

Capacity Scaling of Wireless Networks with Complex Field Network Coding

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Abstract—Network coding in wired networks has been shown to achieve considerable throughput gains relative to traditional routing networks. For wireless multihop networks however, the ergodic capacity is unknown. In this context, the scaling of capacity with the number of nodes (n) has recently received increasing attention. While existing works mainly focus on networks with n source-destination pairs, this paper deals with capacity scaling in any-to-any wireless links, where each node communicates with all other nodes. Complex field network coding (CFNC) is adopted at the physical layer to allow n nodes exchanging information with simultaneous transmissions from multiple sources. As n increases, a hierarchical CFNC-based scheme is developed and shown to achieve asymptotically optimal quadratic capacity scaling in a dense network, where the area is fixed and the density of nodes increases. This is possible by dividing the network into many clusters, with each cluster sub-divided into many sub-clusters, hierarchically. As a result, information is transmitted on multi-input multi-output based multiple access channels and broadcast channels. When generalized to extended networks, where the density of nodes is fixed and the area increases linearly with n , the hierarchical CFNC scheme is shown to scale as $n^{3-\alpha/2}$ for a path loss exponent $\alpha \geq 2$, which is asymptotically optimal when $\alpha < 3$.

Index Terms—Capacity scaling, hierarchical transmission, complex field network coding, multi-input multi-output (MIMO), multiple access channel (MAC), broadcast channel (BC).

I. INTRODUCTION

With the emergence of network science, capacity scaling laws in large ad-hoc wireless networks have attracted growing interest, since the exact ergodic capacity of wireless multihop networks is unknown. Gupta and Kumar first studied the scenario where n nodes are randomly located in the unit disk and each node communicates with a random destination node at a rate $R(n)$ bits/second [1].

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The problem was to assess how fast the total network capacity increases with n , i.e., the maximally achievable scaling of the total capacity $C(n) = nR(n)$. The results in [1] and [2] established that a multihop architecture with conventional single-user decoding and forwarding of packets can achieve $C(n)$ at most $\mathcal{O}(\sqrt{n})$, and the same scaling can be achieved by a scheme using only nearest-neighbor communication.

Different from the *dense* network in [1], where the total area is fixed and the density of nodes increases, many subsequent works dealt with *extended* networks, whose size grows to cover an increasing area with the density of nodes remaining fixed. After successive refinements, the nearest-neighbor multihop scheme was shown to be order-optimal whenever the power path loss exponent α is greater than 3, after bounding the maximum transmit power in the network [3]–[9].

Recently, a scheme based on hierarchical cooperation and distributed multi-input multi-output (MIMO) communication was developed to identify the scaling laws of random ad hoc networks for any path loss exponent $\alpha \geq 2$ [9]. For dense networks, [9] established that the total capacity scales linearly with n . For extended networks, this capacity scales as $n^{2-\alpha/2}$ for $2 \leq \alpha < 3$ and \sqrt{n} for $\alpha \geq 3$. Hence, a better scaling than multihop can be achieved in dense and extended networks under low attenuation.

The Gupta-Kumar model assumes that the signals received from nodes other than the source constitute interference that is regarded as noise degrading the communication link. Under this assumption, direct communication between source and destination pairs is not preferable, as the interference generated discourages most other nodes from communicating. Complex field network coding (CFNC), however, allows multiple users to transmit simultaneously to a destination after precoding, which turns destructive interference into a constructive signal [10]. This motivates the present paper's utilization of CFNC to achieve an improved capacity scaling law.

When traditional Galois field network coding (GFNC) is employed by random networks with n source-destination pairs, compared to the scheme in [1], there is only a constant (as opposed to a scaling) gain [11], [12].

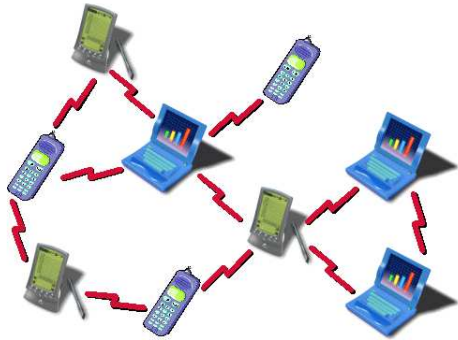


Fig. 1. An ad hoc wireless network.

In contrast, this paper establishes that CFNC achieves asymptotically optimal capacity scaling in a wireless network, where each node transmits to all other nodes. This any-to-any connectivity appears often in both tactical and commercial ad hoc networks, as illustrated in Fig. 1.

The results here apply to both dense and extended networks, which correspond to interference limited and coverage limited regimes, respectively. As the distributed MIMO scheme of [9] does not attain a desirable capacity scaling in a network with n^2 source-destination pairs, the CFNC-based hierarchical scheme here divides the network (or sub-network) into multiple clusters in each layer of the hierarchy. Each layer includes five transmission phases, which entail MIMO multiple access (MAC) and MIMO broadcast (BC) channels [13]–[16]. CFNC is used to overcome the interference during simultaneous transmissions in MIMO-MAC and MIMO-BC. The number of clusters M per layer is critical to the total capacity scaling, which is now defined as $C(n) = n^2 R(n)$ for the n^2 pairing network. With M increasing slowly as n increases, the 5-phase scheme achieves a capacity scaling of order $\mathcal{O}(n^{2-\epsilon})$ in dense networks, for any $\epsilon > 0$. As the capacity scaling is upper-bounded by $\mathcal{O}(n^2 \log n)$, this scheme is nearly optimal. Moreover, the associated capacity scaling exponent approaches the upper bound as n grows large, which justifies the asymptotic optimality claim. For extended networks, a bursty version of the 5-phase scheme achieves a capacity scaling of $\mathcal{O}(n^{3-\alpha/2-\epsilon})$, for any $\epsilon > 0$, which is also asymptotically optimal when $\alpha < 3$.

The rest of this paper is organized as follows. Section II introduces the model and pertinent assumptions; upper and lower bounds of the capacity scaling are developed in Section III; a simple CFNC scheme for information exchange within one cluster is described in Section IV; Section V introduces a hierarchical CFNC scheme which is asymptotically order-optimal in dense networks; Section VI considers extended networks and conclusions are summarized in Section VII.

Notation: Upper and lower case bold symbols denote matrices and column vectors, respectively; $(\cdot)^T$ denotes transpose; $(\cdot)^H$ Hermitian transpose; $\mathcal{CN}(0, \sigma^2)$ the circular symmetric complex Gaussian distribution with zero mean and variance σ^2 ; for a random variable γ , $E[\gamma]$ denotes its mean; $C(n) = \mathcal{O}(n^t)$ means that $\lim_{n \rightarrow \infty} C(n)/n^t = K$, for some bounded constant $K > 0$.

II. MODELING

Consider n nodes uniformly and independently distributed in a square of unit area in dense networks (Sections III–V), or, a square of $\sqrt{n} \times \sqrt{n}$ area when dealing with extended networks (Section VI). Any node can be the source of information to all other nodes, and at the same time, any node can be the destination of all source nodes. Hence, there can be $n(n-1)$ possible source-destination pairs in total. Suppose that each source has the same traffic rate to send to its destination node and a common average transmit power budget of P Joules per symbol. The overall network throughput is $C(n) = n(n-1)R(n)$, where $R(n)$ is the achievable rate per source-destination pair. For simplicity in exposition, suppose that every node is also the destination for itself, that is $C(n) = n^2 R(n)$ from now on.

We assume that wireless communication takes place over a flat channel of bandwidth W Hertz around a carrier frequency f_c with $f_c \gg W$. The complex baseband-equivalent channel gain between node i and node k at time slot m is given by

$$H_{ik}[m] = \sqrt{G} r_{ik}^{-\alpha/2} \exp(j\theta_{ik}[m]) \quad (1)$$

where r_{ik} is the distance between nodes, $\theta_{ik}[m]$ denotes random phase at time m , uniformly distributed in $[0, 2\pi]$ and $\{\theta_{ik}[m]\}_{i,k=1}^n$ is a collection of independent and identically distributed (i.i.d.) random processes. Variables $\theta_{ik}[m]$ and r_{ik} are also assumed independent, while the gain G and the path-loss exponent $\alpha \geq 2$ are assumed constant.

Note that the channel is random and depends on the location of nodes and the channel phases. The locations are assumed to be fixed, while the phases are allowed to vary in a stationary ergodic manner (fast fading). All channel gains are assumed available to all nodes. The signal received by node i at time m is

$$Y_i[m] = \sum_{k=1}^n H_{ik}[m] X_k[m] + Z_i[m] \quad (2)$$

where $X_k[m]$ stands for the symbol sent by node k at time m and $Z_i[m] \sim \mathcal{CN}(0, \sigma^2)$.

The path-loss model applies to a far-field scenario, where the distance is assumed much larger than the carrier wavelength. When the distance is in the order or shorter

than the carrier wavelength, the simple path-loss model no longer holds, since path loss can potentially become path “gain”. The phase $\theta_{ik}[m]$ depends on the distance between the nodes modulo the carrier wavelength. This random-phase model also fits a far-field scenario because node separation is at a much larger spatial scale compared to the carrier wavelength; hence the phases can be modeled as completely random and independent of the actual positions. In addition, a line-of-sight type environment is assumed and multi-path effects are ignored. Nevertheless, the ensuing results can be extended to the multi-path case if multi-carrier modulation is adopted.

III. CAPACITY SCALING

Capacity scaling quantifies how fast the information-theoretic capacity increases with the network size n . The pertinent metric is provided by the scaling exponent $e(n)$, which is defined as

$$e(n) := \lim_{n \rightarrow \infty} \frac{\log C(n)}{\log n}. \quad (3)$$

In networks for which the exact capacity expression is not available, capacity scaling reveals how much throughput gain one can expect as the network size grows. This in turn delineates the tradeoff between throughput gain and deployment cost, which is critical for the network design.

A. Upper Bound

This section provides an information-theoretic upper bound on the achievable scaling law for the aggregate throughput in the network model of Section II. Before pursuing practical communication strategies, the following theorem establishes the best one can hope for.

Theorem 1: *The aggregate throughput in the dense network is bounded above by*

$$C(n) \leq K'n^2 \log n$$

with high probability (i.e., with probability going to 1 as n grows) for some constant K' independent of the number of nodes n .

Proof: For each pair, the transmission rate $R(n)$ from source node to destination node is upper-bounded by the capacity of the single-input multiple-output (SIMO) channel between the source node and the rest of the network. From [9, Thm. 3.1], it follows that $R(n) \leq K' \log n$, with some constant K' independent of n , for all source-destination pairs in the network with high probability. Hence, Theorem 1 follows readily since there are n^2 source-destination pairs in the present setup. ■

Now let us consider an unrealistic example which achieves this upper bound by capitalizing on standard properties of wireless communications, namely: (p1) omnidirectional transmissions, (p2) interference due to simultaneous transmissions from different sources, and (p3)

the half duplex constraint, which disallows simultaneous packet transmission and reception by any node (due to the constraint that nodes are equipped with a single transceiver). If one could bypass constraints p2 and p3, then all n nodes in the network would be allowed to broadcast together, while at the same time, each node would receive the messages from all other nodes. One can easily verify that each source-destination pair in such a scheme freed from p2 and p3, achieves capacity scaling $R(n) = \mathcal{O}(1)$, implying a total capacity scaling of $C(n) = \mathcal{O}(n^2)$, with scaling exponent $e(n) = 2$. We will term this kind of scheme asymptotically optimal, as the scaling exponent difference from the upper bound of Theorem 1 is just $\epsilon(n) = \log_n(\log n)$, which disappears as n increases to infinity.

B. Lower bound

Having envisioned an asymptotically optimal scheme that is too ideal to be true, one is motivated to look also for lower bounds on the capacity scaling. To this end, notice first that any realistic scheme obviously yields an achievable rate scaling, which at the same time provides a lower bound on the capacity scaling of the wireless network. Furthermore, the hierarchical cooperation scheme introduced by [9], which achieves the asymptotically optimal capacity scaling in a network of n pairs, does not lead to an asymptotically optimal capacity scaling in the network of n^2 pairs considered here. Actually, when the hierarchical cooperation scheme in [9] is modified to apply in the network of n^2 pairs, the capacity scaling is still linear: $C(n) = \mathcal{O}(n)$ [17].

For the lower bound, the main result of this paper can be summarized as follows:

Theorem 2: *With $\alpha \geq 2$ and for any $\epsilon > 0$, there exists a constant $K_\epsilon > 0$ independent of n such that with high probability, the aggregate throughput*

$$C(n) \geq K_\epsilon n^{2-\epsilon}$$

is achievable by the dense network model with n^2 source-destination pairs.

Theorem 2 asserts that the achievable capacity scaling can come arbitrarily close to the upper bound of Theorem 1, i.e., one can devise an asymptotically optimal scheme in the wireless network with n^2 pairs. Instrumental to proving Theorem 2 is to show that the interference property p2 can be mitigated with cooperation among nodes using the complex field network coding (CFNC) approach introduced in [10] to achieve high throughput and the maximum diversity gain provided by the wireless network.

The proof of Theorem 2 relies on the construction of an explicit scheme that realizes the promised scaling law. But before that, it is useful to consider the simple transmission scheme of the ensuing section, which is based on CFNC.

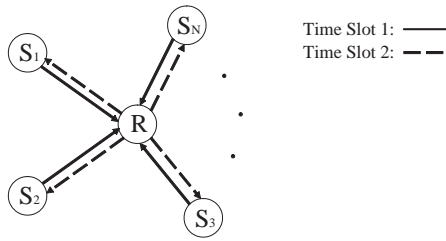


Fig. 2. Information exchange without a common destination in one cluster.

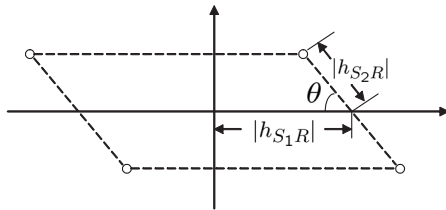


Fig. 3. The constellation received at R in a CFNC-based 2-source 1-relay network with BPSK.

IV. INFORMATION EXCHANGE WITHIN ONE CLUSTER

The information exchange scheme in [10] can be modified to the scenario where there are $M - 1$ nodes $\{S_i\}_{i=1}^{M-1}$, exchanging information with the help of one relay node R yet without any common destination. This motivates the scheme we will develop to achieve the asymptotically optimal capacity scaling in Theorem 2.

A. Information Exchange without a Common D

Consider a cluster of $M - 1$ sources exchanging information without a common destination. As illustrated in Fig. 2, each transmission takes place over two time slots, during which the input-output channel relationships are

$$y_{SR} = \theta_{IE}^T \mathbf{H}_{SR} \mathbf{x} + n_{SR} \quad (4)$$

$$y_{RS_i} = h_{RS_i} \theta_{IE}^T \hat{\mathbf{x}} + n_{RS_i} \quad (5)$$

where $\hat{\mathbf{x}}$ is the estimate of the source's information block \mathbf{x} and the row vector θ_{IE}^T denotes the linear CFNC precoder as in the information exchange (IE) scheme of [10].

In the IE scheme, every source knows its index and the channel state information (CSI) of the $S_i \leftrightarrow R$ link ($h_{S_i R}$) is available at both S_i and R . This is possible e.g., when the $S_i \leftrightarrow R$ channels are reciprocal: $h_{S_i R} = h_{RS_i}$. The transmission of S_i can be channel adaptive in order to e.g., (i) cancel (or control) the phase of $h_{S_i R}$; (ii) cancel (or control) the fading magnitude $|h_{S_i R}|$; and (iii) perform symbol level synchronization with other sources.

Consider the $S_i \rightarrow R$ links in a simple 2-source 1-relay setup, where CFNC is applied to BPSK symbols. Without loss of generality, suppose that $|h_{S_1 R}| \geq |h_{S_2 R}|$

and the phase of $h_{S_1 R}$ is 0 since R can always cancel the phase of $h_{S_1 R}$. The constellation received at R is depicted in Fig. 3, where θ depends only on the phase $\theta_{h_{S_2 R}}$, as $\theta = \pi - (\theta_{h_{S_2 R}} + 3\pi/4)$ [18]. It is easy to see that $\theta = \pi/2$ maximizes the minimum Euclidean distance (MED) between received symbols. Since each source node only knows the link between itself and the relay, the transmit power can be optimized at each individual source accordingly.

For a cluster comprising M^2 pairs, the IE scheme provides a means to achieve scaling of order $\mathcal{O}(M^2/2)$ using CFNC at the physical layer [10]. Since the focus is on capacity scaling and the fading channels are i.i.d., it is possible to set the average power budget equal across nodes. Assuming that symbol level synchronization has been achieved, we are ready to consider the n -node network with multiple clusters.

B. Number of Nodes per Cluster

CFNC is designed to mitigate the effect of the wireless property p_2 , when multiple messages are concurrently received at each node. Since CFNC is also subject to the half duplex constraint, at least two time slots are needed to exchange 1 bit for every transmission pair. This is clearly the case if a multiple access channel (MAC) transmission is used in Time Slot 1, and a broadcast channel (BC) transmission is used in Time Slot 2. Since the BC transmission is upper bounded by the capacity scaling in Theorem 1, focus will be placed henceforth on the MAC transmission.

In a MAC channel with n source nodes and one destination, each equipped with a single antenna, the per-node average power budget will be upper bounded by P/n as opposed to P , for a reason to be clarified in the next section. Then, the sum capacity can be derived from the following lemma (see Appendix A for the proof).

Lemma 1: *The sum mutual information achieved by a MIMO-MAC transmission from M nodes to a single node, each equipped with N antennas, grows at least linearly with N .*

If all n nodes are clumped into one cluster, i.e., $M = n$ and $N = 1$, Lemma 1 asserts that the per node rate is $R(n) = \mathcal{O}(1/n)$ and the aggregate capacity scaling is thus $C(n) = n^2 R(n) = \mathcal{O}(n)$. To improve this capacity scaling law as in Theorem 2, we will rely on the CFNC-based hierarchical transmission described next.

V. HIERARCHICAL TRANSMISSIONS WITH CFNC

The goal of this section is to prove Theorem 2 by constructing a realistic scheme based on hierarchical clustering and CFNC transmissions among clusters. As we have seen in the previous section, to achieve an asymptotically optimal capacity scaling, the number of

nodes (M) transmitting over the MAC should not increase as fast as the network size n . This is possible if the network is split into multiple (M) subnetworks or clusters, each covering a smaller square of area $A = 1/M$. Since there are n nodes uniformly distributed in the network, there will be on average $nA = n/M$ nodes inside each cluster, and each cluster will contain order n/M nodes with probability higher than $1 - Me^{-\Lambda(\delta)n/M}$, where $\Lambda(\delta)$ is independent of n and satisfies $\Lambda(\delta) > 0$ when $\delta > 0$ [9]. While n increases, each cluster should be divided again into another M clusters, each containing n/M^2 nodes. This kind of hierarchical sub-division can be successively performed until each cluster contains less than or equal to M nodes, which results in a total of $\log_M(n)$ layers in the hierarchical clustering.¹

Focusing on the transmission taking place in a particular layer $h+1$ of the hierarchy, consider that layer h has transmission rate R_h , $h = 1, \dots, \log_M(n)$. The last layer $h = 1$ corresponds to the bottom layer of the hierarchy, while $h = \log_M(n)$ denotes the top layer which includes the entire network of size n . In layer $h+1$, each of the M clusters operates at rate R_h and the entire transmission proceeds in five steps.

(s1) Nodes in each cluster exchange information at rate R_h , as detailed in Section IV-A.

(s2) All nodes of each cluster then form a distributed transmit antenna array, so that $M-1$ clusters operate as $M-1$ nodes, each with M^h transmit antennas, for their information bits to be received coherently by the remaining cluster which serves as a relay.

(s3) Each node in the relay cluster obtains one observation from each CFNC transmission in $s2$. This node quantizes and exchanges the observation with the other nodes at rate R_h within the cluster, which can then perform joint MIMO decoding to obtain the transmitted bits using CFNC and multiuser detection as detailed in [10].

(s4) Nodes in the relay cluster re-encode the decoded information from $s3$ as well as their own information, and broadcast to the other $M-1$ clusters together as a distributed transmit antenna array.

(s5) Finally, each of the $M-1$ clusters performs joint MIMO decoding using CFNC and multiuser detection again at rate R_h similar to the relay cluster in $s3$.

From the network point of view, steps $s2$ and $s4$ are MIMO-MAC and MIMO-BC transmissions, respectively, at the cluster level; while steps $s1$, $s3$, and $s5$ include local communications within each cluster and can be carried out in parallel at multiple clusters. This leads to five operating phases of the network:

Phase 1. Information Exchange within Each Cluster: As illustrated in Fig. 4, clusters start communicating

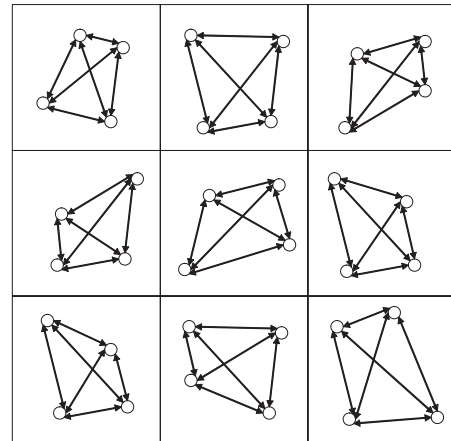


Fig. 4. Phase 1 (also Phase 3 and Phase 5) of the hierarchical transmission scheme using CFNC.

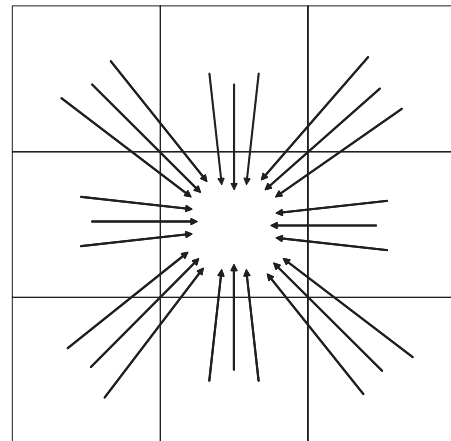


Fig. 5. Phase 2 of the hierarchical transmission scheme using CFNC.

in parallel. Within a cluster, each node distributes B bits to each of the other nodes, so that at the end of this phase, each node has B bits from each of the other nodes in the same cluster. This requires transmitting B bits for each source-destination pair. As each node in the cluster is also the destination of other nodes in the cluster, there is no extra traffic demand introduced by this clustering operation. With this per-cluster transaction occupying T_h time slots, the throughput in Phase 1 is B/T_h , where h denotes the layer in the hierarchy.

Phase 2. MIMO-MAC using CFNC: In this phase, MIMO-MAC transmissions from $M-1$ clusters are directed to the single designated relay cluster. The remaining $M-1$ clusters will be henceforth termed source clusters. The relay cluster is chosen to minimize the total transmit power in Phases 2 and 4. During the MAC transmissions, the bits from the $M-1$ source clusters are transmitted using CFNC and arrive simultaneously at the nodes in the relay cluster, as illustrated in Fig. 5.

¹Unless stated otherwise, it is assumed for simplicity that n is an integer power of M .

Letting $r_{S_i R}$ denote the distance between the midpoints of the source cluster S_i and the relay cluster R , the average transmit power per node is $P(r_{S_i R})^\alpha/M^h$ at layer h . As in the previous section, precoding and symbol synchronization precede each CFNC transmission. The nodes in cluster R quantize and accumulate the signals without decoding the information symbols in this phase. From Section IV, it is clear that this phase requires B time slots, one per bit transmitted from the nodes in each source cluster.

The per-cluster area at layer h is $A_h = 1/M^{\log_M(n)-h}$, and the per-node power is assumed upper bounded by $P(A_h)^{\alpha/2}/M^h$. For the parallel operation to be reliable, it is necessary to further bound the inter-cluster interference as in the following lemma, which is proved in Appendix B.

Lemma 2: For a network of size n , consider clusters of size M^h and area A_h operating as in the 5-phase scheme. Let each node have an average power $P(A_h)^{\alpha/2}/M^h$. For $\alpha > 2$, the interference power received by a node from other simultaneously operating clusters is upper-bounded by MK_{I_1} with a constant K_{I_1} independent of n . For $\alpha = 2$, the interference power is upper-bounded by $MK_{I_2} \log n$ with a constant K_{I_2} independent of n . In addition, the interference signals received by different nodes in the cluster are zero-mean and uncorrelated.

Phase 3. Joint Decoding in the Relay Cluster: Since nodes inside the relay cluster form a distributed receive antenna array, each node receives B MIMO-MAC transmissions during Phase 2. Thus, each node in the cluster receives B observations, one from each MIMO-MAC transmission, and each observation is to be conveyed to all other nodes for decoding. Since these observations are real numbers, nodes in the relay cluster quantize each observation to Q bits; hence, there are now a total of at most QB bits to exchange inside the relay cluster. Using exactly the same scheme as in Phase 1, it is clear that this phase requires QT_h time slots; and the transmission is again illustrated in Fig. 4.

Phase 4. MIMO-BC using CFNC: This phase entails MIMO-BC transmissions from the relay cluster to the source clusters, as depicted in Fig. 6. CFNC is used again as in the previous section, and by analogy it follows that this phase is completed in B time slots.

Phase 5. Joint Decoding in Source Clusters: Since each source cluster receives B MIMO-BC transmissions in Phase 4, each node in the source clusters quantizes and exchanges each observation similar to the relay nodes during Phase 3 using a total of QT_h time slots; see again Fig. 4.

Phases 1, 3, and 5 contain further MIMO-MAC and MIMO-BC transmissions at lower hierarchies, as illustrated in Fig. 7. Therefore, all transmissions in this 5-

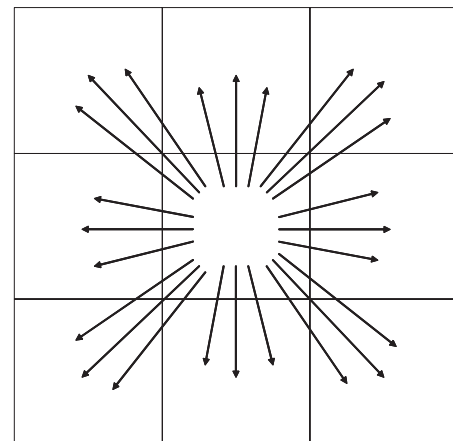


Fig. 6. Phase 4 of the hierarchical transmission scheme using CFNC.

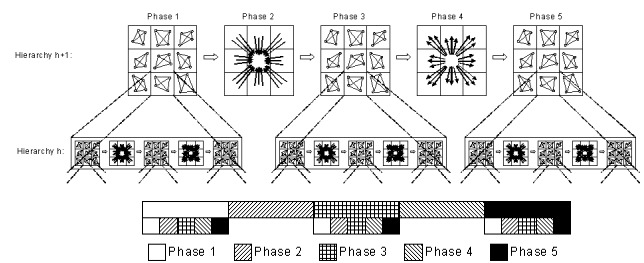


Fig. 7. Hierarchical structure and time division of the 5-phase scheme.

phase scheme take place during Phases 2 and 4 in each layer of the hierarchy. The CFNC scheme described in Section IV provides a concrete physical layer to cope with the interference issues emerging in Phases 2 and 4. As proved in Appendix B, the inter-cluster interference power received at each node in Phase 4 also follows Lemma 2.

With each destination node capable of decoding the source bits from the quantized signals it collects by the end of Phase 5, the total number of time slots used in layer $h + 1$ is

$$T_{h+1} = (2Q + 1)T_h + 2B \tag{6}$$

where $h = 1, 2, \dots, \log_M(n) - 1$ and $T_1 = 2B$. It then follows readily that

$$T_h = B \frac{(2Q + 1)^h - 1}{Q}, \quad h = 1, 2, \dots, \log_M(n) \tag{7}$$

and the total number of time slots used in this 5-phase scheme is

$$T_{total} = T_{\log_M(n)} = B \frac{(2Q + 1)^{\log_M(n)} - 1}{Q}. \tag{8}$$

Before returning to the capacity scaling issue, it is useful to clarify several definitions of the achievable rate. Following the conventional definition in n pairing networks, the total rate is B/T_{total} . While there are n^2

source-destination pairs in the network here, we will focus on the achievable rate for each source-destination pair. The following lemma quantifies the capacity scaling of this 5-phase scheme (see Appendix C for the proof).

Lemma 3: *When $\alpha > 2$, the sum mutual information achieved by the MIMO-MAC from M nodes to one node, each equipped with N antennas, grows at least linearly with N . The other way around, same scaling for the sum mutual information can be achieved in a MIMO-BC transmission. When $\alpha = 2$, the sum mutual information in both MIMO-MAC and MIMO-BC grows at least on the order of $\mathcal{O}(N/\log n)$ for a network of size n .*

Consider first the case of $\alpha > 2$. Recall that all transmissions in this CFNC scheme are either MAC or BC. While joint encoding and decoding is employed in other phases, during MAC and BC transmissions, each cluster is treated as a single node with multiple antennas. At layer $h + 1$, each node in the MAC from $M - 1$ nodes to one node, has $N = M^h$ antennas. From Lemma 3, this leads to a capacity scaling of $\mathcal{O}(N)$ for the MAC. While considering each transmission pair, the achievable capacity scaling per pair is $R(n) = \mathcal{O}(N/(MN)) = \mathcal{O}(1/M)$. As the rate during the BC transmission is $R(n) = \mathcal{O}(1/M)$, the achievable capacity scaling per pair in the 5-phase scheme suffers a penalty of M relative to the conventional definition. Thus, the aggregate capacity scaling per pair in the 5-phase scheme is

$$R(n) = \frac{B}{MT_{total}} = \frac{Q}{M} \frac{1}{(2Q+1)^{\log_M(n)} - 1} \quad (9)$$

and as a result, the capacity scaling of the entire network is

$$C(n) = n^2 R(n) = \frac{Q}{M} \frac{n^2}{(2Q+1)^{\log_M(n)} - 1}. \quad (10)$$

Using (10), the following lemma proved in Appendix D yields the capacity scaling asserted in Theorem 2.

Lemma 4: *There exists a strategy to encode the observations at a fixed rate of Q bits per observation and arrive at a sum mutual information growth rate of $\mathcal{O}(N)$ (when $\alpha > 2$) or $\mathcal{O}(N/\log n)$ (when $\alpha = 2$) for the resultant quantized MIMO-MAC and MIMO-BC channels.*

Having fixed Q , let us turn our attention to M . If M is also fixed, the capacity scaling from (10) is

$$C(n) = \mathcal{O}(n^{2-\log_M(2Q+1)}) \quad (11)$$

which is not as high as asserted by Theorem 2.

To achieve an asymptotically optimal capacity scaling promised by Theorem 2, consider $M = \log n$, which implies that the size of each layer in the hierarchy M increases sufficiently slowly with the network size n . Furthermore, it is prudent to seek an optimal M to maximize the capacity scaling exponent. As M increases

with n , the capacity scaling from (10) is

$$C(n) = \mathcal{O}\left(\frac{n^2}{M(2Q+1)^{\log_M(n)}}\right) \quad (12)$$

$$= \mathcal{O}\left(n^{2-\log_n(M)-\log_M(2Q+1)}\right). \quad (13)$$

To maximize the capacity scaling exponent is equivalent to:

$$\min_M \{\log_n(M) + \log_M(2Q+1)\}. \quad (14)$$

The optimal solution is $2/\sqrt{\log_{2Q+1}(n)}$, when M is chosen as $\log_{2Q+1}(M) = \sqrt{\log_{2Q+1}(n)}$. As a consequence, the capacity scaling of $C(n) = \mathcal{O}\left(n^{2-2/\sqrt{\log_{2Q+1}(n)}}\right)$ is achievable, which proves Theorem 2 for $\alpha > 2$.

When $\alpha = 2$, the per node capacity scaling incurs a penalty of $M \log n$ compared to the conventional case given in Lemma 3. Moreover, the number of transmissions in Phases 2 and 4 will scale as $\log n$, which makes the number of observations at each receiver node also scale as $\log n$. Hence, instead of QT_h , we will have $QT_h \log n$ time slots in Phases 3 and 5. After incorporating these modifications, the overall capacity scaling for $\alpha = 2$ is [cf. (9) and (10)]

$$C(n) = n^2 R(n) = \frac{n^2 B}{MT_{total} \log n} \quad (15)$$

$$= \frac{Q}{M} \frac{n^2}{(2Q \log n + 1)^{\log_M(n)} - 1}. \quad (16)$$

Although it is cumbersome to obtain the optimal M maximizing this capacity scaling, we are ready to complete the proof of Theorem 2. To achieve a capacity scaling of $\mathcal{O}(n^{2-\epsilon})$, it suffices to choose $M = (\log n)^{\log(\log n)}$, which yields a capacity scaling exponent

$$e(n) = 2 - \log_n(M) - \log_M(2Q \log n + 1) \quad (17)$$

$$\geq 2 - \frac{[\log(\log n)]^2}{\log n} - \frac{2}{\log(\log n)}. \quad (18)$$

With $e(n)$ in (18) approaches 2 as n goes to infinity, this completes the proof of Theorem 2.

Remark 1: The power budget P/n is critical to constrain the power of the aggregate interference in Lemma 2, so that the parallel operation of multiple clusters is reliable. Moreover, the P/n budget constraint is instrumental in proving Lemmas 1-4, both for dense as well as for the extended networks treated in the next section.

Remark 2: The 5-phase scheme developed can be further optimized to a 4-phase alternative. When the relay cluster performs joint decoding in Phase 3, the source clusters can exchange information as in Phase 1. Similarly during Phase 5, while the source clusters decode, the relay cluster can exchange information as in Phase 1. Thus, Phase 1 of layer $h + 1$ in the hierarchy can be performed in parallel with Phases 3 and 5 of layer h . The

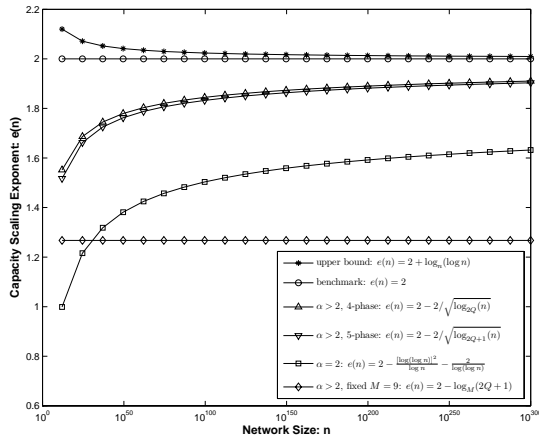


Fig. 8. Capacity scaling exponent under various scenarios in a dense network.

only exception is the bottom layer, which always contains Phases 2 and 4 only. For example, if $\alpha > 2$, the capacity scaling can be improved to $C(n) = \frac{Q}{M} \frac{n^2}{(2Q)^{\log_M(n)} - 1}$, while the optimal capacity scaling exponent becomes $2 - 2/\sqrt{\log_2 Q(n)}$. Achievable capacity scaling exponents under various scenarios are depicted in Fig. 8.

Remark 3: Different from [9], it is not necessary to consider the relative position of the source and destination nodes, simply because each node here is the destination of all other nodes, and no time division multiple access (TDMA) transmissions are needed across clusters in each layer of the hierarchy.

VI. EXTENDED NETWORKS

In the dense networks considered so far, the total geographical area is fixed as the density of nodes increases. Another natural scaling appears in the so termed extended networks, where the density of nodes is fixed while the $\sqrt{n} \times \sqrt{n}$ area increases. This models the situation where the network area is expanded to cover a larger geographical area.

As compared to dense networks, the distance between nodes is increased by a factor of \sqrt{n} ; and hence for the same transmit powers, the received powers are all decreased by a factor of $n^{\alpha/2}$. Equivalently, by re-scaling space, an extended network can be considered as a dense network on a unit area but with the average power constraint per node reduced to $P/n^{\alpha/2}$ instead of P [9].

Theorem 2 established for dense networks carries over to extended networks too, provided that the average power per node in the hierarchical 5-phase scheme is constrained to P/n . This implies that when $\alpha = 2$, the 5-phase scheme applied to extended networks leads to quadratic scaling. For extended networks with $\alpha > 2$, the same

scheme does not satisfy the equivalent power constraint $P/n^{\alpha/2}$. However, it is possible to consider a simple “bursty” modification of the hierarchical CFNC approach which runs the scheme a fraction $1/(n^{\alpha/2-1})$ of the time with power P/n per node and remains silent for the rest of the time. This meets the given average power constraint of $P/n^{\alpha/2}$, while achieving an aggregate throughput of order $\mathcal{O}(n^{2-\epsilon}/n^{\alpha/2-1}) = \mathcal{O}(n^{3-\alpha/2-\epsilon})$.

Recall that the multihop scheme used in n pairing networks achieves capacity scaling $\mathcal{O}(\sqrt{n})$. In a multihop n^2 pairing network, the nodes can transmit for n transmission cycles, with each cycle aiming at a different n pairing. As a result, multihop can be modified to fulfill the any-to-any communications with n times the original time slots. Therefore, the capacity scaling for multihop n^2 pairing is still $\mathcal{O}(\sqrt{n})$. To summarize, the following result holds for extended networks, the counterpart of Theorem 2 for dense networks.

Theorem 3: Consider an extended network on a $\sqrt{n} \times \sqrt{n}$ square. If $\alpha \in [2, 5)$, then for every $\epsilon > 0$, with high probability, an aggregate throughput $C(n) \geq Kn^{3-\alpha/2-\epsilon}$ is achievable, where $K > 0$ is a constant independent of n . If $\alpha \geq 5$, then with high probability, an aggregate throughput $C(n) \geq K\sqrt{n}$ is achievable, where $K > 0$ is a constant independent of n .

An upper bound on the n^2 pairing extended network can be derived from [9], since the only difference between the two network setups is the number of transmission pairs. This implies that our upper bound is n times the bound in [9] as summarized in the following theorem.

Theorem 4: For any $\epsilon > 0$, the aggregate throughput of an extended network with n nodes is bounded above by

$$C(n) \leq \begin{cases} K'n^{3-\alpha/2+\epsilon}, & 2 \leq \alpha \leq 3 \\ K'n^{3/2+\epsilon}, & \alpha > 3 \end{cases}$$

with high probability for a constant $K' > 0$ independent of n .

The lower and upper bounds do not meet when $\alpha > 3$ [cf. Theorems 3 and 4], which calls for either a more sophisticated transmission scheme, or, a tighter upper bound. Both subjects go beyond the scope of the present paper but constitute interesting future research topics.

VII. CONCLUSIONS

We have investigated the capacity limits of wireless ad hoc networks along with physical layer design and processing steps required to meet them. In any-to-any connectivity, the capacity performance of the traditional transmission scheme is limited by the interference among simultaneous transmissions. CFNC can mitigate the interference through judicious precoding. As the network size increases, we have established that a hierarchical

scheme based on CFNC can achieve asymptotically optimal quadratic capacity scaling in a dense network. When applied to extended networks, the hierarchical CFNC scheme is still asymptotically optimal when the path loss of the propagation medium is relatively low, namely $\alpha \in [2, 3)$.²

APPENDIX

A. Proof of Lemma 1

The $MN \times N$ MIMO-MAC between the M source clusters S_i and the relay cluster R is

$$Y = \sum_{i=1}^M H_i X_i + Z \tag{19}$$

where each element of H_i obeys (1), and $Z = (Z_k)$ denotes uncorrelated receiver noise with power N_0 . The transmitted signals are assumed drawn from an i.i.d. $\mathcal{CN}(0, \sigma^2)$ randomly chosen codebook with $\sigma^2 = P(r_{S_i R})^\alpha / (MN)$, where $N = M^h$ for the MAC transmission at layer $h + 1$ of the hierarchy. Recall that $r_{S_i R}$ denotes the distance between the midpoints of the source and relay clusters, and the clusters are split so that the relay cluster always contains the midpoint of the area on the current layer. Since each cluster at layer h occupies a square of area $A_h = M^h/n$ and $r_{S_i R} \leq \sqrt{A_h}$, it follows that $\sigma^2 \leq \frac{P}{M^h} \left(\frac{M^h}{n}\right)^{\alpha/2} = \frac{P}{n} \left(\frac{M^h}{n}\right)^{\alpha/2-1} \leq \frac{P}{n}$.

Assuming that under this power constraint, the noise Z is i.i.d. Gaussian, and perfect CSI is available at both transmit- and receive-ends, the sum capacity of this MIMO-MAC is lower bounded by [19, Chapter 10]

$$C_{\text{sum}} \geq E \left[\log \det \left(I_N + \text{SNR} \sum_{i=1}^M \frac{F_i F_i^H}{MN} \right) \right] \tag{20}$$

where $\text{SNR} := GP/N_0$, $F_{kl} := \rho_{kl} \exp(j\theta_{kl})$, and there exists $b > a > 0$ with a and b independent of n , such that all ρ_{kl} lie in the interval $[a, b]$ specified in [9]. After bounding the eigenvalues of $F_i F_i^H / (MN)$, arguments similar to those in [9] lead to

$$C_{\text{sum}} \geq N \log(1 + M \frac{a}{2} \text{SNR}) \frac{(a^2 - a/2)^2}{2b^4}. \tag{21}$$

Equation (21) shows that C_{sum} grows at least linearly with N , which proves Lemma 1.

B. Proof of Lemma 2

To establish the lemma for MIMO-MAC transmissions, consider a node v in cluster V operating under the 5-phase scheme, as illustrated in Fig. 9 with $M = 9$. The

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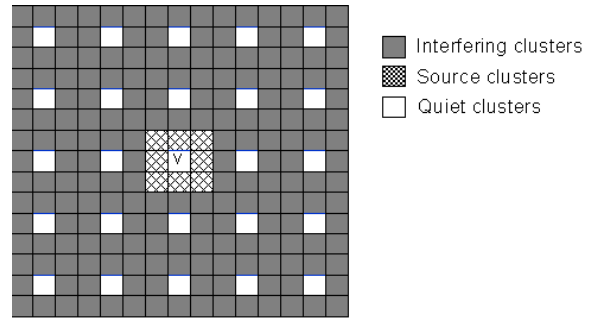


Fig. 9. Interfering clusters during MIMO-MAC transmissions in the 5-phase scheme.

interference present at this node due to a parallel operating cluster set \mathcal{U}_V is given by

$$I_v = \sum_{U \in \mathcal{U}_V} \sum_{j \in U} H_{vj} X_j \tag{22}$$

where H_{vj} is given by (1), and X_j denotes the signal transmitted by a node j belonging to a simultaneously operating cluster U , which contains M^h nodes. Note first that the signals I_v and $I_{v'}$ received by two different nodes v and v' in V are uncorrelated since the channel coefficients H_{vj} and $H_{v'j}$ are independent for all j . The power of I_v is clearly

$$P_I = \sum_{U \in \mathcal{U}_V} \sum_{j \in U} \frac{GP_j}{(r_{vj})^\alpha} \tag{23}$$

because the channel coefficients corresponding to different nodes j are independent. As illustrated by Fig. 9, the interfering clusters \mathcal{U}_V can be grouped based on their distance to V so that each group $\mathcal{U}_V(i)$ contains $(M-1)^2 i$ clusters or less; and each cluster in group $\mathcal{U}_V(i)$ is separated by a distance greater than $(\sqrt{M}i - \sqrt{M} + 1)\sqrt{A_h}$ from V for $i = 1, 2, \dots$, where A_h denotes the cluster area. Since there are n/M^h clusters at layer h of the hierarchy, the number of such groups is upper bounded by n/M^{h+1} . Thus

$$\begin{aligned} P_I &< \sum_{i=1}^{n/M^{h+1}} \sum_{U \in \mathcal{U}_V} \sum_{j \in U} \frac{GP_j}{[(\sqrt{M}i - \sqrt{M} + 1)\sqrt{A_h}]^\alpha} \\ &\leq \sum_{i=1}^{n/M^{h+1}} GP \frac{M^2 i}{[\sqrt{M}(i-1) + 1]^\alpha} \end{aligned} \tag{24}$$

where we have used the fact that $P_j \leq PA_h^{\alpha/2}/M^h, \forall j$. The sum in (24) is convergent for $\alpha > 2$, and is upper-bounded by MK_{I_1} with a constant K_{I_1} independent of n . For $\alpha = 2$, the sum is upper-bounded by $MK_{I_2} \log n$ with a constant K_{I_2} independent of n .

For the MIMO-BC transmissions illustrated in Fig. 10, each node receives interference from less interfering clusters compared to MIMO-MAC. Therefore, the

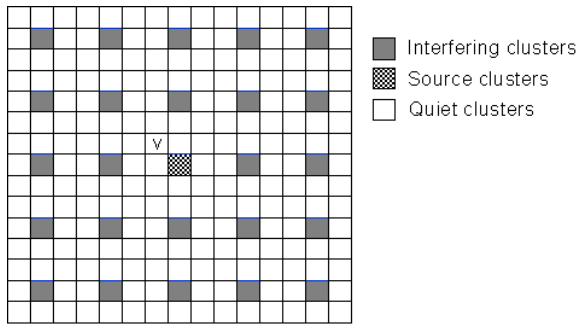


Fig. 10. Interfering clusters during MIMO-BC transmissions in the 5-phase scheme.

interference power in MIMO-BC is upper bounded by MIMO-MAC, and Lemma 2 also applies to MIMO-BC transmissions.

C. Proof of Lemma 3

The MIMO-BC transmission in the 5-phase scheme allows the relay to transmit identical symbols to all sources in the same cluster. Using Lemma 2, it follows that the capacity scaling of MIMO-BC provides an upper bound for MIMO-MAC as well. Hence, it suffices to consider the $MN \times N$ MIMO-MAC between the M source clusters S_i and the relay cluster R . For a network of size n , the input-output relationship of MIMO-MAC is again given by (19), where $Z = (Z_k)$ now denotes uncorrelated noise plus interference at the receiver nodes. Similar to Appendix A, assuming Z is i.i.d. Gaussian, and perfect CSI available at both transmit- and receive-ends, the sum capacity of this MIMO-MAC is lower bounded by

$$C_{\text{sum}} \geq E \left[\log \det \left(I_N + \text{SINR} \sum_{i=1}^M \frac{F_i F_i^H}{MN} \right) \right] \quad (25)$$

where $\text{SINR} := GP/(P_I + N_0)$, and P_I, N_0 denote interference power and noise power, respectively; while all other parameters are as in (20). This leads to

$$C_{\text{sum}} \geq N \log(1 + M \frac{a}{2} \text{SINR}) \frac{(a^2 - a/2)^2}{2b^4} \quad (26)$$

and the capacity scaling depends on the scaling of the interference power as in Lemma 2.

If $\alpha > 2$, then P_I is upper bounded by $\mathcal{O}(M)$, which gives C_{sum} at least a capacity scaling of N from (26). But for $\alpha = 2$, P_I is upper bounded by $MK_{I_2} \log n$ with a constant K_{I_2} independent of n . As a consequence, the sum capacity scales as $\mathcal{O}(N/\log n)$ for n large enough.

D. Proof of Lemma 4

We will establish the lemma for the MIMO-MAC. The proof for the MIMO-BC follows similar steps. We first

consider $\alpha > 2$ and prove that the power received by each node in the relay cluster is bounded below and above by constants P_1 and P_2 , respectively, that are independent of M and n . The signal received by a relay node r located in cluster R during the MIMO-MAC transmission from source clusters S_i is given by

$$Y_r = \sum_{i=1}^M \sum_{s=1}^N H_{ri}^s X_i^s + Z_r \quad (27)$$

where X_i^s denotes the signal sent by a source node $s \in S_i$ with power constrained to $\frac{P(r_{S_i R})^\alpha}{MN}$ and $Z_r \sim \mathcal{CN}(0, N_0)$. The power of Y_r is

$$E[|Y_r|^2] = \sum_{i=1}^M \sum_{s=1}^N |H_{ri}^s|^2 \frac{P(r_{S_i R})^\alpha}{MN} + N_0 \quad (28)$$

$$= \sum_{i=1}^M \sum_{s=1}^N \frac{GP}{MN} \left(\frac{r_{S_i R}}{r_{sr}} \right)^\alpha + N_0 \quad (29)$$

where we used the fact that H_{ri}^s, X_i^s and Z_r are all independent. It follows from [9] that

$$\begin{aligned} P_1 &\equiv \left(\frac{\sqrt{2}}{\sqrt{2} + 1} \right)^\alpha GP + N_0 \\ &\leq E[|Y_r|^2] \\ &\leq \left(\frac{\sqrt{2}}{\sqrt{2} - 1} \right)^\alpha GP + N_0 \equiv P_2 \end{aligned} \quad (30)$$

provided that the transmit power per node to be P/n . Combining (30) with Lemma 2, we deduce that the observations have bounded power $P_2 + MK_{I_1}$. If M is bounded, one can use a fixed number of bits to encode the observations without degrading the scaling performance [9]. If M increases with n , the received signal can be normalized by multiplying it with $q_1 = \sqrt{\frac{P_2}{P_2 + MK_{I_1}}}$ and then proceed with quantization as before.

When $\alpha = 2$, the signals are corrupted by interference of increasing power $MK_{I_2} \log n$ from Lemma 2. In this case, the power received by the destination nodes increases as $P_2 + MK_{I_2} \log n$ as n increases. Similarly, it is also necessary to normalize the received signal by $q_2 = \sqrt{\frac{P_2}{P_2 + MK_{I_2} \log n}}$ before quantization. Notice that the normalization here does not affect the SINR term in the proof of Lemma 3. As a consequence, Lemma 3 still holds true after encoding with a fixed rate of Q bits per observation, which proves Lemma 4.

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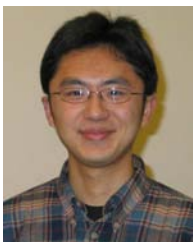
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