Efficient and Reliable Communication in Wireless Relay Networks Using Joint Network Channel Fountain

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Abstract —In the past few years, joint network-channel coding (JNCC) has drawn significant attention for reliable data communication in wireless network. However, it appears that fixed-rate channel coding is ineffective to make the outage probability to zero without having precise channel state information at the transmitter. To enhance the link robustness and the system throughput, this paper presents a joint network-fountain coding (JNFC) scheme which can effectively combat the detrimental effect of wireless fading channel by seamlessly coupling fountain and network paradigms. In particular, we consider a cooperative system with two sources, two relays and one destination where the sources encode the message using fountain code and broadcast to the destination and relays. While the relays first decode the information and then transmit to the destination after network and fountain coding. For information combining at relays we employ Random Linear Network Coding (RLNC) and Modified LT coding (MLT). Simulation results justify that JNFC has significant performance advantage over other schemes and JNFC with Modified LT coding (JNFC-MLT) always outperforms JNFC with Random Linear Network Coding (JNFC-RLNC) in a variety of metrics regardless of network scenarios.

Index Terms—Cooperative communications, fountain codes, network coding

I. Introduction

Cooperative communication is an effective way to improve the throughput, link reliability, power efficiency, and coverage in wireless networks. It utilizes the broadcast nature of wireless channel by considering the neighboring nodes as relays and allows them to transmit the overheard information to the destination. Destination thus receives multiple replicas of the signals from independent fading paths and achieves diversity even though it is equipped with a single antenna [1]. However, the resulting increase diversity comes at the cost of a loss of spectral efficiency. Specially in large networks this relaying strategy becomes bandwidth inefficient due to the allocation of orthogonal channels [2], [3] to different terminal, i.e., inter-user orthogonality.

To overcome this bandwidth bottleneck, network coding technique where coding is employed at the correlative nodes has gained a lot of research attention since its introduction in [4] and has been investigated as a desirable solution for data communication in wireless networks [5]-[7]. Network coding over Galois fields (GFs) is an efficient approach to increase the throughput of multi-source cooperative diversity systems [5], [6]. Several other network coding schemes have been proposed for general multi–source cooperative networks, such as physical layer network coding (PNC) for two way relaying [8] and complex field network coding (CFNC) [9]. Through analysis and simulation, it has been shown that network coding improves link robustness and system capacity significantly.

In recent years, a lot of research efforts have been devoted to unifying network coding with channel coding schemes [10],[11], that shows great potential in deteriorating the detrimental effects of wireless fading channel. The idea behind this is to couple network and channel coding techniques simultaneously in the physical layer so that the redundancy in the network code should be used to support the channel code for better error protection. In [12], to obtain additional diversity gain C. Hausl et al. proposed joint network and channel coding framework for multiple access relay channel (MARC) based on low-density parity-check (LDPC) code. Later in [13], turbo codes based joint network-channel coding was applied to the two-way relay channel [14]. Zheng et al. developed non binary joint network-channel decoding (NB-JNCD) for large networks [15]. It has been shown that NB-JNCD outperforms binary LDPC JNCD. However, these fixed rate codes provides a stable error performance when the channel environment is time-invariant. The decoding failures may occur when the channel degradation exceeds the error-correction capability of the codes specially in the time varying channel. If that occur, an acknowledgement (ACK) is sent to sender after every detection of corrupted message at receiver that increases the end to end delay over heavily impaired channel.

In contrast to typical fixed rate code, fountain code [16]–[20] is a rateless version where the source unconscious of channel state information (CSI) can generate as many encoding symbols as needed by simply performing modulo-2 operation among the source symbols. For its capability in improving the link robustness and reliability [21]–[24], it has been

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incorporated into cooperative relay systems as a desirable solution for data communication. However, there are not many works coupling rateless code and network coding for the multiple access relay system, as opposed to the above works based on fixed-rate code. In [25], Puducheri et al proposed a low complexity combining operation at relays for a multiple access relay system where messages from M sources are encoded by a rateless code that performs like a LT code in data recovery and completes the network coding inherently. In this work, source nodes generate their information using Deconvolved Soliton Distribution (DSD) as in degree distribution and transmit to relay. Relay node then constructs LT codes by merely XOR-ing the received symbols from different sources and transmits to destination. The performance is evaluated in Binary Erasure Channel (BEC) without considering the direct link between source and destination. Later for a multiple access relay system, in [26] Gong et al proposed raptor code based two combining schemes, namely, Raptor coding (RC) and Superposition coding (SC) at the relay that improves the performance gain significantly.

In this paper, we propose a joint network and fountain coding (JNFC) scheme for the cooperative diversity system of two sources and two relays. In our framework, sources generate their data and send to relays and destination using LT codes. We employ the decode-and-forward (DF) strategy at relays where the relays first try to decode the information of both sources. After successfully decoding sources information, relays combine the information from both sources using network and fountain coding and transmit the combined information to the destination. For information combining at relays, we propose two JNFC schemes, namely, JNFC with Random Linear network coding (JNFC-RLNC) and JNFC with modified LT coding (JNFC-MLT) and study their performance in different network scenarios. Moreover, we compare the performance of JNFC with separated network and fountain coding (SNFC) where the redundancy provided by network coding is only useful if channel coding is succeeded.

The rest of the paper is organized as follows. We first introduce the proposed JNFC framework using a two-source two-relay network and describe some preliminaries for the proposed JNFC scheme in Section III. We then present network coding on LT codes with robust solution distribution in Section IV and encoding and decoding schemes of proposed JNFC in Section V. Finally, Section VI demonstrates the simulation results, followed by Section VII to conclude our paper.

II. A Two-Source Two-Relay Network

A. System Model

We consider a five-node cooperative system as depicted Fig. 1, where two sources, i.e., source S_1 and S_2 communicate to a destination D with the help of two relays, i.e., R_1 and R_2.

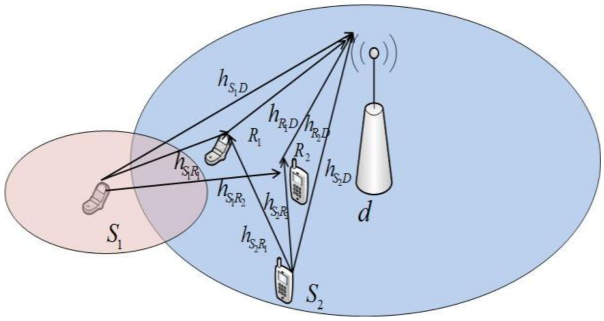

Fig. 1. A two-source two-relay cooperative system.

We assume that all nodes are operated in half-duplex mode, i.e., they can either transmit or receive at a time. Each node in the system adopts binary phase shift keying (BPSK) modulation, where bits 0 and 1 are mapped to +1 and −1 respectively. The channel coefficient of each link is given by $h_{i,j} = \frac{g_{i,j}}{d_{i,j}^{\alpha/2}}$ where $i, j \in \{R_1, R_2, S_1, S_2, D\}$ and $j \neq i$, $g_{i,j}$ represents the small-scale fading gain, $d_{i,j}$ denotes the distance of each node pair, and α is the path loss coefficient. Each $h_{i,j}$ is assumed static during the transmission. The channel quality of the relay link is assumed to be better than the direct link, i.e., $\frac{h_{i,j}}{\sigma_j^2} \geq \frac{h_{i,D}}{\sigma_j^2}$ and $\frac{h_{j,D}}{\sigma_j^2} \geq \frac{h_{i,D}}{\sigma_j^2}$ where $i = S_1, S$ and $j = R_1, R_2$. The transmit power of each transmit node is restricted to $P_i, i = S_1, S_2, R_1, R_2$ and the noise at each receive node is assumed to be white gaussian with a variance of $\sigma_j^2, j = R_1, R_2, D$. The signal-to-noise ratio (SNR) at j for a pair $j-i$ is given by

$$SNR_{j,i} = \frac{P_i \left|h_{j,i}\right|^2}{\sigma_j^2} \tag{1}$$

where P_i denotes the transmitted power of node i.

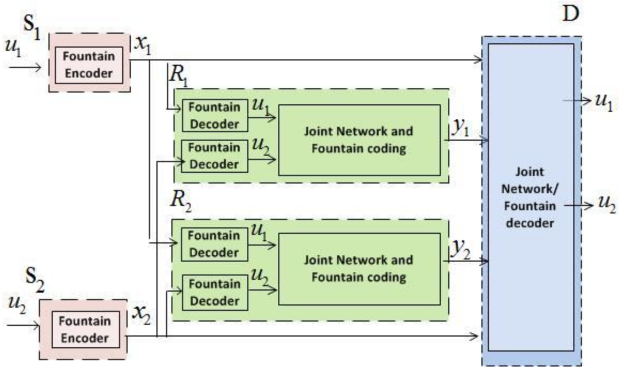

Fig. 2. Block diagram of joint network and fountain coding system.

We assume that orthogonal channels [2], [3], [27] are allocated to different terminals i.e., inter-user orthogonality. Each source transmits to the destination and relays in two different channel and the relays transmit to the destination in other different orthogonal channels. To transmit a k bit message to a destination D, each source generates a large number of code stream using fountain codes. The code stream is then modulated and sequentially transmitted to the destination and relays. In Phase one, both the destination and relays make decoding attempts to recover source messages. Since the relay link is better than that of the source destination link, the relays can almost always successfully decode before the destination does. As soon as the original packets are successfully decoded at relays, they generate new packets using network and fountain coding and send to the destination to provide additional error protection in Phase 2. A block diagram of the system is depicted in Fig. 2. The transmission of one codeword over the relay channel can be divided into two phases: Phase 1 when the relays listen and Phase 2 when the relays transmit.

B. *Network Scenario*

Depending on the operation mode of sources in Phase 2, we consider the following network scenarios:

Scenario A: In this scenario, sources are aware about the existence of relay nodes. Using LT codes, source nodes generate a large number of symbols and broadcast their data to relays and destination until they receive ACK from both relays. After successfully decoding the information of both sources, relays generate new packets from original received packets using network coding and fountain coding. Sources then stop their transmission and relays start to transmit to the destination as shown in Fig. 3 until the destination is able to decode all information.

Scenario B: In this case source nodes transmit their information to destination using LT codes. Relays overhear the direct transmission between source nodes and destination. After successfully decoding information of both source nodes, relays generate new coded symbols to transmit to destination. Both source nodes and relays keep on transmitting until they receive an acknowledgment from the destination indicating that the reception has been successful as shown in Fig. 4.

III. EVOLUTION FROM SNFC TO JNFC

In this section, we first introduce the separated network and fountain coding (SNFC) system that will be used as a reference system to compare the performance of JNFC. We also present background of fountain coding in particular LT coding, and network coding that are used in our proposed JNFC.

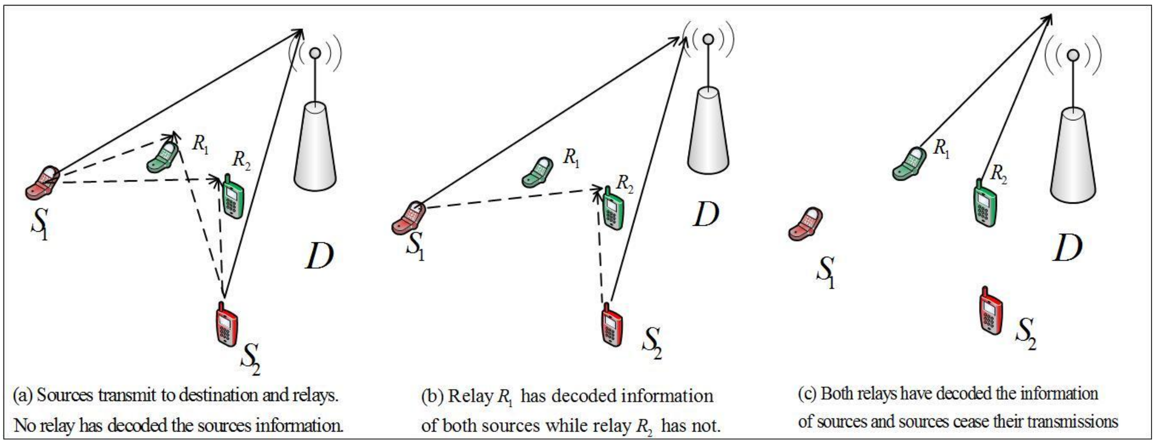

Fig. 3. Transmission phases in network scenario A.

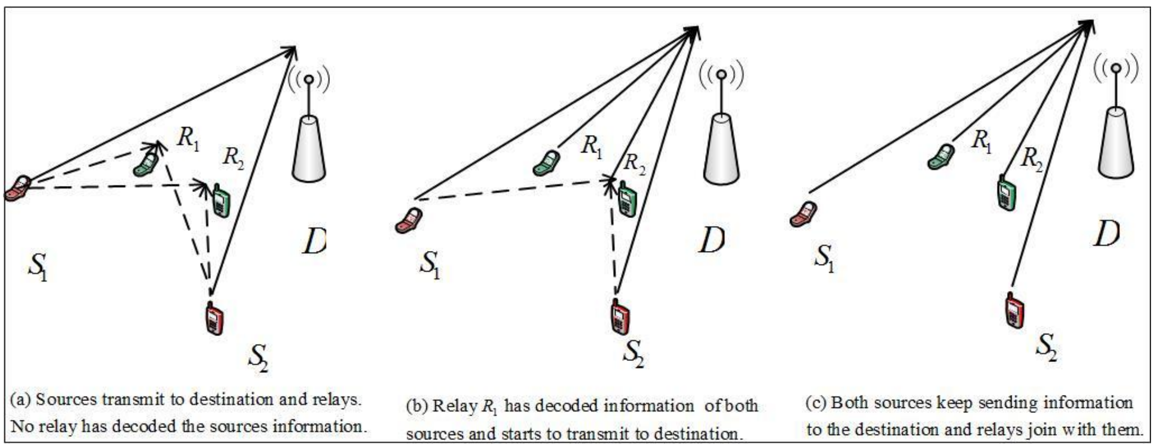

Fig. 4. Transmission phases in network scenario B.

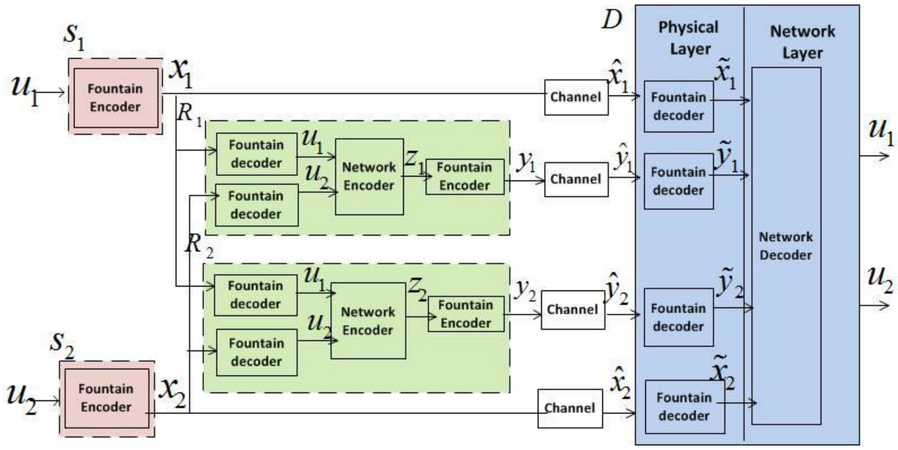

Fig. 5. Block diagram of separated network and fountain coding.

A. Separated Network and Fountain Codes (SNFC)

Spatial diversity is one of the ways to combat fading over wireless channels. One way to gain diversity through network coding in noisy channels is to treat network and channel coding separately where channel coding is used in the physical layer for each transmission then on the network layer network coding is performed on the error free packets provided by the lower layers.

Fig. 5 presents the block diagram of SNFC system where source nodes, S_1 and S_2 encode their data packets u_1 and u_2 respectively using fountain codes and transmit to destination. Relay nodes keep accumulating information from source to destination transmission. As soon as relay Ri decodes sources information correctly, it performs network coding on the original packets u_1 and u_2 at network layer. The network encoder is a modulo-2 addition. Then the output of network encoder z_i is encoded using fountain code and transmitted to destination. The destination collects information from four channels and starts to decode when the received information from each channel is slightly greater than the original symbols. Let destination starts decoding on the four set of symbols $\hat{x}_1, \hat{x}_2, \hat{y}_1$ and $\hat{y}_2$ using fountain decoder. The four decoders make a hard decision and deliver their estimates $\tilde{x}_1, \tilde{x}_2, \tilde{y}_1$ and $\tilde{y}_2$ to the network layer with an indication to indicate whether its estimate is error-free. If one of the two estimates $\tilde{x}_1$, or $\tilde{x}_2$ not error-free and either $\tilde{y}_1$ or $\tilde{y}_2$ are error-free, the network decoder retrieves the corrupted packet by performing module-2 addition between the error free packet among $\tilde{x}_1$, or $\tilde{x}_2$, and the error free packet among $\tilde{y}_1$, or $\tilde{y}_2$. If $\tilde{y}_1$ and $\tilde{y}_2$ both are error free then any one of them is used to retrieve the corrupted packet.

In wireless relay networks, capacity can only be achieved by treating network and channel coding jointly [28]. In SNFC, the redundancy of network coded packets only utilized when the lower layers deliver error-free packets to the network layer. The principle of joint network fountain coding is to efficiently use the redundancy in the network code to support the channel code for better error protection.

B. LT: Encoding and Decoding

The concept of fountain codes was first presented by Byers *et al*. [19]. The most interesting benefit of fountain code is that transmission reliability can be assured without requiring channel state information. Although fountain codes were originally designed for erasure channels, a lot of effort has been dedicated to their extension to general discrete memoryless channels, additive white Gaussian noise (AWGN) channels, and fading channels. A universal fountain codes should have two properties: First, infinite encoded data can be generated from finite source data. Second, the receiver should be able to reconstruct the source data from any set of the encoded data with an efficient decoding process. However, not all fountain codes in use can meet the above two properties. LT codes [17] are the first realization of digital fountain codes.

In LT coding, the symbol length for the code can be arbitrary, from one bit binary symbols to general l-bit symbols. Each LT code symbol is generated by the following encoding process:

a. First a degree d is chosen for an encoding symbol. The degree is chosen randomly from a given degree distribution $\rho(d)$.

b. Choose d distinct information symbols uniformly at random. They will be neighbors of the encoding symbol.

c. Then chosen original symbols are XOR-ed to create an encoded symbol.

The canonical representation of fountain codes are factor graphs [29]. A factor graph is a bipartite graph where nodes in the first set represent original symbols referred as input symbols and the nodes in the second set are the encoded symbols referred as output symbols. The input symbol c is a neighbor of output symbol, v if there is an edge between them. The degree of an output symbol is the number of edges originated from that particular node. A factor graph representation of encoded symbols is shown in Fig. 6 that is truncated to length n. The decoder can recover information symbols with the following three-step process, which is called LT process:

- At first step, the decoder identifies all output symbols of degree one (i.e., those connected to a single input symbol) in the Tanner graph. If there exist no such symbol, the decoding process terminates.
- The input symbols connected to output symbols of degree one are directly decoded and the edges between them is deleted.
- Finally, each the decoded input symbol c is XORed with the every output symbol v to which c is connected and the edge between c and v is deleted.

The decoding process continues iteratively by following the above three steps. The decoding process succeeds if all information symbols are covered by the end.

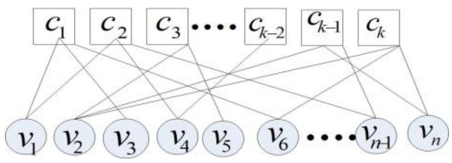

Fig. 6. A factor graph representation of LT code symbols: $v_1 = c_1 \oplus c_2$, v_1 has degree 2 and v_2 has degree 3.

C. Degree Distribution of LT Codes

The probability distribution on the random degree of encoding symbols, $\rho(d)$ is the critical part of the LT codes design to ensure complete recovery of the original data from the minimum number of encoding symbols. In fact, the encoding/decoding complexity and error performance are regulated by the degree distribution of LT code. For better performance, the degree distribution should be such that a small number of encoding symbols must possess high degree, so that all input symbols get connected with output symbols and a large number of output symbols must have low degree, so that the decoding process can get started, and keep going. The optimal distribution of degrees for constructing LT codes is the Robust Soliton Distribution (RSD) [17], proposed by Luby is given bellow:

$$\rho(i) = \frac{\mu(i) + \vartheta(i)}{\beta} \quad \text{for} \quad 1 \le i \le k \tag{2}$$

where

$$\beta = \sum_{i=1}^{k} \left(\mu(i) + \vartheta(i) \right) \tag{3}$$

Here $\mu(i)$, the Ideal Soliton distribution and $\vartheta(i)$ are given by

$$\mu(i) = \begin{cases} \dfrac{1}{k}, & \text{for } i = 1 \\ \dfrac{1}{i(i-1)} & \text{for } 2 \le i \le k \end{cases} \tag{4}$$

$$\vartheta(i) = \begin{cases} \dfrac{R}{ik}, & \text{for} \quad 1 \le i \le \dfrac{k}{R} - 1 \\ R \ln\left(R/\delta\right)/k, & \text{for} \quad \dfrac{k}{R} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

where δ is the allowable failure probability and the parameter R represents the average number of degree one encode symbol and is defined as

$$R = \lambda \ln (k/\delta)\sqrt{k}. \tag{6}$$

It has been shown that for a suitable chosen λ the decoder can recover the data from $n = k\beta = k + \lambda.\sqrt{k}.\ln^2(k\delta)$ encoded symbols with probability at least $1-\delta$ [17]. It is observed that RSD is composed of more than 50% of encoded packets of degree 1 or 2 allowing to bootstrap belief propagation, and an average degree of $\mathrm{O}(\log k)$ resulting in low complexity decoding.

D. Network Coding

In previous studies, it has been shown that random linear network coding is efficient and sufficient [30]-[32]. In RLNC the node linearly combined the received packets using randomly generated coefficients over Galois field $\mathrm{GF}(2^q)$. Let relay receives two set of symbols $x_1(1).....x_1(k)$ and $x_2(1).....x_2(k)$ from two sources. In random linear network coding, relay combines this two sets to compute coded symbols $y_1(1).....y_1(k)$, where $y_1(i)$ is

$$y_1(i) = \sum_{j=1}^{2} g_j . x_j(i) \tag{7}$$

and g_j is coefficient that picked randomly from $\mathrm{GF}(2)$.

IV. Network Coding with Robust Soliton Distribution

One of the attractive features of LT codes is low complexity decoding which is accomplished using Belief Propagation (BP) algorithm that recovers source information k on average $\mathrm{O}\left(k.\log k\right)$ symbol operations. However, the efficiency of BP depends on the statistical properties of encoded symbols degree distribution. Therefore, the degree distribution of the network coded symbols must match RSD to get better performance. More specially, the network node should generate the network coded symbols in such a way so that the structure of LT codes is preserved.

The construction of LT codes from two or more fountain codes is proposed in [25] where each source node encodes its data set onto an LT-like codewords according to a degree distribution $p(.)$ and sequentially transmits to relay node. Relay node then generates new code symbol Y by selectively XOR-ing each pair of symbols it received from S_1 and S_2 then transmits to the destination. The result in sequence of symbols that is referred as a modified LT (MLT) codes follows RSD in degree and has erasure correcting properties similar to those of an LT codes. To determine $p(.)$ the author employed deconvolution of the RSD. In this aim, $\rho(.)$ is split into two distributions $\rho_1(1)$ and $\rho_2(2)$ where

$$\rho_1(i)=\begin{cases}0, & \text{for } i=1\\ \dfrac{\mu(i)+\upsilon(i)}{\beta_1} & \text{for } 2\le i\le \dfrac{k}{R}-1\\ \dfrac{\mu(i)}{\beta_1}, & \text{for } \dfrac{k}{R}\le \mathrm{i}\le \mathrm{k}\end{cases} \tag{8}$$

with the normalization factor β_1 given

$$\beta_1=\sum_{i=1}^{k}\mu(i)+\sum_{i=2}^{k/R-1}\vartheta(i) \tag{9}$$

and

$$\rho_2(i)=\begin{cases}\dfrac{\mu(1)+\vartheta(1)}{\beta_2}, & \text{for } i=1\\ \dfrac{\vartheta\left(\frac{k}{R}\right)}{\beta_2}, & \text{for } i=\dfrac{k}{R}\\ 0, & \text{otherwise,}\end{cases} \tag{10}$$

with the normalization factor β_2 given by

$$\beta_2=\mu(1)+\vartheta(1)+\vartheta(\frac{k}{R}) \tag{11}$$

Finally, the Deconvolved Soliton Distribution (DSD), $p(\cdot)$ is given by [25]

$$p(i)=\gamma.f(i)+(1-\gamma).\rho_2(i), \quad \text{for } 1\le i\le k/2, \tag{12}$$

with the parameter γ where

$$\gamma=\sqrt{\frac{\beta_1}{\beta}} \tag{13}$$

$$f(i)=\begin{cases}\sqrt{\rho_1(2)}, & \text{for } i=1,\\ \dfrac{\rho_1(i+1)-\sum_{j=2}^{i=1}f(j)f(i+1-j)}{2f(1)} & \text{for } 2\le i\le \dfrac{k}{2},\\ 0, & \text{for } \dfrac{k}{2}\le i\le k.\end{cases} \tag{14}$$

The degree distribution of RSD and DSD for δ = .5 and λ = .2 is presented in Table I. It is observed that DSD is dominated by degree 1 while RSD is dominated by degree 2.

TABLE I: THE RSD AND DSD FOR DEGREES $1\le d\le 5$ ($\lambda=0.2$, δ=0.5)

Degree	RSD	DSD
1	.0033	.7028
2	.4915	.1171
3	.1642	.0489
4	.0823	.0271
5	.0495	.0174

Let relay receives two symbols X_1 and X_2 from S_1 and S_2 respectively. Then the network coded symbol Y is generated in the following way [25]:

1) Let d_1 and d_2 are the degree of X_1 and X_2 respectively.

2) The relay generates two independent random variables U_1 and U_2, each uniformly distributed on [0, 1].

3) The relay then generates two binary random variables b_1 and b_2 as follows:

$$b_i=\begin{cases}1, & \text{if }\left(\mathrm{d_i}=1 \text{ and } U_i\le 1-\dfrac{\gamma.f(1)}{p(1)}\right),\\ 1, & \text{if }\left(\mathrm{d_i}=k/R \text{ and } U_i\le 1-\dfrac{\gamma.f(k/R)}{p(k/R)}\right),\\ 0 & \text{otherwise.}\end{cases} \tag{15}$$

4) The relay then transmits the binary random variable Y defined as follows:

$$Y=\begin{cases}X_1\oplus X_2 & \text{if } b_1=b_2=0,\\ X_1, & \text{if } b_1=1 \text{ and } b_2=0,\\ X_2, & \text{if } b_1=0 \text{ and } b_2=1,\\ X_1 \text{ or } \mathrm{X}_2, & \text{if } b_1=1 \text{ and } b_2=1.\end{cases} \tag{16}$$

V. JNFC SCHEME FOR A TWO SOURCE TWO RELAY NETWORK

In this section, we present the encoding and decoding procedures of the proposed JNFC using the topology shown in Fig. 1. We employ LT codes at both sources and the relays.

A. Encoding Scheme at Sources

We assume each source wants to transmit same amount of information to destination D. To transmit a packet u_1 and u_2 with k symbols to D, S_1 and S_2 generate a large number of encoded symbols, $\{x_1(1),x_1(2),....x_1(n)\}$ and $\{x_2(1),x_2(2),....x_2(n)\}$ respectively using LT codes and transmit to the destination and relays using different orthogonal channels. The set of encoded symbols generated by S_1 is given by

$$x_1=u_1G_n^1 \tag{17}$$

where, G_n^1 is the generator matrix of the code symbols that are truncated to length n. Similarly, the set of encoded symbols generated by source S_2 is given by

$$x_2 = u_1 G_n^2 \tag{18}$$

where, G_n^2 is the generator matrix of size $k \times n$.

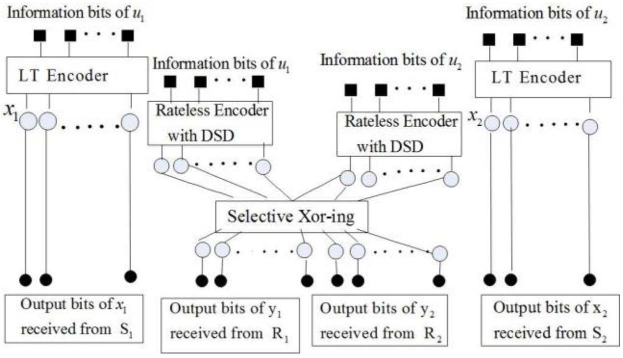

Fig. 7. The decoding graph at the destination D in JNFC-MLT.

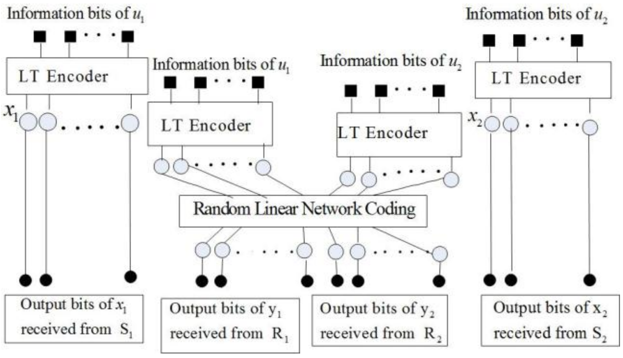

Fig. 8. The decoding graph at the destination D in JNFC-RLNC.

B. JNFC Scheme at Relays

Relays accumulate incoming information from each source destination transmission and attempt to obtain u_1 and u_2 After successfully decoding source symbols, relay nodes generate new codes from u_1 and u_2 and transmit to destination. JNFC at relay nodes is performed by the following ways:

1) JNFC with Modified LT code (MLT): In this scheme, after successfully decoding source packets, relay nodes use DSD-2 [25] to encode each source information. The coded symbols of two sources are then selectively combined in such a way that the result in code symbols follow the degree distribution of LT codes.

2) JNFC with Random linear network coding: After receiving source packets, each relay encodes sources information using LT code. Let $\{x_{i1}(1), x_{i1}(2), ... x_{i1}(n)\}$ and $\{x_{i2}(1), x_{i2}(2), ... x_{i2}(n)\}$ are generated encoded symbols from u_1 and u_2 respectively at relay R_i. Then the output bits of the two LT encoders are bit-wise XORed randomly.

Let y_1 and y_2 are the transmitted symbols from R_1 and R_2 respectively to destination that are represented as

$$y_1 = \alpha_{11} u_1 G_n^3 \oplus \alpha_{12} u_2 G_n^4, \tag{19}$$

$$y_2 = \alpha_{21} u_1 G_n^5 \oplus \alpha_{22} u2 G_n^6, \tag{20}$$

where, the network coding coefficients $\alpha_{ij}(i, j = 1, 2)$ re drawn randomly from GF(2) and the generator matrix $G_n^i (i = 3, 4, 5, 6)$ are assumed to be size $k \times n$. Four packets x_1, x_2, y_1, and y_2 are received at destination. The destination forms a longer code as follows:

$$\begin{bmatrix} x_1 & x_2 & y_1 & y_2 \end{bmatrix} = \begin{bmatrix} u_1 \, u_2 \end{bmatrix} \begin{bmatrix} G_n^1 & 0 & \alpha_{11} G_n^3 & \alpha_{21} G_n^5 \\ 0 & G_n^2 & \alpha_{12} G_n^4 & \alpha_{22} G_n^6 \end{bmatrix} \tag{21}$$

The code in Equation-21 can be viewed as an integrated code with packets and generator matrix G' with size $2k \times 4n$. G' is given by

$$G' = \begin{bmatrix} G_n^1 & 0 & \alpha_{11} G_n^3 & \alpha_{21} G_n^5 \\ 0 & G_n^2 & \alpha_{12} G_n^4 & \alpha_{22} G_n^6 \end{bmatrix} \tag{22}$$

C. Joint Decoding

In fountain encoding the encoded symbols are called output symbols; and the symbols from which these output symbols are calculated are called input symbols. Since we are using (BPSK) modulation each input and output symbols represent each input and output bits respectively. The decoding graph at destination for JNFC-MLT and JNFC-RLNC are illustrated in Fig.7 and Fig.8, respectively where it is assumed that both relays use same generator matrix for fountain coding in each scheme. In both figures, the circles and rectangles represent output symbols nodes and parity-check nodes of the fountain coding, respectively. In noisy channel, the decoding of fountain code is accomplished using the standard BP algorithm on generator matrix G'. At the l^{th} decoding attempt, it performs BP decoding on generator matrix G' by iteratively passing the LLR (log-likelihood ratio) messages from input bits to output bits, and then from output bits back to input bits. Let $\mu_{c_o, v_i}^{j,l}$ and $\upsilon_{v_i, c_o}^{j,l}$ denote the message passed from the output bit c_o to the input bit v_i and input bit v_i to the output bit c_o respectively at the j^{th} iteration of l^{th} decoding attempt. In every iteration, the following message update rules are applied in parallel to all input and output nodes in the factor graph

$$tanh \frac{\mu_{c_o, v_i}^{(j,l)}}{2} = tanh \frac{\left(Z_{c_o}\right)}{2} \prod_{i' \neq i} tanh \frac{\upsilon_{v_i, c_o}^{j,l}}{2} \tag{23}$$

$$\upsilon_{v_i, c_o}^{(j+1,l)} = \sum_{o' \neq o} \mu_{c_o, v_i}^{(j,l)} \tag{24}$$

where Z_{c_o} is log-likelihood ratios (LLR) of the output bit c_o that is calculated based on the channel observation and knowledge of the CSI at the receiver. We use binary phase shift keying (BPSK) as modulation scheme and assume that the transmitted codeword $c_o \in 0,1$ is equal probability. Therefore, in the Rayleigh fading channel while channel state information is available at the receiver, the log likelihood ratio corresponding on the output node c_o can be expressed as

$$Z_{c_o} = \log \frac{\Pr\left(y_o \middle| c_o = 0\right)}{\Pr\left(y_o \middle| c_o = 1\right)} = \frac{2}{\sigma^2} y_o.a \tag{25}$$

where a is the normalized Rayleigh fading factor with $E\left[a^2\right]=1$ and density function $f(a)=2a\exp(-a^2)$ In the end of l^{th} decoding attempt, if the destination is confident that the transmitted packets u_1 and u_2 are decoded successfully, it then sends an ACK through a noiseless feedback channel to the sources and relays to terminate the transmission of the current code words. Otherwise it collects more output symbols from sources and relays and initiates next decoding attempt to decode again.

VI. Simulation Results

In this section, we conduct simulation to investigate the performance of JNFC in Rayleigh fading channel. Two sources S_1 and S_2 generate original packets u_1 and u_2 where each of length $k=500$ bits. Using LT codes then the original packets u_1 and u_2 are encoded into x_1 and x_2 respectively and transmitted to the destination. The decoding failure probability at the decoder, δ is considered as 0.5 and LT design parameter λ is considered 0.1. After correctly decoding the source packets, relay nodes re-encode the packets using a suitable degree distribution and perform network coding on them. We assume that the relay nodes are closer to the destination than the sources. Therefore, the SNR between each relay and destination is higher than the SNR between any source and destination. The SNR of each relay-destination link is given by $SNR_{r_i,d} = SNR_{sd} + 10dB$ where SNR_{sd} is the SNR of source-destination link.

In this paper, we propose two JNFC schemes depending on the coding techniques at relay nodes such as:

- JNFC-RLNC: In this case, relay nodes re-encode source information using LT codes. Then the coded symbols of two packets are randomly chosen to generate network coded symbols. However, in this way the resulting network coded symbols do not follow Robust Soliton Distribution in degree.
- JNFC-MLT: In this scheme, both sources and relays transmit code symbols that follow RSD as in degree distribution. After receiving source packets, relays use DSD to encode source information. The coded symbols of two sources are then selectively combined in such a way that the result in code symbols follows the degree distribution of LT codes.

Fig. 9 presents the bit error rate (BER) of various schemes over the signal to noise ratio (SNR) on the source-destination link. The number of received symbols from each source and relay is considered as $k\beta$.

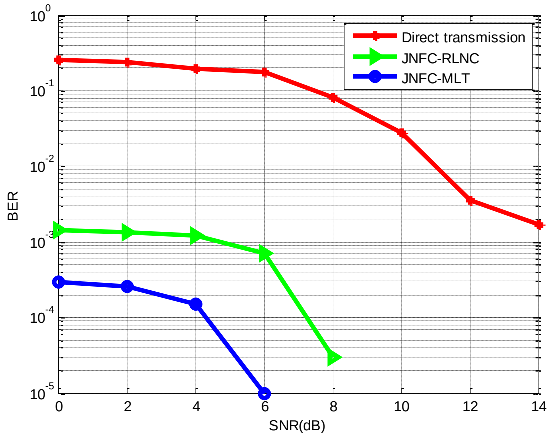

Fig. 9. Bit error rate Vs SNR (dB).

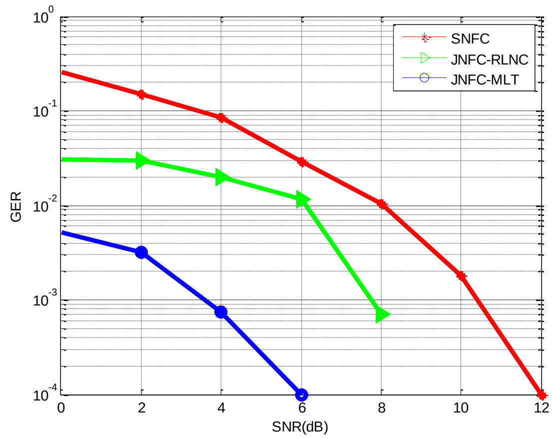

Fig. 10. Generation error rate Vs SNR for large overhead.

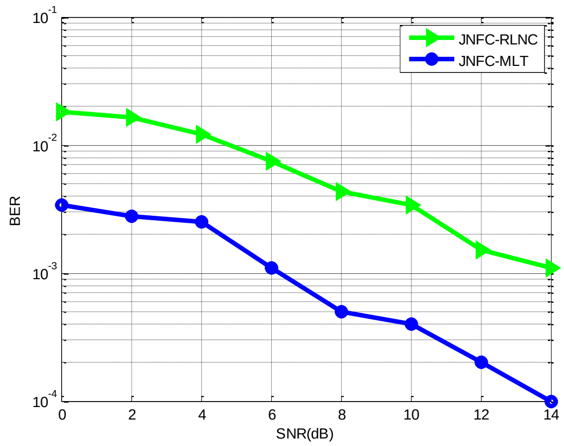

Fig. 11. Bit error rate Vs SNR (dB) for small overhead.

It has been observed that JNFC schemes significantly improve BER performance. This is due to the redundancy and diversity provided by the use of relay nodes and the use of network coding. Moreover it is shown that JNFC-MLT outperforms JNFC-RLNC. The performance of JNFC-MLT is about more than 3 dB compare to JNFCRLNC for a BER of 10^{-4}.

Fig. 10 compares the different JNFC schemes and separated network and fountain coding (SNFC) in terms of Generation Error Rate (GER). Two packets u_1 and u_2 are generated at each generation. The generation error occurs when at least one of the two packets cannot be recovered at the destination correctly. It is shown that SNFC has a performance loss of around 3 dB and 6 dB compared to the JNFC-RLC and JNFC-MLT, respectively for a GER of 10^{-3}. This is because in JNFC the redundancy both in channel coding and network coding are efficiently exploited. However, in SNFC the packets that fail channel decoding can not exploit the redundant packets transmitted by the relay nodes.

One of the salient features of fountain codes is that the decoder starts decoding as soon as the received symbols are slightly more than the original symbols. We are interested to evaluate the performance of JNFC at small overhead. Fig. 11 and Fig. 12 compare the performance of this two JNFC schemes in terms of BER and GER respectively with very small overhead. The number of received symbols from each source and relay is $(k+\alpha)$ where α is 5% of the original symbols. Even with this small overhead, the performance of JNFC-MLT is outstanding than JNFC-RLNC. This is because in JNFC-MLT, the relay nodes transmit LT like codewords that follow the RSD in degree distribution.

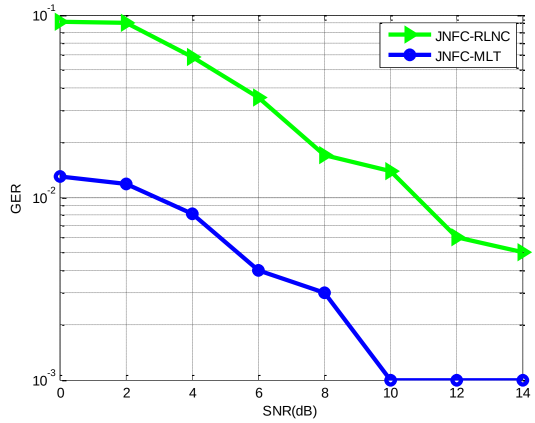

Fig. 12. Generation error rate Vs SNR (dB) for small overhead.

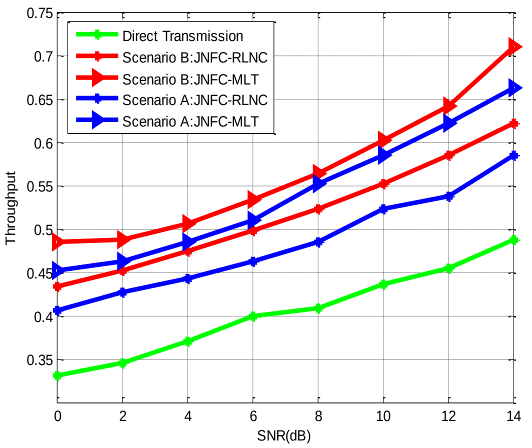

Fig. 13. Throughput Vs SNR (dB).

We evaluate the performance of proposed JNFC schemes in different network scenarios. Fig. 13 compares the performance of proposed JNFC schemes in terms of throughput. Depending on the network scenarios, source nodes or relay nodes continue their transmission until they receive an ACK from destination that the decoding is successful. Here, throughput is calculated by the following equation

$$\eta = \frac{Number\ of\ received\ symbols}{Number\ of\ original\ symbols} \tag{26}$$

It is observed from Fig. 13 that JNFC schemes outperforms the direct transmission where source nodes transmit to destination directly without any help of relay and continue their transmission until they receive ACK from destination. Regardless of network scenarios, the performance of JNFC-MLT is always better than JNFC-RLNC. Moreover, the performance gap between these two schemes is more apparent in Scenarios A. This is because in JNFC-RLNC the destination receives random codewords from both relays, while in JNFC-MLT, the destination receives LT like codewords from both relays. Therefore, the degree distribution of received codewords at destination in JNFC-MLT is actually Robust Soliton distribution that facilitates the decoding process.

VII. Conclusions

In practice, the performance of data transmission often degrades due to the deep fading of wireless channel. To overcome this problem, we presented in this paper a scheme of joint network fountain coding for reliable communication in wireless networks. The proposed JNFC seamless combines fountain and network coding techniques and thus makes use of the redundancy efficiently. Depending on the coding schemes at relay nodes we proposed JNFC-RLNC and JNFC-MLT. Simulations results show that the proposed JNFC outperforms the direct transmission and SNFC in terms of BER, and GER performance. Moreover, regardless of network scenarios JNFC-MLT always outperforms JNFC-RLNC in throughput performance.

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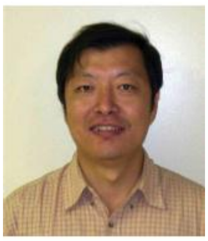

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