Estimation of Channel State Transition Probabilities Based on Markov Chains in Cognitive Radio

Shibing Zhang, Huijian Wang, and Xiaoge Zhang
School of Electronics and Information, Nantong University, China
Email: zhangshb@ntu.edu.cn; 634546090@qq.com; zxg@ntu.edu.cn

Abstract —Prediction of spectrum sensing and access is one of the keys in cognitive radio (CR). It is necessary to know the channel state transition probabilities to predict the spectrum. By the use of the model of partially observable Markov decision process (POMDP), this paper addressed the spectrum sensing and access in cognitive radio and proposed an estimation algorithm of channel state transition probabilities. In this algorithm, the historical statistics information of channel is used to estimate the channel state transition probabilities, and the Least Square (LS) criterion is used to minimize the fitting error. It is showed that the channel state transition process is a special Markov chain, in which the channel state has only one state within each slot. The relationship between estimation precision and the number of converging observation samples is derived. The more the historical statistics information is, the higher the estimation accuracy is. Simulation results showed the estimated error of the LS algorithm is smaller than the linear estimation algorithm.

Index Terms—Cognitive radio, POMDP, channel state transition probability, least Square estimation

I. Introduction

With the development of wireless communication technology and the increase of wireless communication applications, the demand for spectrum is growing. The radio spectrum is becoming increasingly scarce as non-renewable resources. On other hand, the traditional fixed spectrum allocation and the authorized spectrum management lead to the imbalance of spectrum utilizing. Some spectrum bands withstand a large volume of business, such as personal wireless communication spectrum bands. While other spectrum bands are often not in use, such as wireless TV broadcast spectrum bands. The two-dimensional space-time statistics show that the utilization of the allocated spectrum resource is only 15% to 85% in the current allocation scheme [1]. Another survey reports that the licensed spectrum utilization is only 6% in most periods [2]. Cognitive radio (CR) technology can effectively improve the utilization of spectrum resources to alleviate the contradiction between the spectrum allocation and utilization, and has become a focus in wireless communication field [3]-[5].

Manuscript received February 9, 2014; revised June 24, 2014.

This work was supported by the China National Natural Science Foundation under Grant No. 61371112 and the Application Research Project of Ministry of Transport under Grant No. 2014319813220.

Corresponding author: Shibing Zhang, zhangshb@ntu.edu.cn.

doi:10.12720/jcm.9.6.468-474

A key technology in CR is spectrum sensing strategy of cognitive user (CU). It is difficult for CU to sense all channel states in a short period due to the hardware restraints. When primary user's (PU's) channel state is formulated as a Markov model, the spectrum sensing strategy may be designed by a partially observable Markov decision process (POMDP) [6], [7]. But, it is supposed that the channel state transition probabilities are known [8]. However, the channel state transition probabilities will change as PU's behavior change, and are unknown in practical situations.

In this paper, we proposed an algorithm to estimate the channel state transition probabilities by the use of the channel historical information, which is based on the Markov channel model and Least Square criterion. We derived and analyzed the relationship between the estimation accuracy and sample size.

The rest of the paper is arranged as follows. Section II presents the Markov channel model. Section III describes the CR spectrum sensing and access strategy. In Section IV, we proposed the estimation algorithm of the channel state transition probabilities and derived the relationship between the estimation accuracy and sample size. Simulation results are given and analyzed in Section V. Finally, some conclusions are drawn in Section VI.

II. Markov Channel Model

Assuming that the primary users (PUs) in the cognitive network carry out synchronously [9], each channel works in the frequency division multiplexing manner, such as Orthogonal frequency division multiplexing (OFDM) system, and the cognitive network system has tracked the multiplexed clock. There are N independent and identically distributed sub-channels in the primary network, each with bandwidth B_i, $i = 1, 2,\dots, N$. In each slot, the state of every channel is represented by "0" (busy) or "1" (idle), as showed in Fig. 1. Busy state indicates that the channel is occupied by PU in the current slot. Idle state indicates the channel may be used by CU.

The transition of each channel state can be modeled as a discrete-time Markov process, and N sub-channels have 2^N states, as shown in Fig. 2, where P_{01} and P_{10} are the channel state transition probabilities for one step. For convenience, we assume that the statistical properties of the PU spectrum remain unchanged in T slots. Let P_i, $i = 1, 2,\dots, N$, denotes the probability in which the i^{th} channel is available for the CU during a certain slot. From the knowledge of probability theory, we get

$$P_i = \left(1-P_i\right)P_{01} + P_i\left(1-P_{10}\right) \quad (1)$$

By finishing transposition, we can obtain the available probability of the i^{th} channel in the slot

$$P_i = \frac{P_{01}}{P_{01}+P_{10}} \quad (2)$$

Due to the energy consumption limitations and hardware limitations, a CU cannot sense all the N sub-channel states in each slot. It chooses only N_1 channels to sense the spectrum. After sensing, CU chooses N_2 channels to access according to the spectrum sensing. It is obvious that $N_2 \le N_1 \le N$. Since the transition of each channel state system is modeled as a discrete-time Markov process and only limited channels in the system can be observed by CU in each slot, the spectrum sensing and access can be designed as a partially observable Markov decision process (POMDP).

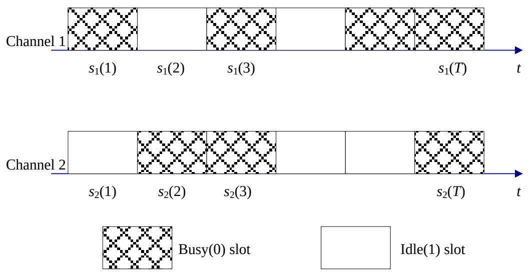

Fig. 1. Channel states in synchronous slot

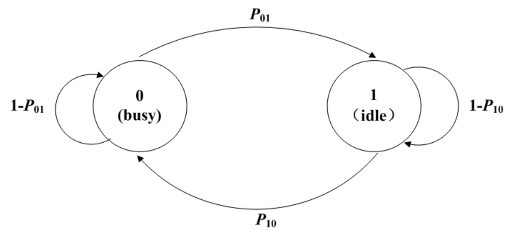

Fig. 2. Markov channel model

III. SPECTRUM SENSING AND ACCESS STRATEGY

A. Channel Revenue and Objective Function

When the internal state of the Markov process in POMDP model is unknown, the internal channel state in slot t can be described by a belief vector [10]

$$\mathbf{\Lambda}(t) = [\lambda_1(t), \lambda_2(t), \cdots, \lambda_M(t)] \quad (3)$$

where $\lambda_m(t)$, $m = 1, 2, \ldots, M$, is the conditional probability of the channel state m in slot t when the channel state transition probabilities, past decision and observational information are known, $M = 2^N$ represent all channel states of N channels.

At the beginning of slot t, the CU chooses N_1 channels to sense, then N_2 channels to access. For any slot t, the vector $\mathbf{\Lambda}(t)$ is a sufficient statistic of the optimal decision $\{N_1, N_2\}$ in slot t [11]. Therefore, a sensing strategy π is given to the CU to determine which channels to be sensed in the given slots under POMDP model. We define the channel state space of N sub-channels as

$$\mathbf{S} = [s_1(t), s_2(t), \cdots, s_N(t)] \in \{0,1\}^N \quad (4)$$

the action selection, which decides which channels to be accessed, as

$$a \in \{1, 2, \cdots, N\} \quad (5)$$

and the available probability of all sub-channels as

$$\mathbf{\Omega} = [P_1, P_2, \cdots, P_N] \quad (6)$$

Generally, the most intuitive revenue is the available bandwidth to be transmitted. Then, the revenue function over channel a, which is chosen by CU, in slot t can be defined as

$$r_a(t) = \sum_{a \in N_2} s_a(t) B_a \quad (7)$$

where $s_a(t) \in \{0,1\}$ is the state of channel a in slot t, B_a is the bandwidth of channel a.

Let ζ represents the maximum probability of collision allowed by the primary network. The access strategy must be designed to maximize the total available bandwidth in T slots, which would be enslaved to ζ. So, the objective function of the optimal POMDP access strategy over channel a in T slots is given by

$$\begin{cases} \pi^* = arg\ max\ E_\pi\left[\sum_{t=1}^{T} r_a(t) \mid \Lambda(1)\right] \\ s.t. \quad P_c < \zeta \end{cases} \quad (8)$$

where E_π is the conditional expectation of the CU revenue obtained in T slots by strategy π, P_c is the probability of collision, $\mathbf{\Lambda}(1)$ is the initial belief vector.

B. Channel State Prediction

In order to improve the utilization efficiency of the idle spectrum on the premise of reducing the collision with the PU, the CU should predict the probability of channel occupied according to the past and present spectrum activities of the PU. Then the reasonable spectrum sensing and access action would be made based on the probability of channel occupied.

Although the vector $\mathbf{\Lambda}(t)$ is a sufficient statistic, it is impossible to get all channel states in the slot t. When the channel states occupied are independent from each other, M dimensions belief vector $\mathbf{\Lambda}(t)$ may be replaced by another N dimensions belief vector (available probability of all sub-channels) $\mathbf{\Omega}(t)$ in the slot t [6], where M is much larger than N.

At the beginning of slot t, if the channel state transition probability P_{01} and P_{10} are known and the channel a is chosen in slot t, the expected revenue is obtained by

$$r_{t,a}^* = [P_a(t)(1-P_{10}) + (1-P_a(t))P_{01}]B_a \quad (9)$$

where $[P_a(t)(1–P_{10})+(1–P_a(t))P_{01}]$ represents the available probability of channel a in slot t. Without considering the effect of the current action on the future revenue, the action in slot t is to maximize the expected immediate revenue with the greedy strategy as

$$\begin{aligned}\pi^*(t) &= arg \max_{a=1,2,\cdots,N} r^*_{t,a} = \\ &arg \max_{a=1,2,\cdots,N} \left\{ \left[P_a(t)(1-P_{10}) + (1-P_a(t))P_{01} \right] B_a \right\}\end{aligned} \tag{10}$$

But the action decision $a^*(t)$ is not always available, which is depended on the observation of channel $a^*(t)$ in slot t $\Theta_{a^*}(t)$, $\Theta_{a^*}(t) \in \{0,1\}$. If $\Theta_{a^*}(t)$ is equal to 1, the channel $a^*(t)$ is available. If $\Theta_{a^*}(t)$ is equal to 0, the channel $a^*(t)$ is unavailable. Therefore, the belief vector $\Omega(t+1)$ in the slot t+1 will be updated based on the information of the channel a^* and the observation $\Theta_{a^*}(t)$ as follows

$$\begin{aligned}\mathbf{\Omega}(t+1) &= [P_1(t+1), \cdots, P_N(t+1)] \\ &= \Gamma\left(\mathbf{\Omega}(t) \mid a^*(t), \Theta_{a^*}(t)\right)\end{aligned} \tag{11}$$

where $P_i(t)$ is updated as follows

$$P_i(t+1) = \begin{cases} 1, \ a^*(t) = i, \Theta_{a^*}(t) = 1 \\ 0, \ a^*(t) = i, \Theta_{a^*}(t) = 0 \\ P_i(t)(1-P_{10}) + (1-P_i(t))P_{01}, \ a^*(t) \neq i \end{cases} \tag{12}$$

From (7) ~ (12), we know that the channel state transition probabilities P_{01} and P_{10} are the key to predict the channel state, sense the spectrum and access the channels in the POMDP for the CU. In the next section, we shall give the estimation algorithm of the channel state transition probabilities.

IV. Estimation Algorithm of Channel State Transition Probabilities

A. Estimation of State Transition Probability

Assume that a channel may be described by L states, which are denoted by $s_0, s_1, \ldots, s_{L-1}$. The probability of the lth state s_l in slot t is presented by $y_l(t)$, $l = 0, 1,\ldots, L–1$. The one step transition probability (the probability that the channel transfers from the jth state to the lth during one slot) is P_{jl}, j, $l = 0, 1,\ldots, L–1$. Because the transition of channel state is a stationary random process, the transition probabilities are independent of the number of slots. The channel state prediction model is given by

$$\mathbf{Y}^T(t) = \mathbf{Y}^T(t-1)\mathbf{P} \tag{13}$$

where $\mathbf{P}=(P_{jl})_{L\times L}$ is the channel state transition probability matrix, $\mathbf{Y}(t)=[y_0(t)\ y_1(t)\ \ldots y_{L-1}(t)]^T$ is the channel state probability vector, and $\sum_{l=0}^{L-1} p_{jl} = 1$, $\sum_{l=0}^{L-1} y_l(t) = 1$. Thus, the number of independent elements in matrix $\mathbf{P}$ is $L(L–1) = L^2–L$.

Assume that the number of existing sets of channel state probability statistics are $(K+1)$ $(K > L^2)$), namely, $y_l(t)$, $l = 0, 1,\ldots, L–1$, $t = 0, 1,\ldots, K$. According to (13), if the estimation of the channel state transition probability $\mathbf{P}$, $\hat{\mathbf{P}} = (\hat{p}_{jl})_{L\times L}$, is known, the fitting error of the l^{th} state probability in slot t is given by

$$e_l(t) = y_l(t) - \sum_{j=0}^{L-1} \hat{p}_{jl} y_j(t-1) \tag{14}$$

The sum of the fitting error squares for the channel l^{th} state probabilities in the entire slots is given by

$$Q_l = \sum_{t=0}^{K} e_l^2(t) \tag{15}$$

And the sum of the fitting error squares for all the state probabilities of the channel is given by

$$Q = \sum_{l=0}^{L-1} Q_l = \sum_{l=0}^{L-1}\sum_{t=0}^{K} e_l^2(t) \tag{16}$$

At this point, the problem of estimation of matrix $\mathbf{P}$ can be formulated by the Least Square problem as follow

$$\begin{aligned}&\min_{p_{jl}}(Q) = \min_{p_{jl}}\left(\sum_{l=0}^{L-1}\sum_{t=0}^{K} e_l^2(t)\right) \\ &s.t. \quad \sum_{l=0}^{L-1} \hat{p}_{jl} = 1 \quad j = 0,1\cdots L-1\end{aligned} \tag{17}$$

Introducing the Lagrange multipliers λ_j (j = 0, 1,…, L–1), we can obtain the Lagrangian function

$$\bar{Q} = \sum_{l=0}^{L-1}\sum_{t=0}^{K} e_l^2(t) + \sum_{j=0}^{L-1} \lambda_j \left(\sum_{l=0}^{L-1} \hat{p}_{jl} - 1\right) \tag{18}$$

Taking the derivative of Lagrangian function, we get

$$\frac{\partial \bar{Q}}{\partial \hat{p}_{jl}} = -2\sum_{t=0}^{K} e_l(t) y_j(t-1) + \lambda_j \tag{19}$$

Letting $\frac{\partial \bar{Q}}{\partial \hat{p}_{jl}} = 0$, we have

$$\lambda_j = 2\sum_{t=0}^{K} e_l(t) y_j(t-1) \tag{20}$$

Taking the constrained condition of the problem in (17) into account, we obtain

$$\begin{aligned}\sum_{l=0}^{L-1} e_l(t) &= \sum_{l=0}^{L-1} y_l(t) - \sum_{l=0}^{L-1}\left[\sum_{j=0}^{L-1} \hat{p}_{jl} y_j(t-1)\right] \\ &= \sum_{l=0}^{L-1} y_l(t) - \sum_{j=0}^{L-1} y_j(t-1) \\ &= 1 - 1 \\ &= 0\end{aligned} \tag{21}$$

Then,

$$\sum_{l=0}^{L-1} \lambda_j = 2\sum_{t=0}^{K}\left(\sum_{l=0}^{L-1} e_l(t)\right) y_j(t-1) = 0 \tag{22}$$

On other hand,

$$\sum_{l=0}^{L-1} \lambda_j = L\lambda_j \tag{23}$$

From equation (22) and (23), we deduce that λ_j is 0 and the Lagrangian function does not make sense. Then, we can take directly the derivative of Q function instead of introduce the Lagrange multipliers λ_j as follows

$$\frac{\partial Q}{\partial \hat{p}_{jl}} = -2\sum_{t=0}^{K} e_l(t) y_j(t-1) \tag{24}$$

Letting $\frac{\partial Q}{\partial \hat{p}_{jl}} = 0$, we have

$$\sum_{t=0}^{K} e_l(t) y_j(t-1) = 0 \tag{25}$$

Substituting (14) into (25), we can get

$$\sum_{t=0}^{K}\left[y_l(t) - \sum_{j=0}^{L-1} \hat{p}_{jl} y_j(t-1)\right] y_j(t-1) = 0 \tag{26}$$

for j, l = 0, 1,…, L–1. They can be expressed in matrix as

$$\mathbf{Y}\hat{\mathbf{P}} = \mathbf{M} \tag{27}$$

where

$$\mathbf{Y} = \mathbf{X}_1^T \mathbf{X}_1 \tag{28}$$

$$\mathbf{M} = \mathbf{X}_1^T \mathbf{X}_2 \tag{29}$$

and

$$\mathbf{X}_1 = \begin{bmatrix} y_0(0) & y_1(0) & \cdots & y_{L-1}(0) \\ y_0(1) & y_1(1) & \cdots & y_{L-1}(1) \\ & \cdots & \cdots & \\ y_0(K-1) & y_1(K-1) & \cdots & y_{L-1}(K-1) \end{bmatrix} \tag{30}$$

$$\mathbf{X}_2 = \begin{bmatrix} y_0(1) & y_1(1) & \cdots & y_{L-1}(1) \\ y_0(2) & y_1(2) & \cdots & y_{L-1}(2) \\ & \cdots & \cdots & \\ y_0(K) & y_1(K) & \cdots & y_{L-1}(K) \end{bmatrix} \tag{31}$$

Then, the Least Square estimation of the state transition probability matrix can be obtained as follow

$$\hat{\mathbf{P}} = \mathbf{Y}^{-1}\mathbf{M} \tag{32}$$

The channel state transition process is a special Markov chain, in which the channel state has only one state within each slot. Then, there is one and only one component, which is "1", in the channel state vector $\mathbf{Y}(t)$, the others are all "0". When the j^{th} component is "1" in $\mathbf{Y}(t)$, i.e., the channel is located in j^{th} state, we have

$$\mathbf{Y}(t) = \left(0,\cdots,0,y_j(t),0,\cdots,0\right)^T = \left(0,\cdots,0,1,0,\cdots,0\right)^T \tag{33}$$

According to (30) and (31), we can obtain the block matrices as follows

$$\mathbf{X}_1 = \left[Y(0), Y(1), \cdots, Y(K-1)\right]^T \tag{34}$$

$$\mathbf{X}_2 = \left[Y(1), Y(2), \cdots, Y(K)\right]^T \tag{35}$$

and

$$\mathbf{X}_1{}^T \mathbf{X}_1 = \sum_{t=0}^{K-1} \mathbf{Y}(t)\mathbf{Y}^T(t) \tag{36}$$

$$\mathbf{X}_1{}^T \mathbf{X}_2 = \sum_{t=0}^{K-1} \mathbf{Y}(t)\mathbf{Y}^T(t+1) \tag{37}$$

If the channel is located in j^{th} state in slot t, we obtain

$$\mathbf{Y}(t)\mathbf{Y}^T(t) = \mathbf{E}_{jj} \tag{38}$$

where $\mathbf{E}_{jj}$ is a L-order matrix, in which the elements in the jth row and jth column are 1, and the other elements are 0.

Assume n_{jl} represent the statistical number of times with which the channel transfers from the jth state to the lth in one slot during observed T, we can obtain the statistics of the channel historical state, as showed in Table I.

Therefore, $\mathbf{X}_1^T\mathbf{X}_1$ is a diagonal matrix as follows

$$\mathbf{X}_1^T\mathbf{X}_1 = \begin{bmatrix} \sum_{l=0}^{L-1} n_{0l} & & \\ & \ddots & \\ & & \sum_{l=0}^{L-1} n_{(L-1)l} \end{bmatrix} \tag{39}$$

where $\sum_{l=0}^{L-1} n_{jl}$ is the frequencies for which the channel is located in jth state from slot 0 to slot K-1.

Further, if the channel is located in jth state in slot t and located in lth state in slot t+1, we will have

$$\mathbf{Y}(t)\mathbf{Y}^T(t+1) = \mathbf{E}_{jl} \tag{40}$$

where $\mathbf{E}_{jl}$ is a L-order matrix, in which the elements in the jth row and lth column are 1, and the other elements are 0.

Then, we obtain

$$\mathbf{X}_1^T\mathbf{X}_2 = \begin{bmatrix} n_{00} & n_{01} & \cdots & n_{0(L-1)} \\ n_{10} & n_{11} & \cdots & n_{1(L-1)} \\ & \cdots & \cdots & \\ n_{(L-1)0} & n_{(L-1)1} & \cdots & n_{(L-1)(L-1)} \end{bmatrix} \tag{41}$$

According to (32), we derived out the statistical estimation of the state transition probability as follows

TABLE I: HISTORICAL STATISTICS OF CHANNEL STATE

	s_0	s_1	…	…	s_{L-1} (states in slot t+1)
s_0 (states in slot t)	n_{00}	n_{01}	…	…	$n_{0(L-1)}$
s_1	n_{10}	n_{11}	…	…	$n_{1(L-1)}$
⋮	⋮	⋮	⋮	⋮	⋮
s_{L-1}	$n_{(L-1)0}$	$n_{(L-1)1}$	…	…	$n_{(L-1)(L-1)}$

$$\hat{P}_{jl} = \frac{n_{jl}}{\sum_{l=0}^{L-1} n_{jl}} \quad j,l = 0,1,\cdots,L-1 \tag{42}$$

If the channel has only two states, i.e., L=1, the statistical estimations of the two state transition probabilities are given by

$$\begin{cases}\hat{P}_{01} = n_{01} / (n_{00} + n_{01}) \\ \hat{P}_{10} = n_{10} / (n_{10} + n_{11})\end{cases} \tag{43}$$

B. Estimated Error of State Transition Probability

Due to the reciprocity of P_{01} and P_{10}, we discussed the estimated error of P_{01} only in the following.

Assuming that we observe the sub-channel for k times during T and there are k_0 times with which the sub-channel state is "0" (s_0). Due to the Markov characteristic of the channel, the stationary probability of the channel state is given by[12]

$$P(s_0) = \lim_{k\to\infty}\frac{k_0}{k} = \frac{P_{10}}{P_{01}+P_{10}} \tag{44}$$

Define

$$Z_n = \begin{cases}0 \quad , \quad s(t) = s_0, s(t+1) = s_0 \\ 1 \quad , \quad s(t) = s_0, s(t+1) = s_1\end{cases} \tag{45}$$

where $t = 1, 2, \ldots, k_0$.

Clearly, Z_n is an independent and identically distributed random variable with the mean P_{01} and variance $P_{01} - P_{01}^2$. Therefore, the estimated value of state transition probability can be formulated as follows

$$\hat{P}_{01} = \frac{1}{k_0}\sum_{n=1}^{k_0} Z_n \tag{46}$$

According to the central limit theorem, the sum of the independent and identically distributed random variables obeys the Normal distribution [13]. Therefore, the probability density function (PDF) of $\hat{P}_{01}$ can be written as follows

$$\hat{P}_{01} \sim N\left(P_{01}, \frac{P_{01} - P_{01}^2}{k_0}\right) \quad \text{when } k_0 \to \infty \tag{47}$$

where $N(\bullet)$ is a Normal distribution function.

Define the relative estimated error of transition probability P_{01} as

$$\xi = \frac{\hat{P}_{01} - P_{01}}{P_{01}} \tag{48}$$

When the upper bound of the relative estimated error ε is given, the corresponding confidence P_ε can be obtained from

$$\begin{aligned} P_\varepsilon &= P\{\xi < \varepsilon\} = 2\varphi\left(\varepsilon\frac{\sqrt{k_0}}{\sqrt{(1/P_{01})-1}}\right) - 1 \\ &\approx 2\varphi\left(\varepsilon\frac{\sqrt{k_0}}{\sqrt{\left(1/\hat{P}_{01}\right)-1}}\right) - 1 \end{aligned} \tag{49}$$

where $\phi(\bullet)$ is the normal cumulative distribution function.

When the confidence P_ε is given, the statistical upper bound of the relative estimated error ε can be obtained from

$$\begin{aligned} \varepsilon &= \varphi^{-1}\left(\frac{P_\varepsilon+1}{2}\right)\sqrt{\frac{(1-P_{01})\left(\frac{1}{P_{01}}+\frac{1}{P_{10}}\right)}{k}} \\ &\approx \varphi^{-1}\left(\frac{P_\varepsilon+1}{2}\right)\sqrt{\frac{(1-\hat{P}_{01})\left(\frac{1}{\hat{P}_{01}}+\frac{1}{\hat{P}_{10}}\right)}{k}} \end{aligned} \tag{50}$$

On other hand, when the upper bound of the relative estimated error ε and the corresponding confidence P_ε are given, the sample size is given by

$$k = \frac{\left(1-\hat{P}_{01}\right)\left(\frac{1}{\hat{P}_{01}}+\frac{1}{\hat{P}_{10}}\right)\left[\varphi^{-1}\left(\frac{P_\varepsilon+1}{2}\right)\right]^2}{\varepsilon^2} \tag{51}$$

It shows the relationship between sample size, the upper bound of the relative error, the confidence and the channel state transition probabilities estimated. The larger the sample size is, the smaller the estimated error is, and the higher the confidence is.

C. Estimation Algorithm

The estimation of channel state transition probabilities is summed as follows:

(1) Initializing the channel state $s_j(t) \in (0,1)$, where j = 0, 1.

(2) Observing the channel state information in the past.

(3) Counting the number of channel state transferring from the jth state to the lth state n_{jl} in one step in the historical channel state, where j, l = 0, 1.

(4) Calculating the channel state transition probabilities $\left(\hat{P}_{01}, \hat{P}_{10}\right)$ according (43).

(5) Calculating the sample size k required for the given upper bound of the relative estimated error and the confidence according (51).

(6) Comparing the size of historical channel state information with k. If the sample size required k is smaller than the size of historical channel state information, $\left(\hat{P}_{01}, \hat{P}_{10}\right)$ are the state transition probabilities estimated. Otherwise, grow the size of historical channel state information to k and go back to step (2), estimate the channel state transition probabilities again until the sample size required is smaller than the size of historical channel state information.

V. Simulation Results

In this section, we provided some simulation results of the LS estimation algorithm proposed and compared it with the linear estimation algorithm[14].

Fig. 3 and Fig. 4 show the estimation of the channel state transition probabilities with LS estimation algorithm under different channel state transition probabilities. Along with the sample size increases, the values estimated are close to the channel state transition probabilities given.

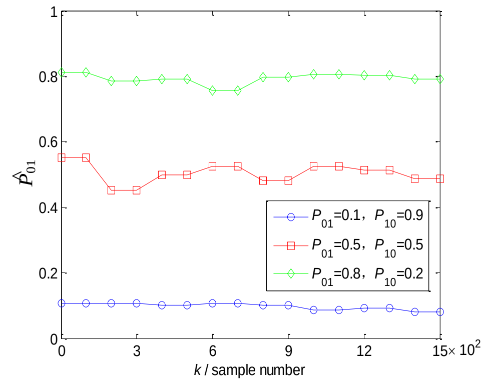

Fig. 3. Estimation of transition probability P_{01}

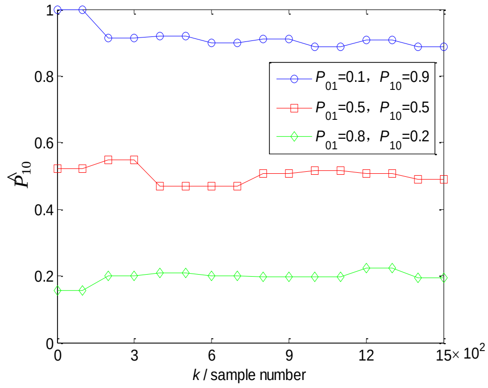

Fig. 4. Estimation of transition probability P_{10}

Fig. 5 and Fig. 6 give the estimated values when the channel state transition probabilities vary from 0 to 1. The curves are lines with slope 1 approximately. It means that the estimated values are almost same to the preset values.

Fig. 7 quantifies the relative estimated error of the channel state transition probability P_{01} with LS estimation algorithm. As the increasing of the sample number, the estimated value of the channel state transition probabilities gradually convergences to the preset value. Suppose that the upper limit of the estimation error ε is 10%, the corresponding confidence P_{ε} is 95%. The sample numbers required for (P_{01}, P_{10}) = (0.1, 0.9), (0.5, 0.5), (0.8, 0.2) are 3841, 768, 480 respectively according to (51). They are close to the sample numbers.

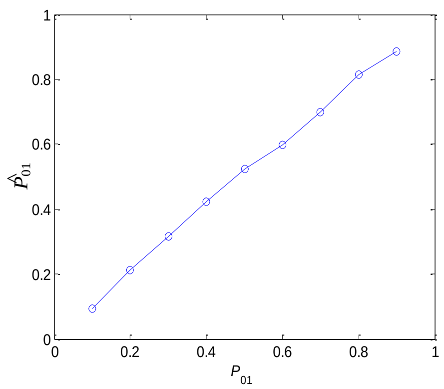

Fig. 5. Estimated value when P_{01} varies from 0 to 1

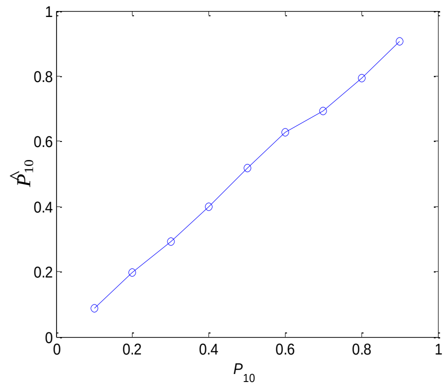

Fig. 6. Estimated value when P_{10} varies from 0 to 1

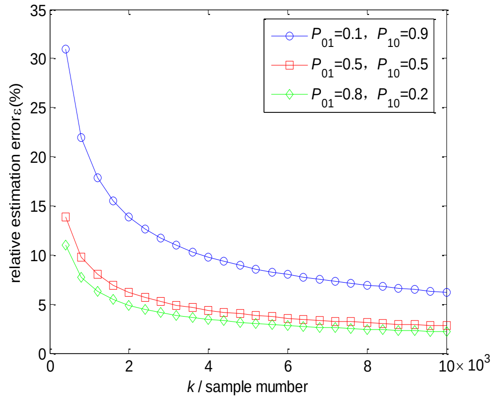

Fig. 7. Relative estimation error of P_{01}

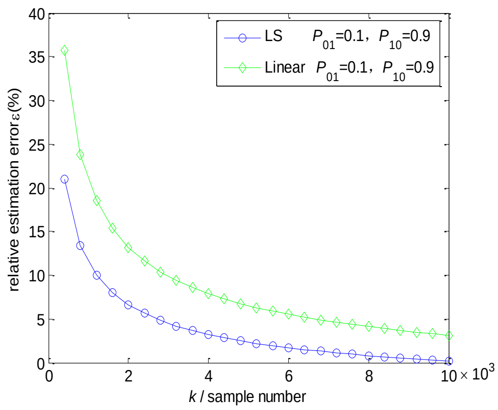

Fig. 8. Comparison of estimation error of P_{01} between LS algorithm and linear

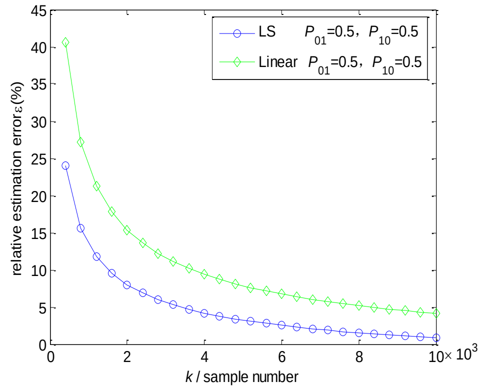

Fig. 9. Comparison of estimation error of P_{10} between LS algorithm and linear

Fig. 8 and Fig. 9 present the comparisons of the relative estimation error of channel state transition probabilities between LS estimation algorithm and linear algorithm. They show that the error of the LS estimation algorithm proposed is smaller than the linear estimation algorithm. It implies that the convergence speed of LS algorithm is faster than linear algorithm. For example, when we require the relative estimation error of P_{01} is smaller than 5%, the sample number required in LS algorithm is about 2.6×10^3. But the sample number in linear algorithm is about 7.2×10^3. the sample number in LS algorithm is only 36% of the one in linear algorithm.

VI. Conclusions

The probabilities of channel state transition are very important for sensing and accessing spectrum in CR networks. In this paper, we used the historical statistics information of channel to estimate the channel state transition probabilities and the LS algorithm to minimize the estimating error. The relationship between the estimated precision of the channel state transition probabilities and sample number of the historical statistics information of channel is analyzed. Simulation results have shown that the LS algorithm estimates the channel state transition probabilities more accurate and faster than linear algorithm.

References

[1] FCC, "ET Docknet NO. 03-103 Notice of proposed rule making and order," December 2003.

[2] M. McHenry, "Spectrum white space measurements," *New American Foundation Broadband Forum*, June 2003.

[3] Y. Song and J. Xie, "A proactive spectrum handoff framework for cognitive radio Ad hoc networks without common control channel," *IEEE Transactions on Mobile Computing*, vol. 11, no. 7, pp. 1127-1139, July 2012.

[4] M. Hasegawa, H. Hirai, K. Nagano, H. Harada, and K. Aihara, "Optimization for centralized and decentralized cognitive radio networks," in *Proc. IEEE*, vol. 102, no. 4, pp. 574-584, April 2014.

[5] J. Noh and S. Oh, "Cognitive radio channel with cooperative multi-antenna secondary systems," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 3, pp. 539-549, March 2014.

[6] Q. Zhao, L. Tong, A. Swami, and Y. Chen, "Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 3, pp. 589-600, April 2007.

[7] Y. Che, R. Zhang, and Y. Gong, "On design of opportunistic spectrum access in the presence of reactive primary users," *IEEE Transactions on Communications*, vol. 61, no. 7, pp. 2678-2691, July 2013.

[8] X. Long, X. Y. Gan, Y. Y. Xu, *et al.*, "An estimation algorithm of channel state transition probabilities for cognitive radio systems," in *Proc. 3rd International Conference on Cognitive Radio Oriented Wireless Networks and Communications*, Singapore: IEEE, 345 E 47TH ST, New York, NY 10017 USA, 2008, pp. 296–299.

[9] D. V. Djonin and Q. Zhao, "Optimality and complexity of opportunistic spectrum access: A truncated markov decision process formulation," in *Proc. IEEE Communications Society Subject Matter Experts for Publication in ICC 2007 Proceedings*, Glasgow, Scotland: IEEE, 345 E 47TH ST, New York, NY 10017 USA, 2007, pp. 5787-5792.

[10] H. Zhao, "Opportunistic spectrum access technology research in cognitive radio system," Beijing: Beijing University of Posts and Telecommunication, 2010.

[11] R. Smallwood and E. Sondik, "The optimal control of partially observable Markov process over a finite horizon," *Operations Research*, pp. 1071-1088, 1971.

[12] H. Kim and K. G. Shin, "Efficient discovery of spectrum opportunities with MAC-layer sensing in cognitive radio networks," *IEEE Transactions on Mobile Computing*, vol. 7, no. 5, pp. 533-545, May 2008.

[13] C. Jinwoo and N. B. Shroff, "A Central-limit-theorem-based approach for analyzing queue behavior in high-speed networks," *IEEE/ACM Transactions on Networking*, vol. 6, no. 5, pp. 659-671, October 1998.

[14] T. Wu and J. Zhang, "The channel quality estimation algorithm based on linear model," *Computer Engineering*, vol. 37, no. 1, pp. 86-89, January 2011.

Shibing Zhang was born in Haimen, China in 1962. He received his B.S. degree in wireless communications from Dalian Maritime University, China in 1983, M.S. degree and Ph.D. degree in wireless communications from Nanjing University of Posts and Telecommunications, China in 1989, 2007 respectively. He worked as an associate engineer in the Nantong Changjiang Communication and Navigation Management Section from Sept. 1983 to Aug. 1986, engineer in the Yancheng Electronic Equipment Manufactory from Feb. 1989 to Apr. 1997 and senior engineer in the Haimen Economical Information Centre from May. 1997 to Mar. 2001 respectively. Since Apr. 2001, he has been working in Nantong University as a professor. His current research interests include wireless communications and networking, OFDM system, especially on ultrawideband communications, cognitive radio.
From Jul. 2009 to Mar. 2010, Dr. Zhang was a visiting scholar with the Department of Electrical and Computer Engineering, University of Victoria.

Huijian Wang was born in Taizhou, China in 1987. He received his B. S. degree in communications engineering from Nantong University China in 2011. He is currently a graduate student of Nantong University. He is engaged in the research activities in the areas of communication signal processing and cognitive wireless networks.

Xiaoge Zhang was born in Haimen, China in 1975. He received his Ph.D. degree in wireless communications from Nanjing University of Posts and Telecommunications, China in 2010. He is currently an assistant professor of Nantong University. He is engaged in the research activities in the areas of relay, OFDM and cognitive wireless networks.