\ &= \mathrm{tr}(E[(\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})^H (\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})\mathbf{h}_{li}\mathbf{h}_{li}^H]) \\ &\quad + \mathrm{tr}(E[\mathbf{F}_P (\mathbf{R}_n^{-1})^H \mathbf{R}_n^{-}\mathbf{F}_P^H \mathbf{V}_l \mathbf{V}_l^H]) \\ &= \mathrm{tr}((\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})^H (\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})E[\mathbf{h}_{li}\mathbf{h}_{li}^H]) \\ &\quad + \mathrm{tr}(\mathbf{F}_P (\mathbf{R}_n^{-1})^H \mathbf{R}_n^{-1}\mathbf{F}_P^H E[\mathbf{V}_l \mathbf{V}_l^H]) \\ &= \mathrm{tr}((\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})^H (\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})E[\mathbf{h}_{li}\mathbf{h}_{li}^H]) \\ &\quad + \sigma_v^2 \mathrm{tr}(\mathbf{F}_P (\mathbf{R}_n^{-1})^H \mathbf{R}_n^{-1}\mathbf{F}_P^H) \end{aligned} \tag{15}$$

where, σ_v^2 is the noise average energy of a subcarrier on the frequency domain, and such item is easily estimated at the receiver; in addition, when each path of the channel is independent, and the individual mean is 0, $E[\mathbf{h}_{li}\mathbf{h}_{li}^H]$ is a diagonal matrix, and if such item is known, we can draw the curve of α to E_h to determine the best α value to minimize E_h. However, the statistical characteristics of the channel must be known for such an item, and there is no simple method to estimate $E[\mathbf{h}_{li}\mathbf{h}_{li}^H]$. In [11], as the statistical characteristics of the channel must be known for calculating α, the method in [11] is very hard to realize in the receiver.

In (15), E_l is the sum of two items. The former equation $\mathrm{tr}((\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})^H (\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})E[\mathbf{h}_{li}\mathbf{h}_{li}^H])$ is the error item caused by the substitution of $\mathbf{R}$ with $(\alpha\mathbf{I}+\mathbf{R})$; thus, it is obvious that when α=0, such part will be equal to 0, and the larger α is, the larger the value of such part will be. The latter equation $\sigma_v^2\mathrm{tr}(\mathbf{F}_P (\mathbf{R}_n^{-1})^H \mathbf{R}_n^{-1}\mathbf{F}_P^H)$ is caused by noise and related to α, when α is closer to 0, $\mathbf{R}_n$ will be closer to a singular matrix, and the value of such item will be much larger. However, when α slowly becomes larger, as $\mathbf{R}_n$ will reduce the nearly singular phenomenon, the value of such item will be correspondingly smaller. It is discovered from the system simulation in (15) that, as α gradually increases from 0, the value increase of the former equation will be much slower than the value decrease of the latter equation.

As (15) cannot be directly applied, this study discusses an approximate equation to replace (15). First, it is ordered that tr{$E[\mathbf{h}_{li}\mathbf{h}_{li}^{H}]$} is A, that is, A= $\sum_{n=0}^{M-1} E[|h_{li}[n]|^2]$, when the average energy of the transmitted data on all sub-carriers (including the pilot) is 1, the average energy of received signals $y_l[n]$ at receiver in time domain is $N_d A/N^2$, where, N_d is the sum of the number of the data sub-carrier and the pilot sub-carrier; in addition, the average energy of noise in time domain is $\sigma_w^2 = \sigma_v^2 / N$; therefore, the signal-to-noise ratio SNR is obtained, as follows:

$$\begin{aligned}\mathrm{SNR} &= 10\log_{10}(E[|y[n]|^2]/\sigma_w^2) \\ &= 10\log_{10}(N_d \mathrm{A}/(N\sigma_v^2))\end{aligned}$$

Thus, $\sigma_v^2 = N_d \mathrm{A} 10^{-\mathrm{SNR}/10} / N$. Then, it is ordered that the channel impulse response of normalized energy $h_{lN}[n] = h_{li}[n]/\mathrm{A}^{0.5}$; then Eq. (15) can be simplified, as follows:

$$\begin{aligned}E_l = \mathrm{A}\{&\mathrm{tr}((\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})^H(\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})E[\mathbf{h}_{lN}\mathbf{h}_{lN}^H]) \\ &+ N_d 10^{-\mathrm{SNR}/10}\mathrm{tr}(\mathbf{F}_p(\mathbf{R}_n^{-1})^H\mathbf{R}_n^{-1}\mathbf{F}_p^H)/N\} \quad (16)\end{aligned}$$

In (16), A is independent on the value of α which can minimize E_l. Therefore, we only need to observe the item in brace, and the value in brace is affected by α, $E[\mathbf{h}_{li}\mathbf{h}_{li}^H]$ and SNR; as SNR can be simply estimated at the receiver, we hope to draw the curve of $E_l(\alpha)$ when SNR is fixed at a certain value, in order to determine the α value, which can minimize E_l. Although $\mathrm{tr}((\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})^H(\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})E[\mathbf{h}_{lN}\mathbf{h}_{lN}^H])$ cannot be obtained, as the channel statistical characteristics are unknown at the receiver, this study discovered that, if $\mathrm{tr}((\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})^H(\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})/M)$ is used instead, a suitable substitution will be achieved; thus, we write (17), as follows:

$$\begin{aligned}\hat{E}_l &= \mathrm{tr}((\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I})^H(\mathbf{R}_n^{-1}\mathbf{R}-\mathbf{I}))/M \\ &\quad + N_d 10^{-\mathrm{SNR}/10}\mathrm{tr}(\mathbf{F}_P(\mathbf{R}_n^{-1})^H\mathbf{R}_n^{-1}\mathbf{F}_P^H)/N \quad (17)\end{aligned}$$

It is discovered from system simulations that, when we draw the curves of $E_l(\alpha)$ and $\hat{E}_l(\alpha)$ under the condition of a fixed SNR, the α values that minimize $E_l(\alpha)$ and $\hat{E}_l(\alpha)$ are very close, and there is an example to verify this point in the next section.

IV. Simulation and Discussion

A. Time-Invariant Channel Simulation

In the simulation condition of example 1, we use the exponential decay channel for simulation. As the exponential decay channel is time-invariant channel, the channel frequency response can be determined by the channel impulse response. Such a 'perfect" channel frequency response is taken as the best boundary curve, and is compared with the method proposed in this study. The exponential decay channel model is shown, as follows:

$$h[n] = N(0,\frac{\sigma_n^2}{2}) + jN(0,\frac{\sigma_n^2}{2}),\ \text{where } \sigma_n^2 = e^{-n\tau} \quad (18)$$

where, τ is the attenuation parameter, $N(0,\sigma_n^2/2)$ is the Gaussian distribution with zero mean and a variance $\sigma_n^2/2$; here, we order τ=0.05, the maximum channel delay is 30 sampling points; however, as the maximum channel delay is unknown at the receiver, in the simulation, M is much larger than 30; besides, 100 channels have been generated randomly with the exponential decay channel model, and only one OFDM symbol is transmitted in each channels in simulation. The energy of these 100 channels is normalized to 1, that is A=1 in (16), and then, $E[\mathbf{h}_{lN}\mathbf{h}_{lN}^H]$ in (16) is estimated by the time average with these 100 channels, and the estimated channel impulse response will be substituted into (12) to estimate E_l with time-averaging. Fig. 1 shows the curves of α versus (12), (16), and (17), where it can be discovered that (12) and (16) are almost overlaid, which can prove that the derivation of (16) is correct. In addition, we observe from Fig. 1. The α value at the lowest point of the curve (17) is very close to the ones of the curves (12) and (16), which indicates that it is feasible to replace (16) with (17) to determine the proper α value. It is discovered in our simulation that, the value of the second item in (16) will reduce rapidly and dominate the curve as α slowly increases from 0. Thus, the value of the first item will slowly increase and dominate the curve for 0.05**R**(0,0)<α< **R**(0,0). Such phenomenon can be seen from Fig. 1.

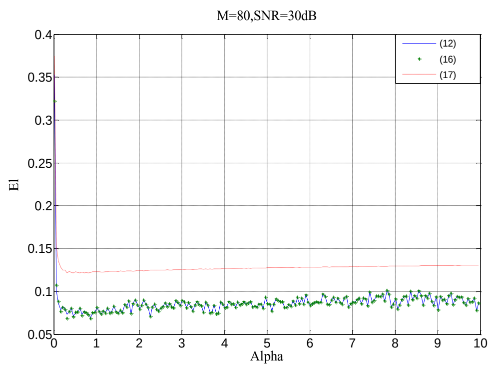

Fig. 1. Three simulation curves of (12), (16), and (17) when M=80 and SNR=30dB.

Next, this study simulates the BER curve for different α. Regarding the above simulation conditions, the number of OFDM symbols transmitted each time is changed to 100, while others are kept unchanged. Fig. 2a and Fig. 2b are given when SNR=30dB. It can be seen from Fig. 2a that, when α slowly increases from 0, BER initially reduces rapidly, while when α increases a little, BER almost reaches a stable state; at this time, although α increases, BER reduces very slowly. It can be seen from Fig. 2b that, we can find minimum BER at α=0.04, which is consistent with the result shown in Fig. 1. Near the position of α=0.04, the BER values are close. During simulation, BER curve may fluctuate up and down a little, and even the simulation of the 100-run average is based on different channels. Fig. 3a and Fig. 3b are given when SNR=10dB, with a phenomenon similar to Fig. 2a and Fig. 2b. The BER values are close near the position of α=1, which is influenced by noise in each simulation, and even though the simulation of the 100-run average has been conducted, it is difficult to correctly determine the α value which minimizes BER. Therefore, in actual application, it is less important to determine the best α value. It is good enough to estimate the best α value roughly. Therefore, it is able to determine the best α value under several specific SNR (for example, every 5dB) by using (17), and then determine the corresponding $(\alpha\mathbf{I}+\mathbf{R})^{-1}$. In the receiver, such inverse matrixes can be stored in the memory, and the LS estimation method can be realized by estimating the SNR and adopting the corresponding inverse matrix.

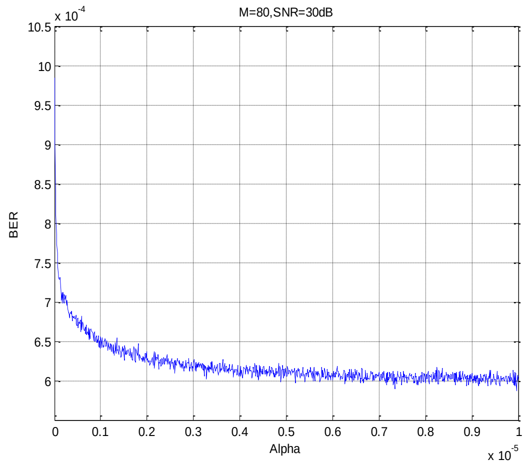

Fig. 2a. BER curve for different α in LS method when M=80 and SNR=30dB.

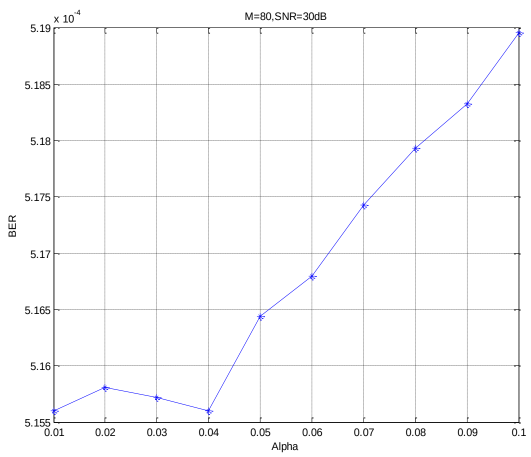

Fig. 2b. BER curve for different α in LS method when M=80 and SNR=30dB.

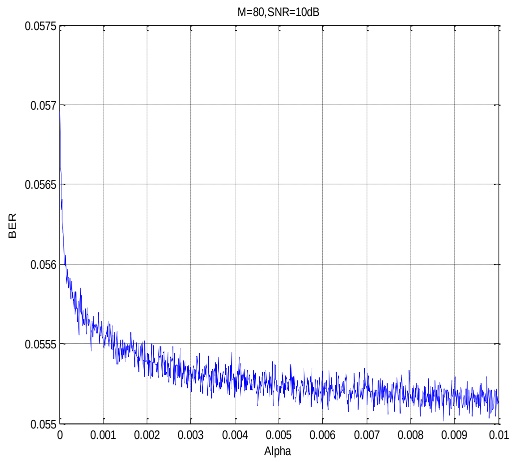

Fig. 3a. BER curve for different α in LS method when M=80 and SNR=10dB.

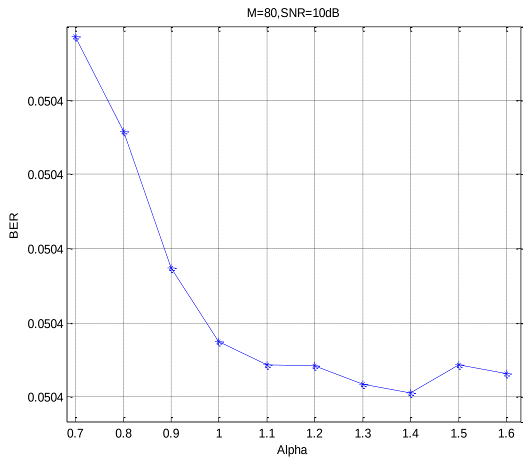

Fig. 3b. BER curve for different α in LS method when M=80 and SNR=10dB.

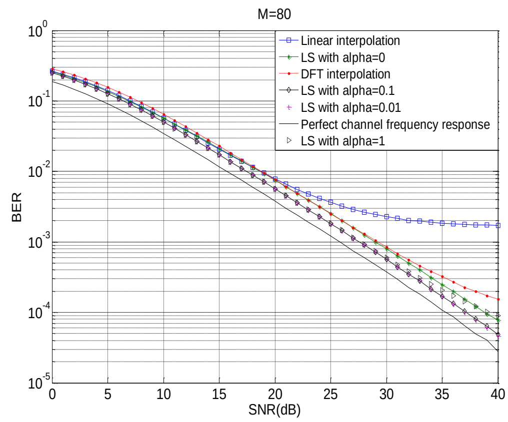

Fig. 4. BER curves for different SNR when M=80, with linear interpolation, perfect channel frequency response, and LS method.

Fig. 4 shows the curves of BER for different SNR; with the simulation results of one-dimensional linear interpolation, perfect channel frequency response, and the FDLS method, in which there are four conditions: α=0, 0.01, 0.1, and 1, and it is obvious that the BER curve of α=0 is the worst of the four, which is mainly because the noise is over amplified due to the matrix being close to singular. In Fig. 4, when SNR is higher, the BER curve of α=1 will become worse, which is mainly caused by errors in first item in (16). Therefore, the overall simulation trend is in line with the previous discussion.

B. Time-Variant Channel Simulation

Simulation environment: carrier frequency f_c=862MHz, sampling frequency f_s=8MHz, relative speed v=40km/hr.

Channel model: six-path (Jakes model [16]) are positioned at 0, 1, 3, 12, 17, 39 sampling points, with powers equal to 0.1, 0.63, 1, 0.5, 0.1, 0.01, respectively. Twenty channels have been randomly generated with the above channel model for different SNR values. Other simulation parameters, such as FFT size and modulation are same as those given in example 1. Fig. 5 shows the curves of BER for different SNR under the time-variant channel, according to the simulation results of one-dimensional linear interpolation and the FDLS estimation method, in which, α has four conditions, including α=0, α=0.01, α=0.1, and α=1. Although channel model is different from that in example 1, the trend of simulation results given in Fig. 5 is same as that in Fig. 4. It indicates that the approximate formula in Eq. (17) is feasible in actual application.

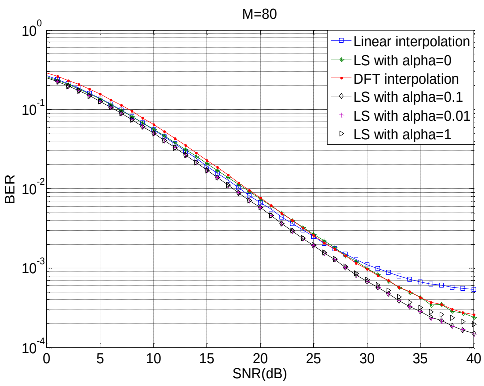

Fig. 5. BER curves for different SNR when M=80 under time-variant channel, with linear interpolation and LS method.

V. Conclusion

In the LS estimation method, there exists the problem that the matrix would be close to singular. We have carried out detailed mathematics discussion on this problem, discussed key parameter α in [11], and put forward (17) an approximate formula to determine the proper α value under different SNRs, in order to solve the problem of singular matrix in the LS estimation method. The feasibility of (17) is verified in time-variant and time-invariant channel simulations. Therefore, only when the SNR value is estimated at the receiver, we can use the LS estimation method to conduct efficient channel compensation.

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