

# Multi-user Interference and Cross Correlation Effects of Spline Multiwavelet based Cognitive Radio Network

Manju Mathew, A. B. Premkumar and C.T. Lau

School of Computer Engineering, Nanyang Technological University, Nanyang Avenue, Singapore

Email: {manj0001, asannamalai, asctlau}@ntu.edu.sg

**Abstract**—A multi-user Cognitive Radio (CR) network resolves the problem of spectrum under utilization and enables the deployment of new wireless applications in the fast growing wireless communication area. The unique feature of CR is its ability to use the vacant band while insuring minimum interference to adjacent users. So modulation and multiple access schemes insuring minimum interference are required for CR transmission. Multiwavelets are rarely considered in the field of wireless communication. They are orthogonal and symmetric waveforms with short support and can insure lower multiple access interference in a multi-user environment. Hence a multi-user CR network using B-spline multiwavelets of different orders is considered in this paper for evaluating the performance characteristics. The cross correlation and multi-user interference are derived and its effect on error performance of linear, quadratic and cubic wavelets are analyzed. Simulation results prove that the number of users within a network can be increased by using spline wavelets of different orders without compromising on error performance.

**Index Terms**—Wavelet Transform, multi-user, multiwavelets, compact support, cross correlation, B-splines.

## I. INTRODUCTION

The key feature of Cognitive Radio (CR) is its ability to locate and use unoccupied spectrum at different times and in different geographical locations [1]. Extensive work has been done addressing issues of single user CR environment. To efficiently exploit the spectrum holes, CR should have multi-user support at adequate data rates. Even though OFDM based systems have been proposed to be the most promising candidate for CR transmission [2]- [5], recent research works pertain to the capabilities of wavelet based schemes to be an alternative modulation technique in terms of flexibility, adaptivity, and spectrum efficiency [6]- [14]. But the pros and cons of wavelets based modulation scheme for next generation multi-user networks is least explored. Till date, to the best knowledge of the authors, other than the preliminary work in [15], a detailed investigation on the performance of multiwavelets in CR environment has not been addressed in literature. Hence the unique features of spline multiwavelets and their cross correlation characteristics are exploited to get a CR network with lower interference.

The rest of the paper is organized as follows: Section

II portrays the existing and ongoing wavelet based research in digital communication and the unique features of splines. The proposed system model and necessary equations of spline based CR network are described in section III. The bit error rate (BER) and cross correlation values are derived in section IV. Simulation results are described in section V. The computational complexity involved in spline wavelet formulation is presented in section VI. Conclusion and future work are outlined in section VII.

## II. A BRIEF REVIEW ON WAVELET BASED COMMUNICATION SYSTEMS AND MULTIWAVELETS

A brief introduction to the use of scalar wavelets in the field of wireless communications and attractive features of multiwavelets are described in the following sections to support the proposed work.

### A. Wavelets in Digital Communications

The use of wavelet bases for wireless communication has been explored in the past [6]- [14]. It has been demonstrated by Daneshgaran et al that the use of scaling functions and wavelets as envelope waveforms for modulation is natural and is quite useful in transmitting data in adjacent frequency bands that overlap in frequency domain [6]. But no performance characteristics of such a system has been included in their study. Wavelet packet multiple access (WPMA) and wavelet packet division multiplexing (WPDM) were proposed in [7] and [8]. In WPMA scheme a suboptimal detector using recursive joint detection is used whereas WPDM system use the coherent matched filter detection as a conventional receiver. Both systems prove that scalar wavelet packets can be used in multi-user communication. Lindsey has introduced wavelet packet based multicarrier modulation in 1997 [9] without giving a detailed performance evaluation. Later, the advantages of wavelets and wavelet packets are outlined by Jamin and Mahonen [10]. They have made a thorough investigation on the application of wavelet packets in wireless communication and have shown that flexible multicarrier modulation waveforms can be generated using wavelet packet transforms with moderate complexity and optimum performance with acceptable error levels [10]. In their review of wavelets for digital wireless communication,

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Lakshmanan and Nikookar have validated the findings [11]. Wavelet packet multicarrier modulation using maximally frequency selective filter banks for CR applications have been proposed by Lakshmanan et al [12]. They have considered the effects of phase and frequency offset in wavelet packet multicarrier system and is compared with OFDM [13]. Scalar wavelet multicarrier multiple access scheme for CR has been described in [14]. All of the above research and results are limited to scalar wavelets. As CR transmission scheme demands high flexibility, adaptivity and maximum throughput with minimum adjacent band interference, it is required to explore the efficacy of multiwavelets for next generation wireless systems. Hence a multiwavelet based multi-user CR network was first proposed in [15]. Simulations were done only using linear splines - the simplest spline wavelets available for waveform modulation. Results were not supported with theoretical analysis and the computational complexity involved in spline wavelet generation was not presented. These issues are addressed in detail for spline wavelets of different orders in this paper.

### B. Multiwavelets - several wavelets and several scaling functions

Multiwavelets can be considered as the vector extension of scalar wavelet theory or as the special case of scalar wavelets [16]. That is, multiwavelets occur when single wavelet function cannot compose the basis of  $L^2(R)$  by its dilates and translates but a set of functions are required. ( $L^2(R)$  is the set of all functions whose squared value has a finite integral). Therefore, unlike the standard scalar wavelets, multiwavelet families have more than one scaling and wavelet function and the number of wavelet functions is dubbed as their multiplicity. A set of functions  $\psi_1, \psi_2, \dots, \psi_r \in L^2(R)$  are called orthogonal multiwavelets of multiplicity  $r$  if  $\psi_1(2^j x - k), \psi_2(2^j x - k), \dots, \psi_r(2^j x - k), j, k \in Z$  form an orthonormal basis of  $L^2(R)$  [17]. A multiwavelet system can simultaneously provide perfect reconstruction while preserving length (orthogonality), good performance at the boundaries (linear-phase symmetry) and a high order of approximation (vanishing moments) together with short support. Therefore multiwavelets offer superior performance for image processing applications, compared to scalar wavelets [18]. It would be interesting to explore the communication aspects of multiwavelets and hence B-spline based single carrier multi-user communication system is considered in this work. B-splines belong to the class of piecewise polynomials and can be generated by repeated convolution of the box function [19]. It is possible to obtain scaling and wavelet functions from B-splines of any order [20]. In the past, spline wavelets were not considered for major applications due to the high computational complexity involved in their formulation. Recently, Okkyung Cho and M. J. Lai [21] have proposed a simple method to obtain the spline scaling functions by using numerical approximation for the factorization of Laurent polynomial matrices and associated wavelets. Hence in the proposed

work, spline wavelets using analytical method described in [21] is discussed and their performance in a multi-user CR network is evaluated. In the context of CR, spline based multi-user system insures a few advantages. Firstly the orthonormality and compact support of spline waveforms provide minimum MAI and simple receiver structure. Secondly the adaptivity and reconfigurability in implementation is possible due to the unique properties of B-splines. Spline of order  $N$  can be easily obtained by convolving spline of order  $N - 1$  with a box function. So splines of higher order can be obtained easily from lower order splines. Spline multiwavelets of same order can be generated from the scaling functions by simply changing the coefficients and shifts that makes the overall system easily reconfigurable. As the order of the spline increases certain unique features like regularity, auto correlation and cross correlation will be improved which can be exploited in a changing radio environment at the cost of moderate complexity.

### III. SINGLE CARRIER MULTI-USER CR NETWORK WITH MULTIWAVELETS.

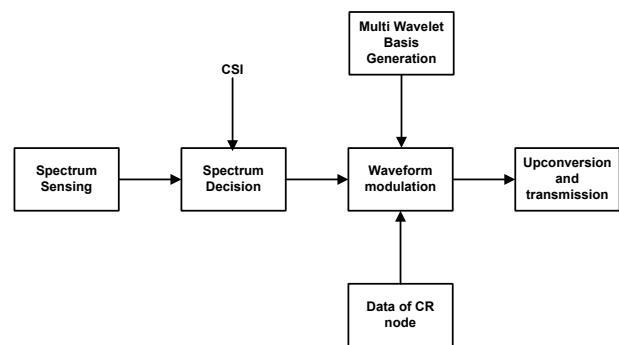


Figure 1. Single carrier CR uplink transmitter

The basic multi-user CR network using multiwavelet concept is described in [15]. It is assumed to have a number of CR nodes communicating with a central base station (BS). The uplink and downlink schematic are shown in Fig. 1 - 4. Each CR node within a network performs identical spectrum scanning which is made available at BS. The best free band is chosen based on the channel state information (CSI) obtained. The number of users that can be accommodated in a network depends on the type of wavelet and the modulation scheme. In general, for single carrier network the maximum number of users is  $w$  where  $w$  is the number of multiwavelet bases that can be generated at the base station. CR nodes within the same network shares the same vacant band but the wavelet base used for modulation will be different and due to inherent orthogonality minimum interference is insured. This work mainly focus on the detailed analysis of the downlink channel. At the BS each user data is separated using the corresponding wavelet base. Data of all CR nodes are summed up and transmitted. The signal

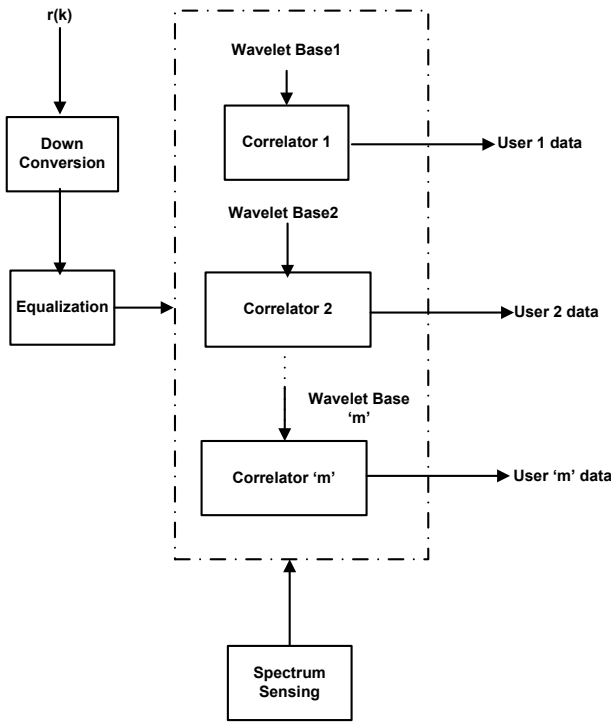


Figure 2. Single carrier CR uplink receiver

transmitted from the BS is

$$s_{bs}(k) = \sum_n \chi_m(n) e^{j2\pi k f_c} \quad (1)$$

where  $\chi_m(n)$  is the  $m^{th}$  user baseband data stream modulated with the  $m^{th}$  orthogonal wavelet base. The signal

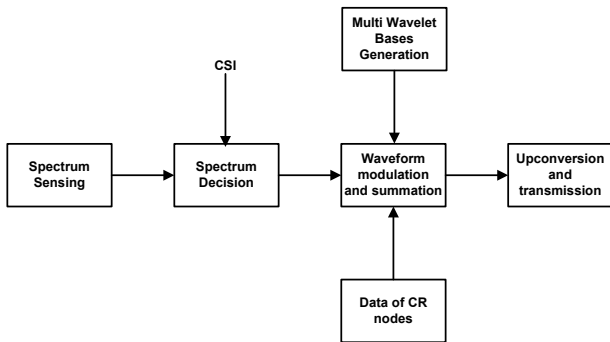


Figure 3. Single carrier CR downlink transmitter

received at the user terminal will be the sum of all user waveforms distorted with channel impairments and noise. At the receiver, each CR node generates its own unique wavelet base to correlate with the received signal. The signal is downconverted, compensated for channel impairments and correlated with the wavelet base corresponding to each user to extract the information. BPSK or QPSK scheme can be used to map the baseband data stream to the wavelet base. In this work, it is assumed that spectrum sensing is already performed and a contiguous free band is available for transmission. B spline multiwavelets of order two, three and four are considered for waveform modulation. The basic spline wavelet equations, their

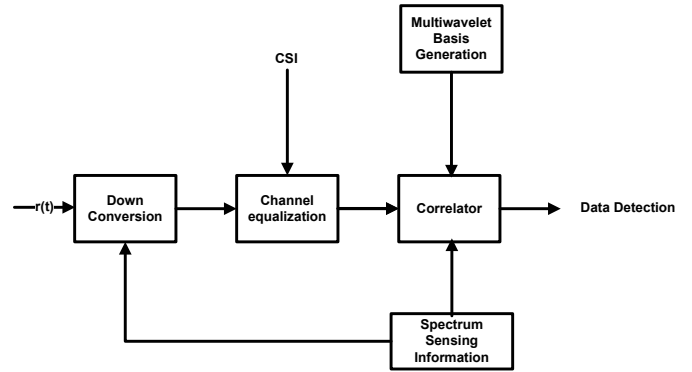


Figure 4. Single carrier CR downlink receiver

correlation features, multi-user interference and the steps to derive the BER of a spline based network are outlined briefly in the following sections.

### A. B Spline Equations

Orthogonal B spline wavelets with compact support are considered in this work for simulation purpose. The steps in [21] are followed to generate B splines of three different degrees starting from 1. Order of a basis spline is one higher than its degree. As B-splines are generated by repeated convolution of box function, it is convenient to represent them in Fourier domain. The Fourier transform (FT) of the B-spline of order  $m$  can be obtained by  $m$  times multiplication of the lowest order B-spline FT. The normalized spline function of order  $m$  is usually denoted as  $N_m(x)$  and its Fourier transform is given by

$$N_m(f) = \left( \frac{1 - e^{-jf}}{jf} \right)^m \quad (2)$$

There are three scaling functions and three wavelet functions for spline of any order. The scaling and wavelet equations are discussed in this paper.

1) *Linear Spline Equations:* Linear spline wavelets are the simplest and of the lowest order waveforms among the B-spline multiwavelets. The basis spline function is a triangular function and hence called as linear splines. The complexity involved in generating the scaling and wavelet functions are lower compared to other spline functions described in this work. The scaling functions are

$$\phi_1(x) = \sqrt{3}N_2(2x) \quad (3)$$

$$\phi_2(x) = \frac{\sqrt{165}}{11}N_2(2x) - \frac{4(2 + \sqrt{5})}{\sqrt{33}}N_2(4x) + \frac{4(2 - \sqrt{5})}{\sqrt{33}}N_2(4x - 2) \quad (4)$$

$$\phi_3(x) = \sum_{j=0}^2 \alpha_j N_2(2x - j) + \sum_{k=0}^3 \beta_k N_2(2x - k) \quad (5)$$

The three associated wavelets of linear B-spline are the linear combinations of scaling functions and hence the coefficients for each will be different. The equations are

as given below.

$$\psi_1(x) = \sum_{j=0}^3 \alpha_j \sqrt{2} \phi_1(2x-j) + \sum_{k=0}^3 \beta_k \sqrt{2} \phi_2(2x-k) + \sum_{l=0}^3 \gamma_l \sqrt{2} \phi_3(2x-l) \quad (6)$$

$$\psi_2(x) = \sum_{j=0}^3 \alpha'_j \sqrt{2} \phi_1(2x-j) + \sum_{k=0}^3 \beta'_k \sqrt{2} \phi_2(2x-k) + \sum_{l=0}^3 \gamma'_l \sqrt{2} \phi_3(2x-l) \quad (7)$$

$$\psi_3(x) = \sum_{j=0}^3 \alpha''_j \sqrt{2} \phi_1(2x-j) + \sum_{k=0}^3 \beta''_k \sqrt{2} \phi_2(2x-k) + \sum_{l=0}^3 \gamma''_l \sqrt{2} \phi_3(2x-l) \quad (8)$$

The constant values of the coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  for both scaling and wavelet functions are given in [21]

2) *Quadratic Spline Equations*: Quadratic splines are one order higher than linear splines. They are smooth waveforms but of higher complexity. The basis spline is a Gaussian shaped waveform. Scaling functions and the associated wavelet functions are given as

$$\phi_1(x) = \sum_{j=0}^6 \alpha_j N_3(2x-j) + \sum_{k=4}^{15} \beta_k N_3(4x-k) \quad (9)$$

$$\phi_2(x) = \sum_{j=0}^6 \alpha'_j N_3(2x-j) + \sum_{k=4}^{15} \beta'_k N_3(4x-k) \quad (10)$$

$$\phi_3(x) = \sum_{j=0}^6 \alpha''_j N_3(2x-j) + \sum_{k=4}^{15} \beta''_k N_3(4x-k) \quad (11)$$

$$\psi_1(x) = \sum_{j=0}^7 \alpha_j \sqrt{2} \phi_1(2x-j) + \sum_{k=0}^7 \sqrt{2} \beta_k \phi_2(2x-k) + \sum_{l=0}^7 \gamma_l \sqrt{2} \phi_3(2x-l) \quad (12)$$

$$\psi_2(x) = \sum_{j=0}^7 \alpha'_j \sqrt{2} \phi_1(2x-j) + \sum_{k=0}^7 \beta'_k \sqrt{2} \phi_2(2x-k) + \sum_{l=0}^7 \gamma'_l \sqrt{2} \phi_3(2x-l) \quad (13)$$

$$\psi_3(x) = \sum_{j=0}^7 \alpha''_j \sqrt{2} \phi_1(2x-j) + \sum_{k=0}^7 \beta''_k \sqrt{2} \phi_2(2x-k) + \sum_{l=0}^7 \gamma''_l \sqrt{2} \phi_3(2x-l) \quad (14)$$

The scaling and wavelet coefficients are constants and are tabulated in [21].

3) *Cubic Spline Equations*: The basis spline of cubic wavelets is of order four and associated wavelets have good regularity and cross correlation. Scaling function

equations are

$$\phi_1(x) = \sum_{j=0}^8 \alpha_j N_4(2x-j) + \sum_{k=4}^{20} \beta_k N_4(4x-k) \quad (15)$$

$$\phi_2(x) = \sum_{j=0}^9 \alpha'_j N_4(2x-j) + \sum_{k=4}^{20} \beta'_k N_4(4x-k) \quad (16)$$

$$\phi_3(x) = \sum_{j=0}^8 \alpha''_j N_4(2x-j) + \sum_{k=4}^{20} \beta''_k N_4(4x-k) \quad (17)$$

The associated wavelet functions are

$$\psi_1(x) = \sum_{j=1}^9 \alpha_j \sqrt{2} \phi_1(2x-j) + \sum_{k=1}^9 \beta_k \sqrt{2} \phi_2(2x-k) + \sum_{l=1}^9 \gamma_l \sqrt{2} \phi_3(2x-l) \quad (18)$$

$$\psi_2(x) = \sum_{j=1}^9 \alpha'_j \sqrt{2} \phi_1(2x-j) + \sum_{k=1}^9 \beta'_k \sqrt{2} \phi_2(2x-k) + \sum_{l=1}^9 \gamma'_l \sqrt{2} \phi_3(2x-l) \quad (19)$$

$$\psi_3(x) = \sum_{j=1}^9 \alpha''_j \sqrt{2} \phi_1(2x-j) + \sum_{k=1}^9 \beta''_k \sqrt{2} \phi_2(2x-k) + \sum_{l=1}^9 \gamma''_l \sqrt{2} \phi_3(2x-l) \quad (20)$$

### B. Auto Correlation, Cross Correlation and Multi-user Interference

The error performance of a multi-user CR system depends on the interference caused by neighbouring users who share the same spectrum. In the single carrier orthogonal waveform based system this Multi User Interference (MUI) is directly proportional to cross correlation between the waveforms. In the proposed spline based system the information is mapped on the orthogonal wavelets and the data is retrieved by generating the exact wavelet base at the receiver. Hence better error performance can be achieved only if the auto correlation is maximum and cross correlation value is minimum at the sampling instants. It is well known that the auto correlation function of a basis spline of degree  $n$  is a higher degree spline of degree  $2n+1$  [22]. Moreover the orthonormal feature of the wavelets are invariant for integer shifts and hence even when user signals are nearly synchronized autocorrelation value will be fairly good. Cross correlation among spline multiwavelets can be of two types. The cross correlation between wavelets of same order can be termed as intra cross correlation and that between wavelets of different order can be called inter cross correlation. If the inter cross correlation is good enough, splines of different order can be used within a network to increase the number of users without compromising on the performance. In this work, auto correlation and intra cross correlation of different spline scaling functions are simulated and plotted. Inter cross correlation has three different cases:(i) between quadratic and linear, (ii) between cubic and quadratic and (iii) between linear and cubic. In each case, nine different

combinations are possible. These graphs are excluded from the paper as merging of too many graphs in a single plot is required. But it is shown that inter cross correlation between quadratic and cubic is at the desirable levels as seen in the plots for BER characteristics.

IV. ERROR PERFORMANCE AND INTERFERENCE ANALYSIS

To explore the efficacy of spline based multi-user system it is required to evaluate the error performance of the proposed network which is outlined in the following subsections.

A. BER calculation in AWGN Channel

To evaluate the BER of the proposed system BPSK modulated CR network is considered. The signal transmitted from the  $m^{th}$  user can be written as

$$s^{(m)}(t) = \sqrt{\frac{2E_b}{T}} \sum_k a^{(k)} p_m(t - kT) \tag{21}$$

where  $E_b$  is the bit energy,  $T$  is the symbol period,  $p_m(t)$  is the wavelet base of the  $m^{th}$  CR node, 'a' is the symbol variable that takes the values +1 or -1 depending on the transmitted symbol and  $k$  denotes the symbol sampling instant. Received signal at the Base Station(BS) is a composite signal of all the  $N_u$  users present in the network along with noise and channel impairments. Similarly signal received on each CR mobile terminal also would be a composite of all user signals distorted by the channel impairments. For analytical simplicity, AWGN channel is considered. The baseband signal received at any user terminal can be given as

$$r(t) = \sum_{m=1}^{N_u} s^{(m)}(t) + n(t) \tag{22}$$

The correlator extracts the  $k^{th}$  symbol of the desired user data and let  $m=1$  be the desired user. Hence the output of the correlator is

$$R = \int_0^T r(t) p_1(t - kT) dt \tag{23}$$

$$R = \int_0^T \left[ \sum_{m=1}^{N_u} s^{(m)}(t) + n(t) \right] p_1(t - kT) dt \tag{24}$$

This can be expanded as shown below

$$R = \int_0^T \sum_{m=2}^{N_u} s^{(m)}(t) p_1(t - kT) dt + \int_0^T n(t) p_1(t - kT) dt + \int_0^T \left[ \sqrt{\frac{2E_b}{T}} \sum_k a^{(k)} p_1(t - kT) \right] p_1(t - kT) dt \tag{25}$$

First term is the multi user interference (MUI) caused by  $N_u - 1$  users, second term is the noise component and

last term is the useful signal component. The MUI can be expressed as

$$\begin{aligned} MUI &= \sum_{m=2}^{N_u} \int_0^T \sqrt{\frac{2E_b}{T}} \sum_{k'} a^{(k')} p_m(t - k'T) p_1(t - kT) dt \\ &= \sum_{m=2}^{N_u} \sqrt{\frac{2E_b}{T}} \sum_{k'} a^{(k')} \int_0^T p_m(t - k'T) p_1(t - kT) dt \\ &= \sum_{m=2}^{N_u} \sqrt{\frac{2E_b}{T}} \sum_{k'} a^{(k')} C_{m,d} \end{aligned} \tag{26}$$

where  $C_{m,d}$  represents the cross correlation between the wavelet base of the desired user and the  $m^{th}$  user. It is evident that the interference is a measure of the cross correlation between the wavelet bases used in the network. Hence the interference and BER will be minimum in a network where the cross correlation between wavelets is the lowest. As the correlator output expression is similar to that of a BPSK based Direct Sequence Spread Spectrum (DS-SS) system, BER of the proposed system with BPSK modulation can be written as

$$P_B = Q \left( \sqrt{\frac{2E_b}{(MAI + N_0)}} \right) \tag{27}$$

MAI denotes the multiple access interference or MUI which is given by equation (26) and  $N_0$  is the two sided noise power spectral density. It is assumed that MAI is also Gaussian in nature since it depends on cross correlation of splines. It is known that splines of order higher than 2 are Gaussian shaped waveforms [21]. To get the exact expression of MUI cross correlation under different conditions are to be derived and have been done in the subsequent section.

B. Cross correlation value in different cases

Case (I) For spline wavelets of the same order (Intra cross correlation) - If each user modulate the waveform with multiwavelets of the same order the cross correlation will be zero upon perfect synchronization. As splines are shift orthogonal for all integer translates up to its support length, the orthogonality will be maintained even when for certain integer shifts. Hence in the absence of noise the MAI will be zero. Under fading channel conditions the time shifts would not be integer and the hence orthogonality could be lost even in the absence of white noise. The amount of cross correlation depends on the type of wavelet. As cubic wavelets are the highest order wavelet used in our simulation, steps to calculate cross correlation between two cubic spline wavelet is discussed below.

Let  $C_{m,n}$  denotes cross correlation between  $m^{th}$  and  $n^{th}$





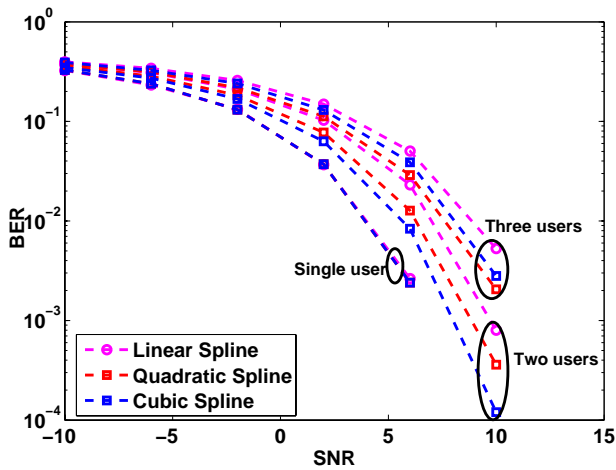


Figure 8. Variation of BER in flat fading Channel showing the effect of intra cross correlation

#### A. BER in single path fading channels

In section IV it is shown that the BER of the proposed system depends on MUI and hence the cross correlation properties of wavelets. Performance curves showing the effect of intra and inter cross correlation in different channel conditions are obtained. Fig. 8 shows the effect of intra cross correlation of different spline wavelets on BER in a flat fading channel. The maximum number of nodes is limited to three as wavelet bases of same order are used within the network. The BER curves of single user splines of any order are almost similar. With increasing order, as the intra cross correlation of spline improves, the BER also improves in a multi-user environment. Cubic spline based multi-user system shows lower BER compared to quadratic and linear based spline systems in the case of two user and three user networks.

As CR mainly targets wide band Radio Frequency (RF) spectrum the channel will be of frequency selective nature rather than simple AWGN. Hence to obtain the error performance in a more realistic channel environment, frequency selective Rayleigh fading together with AWGN channel is simulated for a Doppler shift of 60Hz and the results are plotted in Figs 9 - 11. The channel impulse response coefficients are generated such that baseband symbols of one signal block undergoes different fades. Fig. 9 shows the effect of intra cross correlation in the frequency selective Rayleigh fading channel with AWGN. The error curves validates the cross correlation properties of splines as cubic wavelets gives the minimum BER and linear wavelets produces the maximum BER. Fig. 10 gives the effect of inter cross correlation of quadratic and cubic spline wavelets on BER for the frequency selective fading condition. To achieve any specific BER, slight SNR enhancement is required with increase in number of users. In order to examine the cost of expanding the network from three users to six users, the three user cubic spline graph is also plotted. For a BER of  $10^{-2}$  the SNR enhancement required for a six user network compared to three user is approximately 4dB. Hence simulation

results support the concept of exploiting inter cross correlation for the expansion of network. To compare the

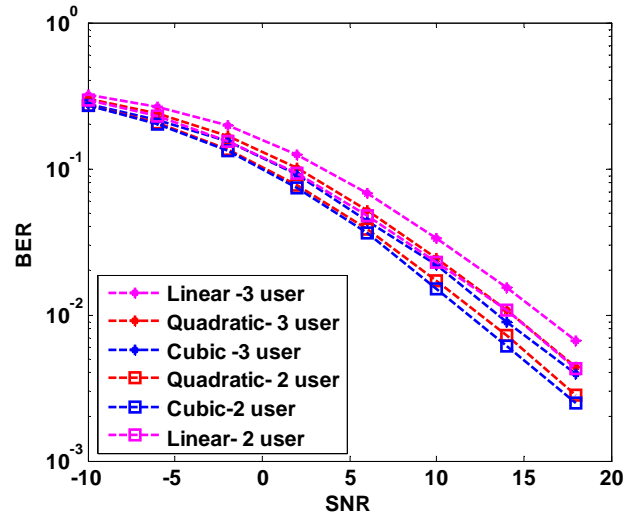


Figure 9. BER curves showing the effect of intra cross correlation (Frequency selective fading and AWGN channel)

performance of the proposed system with an existing scheme, a sixteen subcarrier OFDMA system with four users is simulated. To obtain a four user network with multiwavelets it is required to consider B-spline wavelets of different orders since there are only three wavelet bases for spline of any order. In this paper, quadratic and cubic wavelets are used for implementing four user CR network. BER comparison of different four user networks of spline based system incorporating quadratic and cubic splines with a four user OFDMA system in a single path frequency selective fading channel is given in Fig. 11. It is evident that the single carrier spline based system gives a moderate error performance compared to the multicarrier OFDMA scheme in a frequency selective fading channel. In the worst case scenario the spline based system requires an SNR enhancement of 1dB to attain the same BER characteristics of OFDMA.

#### B. BER in multi-path fading channels

To evaluate the capability of the system to mitigate multi-path fading channel impairments, three path fading channel with fixed and Gaussian fading coefficients are considered. The channel delay is kept constant and is equal to three in both cases. Cyclic Prefix (CP) of the simulated OFDMA system is 25 percent of the total number of subcarriers which is equal to four. Thus CP of the OFDMA system is always longer than the channel delay, thereby reducing the equalizer complexity at the receiver side. The wavelet based receiver uses a time domain zero forcing equalizer with three taps per sample as given in [23] and OFDMA uses a frequency domain equalizer with one tap per subcarrier to compensate for channel distortions. It is assumed that the channel impulse response is known at the receiver. The fixed channel impulse response used for simulation is  $h = [0.407 \ 1 \ 0.407]$ . These coefficients are chosen such that



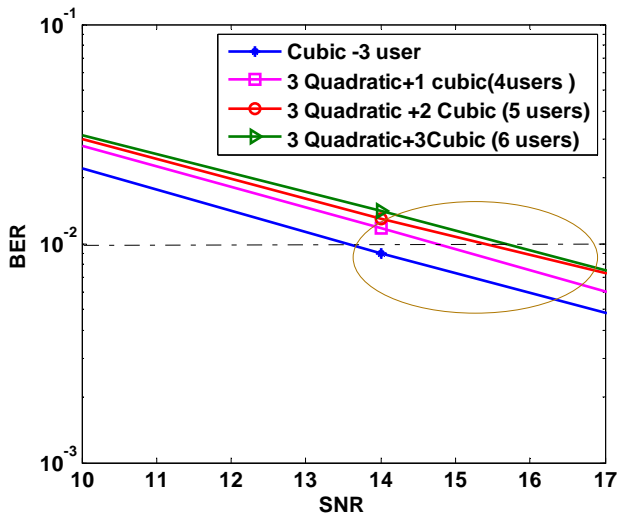


Figure 10. BER curves showing the effect of inter cross correlation (Frequency selective fading and AWGN Channel)

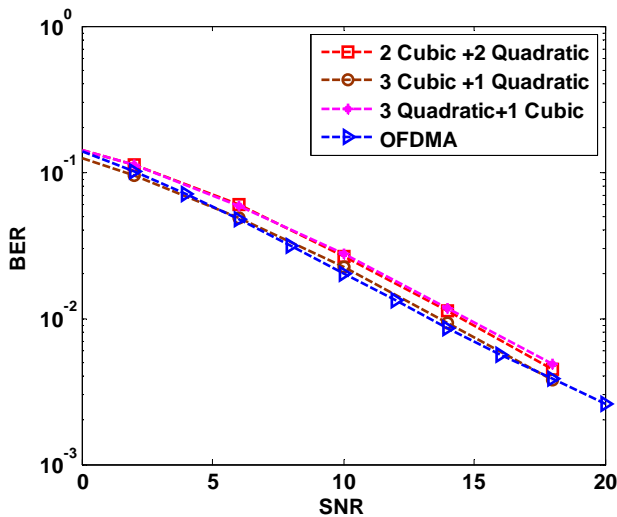


Figure 11. BER comparison of different four user networks (Frequency selective fading and AWGN Channel)

the frequency response of the channel represents low pass filter characteristics with frequency selective nature. The error curves of both systems are plotted in Fig. 12. When the channel coefficients are fixed the wavelet based system outperforms OFDMA system since the equalizer weights of the wavelet scheme can be designed such that the combined channel and equalizer impulse response will be zero at all sample points except the desired sample point. All four user networks using the proposed wavelet system show better performance than OFDMA. Among the simulated wavelet systems the network incorporating three cubic splines and one quadratic spline gives the best performance since the inter cross correlation is minimum in that typical network. But when the channel coefficients are normally distributed Gaussian random variables the proposed scheme gives a very poor performance due to the Rayleigh fading effect of the channel. In the absence of line of sight (LOS) component the time domain equalizer fail to combat the channel distortions. Only wavelet

based network with minimum inter cross correlation is plotted. Other spline networks also behave in a similar manner are not shown in the diagram. OFDMA system gives a fairly good performance because of the presence of CP. As CP cannot be incorporated with spline wavelet systems, it is required to design a better equalization technique in order to improve the error performance in a multi-path Rayleigh fading channel.

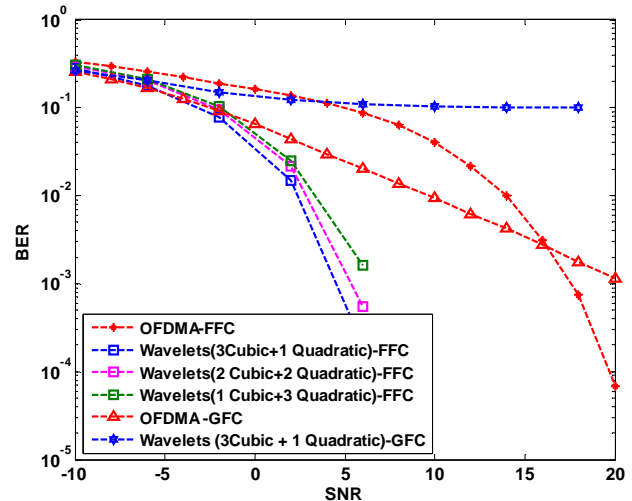


Figure 12. BER comparison of different four user networks (Multi-path fading Channel where FFC-Fixed Fading Coefficients and GFC-Gaussian Fading Coefficients)

## VI. COMPLEXITY ANALYSIS

In this section the computational complexity in generating different spline wavelets is described. As B-spline wavelets have explicit expressions, calculating the number of multiplications and additions in each case is an easy task. Generating spline wavelets involve only real multiplications and additions. The equations given in Section III prove that multiwavelets of the same order require the same number of computations. Mathematical operations involved in the final stage of wavelet generation is considered to obtain the values given in this paper. Linear, quadratic and cubic splines are obtained by the linear combinations of their respective scaling functions and hence there will be three groups of summation for all wavelet equations. In the case of linear wavelets, each summation includes 4 multiplications and 3 additions. Therefore to generate the wavelet function total number of multiplications will be  $12(3 \times 4)$  and total number of additions will be  $11(3 \times 3 + 2)$ . For quadratic wavelets in each group there are 8 multiplications and 7 additions. Hence to generate each quadratic wavelet 24 multiplications and 23 additions are required. Lastly cubic wavelets require 9 multiplications and 8 additions in each group. The total calculations required are 27 multiplications and 26 additions for cubic wavelets. In general, number of multiplications is equivalent to  $N_c$  which corresponds to the total number of coefficients involved and the number of additions is equal to  $N_c - 1$ . It is evident that linear

wavelets have the lowest complexity. As the order of the spline wavelet increases, complexity also increases. Different wavelets of the same order can be generated by changing the coefficient values of the same equation which demands a simple and reconfigurable hardware circuitry. Hence the overall complexity of spline based CR system is comparable with existing schemes. The number of computations required for different four user multiwavelet systems is tabulated and is compared with OFDMA system with four users.

Network type	Number of real multiplications	Number of real additions
3 cubic + 1 quadratic	105	101
2 cubic + 2 quadratic	102	98
1 cubic + 3 quadratic	99	95
OFDMA	96	128

TABLE I.  
COMPLEXITY ANALYSIS

## VII. CONCLUSION AND FUTURE WORK

In this work, a novel multi-user CR network based on B-spline multiwavelets is described in detail and the auto correlation and cross correlation features of spline waveforms are analyzed. Exploiting the adaptivity and orthogonality of B-spline multiwavelets of different orders for a multi-user CR system is considered for the first time in literature. The single carrier spline based CR system is simulated and various performance curves are plotted. The computational complexity of the proposed system is briefly discussed. The BER of the proposed system is compared with OFDMA. It is shown that comparable error performance can be achieved with the proposed system in single path fading channels. The MUI of the proposed system depends on the correlation characteristics of different splines and hence waveforms with minimum cross correlation are to be chosen to get better error performance. The proposed system fail to combat multi-path fading when the amplitude and phase changes in random. Moreover the number of users within a network is limited by the multiplicity of splines. Hence it is required to design more robust multi-user systems capable of incorporating adequate number of users with better error performance and higher spectrum efficiency and is taken as our future work. It will incorporate orthonormal multiwavelet filter banks to generate a new multicarrier multiple access scheme for CR.

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reviewers for their detailed comments.

**Manju Mathew** is currently a Ph.D. candidate at Nanyang Technological University, Singapore. She received her B.Tech degree in Electrical and Electronics Engineering from the Mahatma Gandhi University of Kerala, India, in 2002 with Third Rank and Gold Medal.

She was a lecturer at the Cochin University of Science and Technology, India and in various engineering colleges during 2002-2006. Her research interests include wavelets for modulation and multiple access, software defined radio, spectrum sensing and Cognitive Radio.

**Benjamin Premkumar** received his Bachelor of Science and Bachelor of Engineering in Electrical communication in India. He briefly worked in large communication industry in Bangalore (India) in their Research and Development division before proceeding to the US to earn his M.S. and PhD both in Digital Signal processing from the University of Idaho. He has held various teaching positions since 1991 both in the US and Singapore. He was the recipient of Erskine Fellowship at the University of Canterbury in 2004. Currently he is an Associate Professor in the school of Computer Engineering, Nanyang Technological University, Singapore.

His research interests are in the areas of Digital filters and its applications in Wireless Communication, Software Defined Radio, Ultra wide Band Radio and Cognitive Radio. He also works in the area underwater tracking and localization of sources in shallow waters. He has worked on several different number systems for implementing digital filters efficiently in hardware and has successfully applied these systems in communications systems for error detection and correction. He has over 150 publications in international journals and conferences and has authored and co authored three text books, two on mobile communications and one on number theory. He has been a senior member of IEEE since 1999.

**Chiew-Tong Lau** received his B.Eng. degree from Lakehead University in 1983 and M.A.Sc. and Ph.D. degrees in Electrical Engineering from the University of British Columbia in 1985 and 1990 respectively. He is currently an Associate Professor in the School of Computer Engineering, Nanyang Technological University, Singapore. His main research interests are in wireless communications and in Signal Processing.