Channel Correlation Properties in OFDM by using Time-Varying Cyclic Delay Diversity

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***Abstract*— This paper analyzes the influence of time-varying cyclic delay diversity (TV-CDD) on the channel fading correlation properties in orthogonal frequency division multiplexing (OFDM) based systems. The underlying transmit diversity technique CDD only increases the frequency diversity at the receiver. In contrast, TV-CDD introduces additionally time diversity which can be exploited without the need of additional complexity at the receiver. This paper gives investigations regarding the resulting channel characteristics from TV-CDD and the impact on the system performance. Due to the increased frequency and time selectivity, an unintended higher channel estimation effort is possible. Therefore, we analyze the impact of choice of the maximum cyclic delay. We show that the resulting channel for TV-CDD can be seen as an uncorrelated Rayleigh fading channel (except for the first sub-carrier) for a large maximum cyclic delay. Furthermore, analysis and simulation results demonstrate a feasible choice of small time-varying cyclic delays for guaranteeing the standard conformability of the TV-CDD technique at the receiver without significant performance degradations.**

I. Introduction

Multiple-antenna concepts for communications systems offer high spectral efficiency. Since these techniques increase the achievable data throughput, they have become desirable in the last decade. One of these concepts, delay diversity (DD) [2], is based on increasing the frequency diversity by using several transmit (TX) antennas and sending modified replicas of the desired transmitted signal. Due to the specific modification, i.e., introducing a time delay, the transmitted signal can be processed at the receiver (RX) without any additional antennas and processing complexity. Signal delays in DD may cause intersymbol interference (ISI). This scheme was also took up for orthogonal frequency division multiplexing (OFDM) based systems and the new scheme, namely cyclic delay diversity (CDD) [3], [4], introduces cyclic delays in the signal replicas to avoid additional ISI. A further approach to additionally increase the time diversity was given by time-varying cyclic delays, i.e., time-varying CDD (TV-CDD) [5].

Typically, multi TX/RX-antenna techniques like space-time coding [6], [7] require signal processing in both the transmitter and the receiver. However, CDD as well as TV-CDD can be implemented solely at the transmitter. The fact that the counterpart needs not to be aware of the implementation makes these techniques standard compatible, i.e., they can be implemented as an extension for already existing systems without changing the standard.

Transmit diversity schemes increase the frequency and/or time selectivity of the resulting channel seen at the receiver. Furthermore, the overall channel delay is larger. Therefore, it is necessary to investigate the influence of the choice of cyclic signal delays on the performance for maintaining the standard conformability of the applied diversity technique.

In principle, a system standard does not necessarily have to be changed when CDD is going to be implemented. Nevertheless, this TX-antenna technology has attracted interest in present standardization activities. The Draft of the IEEE 802.11n WLAN standard [8] includes CDD under the term 'Cyclic Shift Diversity (CSD)'. In the framework of 3GPP LTE (3^{rd} Generation Partnership Project – Long Term Evolution) [9], CDD is used as a special case of precoding technology.

In this paper, we investigate transmit diversity techniques based on the frequency domain for OFDM based

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systems. This paper, which is an expanded version of [1], extends previous work by a more detailed description of the state-of-the-art and the given problem to investigate. We introduce briefly different variants of transmit diversity techniques. Then, the focus will be on the time-varying cyclic delay diversity principle. We will give first analyzes about the influence of TV-CDD on the resulting channel fading correlations. Furthermore, the choice of the maximum random delay shift for TV-CDD is analyzed to avoid additional channel estimation requirements for TV-CDD systems. Finally, simulation results are presented which confirm the analyzes.

II. Frequency Domain Diversity Techniques for OFDM

Required reliable link performances for future communications systems can be established by transmit antenna diversity techniques [10]. Modified replicas of the original signal are sent from additional implemented transmit antennas. For OFDM based systems, shifts in time domain are possible signal modifications. Since the additional time domain shift influences the signal spectrum, we refer to these schemes as frequency domain diversity techniques. The goal of these techniques is to increase the frequency selectivity of the channel, and therefore, to decrease the coherence bandwidth. To exploit the additional diversity in an OFDM system, forward error correction (FEC) is needed. The elementary diversity method, namely delay diversity [2], transmits delayed replicas of a signal from several transmit antennas N_{T} with delays $\delta_n, n = 0, \ldots, N_{\mathrm{T}} - 1$, where δ_n is given in samples. In DD inter-symbol interference (ISI) can occur if the maximum possible delay exceeds the guard interval length N_{G} of the OFDM system:

$$N_{\mathrm{G}} \geq \tau_{\max} + \max_n \delta_n \,, \tag{1}$$

where $\tau_{\max}$ denotes the maximum channel delay in samples.

A. Cyclic Delay Diversity

A neat solution to provide DD without exceeding the guard interval, and therefore, without reducing the bandwidth efficiency, is the cyclic delay diversity technique which was proposed in the year 2001 [3], [4]. By applying CDD no changes at the receiver are needed, there exists no rate loss for higher number of transmit antennas, and there are no requirements regarding constant channel properties over several sub-carriers or symbols and transmit antenna numbers. This is an advantage over already established diversity techniques, e.g., orthogonal space-time block codes [6]. Figure 1 shows the front end of a CDD OFDM transmitter. For simplicity of the notation, we consider the transmission of one OFDM symbol. N_{FFT} data symbols $S(k), k = 0, \ldots, N_{\mathrm{FFT}} - 1$ are obtained from a precedent coding, modulation, and framing part. These complex valued symbols are transformed into the time domain by the OFDM entity using an inverse fast Fourier transform

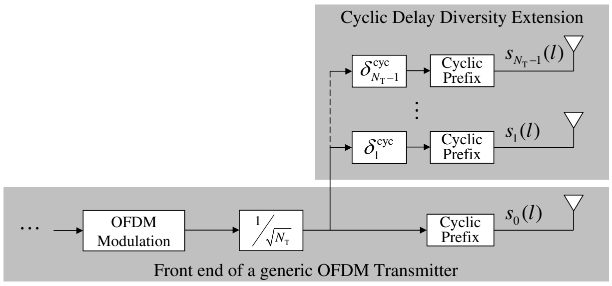


Figure 1. Principle of cyclic delay diversity.

(FFT). This results in N_{FFT} time domain OFDM symbols, represented by the samples

$$s(l) = \frac{1}{\sqrt{N_{\mathrm{FFT}}}} \sum_{k=0}^{N_{\mathrm{FFT}}-1} S(k) \cdot e^{j \frac{2\pi}{N_{\mathrm{FFT}}} kl} \,, \tag{2}$$

where l and k denote the discrete time and frequency. Before inserting a cyclic prefix as guard interval, the time domain OFDM symbol is shifted cyclically, which results in the signal

$$s(l - \delta_n^{\mathrm{cyc}} \bmod N_{\mathrm{FFT}}) = \frac{1}{\sqrt{N_{\mathrm{FFT}}}} \sum_{k=0}^{N_{\mathrm{FFT}}-1} e^{-j \frac{2\pi}{N_{\mathrm{FFT}}} k \delta_n^{\mathrm{cyc}}} S(k) e^{j \frac{2\pi}{N_{\mathrm{FFT}}} kl} \,. \tag{3}$$

The antenna specific TX-signal is given by

$$s_n(l) = \frac{1}{\sqrt{N_{\mathrm{T}}}} \cdot s(l - \delta_n^{\mathrm{cyc}} \bmod N_{\mathrm{FFT}}) \,, \tag{4}$$

where the signal is normalized by $1/\sqrt{N_{\mathrm{T}}}$ to keep the average transmission power independent of the number of transmit antennas. To avoid ISI within CDD, the guard interval length N_{G} has to fulfill

$$N_{\mathrm{G}} \geq \tau_{\max} \,. \tag{5}$$

Therefore, the length of the guard interval for CDD does not depend on the cyclic delays δ_n^{cyc}, where δ_n^{cyc} is given in samples. Furthermore, the cyclic delays avoid delayed transmitted replica signals compared to DD which is beneficial for synchronization processes at the receiver. Therefore, δ_n^{cyc} does not delay the overall OFDM symbol but the influence of δ_n^{cyc} can be seen as a delay on each sub-carrier due to the corresponding phase shift in frequency domain by the factor $e^{-j \frac{2\pi}{N_{\mathrm{FFT}}} k \delta_n^{\mathrm{cyc}}}$.

B. Phase Diversity

The time domain cyclic shifts can be also transformed in the frequency domain by including the delays as a phase multiplication before the inverse FFT, which results in phase diversity [11]. This techniques offers the flexibility of an arbitrary choice of the phase factor ϕ_n with its phase increment

$$\Delta\phi_n = \frac{2\pi}{N_{\mathrm{FFT}}} \cdot \delta_n^{\mathrm{cyc}} \, [\mathrm{rad}] \,. \tag{6}$$

This flexibility has to be payed by $N_{\mathrm{T}} - 1$ additional inverse FFT and cyclic prefix processings in the transmitter.

C. Soft Cyclic Delay Diversity

CDD introduces additional propagation paths. As long as non-line-of-sight propagation is considered by assuming Rayleigh fading processes, wireless communications systems usually benefit from an increased amount of diversity offered by the effective channel. The situation changes when there is line-of-sight, which is usually modeled by Ricean fading processes. Here CDD causes deterministic shaping of the spectrum for the constant part of the Ricean fading process. This decreases the SNR gain or even turns the SNR gain into an SNR loss when the Ricean factor is high, i.e., the constant (line-of-sight) part of the channel exceeds the Rayleigh fading (non-line-of-sight) paths. One approach is to use unbalanced TX powers for the different TX-antenna branches in CDD. Investigations in [12] have shown that an unbalanced TX power decreases the SNR loss drastically in case of line-of-sight. The price to pay is a slightly reduced SNR gain in case of non-line-of-sight (Rayleigh fading) propagation.

D. Time-Varying Cyclic Delay Diversity

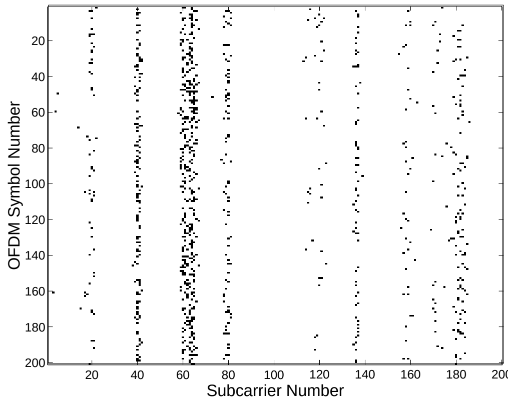
The channel seen by the receiver for the CDD concept is transformed from a multiple-input single-output (MISO) channel to a single-input single-output (SISO) channel, i.e., the spatial diversity is transformed into frequency diversity. Nevertheless, it is also possible to influence the time diversity in such a system by applying the time-varying CDD (TV-CDD) technique introduced in 2006 [5]. In OFDM multi-user systems several users suffer from deep fades on their sub-carriers and others do not. To achieve a higher fairness among the users, a time-varying component for CDD can break the long deep fades to shorter ones which are scattered to the adjacent sub-carriers. Since good sub-carriers can help the weak sub-carriers, the outer FEC can exploit the additional time diversity [5], [13].

The time-varying component is introduced to CDD by cyclic shifts which are a function of the time or the discrete time value t of a transmitted OFDM symbol. The cyclic shifts $\delta_n^{\mathrm{cyc}}(t)$ are elements of the integer interval $\mathbb{S} = [0, \ldots, N_{\mathrm{FFT}} - 1]$. The cyclic shifts are randomly chosen for each OFDM symbol. Therefore, the TV-CDD signal at the transmit antennas is given by

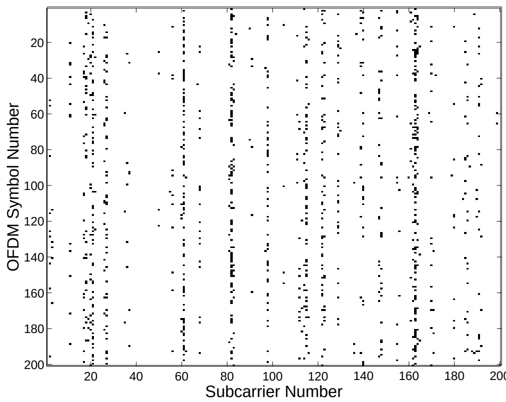
$$s_n(l,t) = \frac{1}{\sqrt{N_{\mathrm{T}} N_{\mathrm{FFT}}}} \sum_{k=0}^{N_{\mathrm{FFT}}-1} e^{-j\frac{2\pi}{N_{\mathrm{FFT}}}k\delta_n^{\mathrm{cyc}}(t)} S(k) e^{j\frac{2\pi}{N_{\mathrm{FFT}}}kl} . \tag{7}$$

The resulting TV-CDD concept preserves the frequency diversity of pure CDD and adds additional time diversity to the resulting channel. We chose the start of the interval $\mathbb{S}$ at 0 instead of 1 to ease the calculations and notational convenience without loss of generality in the following section. The Appendix provides the basic calculations for $\mathbb{S} = [1, \ldots, N_{\mathrm{FFT}} - 1]$.

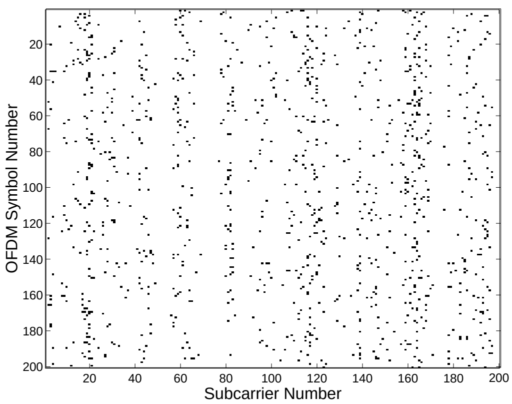
Figure 2 shows different error patterns for a transmission of an uncoded OFDM frame over a time-invariant multi-path channel. It is visible, that no applied transmit diversity results in burst errors for deep faded sub-carriers (cf. Figure 2(a)). Including the CDD technique, the frequency selectivity increases and the error bursts reduces consequentially (cf. Figure 2(b)). Finally, the TV-CDD scatters the errors to neighboring sub-carriers and no deep fades over a whole sub-carrier exist anymore (cf. Figure 2(c)). Therefore, an applied FEC will gain from more distributed error patterns and the coding gain for TV-CDD is larger.



(a) Without CDD or TV-CDD



(b) With CDD



(c) With TV-CDD

Figure 2. Example of error patterns for uncoded OFDM frame transmission.

III. Resulting Channel Characteristics

The influence of CDD based transmit diversity techniques on the system can be observed at the receiver as a change of the channel conditions [14], [15]. In the following, we will investigate this modified channel in terms of its channel transfer functions (CTF) and fading correlation in time and frequency direction.

We assume for the channel fading a quasi-static fading process, i.e., the fading is constant for the duration of several OFDM symbols. With this quasi-static channel assumption the well-known description of OFDM in the frequency domain is given by the multiplication of the transmitted data symbol $S'_n(k,t) = 1/\sqrt{N_\mathrm{T}} \cdot S(k) e^{-j\frac{2\pi}{N_\mathrm{FFT}} k \delta_n^\mathrm{cyc}(t)}$ and a complex valued fading coefficient $H_n(k,t)$. Therefore, the received signal at the receiver for TV-CDD is

$$R(k,t) = \sum_{n=0}^{N_\mathrm{T}-1} S'_n(k,t) \cdot H_n(k,t) + n(k,t)\,. \quad (8)$$

The frequency domain fading processes for different propagation paths are uncorrelated in the assumed quasi-static channel. Since the number of sub-carriers is larger than the number of propagation paths, there exists correlation between the sub-carriers in the frequency domain.

Since the interest is based on the fading and signal characteristics observed at the receiver, the additive white Gaussian noise (AWGN) term $n(k,t)$ with zero mean is skipped for notational convenience. Formally the cyclic shift can be assigned to the channel transfer function, and therefore, the overall channel transfer function $H'(k,t)$ can be displayed in the received signal

$$R(k,t) = S(k) \cdot \underbrace{\frac{1}{\sqrt{N_\mathrm{T}}} \sum_{n=0}^{N_\mathrm{T}-1} e^{-j\frac{2\pi}{N_\mathrm{FFT}} k \delta_n^\mathrm{cyc}(t)} H_n(k,t)}_{H'(k,t)}\,. \quad (9)$$

The expectation

$$\mathbf{R}(k_1,k_2,t_1,t_2) = E\{H'(k_1,t_1) H'^*(k_2,t_2)\} \quad (10)$$

yields the correlation properties of the frequency domain channel fading, where $(\cdot)^*$ means complex conjugate.

A. Fading Correlation Properties for TV-CDD

The fading correlation properties can be divided in three cases. The first represents the autocorrelation respectively power, the second investigates the correlation properties between the OFDM symbols (time direction), and the third examines the correlation properties between the sub-carriers (frequency direction).

Case 1: Since we assume uncorrelated sub-carriers the autocorrelation of the CTF $(k_1 = k_2 = k, t_1 = t_2 = t)$ is

$$\mathbf{R}(k,t) = \frac{1}{N_\mathrm{T}} \sum_{n=0}^{N_\mathrm{T}-1} E\{|H_n(k,t)|^2\} = 1\,. \quad (11)$$

Case 2: The correlation properties in time direction are given by $k_1 = k_2 = k$ and $t_1 \neq t_2$. We get

$$\mathbf{R}(k, t_1 \neq t_2) = \frac{1}{N_\mathrm{T}} \sum_{n=0}^{N_\mathrm{T}-1} E\{e^{-j\frac{2\pi}{N_\mathrm{FFT}} k \delta_n^\mathrm{cyc}(t_1)}\} \cdot E\{e^{+j\frac{2\pi}{N_\mathrm{FFT}} k \delta_n^\mathrm{cyc}(t_2)}\} E\{H_n(k,t_1) H_n^*(k,t_2)\}\,. \quad (12)$$

The probability of the uniformly distributed random cyclic shift $\delta_n^\mathrm{cyc}(t) \in \mathbb{S}$ is given by

$$P(\delta) = 1/N_\mathrm{FFT}\,. \quad (13)$$

The first expectation value in (12) can be developed from a geometric series to

$$E\{e^{-j\frac{2\pi}{N_\mathrm{FFT}} k \delta_n^\mathrm{cyc}(t_1)}\} = \sum_{\delta=0}^{N_\mathrm{FFT}-1} e^{-j\frac{2\pi}{N_\mathrm{FFT}} k\delta} \cdot P(\delta) = \frac{1}{N_\mathrm{FFT}} \cdot \frac{1 - e^{-j2\pi k}}{1 - e^{-j\frac{2\pi}{N_\mathrm{FFT}} k}} = \begin{cases} 1 & \text{for } k = 0 \\ 0 & \text{for } k \neq 0 \end{cases}\,. \quad (14)$$

Note the range of k is $0, \ldots, N_\mathrm{FFT}-1$. Since $\delta_n^\mathrm{cyc}(t_1)$ and $\delta_n^\mathrm{cyc}(t_2)$ have the same statistical properties, the second expectation term can be expanded in the same manner. In the case of $k \neq 0$, the resulting channel for TV-CDD can be seen as an uncorrelated Rayleigh fading channel. Thus,

$$\mathbf{R}(k \neq 0, t_1 \neq t_2) = 0\,. \quad (15)$$

In the case of no Doppler shift in the channel, $E\{H_n(k,t_1) \cdot H_n^*(k,t_2)\} = 1$, and therefore, the resulting channel for the first sub-carrier ($k = 0$) is fully correlated:

$$\mathbf{R}(k = 0, t_1 \neq t_2) = 1\,. \quad (16)$$

Otherwise (Doppler shift is unequal zero), the channel characteristics are given by $\mathbf{R}(k = 0, t_1 \neq t_2) = E\{H_n(k,t_1) \cdot H_n^*(k,t_2)\}$. For further investigations of the time-direction correlations we assume no Doppler shift.

Case 3: In frequency direction ($k_1 \neq k_2$ and $t_1 = t_2 = t$) the correlation properties are given by

$$\mathbf{R}(k_1 \neq k_2, t) = \frac{1}{N_\mathrm{T}} \sum_{n=0}^{N_\mathrm{T}-1} E\{e^{-j\frac{2\pi}{N_\mathrm{FFT}} (k_1 - k_2) \delta_n^\mathrm{cyc}(t)}\}. E\{H_n(k_1,t) \cdot H_n^*(k_2,t)\} = 0\,. \quad (17)$$

Therefore, the TV-CDD technique generates in frequency direction for all t an uncorrelated Rayleigh fading channel.

B. Impact of Random Cyclic Delays

By introducing cyclic shifts, and therefore, generating a more frequency selective channel, the effective maximum delay of the resulting channel τ'_max becomes larger. An upper bound can be given by

$$\tau'_\mathrm{max} = \tau_\mathrm{max} + \delta_\mathrm{max}^\mathrm{cyc}\,, \quad (18)$$

where $\delta_\mathrm{max}^\mathrm{cyc}$ represents the maximum cyclic shift of the interval $\mathbb{S}$. For an appropriate channel estimation process at the receiver the guard interval length N_G is set to be larger than τ_max. If the maximum resulting channel delay

$\tau'_{\max}$ does not intensively exceed the length of N_{G}, there is no configuration at the receiver needed regarding the channel estimation. For $\tau'_{\max} \gg N_{\mathrm{G}}$, the receiver needs the additional information of the modified pilot grid for the channel estimation process [16], and therefore, TV-CDD is not standard conformable anymore. This can be circumvented by using differential modulation [17] which is not in the focus of this paper.

The impact of different $\delta^{\mathrm{cyc}}_{\max}$ to the resulting channel correlation properties is investigated in the following to optimize the choice of $\delta^{\mathrm{cyc}}_{\max}$ and to endeavor a standard conformable technique.

We assume an interval $\mathbb{S}_a = [0, \ldots, \delta^{\mathrm{cyc}}_{\max}]$ with integer values, where $\delta^{\mathrm{cyc}}_{\max} = \frac{N_{\mathrm{FFT}}}{a} - 1$ with $a \in 2^m, m = [1, \ldots, \log_2(N_{\mathrm{FFT}})]$. Again, there are three cases for the channel correlation properties.

Case 1:

$$\mathbf{R}(k,t) = 1\,. \tag{19}$$

Case 2: The first expectation of (12) has now the probability

$$P(\delta) = a/N_{\mathrm{FFT}} \tag{20}$$

for $\delta = 0, \ldots, N_{\mathrm{FFT}}/a - 1$:

$$\begin{aligned} E\{e^{-j\frac{2\pi}{N_{\mathrm{FFT}}}k\delta^{\mathrm{cyc}}_n(t_1)}\} &= \sum_{\delta=0}^{\frac{N_{\mathrm{FFT}}}{a}-1} e^{-j\frac{2\pi}{N_{\mathrm{FFT}}}k\delta} \cdot \frac{a}{N_{\mathrm{FFT}}} \\ &= \frac{a}{N_{\mathrm{FFT}}} \cdot \frac{1 - e^{-j\frac{2\pi}{a}k}}{1 - e^{-j\frac{2\pi}{N_{\mathrm{FFT}}}k}}\,. \end{aligned} \tag{21}$$

In the case of the first sub-carrier and no Doppler shift, the channel is fully correlated in the time direction, i.e.,

$$\mathbf{R}(k = 0, t_1 \neq t_2) = 1\,. \tag{22}$$

Since we assume $E\{H_n(k,t_1) \cdot H^*_n(k,t_2)\} = 1$ and $\delta^{\mathrm{cyc}}_n(t_1)$, $\delta^{\mathrm{cyc}}_n(t_2)$ have the same statistical properties,

$$\mathbf{R}(k \neq 0, t_1 \neq t_2) = \frac{1}{N_{\mathrm{T}}} \sum_{n=0}^{N_{\mathrm{T}}-1} E\{|e^{-j\frac{2\pi}{N_{\mathrm{FFT}}}k\delta^{\mathrm{cyc}}_n(t_1)}|^2\}\,. \tag{23}$$

Figure 3 illustrates the correlation characteristics of (23) versus the sub-carriers and $\delta^{\mathrm{cyc}}_{\max}$ for $N_{\mathrm{FFT}} = 512$. From Figure 4, we see that most of the sub-carriers are sufficient uncorrelated for $\delta^{\mathrm{cyc}}_{\max} \geq 7$ or $a \leq 64$. Note, the correlation properties do not depend on the time difference between the considered OFDM symbols because the delays are chosen randomly for each consecutive OFDM symbol.

Case 3: The frequency-direction properties of the resulting channel are

$$\mathbf{R}(k_1 \neq k_2, t) = \frac{1}{N_{\mathrm{T}}} \sum_{n=0}^{N_{\mathrm{T}}-1} E\{|e^{-j\frac{2\pi}{N_{\mathrm{FFT}}}(k_1-k_2)\delta^{\mathrm{cyc}}_n(t)}|^2\}\,, \tag{24}$$

which are similar to $\mathbf{R}(k \neq 0, t_1 \neq t_2)$.

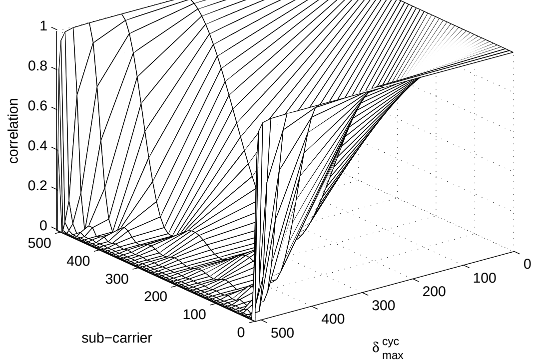


Figure 3. Correlation properties in time direction with $N_{\mathrm{FFT}} = 512$ for varying $\delta^{\mathrm{cyc}}_{\max}$.

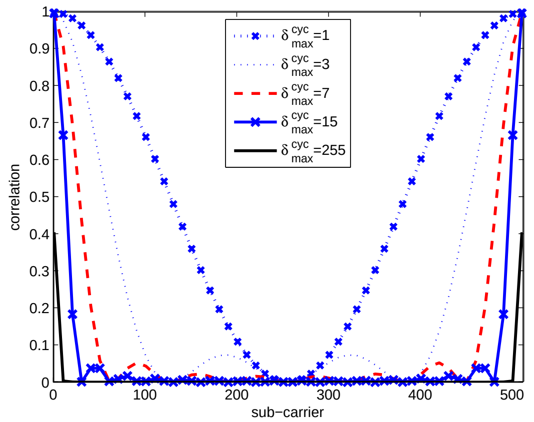
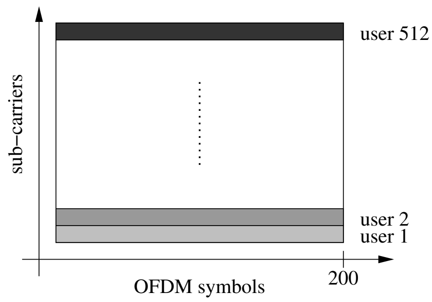
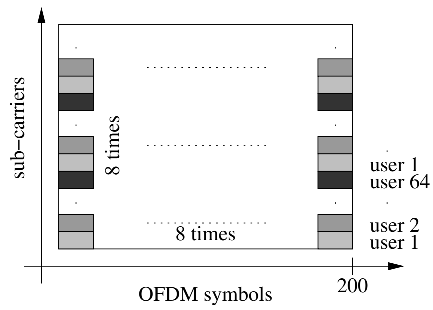


Figure 4. Correlation properties for different $\delta^{\mathrm{cyc}}_{\max}$ [samples] in time direction with $N_{\mathrm{FFT}} = 512$.



(a) Scheme 1



(b) Scheme 2

Figure 5. Mapping schemes for OFDM transmission.

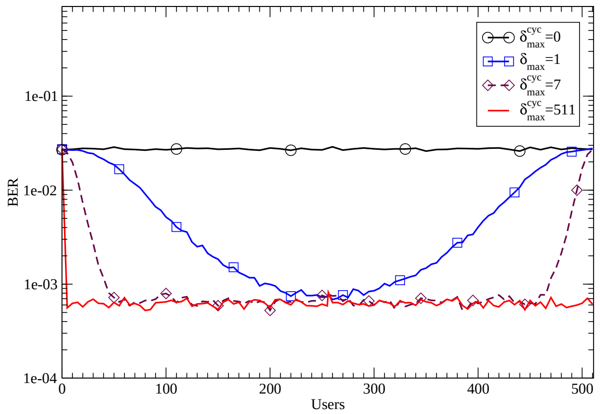


Figure 6. BER for users of *Scheme 1* with different $\delta_{\max}^{\text{cyc}}$ and $N_{\text{T}} = 4$.

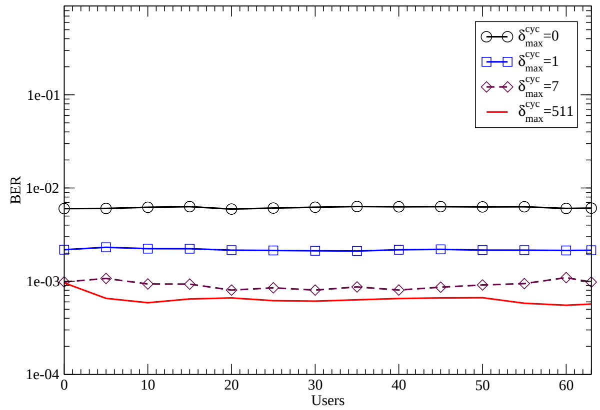


Figure 7. BER for users of *Scheme 2* with different $\delta_{\max}^{\text{cyc}}$ and $N_{\text{T}} = 4$.

IV. Simulation Results

The simulation results for verifying the derived channel analysis of TV-CDD are based on the following parameters. The carrier frequency is 5 GHz, bandwidth is 20 MHz, $N_{\text{FFT}} = 512$, $N_{\text{G}} = 128$, and BPSK modulation. All 512 sub-carriers are used for data transmission. To exploit the received diversity at the receiver a $(171, 133)_{\text{oct}}$ convolutional code is used. The codeword length is set to 200 code bits. The channel has an exponential decaying power delay profile with 25 taps, has a maximum channel delay $\tau_{\max} = 5\ \mu s$, and remains constant over one OFDM frame (quasi-static). Perfect channel knowledge is assumed at the receiver.

Two mapping schemes for the users onto the OFDM frame (consisting of 200 OFDM symbols) will be investigated, see also Figure 5. First, *Scheme 1* is the extreme case by allocating on each sub-carrier only one user, and therefore, this scheme has 512 users. *Scheme 2* is a more realistic approach by distributing each user over 25 consecutive OFDM symbols and 8 sub-carriers which are periodically interleaved over the available sub-carriers. The second scenario serves 64 users.

Figure 6 shows the bit error rat (BER) performances for each user for the first scheme with different $\delta_{\max}^{\text{cyc}}$, 4 transmit antennas and a signal-to-noise ratio (SNR) of 11 dB. Since each user allocates only one sub-carrier, the performance results correlate directly with the analytical results of Figure 4 and (23). For no additional cyclic delay, the system does not increase the frequency/time selectivity, and therefore, the BER has the worst performance constant over all users and marks an upper performance bound. The larger the cyclic delay the more sub-carriers are uncorrelated. With the maximum time-varying cyclic delay of 511, the best performance over all sub-carriers (except the first, see also (16) and (22)) can be achieved. Since the performances are based on 4 used transmit antennas, $\delta_{\max}^{\text{cyc}} \leq 7$ guarantees a standard conformable system ($\tau'_{\max} \leq N_{\text{G}}$ by using (18)) and a preferred maximum performance over most sub-carriers.

By using the mapping of *Scheme 2* a constant performance over all users can be achieved. These results are shown in Figure 7 for an SNR of 7 dB. Even with lower SNR and no introduced cyclic delay we can outperform *Scheme 1* due to the exploitation from the mapping of the frequency and time diversity. There is a large performance gain by increasing the maximum cyclic delay from 0 to 7 and for $\delta_{\max}^{\text{cyc}} = 7$ the performance of $\delta_{\max}^{\text{cyc}} = 511$ is almost reached. Therefore, small $\delta_{\max}^{\text{cyc}}$ can already achieve performances close to the maximum possible performance.

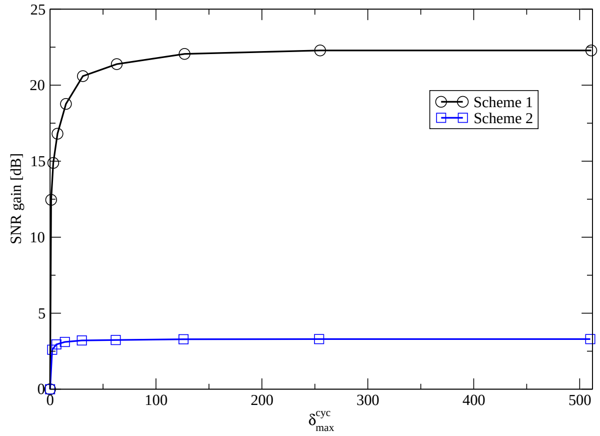


Figure 8. SNR gain in dB for a traget BER of 10^{-4} for varying $\delta_{\max}^{\text{cyc}}$ and $N_{\text{T}} = 4$.

The overall performance differences averaged over all users for both schemes is pictured in Figure 8. The performance differences are measured in a SNR gain in dB for a target BER of 10^{-4}. Through the averaging concept, *Scheme 2* has a smaller SNR gain than *Scheme 1*. The first scenario can gain 22.3 dB by using the maximum possible cyclic delay and *Scheme 2* gains 3.3 dB compared to a system without introduced cyclic delays. Both performances show a fast convergence to the maximum SNR gain for small $\delta_{\max}^{\text{cyc}}$ ($\delta_{\max}^{\text{cyc}} \leq 31$ for *Scheme 1* and $\delta_{\max}^{\text{cyc}} \leq 7$ for *Scheme 2*). These results substantiate the possible choice of small $\delta_{\max}^{\text{cyc}}$ to preserve the standard compatibility of communications systems by using TV-CDD without a larger performance degradation.

V. CONCLUSIONS

In this paper, analytical studies investigate the resulting channel characteristics by using time-varying cyclic delay diversity in an OFDM based transmission system. TV-CDD generates an uncorrelated Rayleigh fading channel over all sub-carriers (except the first) for a maximum chosen interval of the time varying cyclic delay $\delta_{\max}^{\mathrm{cyc}}$. To endeavor a standard conformable receiver structure the use of smaller $\delta_{\max}^{\mathrm{cyc}}$ is necessary. Small performance degradations approved by analytical results and verified by simulation results give the possibility to use TV-CDD in a standard conformable manner for increasing frequency and time diversity.

APPENDIX

To avoid a second transmit antenna branch with no delay $\delta_n^{\mathrm{cyc}}(t) = 0$, we set the interval $\mathbb{S}$ to $[1, \ldots, \delta_{\max}^{\mathrm{cyc}}]$. In future OFDM based communications systems large FFT sizes ($N_{\mathrm{FFT}} \geq 512$) will be chosen to achieve a high spectral efficiency. Therefore, for large N_{FFT} and $P(\delta) = \frac{1}{N_{\mathrm{FFT}}-1}$ the expectation value in (14) is now

$$E\{e^{-j\frac{2\pi}{N_{\mathrm{FFT}}}k\delta_n^{\mathrm{cyc}}(t_1)}\} = \sum_{\delta=1}^{N_{\mathrm{FFT}}-1} e^{-j\frac{2\pi}{N_{\mathrm{FFT}}}k\delta} \cdot P(\delta)$$

$$= \frac{1}{N_{\mathrm{FFT}}-1} \cdot \frac{(e^{-\frac{1}{N_{\mathrm{FFT}}}} - 1)e^{-j2\pi k}}{1 - e^{-j\frac{2\pi}{N_{\mathrm{FFT}}}k}}$$

$$= \begin{cases} \approx 1 & \text{for} \quad k = 0 \text{ and large } N_{\mathrm{FFT}} \\ \approx 0 & \text{for} \quad k \neq 0 \text{ and large } N_{\mathrm{FFT}} \end{cases} . \qquad (25)$$

Consequently, the fading correlation properties are identical to (15), (16), and (17).

In the case of varying $\delta_{\max}^{\mathrm{cyc}}$, the probability of the delays is given by $P(\delta) = \frac{a}{N_{\mathrm{FFT}}-1}$, and therefore, the expectation value of (21) is

$$E\{e^{-j\frac{2\pi}{N_{\mathrm{FFT}}}k\delta_n^{\mathrm{cyc}}(t_1)}\} = \frac{a}{N_{\mathrm{FFT}}-1} \cdot \frac{e^{-j\frac{2\pi}{N_{\mathrm{FFT}}}k} - e^{-j\frac{2\pi}{a}k}}{1 - e^{-j\frac{2\pi}{N_{\mathrm{FFT}}}k}} . \qquad (26)$$

As before, for large FFT sizes the correlation properties approximate the results of (23) and (24) in Section III.

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