ICI Mitigation Schemes for Uncoded OFDM over Channels with Doppler Spreads and Frequency Offsets – Part I: Review

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***Abstract*— Orthogonal frequency-division multiplexing (OFDM) suffers from inter-carrier interference (ICI) if the channel has the Doppler spread and/or frequency offset. Polynomial cancellation coding (PCC), symmetric cancellation coding (SCC), and windowing are simple, but effective, schemes to mitigate the ICI in the OFDM system without error-correcting coding. In this two-part paper, we derive approximations to the signal-to-interference plus noise power ratio (SINR) attainable by PCC, SCC, and windowing and compare their performances in terms of SINR. To this end, in this first part, we review these known schemes and discuss some of interrelationships between them.**

I. INTRODUCTION

Main causes of intercarrier interference (ICI) in a mobile system employing orthogonal frequency-division multiplexing (OFDM) are the frequency spread due to Doppler effects and carrier frequency offset (CFO) [1], both of which severely affect the bit error rate (BER) behavior of the system. Although equalization with and without partial response precoding [2], [3] over subcarriers and spatial processing [4] can considerably reduce the effects of ICI, these sophisticated schemes make the receiver too costly, and we may have recourse to simpler ICI suppression schemes, including windowing [5]-[14], polynomial cancellation coding (PCC) [14]-[25], symmetric cancellation coding (SCC) [26]-[31], and some related schemes [32]-[34]. There are not, however, enough guidelines for which one is useful for the given situation.

While time-domain Nyquist windowing is commonly used at the transmitter to suppress the out-of-band (OOB) spectral radiation [1], windowing at the receiver or both at the transmitter and receiver is also known to be effective to suppress ICI [5],[6]. For this purpose, Nyquist windows are used since they are ICI-free if no impairments exist in the channel. A Nyquist window with non-zero roll-off has a prolonged window length and hence requires a cyclic prefix (CP) long enough to match the window length. Then, changing the roll-off, we may realize an OFDM system with variable robustness to channel impairments. In [6], such an adaptive windowing system employing the raised-cosine (RC) window is proposed to mitigate the ICI caused by CFO or by carrier wave spurious. Efforts to get better windows have revealed that the RC window, which is commonly used for OOB spectral suppression, is not nice for ICI suppression [7]. Window optimization has been carried out in restricted classes of windows [7], [8], [9], [10]. From these works, we can notice that windows with discontinuities or piece-wise linearity [8] generally work better than the RC window. Especially, optimization over a class of windows constructed from quadratic functions in [9] reveals that it tends to give a triangular window for a sufficiently small Doppler spreads and a theorem in [10] shows that the triangular window is exactly optimal when the channel path coefficients vary linearly. In [11] and [12], more general classes of windows are considered and the associated SINR is calculated. However, the derived SINR form is complex or optimization is carried out only numerically. In [13], a numerical optimization scheme is proposed to find the best Nyquist window that has the largest SINR for a given CFO. However, the scheme is useful only for non-fading, frequency non-selective channels.

Self-ICI-cancellation is another simple means to suppress ICI. The first self-ICI-cancellation scheme was proposed by Zhao *et al.* in [15], where each data is transmitted over two adjacent subcarriers, say, the nth and the $(n+1)$ st subcarriers, with opposite polarities and, at the receiver, ICI is made "self-canceled" with subtraction combining. The scheme is subsequently classified as the first order polynomial cancellation coding (PCC) and higher order PCC schemes are discussed in [14] (see also [16]). Although the first-order PCC, or simply PCC, can send only $N/2$ data over N subcarriers, it is shown in [18] that, when differential modulation is assumed, PCC is more robust against Doppler spreads than un-interleaved convolutionally coded OFDM of the same rate. PCC is also considered for channels with phase noise in [19] and [20] and for a multi-antenna system in [21]. In [14], it is suggested that the scheme is actually a windowing scheme with square-root RC Nyquist windowing of roll-off one at the transmitter and receiver. Thus, it is also useful to reduce out-of-band (OOB) radiation [17], [22].

Recently, in [23] (also see [25]), an exact closed-form expression of the signal-to-interference-plus-noise power ratio (SINR) over a multipath Rayleigh fading channel is given for PCC and the bit error rate (BER) calculated under Gaussian ICI assumption is shown to match simu-

doi:10.4304/jcm.7.9.676-685

lation results well. The above SINR expression, which is exact and closed-form, is complicated and may be calculated only numerically. Independently, in [24], a simple approximation for the ICI power in PCC-OFDM is derived based on an approximation technique used for the normal OFDM [35]. However, the derivation has flaws as shown in Appendix I.

In spite of its good features, PCC has a crucial disadvantage that it amplifies the peak-to-average power ratio (PAPR) substantially. In [25], the complementary cumulative distribution function of PAPR is derived for PCC with Gaussian approximation and is shown to have a prolonged tail.

Symmetric cancellation coding (SCC) is introduced in [26] and [27] as another self-ICI-cancellation scheme where each data symbol is transmitted over two subcarriers at the symmetric positions, say, the n th and $(N-1-n)$ th subcarriers, with opposite polarities for N subcarriers. It is shown by simulation in these works that SCC generally performs better than PCC. We need to be careful on this point, however. It is shown in [30] that the ICI power of SCC is quit small when fading is frequency-flat but that the ICI power increase considerably at the increase of frequency-selectivity. Given a complete channel state information (CSI) at the receiver, however, the frequency-selectivity of the channel then allows SCC to realize frequency diversity with the use of maximal ratio combining (MRC) at the receiver. Thus, SCC is a self-ICI-cancellation scheme for flat fading and, at the same time, a frequency diversity scheme for frequency-selective fading as well. In this respect, we should note the similarity between SCC-OFDM and OFDM based on discrete-cosine transform (DCT-OFDM) [28], [29] (and the reference therein). Conjugate cancellation discussed in [33] has a similar flavor as SCC and may be considered not as a specific coding scheme but a basic principle behind SCC. The self-ICI-cancellation capability of conjugate cancellation over flat fading channels is further enhanced with the introduction of phase rotation in [34]. Interestingly, it is shown in [30] and [31] that, when differential modulation in frequency domain is used, SCC without MRC still performs better than PCC for a wide range of SNRs. However, we are not going to discuss differential modulation in this paper.

In the light of the diversity effects of SCC, cyclic cancellation coding (CCC) [32] is proposed as an its enhancement, where the same data symbol is transmitted over subcarriers in cyclically opposite positions. The transmit diversity scheme in [36] may be another enhancement. The vector OFDM (V-OFDM) [37] may be considered as a further enhancement of CCC, although it was not proposed as a scheme to mitigate ICI. The diversity attainment with V-OFDM is not perfect and the constellation-rotated V-OFDM (CRV-OFDM) is proposed in [38] as a full-diversity extension.

As we have seen, there are many ICI mitigation schemes which are simple, effective, and mutually related. They are self-ICI-cancellation schemes, diversity schemes, or both. However, there are almost no comprehensive discussions nor comparisons of all of these schemes.

In the first part of this two-part paper, we review PCC and SCC as self-ICI-cancellation schemes as well as windowing and consider their inter-relationships. Additionally, we also study, with memoryless Gaussian process assumption, PAPR characteristics of PCC-OFDM and SCC-OFDM signals. We then proceed to review SCC and CCC as diversity schemes and discuss relationships to DCT-OFDM and to (CR)V-OFDM. Performance analysis based on the Gaussian ICI assumption is given in the second part, where we consider signal-to-interference plus noise power ratio (SINR) and BER expressions for these schemes.

The rest of this first part of the two-part paper is as follows. In Section II, we give some background for ICI and performance analysis and also give some history of ICI analysis for the normal OFDM. In Section III, we review self-ICI-cancellation schemes and windowing schemes. In Section IV, we discuss the effect of PCC and SCC on the PAPR of the transmitted signals. In Section V, we consider SCC and CCC as diversity schemes and give approximate expressions for error probability using Gaussian approximation. Section VI is the conclusion. In the second part of the two-part paper, we give approximate analysis of the performance of these schemes discussed in this first part.

II. BACK GROUND

With appropriate zero-valued guard subcarriers and the finite bandwidth assumption [1], an OFDM signal of symbol period T (sec) with cyclic prefix (CP) of period T_{G} (sec) may be expressed in a discrete-time, complex-valued baseband form, with sampling period $T_{\mathrm{s}} = \frac{T}{N}$,

$$x_k = \sum_{m=0}^{N-1} a_m e^{j\frac{2\pi mk}{N}}, \qquad \text{for } -N_{\mathrm{G}} \le n \le N-1, \quad (1)$$

where N is the number of subcarriers, a_n, $n = 0, 1, \cdots, N-1$, are the subcarrier values, and $N_{\mathrm{G}} = \frac{T_{\mathrm{G}}}{T_{\mathrm{s}}}$ is the length of the guard interval in samples. The expression (1) is nothing but an N-point inverse discrete Fourier transform (IDFT). We always suppose that N is an even integer.

In the same manner, the channel is represented as a discrete-time, time-variant multipath Rayleigh fading channel

$$y_k = \sum_{\ell=0}^{L-1} h_{k,\ell} x_{k-\ell} + w_k, \quad (2)$$

where w_k is the discrete-time, complex-valued white Gaussian noise with variance $\sigma_{\mathrm{n}}^2 = NN_o$. We suppose $L-1 \le N_{\mathrm{G}}$.

Discrete Fourier transform (DFT) of y_k at the receiver gives

$$\begin{aligned} R_n &= \frac{1}{N}\sum_{k=0}^{N-1} y_k e^{-j\frac{2\pi nk}{N}} \\ &= \frac{1}{N}\sum_{k=0}^{N-1}\sum_{\ell=0}^{L-1} h_{k,\ell}x_{k-\ell}e^{-j\frac{2\pi nk}{N}} + W_n \\ &= A(n)a_n + I(n) + W_n, \end{aligned}$$

where $W_n = \frac{1}{N}\sum_{k=0}^{N-1} w_k e^{-j\frac{2\pi nk}{N}}$ is a white Gaussian noise with mean-zero and variance N_o and, with the convention

$$A_{m,n} \triangleq \frac{1}{N}\sum_{k=0}^{N-1}\sum_{\ell=0}^{L-1} h_{k,\ell}e^{j\frac{2\pi(m-n)k}{N}}e^{-j\frac{2\pi m\ell}{N}}, \quad (3)$$

$A(n) = A_{n,n}$ is the (complex) amplitude of the nth subcarrier and $I(n) = \sum_{m\neq n} a_m A_{m,n}$ is the intercarrier interference (ICI).

We assume the wide-sense stationary and uncorrelated scattering (WSSUS) model for the fading processes [39], [40], [41]. In practice, however, there is a CFO between the transmitter and receiver local oscillators, and we assume that the channel coefficients $h_{k,\ell}$ are circularly symmetric (CS) Gaussian random processes [42] satisfying

$$\mathrm{E}[h_{k,\ell}h^*_{k',\ell'}] = p_\ell e^{j\omega_O T_s[k-k']}J_0(\omega_D T_s[k-k'])\delta_{\ell-\ell'},$$

where p_ℓ is the power of the ℓth path, $\omega_D \triangleq 2\pi f_D$ (rad/sec) is the (maximum) Doppler frequency, $\omega_O \triangleq 2\pi f_O$ (rad/sec) is the CFO, $J_0(x)$ is the zeroth-order Bessel function of the first kind, and δ_k is Kronecker's delta function. Without loss of generality, we assume $\sum_{\ell=0}^{L-1} p_\ell = 1$.

If a_n are mutually independent random variables with zero mean and variance $\mathrm{E}\left[|a_n|^2\right] = E_{\mathrm{s}}$, then SINR is given as

$$\Gamma(n) = \frac{E_{\mathrm{s}}\cdot \mathrm{E}\left[|A(n)|^2\right]}{\mathrm{E}\left[|I(n)|^2\right] + \mathrm{E}\left[|W_n|^2\right]}. \quad (4)$$

We employ, in the following discussions, the Gaussian interference assumption that the interference $I(n)$ is well approximated by a CS Gaussian random variable independent of $A(n)$. Moreover, different schemes are compared on the basis of the fixed total transmit power for the same bandwidth.

It is known that, in the case of an additive white Gaussian noise (AWGN) channel, the bit error rates (BERs) P_{b} of BPSK and QPSK are given, respectively, by $Q(\sqrt{2\gamma})$ and by $Q(\sqrt{\gamma})$ for signal-to-noise power ratio (SNR) per symbol γ with the use of the Gaussian Q-function $Q(\,\cdot\,)$. In the case of a Rayleigh fading channel, the average BER $\bar{P}_{\mathrm{b}}$ is known to be given by, for both BPSK and QPSK,

$$\bar{P}_{\mathrm{b}} = \frac{1}{2}\left[1 - \sqrt{\frac{\gamma_{\mathrm{b,av}}}{1+\gamma_{\mathrm{b,av}}}}\right] \approx \frac{1}{4\gamma_{\mathrm{b,av}}}, \quad (5)$$

where $\gamma_{\mathrm{b,av}}$ is the average SNR per bit [43].

Then, for a multipath fading channel with Doppler spread and CFO, the average BER $\bar{P}_{\mathrm{b}}$ in the nth subcarrier is approximated by $\frac{1}{4\Gamma(n)}$ and by $\frac{1}{2\Gamma(n)}$, respectively, for BPSK and QPSK.

A brief review of performance analysis for the normal OFDM may be helpful for the subsequent discussions.

In [44], ICI power is calculated for a channel with CFO and a simple approximation to the signal-to-interference power ratio (SIR) is given. In [45], the exact BER is calculated and is compared with the Gaussian approximation. For frequency-selective (or frequency-flat) fast fading channels, in [46], approximate BER is calculated by Gaussian approximation and is compared with simulation results. These results are contrasting: the Gaussian approximation shows a good match for the fast fading channel while it does not for the Gaussian channel with CFO only. This may be because the ICI due to CFO has a probability distribution quite different from the Gaussian distribution. According to a more thorough analysis in [47], Gaussian approximation gives slightly different BER from the true value by about one or more dBs for channels with Doppler spreads. ICI power is also calculated in [48] under the assumption of infinitely many subcarriers and in [49] under a more general condition.

In the following discussions, we do not use the infinite subcarrier approximation but directly approximate the ICI power.

III. Cancellation coding schemes

We review cancellation coding schemes and the behavior of the generated OFDM signals.

A. Polynomial cancellation coding (PCC)

The dth-order PCC is a mapping scheme where each data symbol is mapped over contiguous $(d+1)$ subcarriers with weighting coefficients determined from the dth-order polynomial $(1-D)^d$ in D. In the first-order ($d=1$) PCC, $(1-D)^d = (1-D)$ and the mth data symbol a'_m of $\mathrm{E}\left[|a'_m|^2\right] = E'_{\mathrm{s}}$ ($m = 0, 1, \cdots, \frac{N}{2}-1$) is located both at the $2m$th and $(2m+1)$st subcarriers as[1]

$$a_n = \begin{cases} \frac{1}{\sqrt{2}}a'_m, & \text{if } n = 2m, \\ -\frac{1}{\sqrt{2}}a'_m, & \text{if } n = 2m+1. \end{cases}$$

At the receiver, decision is made for the result of subtraction combining

$$R^{\mathrm{P}}(n) = \frac{1}{\sqrt{2}}R_{2n} - \frac{1}{\sqrt{2}}R_{2n+1}. \quad (6)$$

Although there are higher-order PCC schemes, we only consider the above first-order PCC because of spectral efficiency and call the first-order PCC just PCC in the following discussions.

When PCC is used, the discrete-time baseband OFDM signal (1) has the form

$$x_k = \frac{1}{\sqrt{2}}\sum_{m=0}^{\frac{N}{2}-1} a'_m\left\{e^{j\frac{2\pi(2m)k}{N}} - e^{j\frac{2\pi(2m+1)k}{N}}\right\} \quad (7)$$

[1]The factor $\frac{1}{\sqrt{2}}$ is used for normalization.

and the FFT output at the receiver is expressed, with the use of (3), as

$$R_n = \frac{1}{\sqrt{2}} \sum_{m=0}^{\frac{N}{2}-1} a'_m (A_{2m,n} - A_{2m+1,n}) + W_n.$$

Then, subtraction combining (6) gives, for $0 \leq n < \frac{N}{2}$,

$$R^{\mathrm{P}}(n) = A^{\mathrm{P}}(n) a_n + I^{\mathrm{P}}(n) + W^{\mathrm{P}}(n),$$

where we let $W^{\mathrm{P}}(n) \triangleq \frac{1}{\sqrt{2}} W_{2n} - \frac{1}{\sqrt{2}} W_{2n+1}$ and, for the convention

$$\begin{aligned} A^{\mathrm{P}}_{m,n} \triangleq\ & \frac{1}{2}(A_{2m,2n} - A_{2m+1,2n}) \\ & -\frac{1}{2}(A_{2m,2n+1} - A_{2m+1,2n+1}), \end{aligned}$$

the complex signal coefficient and the interference are given, respectively, as

$$A^{\mathrm{P}}(n) \triangleq A^{\mathrm{P}}_{n,n} \quad \text{and} \quad I^{\mathrm{P}}(n) \triangleq \sum_{\substack{m=0 \\ m \neq n}}^{\frac{N}{2}-1} a'_m A^{\mathrm{P}}_{m,n}.$$

1) PCC and Nyquist windowing: The PCC-OFDM signal (7) is also written as

$$x_k = -j e^{j\frac{\pi k}{N}} \sqrt{2} \sin\left(\frac{\pi k}{N}\right) \sum_{m=0}^{\frac{N}{2}-1} a'_m e^{j\frac{2\pi m k}{N/2}} = \sqrt{2 g_k}\, \tilde{x}_k, \tag{8}$$

where $\tilde{x}_k = -j e^{j\frac{\pi k}{N}} \sum_{m=0}^{\frac{N}{2}-1} a'_m e^{j\frac{2\pi m k}{N/2}}$ is the phase rotated $N/2$-point IDFT of $\{a'_m\}$ and $g_k = \frac{1+\cos\left(2\pi\left[\frac{k}{N}-\frac{1}{2}\right]\right)}{2}$. Then, we can see that PCC is nothing but a windowing method using the square-root raised-cosine (RC) window $\{\sqrt{g_k}\}$ of roll-off $\rho = 1$ both at the transmitter and the receiver.[2]

The square-root RC shaping explains the small OOB radiation of PCC [17], [22] and some robustness against Doppler spreads. However, this kind of transmitter windowing intensifies signal peak powers at the center of each OFDM symbol and hence yields a large peak-to-average power ratio (PAPR) [30] as discussed in Section IV. Moreover, windowing based on the RC function is considerably suboptimal when ICI reduction is considered [7] as discussed for the intersymbol-interference (ISI) problem due to timing jitter for pulse amplitude modulation (PAM) [50]. It is generally known that, for a relatively small Doppler spread or CFO, piece-wise linear window functions give better results [8], [9], [10], which has been also known for PAM [51].

2) On higher-order PCC: It may be instructive to compare higher-order PCC and windowing. The dth order PCC-OFDM signal can written, with an appropriate normalization, as

$$x_k = \frac{\left(-j e^{j\frac{\pi k}{N}}\right)^d}{\sqrt{d+1}} \cdot \sqrt{(2 g_k)^d} \sum_{m=0}^{\frac{N}{d+1}-1} a'_m e^{j\frac{2\pi m k}{N/(d+1)}}$$

The window $\{g^d_k\}$ is not a Nyquist window for $d > 1$. Thus, we need guard subcarriers to avoid ICI introduced by windowing, which explain that the dth order PCC requires $d+1$ subcarriers for each a'_m. Moreover, the window peak value $(2 g_{\frac{N}{2}})^d = 2^d$ certainly increase the PAPR of PCC-OFDM. Thus, there is practically no merit in considering a higher-order PCC-OFDM.

B. Symmetric cancellation coding (SCC)

In SCC [26], [27], for given data symbols a'_n of $\mathrm{E}\left[|a'_n|^2\right] = E'_{\mathrm{s}}$, $n = 0, 1, \cdots, \frac{N}{2} - 1$, subcarrier values a_n are determined as

$$a_n = \begin{cases} \frac{1}{\sqrt{2}} a'_n, & \text{for } 0 \leq n < \frac{N}{2}, \\ -\frac{1}{\sqrt{2}} a'_{N-n-1}, & \text{for } \frac{N}{2} \leq n < N. \end{cases}$$

Thus, the SCC-OFDM signal is given, for $-N_{\mathrm{G}} \leq k < N - 1$, as

$$x_k = \frac{1}{\sqrt{2}} \sum_{m=0}^{\frac{N}{2}-1} a'_m \left\{ e^{j\frac{2\pi m k}{N}} - e^{-j\frac{2\pi(m+1)k}{N}} \right\}. \tag{9}$$

The FFT output at the receiver is written as

$$R_n = \frac{1}{\sqrt{2}} \sum_{m=0}^{\frac{N}{2}-1} a'_m (A_{m,n} - A_{-1-m,n}) + W_n. \tag{10}$$

The subcarrier values at the symmetric positions are combined as

$$R^S(n) = \phi^{(1)}_n R_n + \phi^{(2)}_n R_{N-n-1},$$

where $\phi^{(1)}_n$ and $\phi^{(2)}_n$ are combining coefficients satisfying $|\phi^{(1)}_n|^2 + |\phi^{(2)}_n|^2 = 1$.

In [26] and [27], it is discussed that, for a flat fading channel, the ICI due to CFO is efficiently suppressed at the receiver if subtraction combining $\phi^{(1)}_n = -\phi^{(2)}_n$ is used. For frequency selective fading channels, however, ICI cancellation by subtraction combining is not effective since the pair of subcarriers subject different fading, and the frequency diversity effects obtained with maximal ratio combining (MRC) become more important [26], [27].

The combiner output is given as

$$R^{\mathrm{S}}(n) = A^{\mathrm{S}}(n) a'_n + I^{\mathrm{S}}(n) + W^{\mathrm{S}}(n),$$

where we let $W^{\mathrm{S}}(n) \triangleq \phi^{(1)}_n W_n + \phi^{(2)}_n W_{N-1-n}$ and, for[3]

$$\begin{aligned} A^{\mathrm{S}}_{m,n} \triangleq\ & \phi^{(1)}_n (A_{m,n} - A_{-1-m,n}) \\ & + \phi^{(2)}_n (A_{m,-1-n} - A_{-1-m,-1-n}), \end{aligned}$$

the complex signal coefficient and and the interference are given, respectively, as

$$A^{\mathrm{S}}(n) \triangleq A^{\mathrm{S}}_{n,n} \quad \text{and} \quad I^{\mathrm{S}}(n) \triangleq \sum_{\substack{m=0 \\ m \neq n}}^{N-1} a'_m A^{\mathrm{S}}_{m,n}$$

[2]Since the cyclic prefix creates a sharp edge, zero-padding may be more suitable as a guard in order to suppress the OOB spectral radiation.

[3]Because of periodicity, $A_{m,N-n} = A_{m,-n}$.

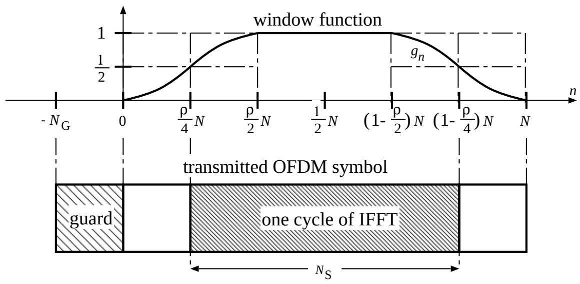

Figure 1. Nyquist windowing at the transmitter

There is a certain relationship between SCC and OFDM with discrete-cosine transform (DCT) [28], that is, DCT-OFDM [29]. In fact, if we let $a''_m = \sqrt{2}e^{\pi m}a'_m$, then we can also write the SCC-OFDM signal (9) as

$$x_{k-\frac{N}{2}} = -e^{-j\frac{\pi k}{N}} \cdot \sum_{n=0}^{\frac{N}{2}-1} a''_n \cos\frac{\pi(2n+1)k}{N}. \quad (11)$$

This shows that SCC-OFDM is a special case of the time- and frequency-shifted version of DCT-OFDM. The relationships to DCT-OFDM is discussed in Section V further.

C. Nyquist windowing

For a given ρ such that $0 \le \rho \le 1$, the window $\{g_k\}$ of roll-off ρ and an OFDM symbol are related as shown in Fig. 1, where the window has the total length N and the effective length $N_S = (1-\frac{\rho}{2})N$. This window is multiplied with a N_S-point IDFT output and its cyclic prefix/postfix of total length $\frac{\rho}{2}N$. Moreover, a guard interval is also necessary to protect the current OFDM symbol from the previous one.

We suppose that the window weights satisfy the non-negativity and symmetry

$$\begin{cases} g_k \ \ge \ 0, & \text{for } 0 \le k < N, \\ g_k \ = \ g_{N-k}, & \text{for } 0 \le k < \frac{N}{2}. \end{cases}$$

Then, the window is a Nyquist window if it satisfies the condition

$$\begin{cases} g_{\frac{\rho}{4}N} \ = \ \frac{1}{2}, & \\ g_k + g_{(1-\frac{\rho}{2})N+k} \ = \ 1, & \text{for } 0 \le k < \frac{\rho}{2}N. \end{cases} \quad (12)$$

From the second condition in (12), we have the identity,

$$2\sum_{k=0}^{\frac{\rho}{2}N-1} g_k g_{N_S+k} = N_S - \sum_{k=0}^{N-1} .g_k^2 \quad (13)$$

For the Nyquist window, we have identities $\frac{\rho}{4}N = \frac{1}{2}(N-N_S)$, $\frac{\rho}{2}N = N - N_S$, $(1-\frac{\rho}{2})N = N_S$, and $(1-\frac{\rho}{4})N = \frac{1}{2}(N+N_S)$, which are assumed in the following discussions.

For the choice of ρ, there is much freedom [6]-[9]. We frequently consider $\rho = 1$ in the later discussions and comparisons since it allows the simplest implementation, provides the best result [9], [10], and allows direct comparison with PCC.

Windowing may be applied at the transmitter as well as at the receiver.

1) Windowing at the receiver only: In this windowing scheme, the N_S-point IDFT output of length $N + N_G$,

$$x_k = \sum_{m=0}^{N_S-1} a_m e^{j\frac{2\pi mk}{N_S}}, \qquad \text{for } -N_G \le k \le N-1,$$

is transmitted over the channel (2). At the receiver, the part corresponding to the guard is omitted from the received signal and the resultant signal of length N is multiplied with w_k and is applied with N_S-point DFT. The result is expressed, for $0 \le n \le N_S - 1$, as

$$\begin{aligned} R^W(n) &= \frac{1}{N}\sum_{k=0}^{N-1} g_k y_k e^{-j\frac{2\pi kn}{N_S}} \\ &= A^W(n)a_n + I^W(n) + W^W(n), \end{aligned}$$

where the signal coefficient, interference, and noise are, respectively,

$$A^W(n) \triangleq \frac{1}{N}\sum_{k=0}^{N-1}\sum_{\ell=0}^{L-1} g_k h_{k,\ell} e^{j\frac{-2\pi\ell n}{N_S}}, \quad (14)$$

$$W^W(n) \triangleq \frac{1}{N}\sum_{k=0}^{N-1} g_k w_k e^{-j\frac{2\pi kn}{N_S}}, \quad (15)$$

$$I^W(n) \triangleq \frac{1}{N}\sum_{\substack{m=0 \\ m\neq n}}^{N_S-1} a_m \sum_{k=0}^{N-1}\sum_{\ell=0}^{L-1} g_k h_{k,\ell} \times e^{j\frac{2\pi\ell m}{N_S}} e^{-j\frac{2\pi k(n-m)}{N_S}}. \quad (16)$$

2) Matched windowing: The receiver windowing scheme is against the principle of matched receiver (or filtering) [42] and yields a certain power loss since the receiver is not "matched" to the transmitted signal. A remedy for the SNR loss is to use a square-root Nyquist window at both transmitter and receiver. That is, for a Nyquist window g_k satisfying $g_k \ge 0$ and $\frac{1}{N}\sum_{k=0}^{N-1} g_k = 1$, the transmitted signal is shaped as

$$x_k = g_k^{\frac{1}{2}} \sum_{m=0}^{\frac{N}{2}-1} a_m e^{j\frac{2\pi mk}{N/2}}, \qquad \text{for } 0 \le k \le N-1,$$

and, at the receiver, the channel output is also shaped as

$$\begin{aligned} R^{MW}(n) &= \frac{1}{N}\sum_{k=0}^{N-1} g_k^{\frac{1}{2}} y_k e^{-j\frac{2\pi kn}{N/2}} \\ &= A^{MW}(n)a_n + I^{MW}(n) + W^{MW}(n) \end{aligned}$$

where, for

$$A^{MW}_{m,n} \triangleq \frac{1}{N}\sum_{k=0}^{N-1}\sum_{\ell=0}^{L-1} h_{k,\ell} g_k^{\frac{1}{2}} g_{k-\ell}^{\frac{1}{2}} e^{j\frac{2\pi(m-n)k}{N/2}} e^{-j\frac{2\pi m\ell}{N/2}},$$

the signal amplitude and interference are, respectively,

$$A^{MW}(n) = A^{MW}_{n,n}, \quad (17)$$

$$I^{MW}(n) = \sum_{m=0, m\neq n}^{N/2} A^{MW}_{m,n} a_m, \quad (18)$$

$$W^{MW}(n) = \frac{1}{N}\sum_{k=0}^{N-1} g_k^{\frac{1}{2}} w_k e^{-j\frac{2\pi kn}{N/2}}. \quad (19)$$

As previously discussed, PCC can be considered as a matched windowing scheme based on the RC window.

IV. PAPR CHARACTERISTICS OF PCC AND SCC

PAPR is, although it is not the main topic of this paper, an important characteristic which may determine the applicability of ICI mitigation schemes. Since CCC has the same behavior except that half the samples are zero, we briefly consider the peak behavior of PCC and SCC employing Gaussian approximation.

Given a continuous time signal $x(t)$, $0 \le t \le T$, we let $x_k^A = x(k\Delta)$, $0 \le k \le AN$, for $\Delta = \frac{T}{NA}$ and for a positive integer A and consider the PAPR with oversampling factor A given by

$$\mathrm{PAPR} = \frac{\max_k |\, x_k^{(A)} \,|^2}{\frac{1}{NA}\sum_k |\, x_k^{(A)} \,|^2}.$$

Although oversampling factor $A = 4$ is used to evaluate accurate PAPR [1], we let $A = 1$ and hence let $x_k = x_k^{(1)}$ to make discussions simple. Then, the complementary cumulative distribution function (CCDF) of PAPR for the normal OFDM is known to be given as [1]

$$\Pr(\mathrm{PAPR} \ge z) = 1 - \left(1 - e^{-z}\right)^{\kappa N}, \qquad (20)$$

where κ is a parameter generally determined by computer simulation. If $\{x_k\}$ is a series of mutually independent Gaussian random variables, then $\kappa = 1$.

To calculate the CCDF for the PCC-OFDM, we rewrite (8) as

$$\begin{aligned} x_k &= -je^{j\frac{\pi k}{N}} \sin\left(\frac{\pi k}{N}\right) \sqrt{2} \sum_{n=0}^{\frac{N}{2}-1} a_n' e^{j\frac{2\pi nk}{N/2}} \\ &= -je^{j\frac{\pi k}{N}} \sin\left(\frac{\pi k}{N}\right) \widehat{x}_k, \end{aligned}$$

where $\widehat{x}_k$ is a normal OFDM signal with $N/2$ period and with average power $2E_\mathrm{s}$. Assuming that $\widehat{x}_k$ are mutually independent, circularly symmetric (SC) and have Gaussian distributions, we have, for a given real number z,

$$\begin{aligned} &\Pr\left\{|x_k|^2 \le \frac{zE_\mathrm{s}}{2} \text{ for } k = 0, 1, \cdots, N-1\right\} \\ &= \Pr\left\{\sin^2\left(\frac{\pi k}{N}\right) |\widehat{x}_k|^2 \le \frac{zE_\mathrm{s}}{2} \right. \\ &\qquad\qquad \left. \text{for } k = 0, 1, \cdots, N-1\right\} \\ &= \Pr\left\{\left(1 + \sin\frac{2\pi k}{N}\right) |\widehat{x}_{k+\frac{N}{4}}|^2 \le zE_\mathrm{s} \right. \\ &\qquad\qquad \left. \text{for } k = 0, 1, \cdots, \tfrac{N}{2} - 1\right\}, \end{aligned}$$

where we used the fact that $\widehat{x}_k$ has period $\frac{N}{2}$. Thus, we have

$$\begin{aligned} &\Pr(\mathrm{PAPR} \ge z) \\ &= 1 - \Pr\left\{\left(1 + \sin\frac{2\pi k}{N}\right) |\widehat{x}_{k+\frac{N}{4}}|^2 \le zE_\mathrm{s} \right. \\ &\qquad\qquad \left. \text{for } k = 0, 1, \cdots, \tfrac{N}{2} - 1\right\} \\ &= 1 - \prod_{k=0}^{\frac{N}{2}-1} \left[1 - \exp\left(-\frac{z}{1 + \sin\frac{2\pi k}{N}}\right)\right]. \qquad (21) \end{aligned}$$

The SCC-OFDM signal (9) is, on the other hand, written, for $k = 0,\ 1,\ \cdots,\ \frac{N}{2} - 1$, as

$$\begin{aligned} x_k &= \frac{1}{\sqrt{2}} \sum_{n=0}^{\frac{N}{2}-1} a_n' \left\{e^{j\frac{2\pi nk}{N}} - e^{-j\frac{2\pi(n+1)k}{N}}\right\} \\ &= j\sqrt{2} e^{-j\frac{\pi k}{N}} \sum_{n=0}^{\frac{N}{2}-1} a_n' \sin\left(\frac{2\pi(n+\frac{1}{2})k}{N}\right) \qquad (22) \end{aligned}$$

and

$$x_{N-k} = -e^{-j\frac{2\pi nk}{N}} x_k. \qquad (23)$$

Thus, we have

$$\begin{aligned} &\Pr(\mathrm{PAPR} \ge z) \\ &= 1 - \Pr\left\{|x_k|^2 \le zE_\mathrm{s} \text{ for } k = 0, 1, \cdots, \tfrac{N}{2} - 1\right\} \\ &= 1 - \left(1 - e^{-z}\right)^{\frac{\kappa N}{2}}, \qquad (24) \end{aligned}$$

where we put the coefficient κ to modified the result appropriately.

In Fig. 2, we show the CCDFs of the normal OFDM, PCC-OFDM, and SCC-OFDM obtained by simulation for $N = 512$ and QPSK modulation. From the figure, we can see that SCC-OFDM has gained 1 dB compared to the normal OFDM and 11 dB compared to PCC-OFDM at CCDF$= 0.001$. The figure also show the approximations (20) for $\kappa = 1.2$ in the case of normal OFDM,[4] (21) in the case of PCC-OFDM, and (24) for $\kappa = 1.2$ in the case of SCC-OFDM. Suboptimality of PCC-OFDM in terms of PAPR is clearly seen. We can consider the PAPR of an OFDM system with transmitter windowing in a similar manner.

V. DIVERSITY SCHEMES

In [29], it is shown, by analysis, that the DCT-OFDM is more robust against CFO than the conventional OFDM and, by simulation, that the DCT-OFDM with frequency-domain equalization outperforms the conventional OFDM over a frequency selective fast fading channel. These results show good agreements with the results observed for SCC-OFDM in [26] and [27].

It is claimed in [29] that the good performance of DCT-OFDM for a flat fading channel explains its good performance for a frequency-selective fading channel.

[4]It is generally conceived, for the normal OFDM, that $\kappa = 2.8$ gives the best match [1]. However, as discussed in [52], $\kappa = 2.8$ does not give a good approximation for a large N. For $N = 512$ and for this range of z, $\kappa = 1.2$ seems to give good match.

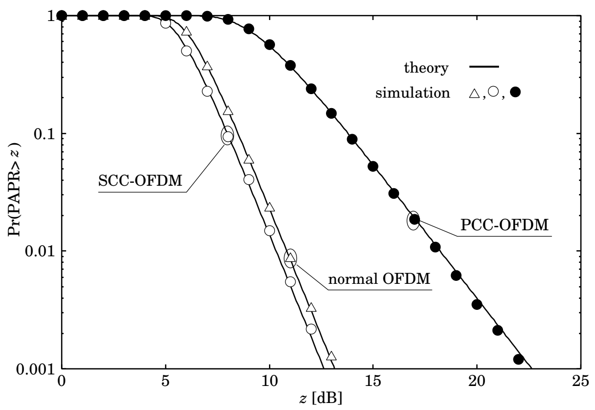

Figure 2. CCDF versus PAPR threshold for the normal OFDM, PCC-OFDM, and SCC-OFDM, ($N = 512$, $\kappa = 1.2$, QPSK)

Contrary to this assertion, however, its good performance for a flat fading channel is ascribed to its characteristic as a self-ICI-cancellation coding scheme and that for a frequency-selective fading channel is ascribed to its characteristic as a frequency-diversity scheme. To see this analytically, we consider SCC-OFDM in conjunction with optimal detection under the assumption that the receiver know the channel completely.

A. SCC-OFDM with optimized combining

1) Basic formulation: Let n be such that $0 \le n < \frac{N}{2}$. Then, for ξ which is either n or $N-1-n$, the DFT output is written, from (10), as

$$R_\xi = \tilde{A}^{\mathrm{S}}(\xi)a'_n + \tilde{I}^{\mathrm{S}}(\xi) + W_\xi,$$

where we let

$$\tilde{A}^{\mathrm{S}}(\xi) \triangleq \frac{1}{\sqrt{2}}\left(A_{n,\xi} - A_{-1-n,\xi}\right) \tag{25}$$

$$\tilde{I}^{\mathrm{S}}(\xi) \triangleq \sum_{m=0,\, m\neq n}^{\frac{N}{2}-1} a'_m \left(A_{m,\xi} - A_{-1-m,\xi}\right) \tag{26}$$

Let

$$\boldsymbol{r}_n \triangleq \begin{bmatrix} R_n \\ R_{N-1-n} \end{bmatrix},\ \boldsymbol{h}_n \triangleq \begin{bmatrix} \tilde{A}^{\mathrm{S}}(n) \\ \tilde{A}^{\mathrm{S}}(-1-n) \end{bmatrix},\ \text{and}$$
$$\boldsymbol{v}_n \triangleq \begin{bmatrix} \tilde{I}^{\mathrm{S}}(n) + W_n \\ \tilde{I}^{\mathrm{S}}(-1-n) + W_{N-1-n} \end{bmatrix}. \tag{27}$$

Then the above SCC-OFDM channel model is written as

$$\boldsymbol{r}_n = \boldsymbol{h}_n\, a'_n + \boldsymbol{v}_n$$

We assume that subcarrier values a'_m are mutually independent and CS, that is, QPSK- or 16QAM-modulated signals. Since $h_{k,\ell}$ are CS too, $\tilde{I}^{\mathrm{S}}(n)$ and $\tilde{I}^{\mathrm{S}}(-1-n)$ are CS random variables uncorrelated to $\tilde{A}^{\mathrm{S}}(n)$ and $\tilde{A}^{\mathrm{S}}(-1-n)$. We assume, for a sufficiently large N, that $\boldsymbol{v}_n$ is a CS Gaussian random vector independent of $\boldsymbol{h}_n$, and let

$$\boldsymbol{H}_n = \mathrm{E}\left[\boldsymbol{h}_n\boldsymbol{h}_n^H\right] \text{ and } \hat{N}_o\boldsymbol{V}_n = \mathrm{E}\left[\boldsymbol{v}_n\boldsymbol{v}_n^H\right], \tag{28}$$

where $\hat{N}_o$ and $\boldsymbol{V}_n$ are determined so that $|\boldsymbol{V}_n| = 1$. We assume that both $\boldsymbol{H}_n$ and $\boldsymbol{V}_n$ are non-singular.

For a unitary matrix $\boldsymbol{Q}_n$ such that $\boldsymbol{Q}_n\boldsymbol{V}_n\boldsymbol{Q}_n^H = \boldsymbol{I}$, we let $\boldsymbol{\eta}_n = \boldsymbol{Q}_n\boldsymbol{h}_n$ and $\boldsymbol{w}_n = \boldsymbol{Q}_n\boldsymbol{v}_n$. Then, we have

$$\boldsymbol{r}'_n \triangleq \boldsymbol{Q}_n\boldsymbol{r}_n = \boldsymbol{\eta}_n\, a_n + \boldsymbol{w}_n,$$

where components of $\boldsymbol{w}_n$ are mutually independent random variables with variance $\hat{N}_o$ and the coefficient vector $\boldsymbol{\eta}_n$ has the covariance matrix $\hat{\boldsymbol{H}}_n = \mathrm{E}\left[\boldsymbol{\eta}_n\boldsymbol{\eta}_n^H\right] = \boldsymbol{Q}_n\boldsymbol{H}_n\boldsymbol{Q}_n^H$. Then, for a given $\boldsymbol{\eta}_n$, the optimal decision is made based on the matched filter output

$$y_n = \boldsymbol{\eta}_n^H\boldsymbol{r}'_n = \boldsymbol{\eta}_n^H\boldsymbol{\eta}_n a'_n + \boldsymbol{\eta}_n^H\boldsymbol{w}_n$$

and the (instantaneous) SNR per symbol at the decision is

$$\gamma_n = \frac{E'_{\mathrm{s}}}{\hat{N}_o}\boldsymbol{\eta}_n^H\boldsymbol{\eta}_n = \frac{2E_{\mathrm{s}}}{\hat{N}_o}\boldsymbol{\eta}_n^H\boldsymbol{\eta}_n$$

where we let $E'_{\mathrm{s}} = \mathrm{E}\left[|a'_n|^2\right] = 2E_{\mathrm{s}}$.

2) BER expression: We restrict our attention to QPSK signals. For a given $\boldsymbol{\eta}_n$, the BER is $P_{\mathrm{b}}(\gamma_n) = Q(\sqrt{\gamma_n})$ for the Gaussian Q-function $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}\,dt$ [43].

The CS Gaussian random variable $\boldsymbol{\eta}_n$ has a pdf given by

$$p(\boldsymbol{\eta}) = \frac{1}{\pi^2|\hat{\boldsymbol{H}}_n|}\exp\left(-\boldsymbol{\eta}^H\hat{\boldsymbol{H}}_n^{-1}\boldsymbol{\eta}\right)$$

Thus, employing Craig's expression for the Gaussian Q-function [53]

$$Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\exp\left(-\frac{x^2}{2\sin^2\theta}\right)\,d\theta,$$

we can calculate the average of $P_{\mathrm{b}}(\gamma_n)$ with respect to γ_n as

$$\begin{aligned} P_{\mathrm{b}}(n) &= \mathrm{E}\left[Q\left(\sqrt{\frac{2E_{\mathrm{s}}}{\hat{N}_o}\boldsymbol{\eta}_n^H\boldsymbol{\eta}_n}\right)\right] \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left|I + \frac{E_{\mathrm{s}}}{\hat{N}_o\sin^2\theta}\boldsymbol{H}_n\boldsymbol{V}_n^{-1}\right|^{-1}\,d\theta \end{aligned}$$

If $\frac{E_{\mathrm{s}}}{\hat{N}_o}$ is sufficiently large, the right-hand side is approximated as

$$\begin{aligned} P_{\mathrm{b}}(n) \approx &\approx \frac{1}{\pi}\int_0^{\frac{\pi}{2}}4\sin^4\theta\,d\theta\cdot\left|\frac{2E_s}{\hat{N}_o}\boldsymbol{H}_n\boldsymbol{V}_n^{-1}\right|^{-1} \\ &= \frac{3}{4}\left|\frac{\hat{N}_o}{2E_{\mathrm{s}}}H_n^{-1}V_n\right| \end{aligned} \tag{29}$$

The above expressions hold under the assumption that $\boldsymbol{H}_n$ and $\boldsymbol{V}_n$ are nonsingular, which holds, from (27) and (28), only when the channel is frequency selective. Thus, SCC acts as a ICI cancellation scheme over a flat fading channel while it acts as a diversity scheme only over a multipath fading channel. We note, however, that the BER performance of SCC-OFDM is position-dependent.

3) Relationships to DCT-OFDM: In [29], the following DCT-OFDM signal representation is considered.

$$x_k = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} d_n a_n \cos \frac{\pi(2n+1)k}{2N},$$

where $d_n = \frac{1}{sqrt2}$ for $n = 0$ and $d_n = 1$ otherwise. Ignoring terms irrespective of DCT transform (and d_n), the SCC-OFDM signal (11) is a DCT-OFDM signal with a doubled subcarrier space. This is because SCC halves the symbol rate. Thus, except for this difference, SCC-OFDM and DCT-OFDM are basically the same modulation and results obtained for one are also expected to hold for the other.

B. CCC-OFDM: another diversity scheme

1) Basic formulation: Motivated by SCC, we introduced cyclic cancellation coding (CCC) in [32] as a scheme where each of the $\frac{N}{2}$ data symbols a'_n is assigned to a pair of subcarriers at cyclically symmetric positions as

$$a_n = \begin{cases} \frac{1}{\sqrt{2}} a'_n, & \text{if } 0 \le n < \frac{N}{2}, \\ -\frac{1}{\sqrt{2}} a'_{n-\frac{N}{2}}, & \text{if } \frac{N}{2} \le n < N. \end{cases}$$

The CCC-OFDM signal is then given by

$$x_k = \frac{1}{\sqrt{2}} \sum_{m=0}^{\frac{N}{2}-1} a'_m \left\{ e^{j\frac{2\pi mk}{N}} - e^{j\frac{2\pi(m+\frac{N}{2})k}{N}} \right\}.$$

The merit of CCC is that it is more effective in realizing frequency diversity effects than SCC since every data symbol is transmitted over a subcarrier pair separated by $\frac{N}{2}$. If we let $a''_m = a'_m e^{j\frac{2\pi(m-N/8)}{N}}$, then the CCC-OFDM signal is also written as the up-sampled $\frac{N}{2}$-IDFT

$$x_k = \begin{cases} \sum_{m=0}^{\frac{N}{2}-1} a''_m e^{j\frac{2\pi mk}{N/2}}, & \text{for } k \text{ odd}, \\ 0, & \text{for } k \text{ even}. \end{cases} \quad (30)$$

Since a half the samples are zero, the average peak power is increased by 3 dB.

Actually, CCC is not "cancellation coding" and is rather a special case of a frequency-diversity scheme called the single-antenna vector OFDM (V-OFDM) first introduced in [37]. In V-OFDM, samples from another $\frac{N}{2}$-point DFT are transmitted at even-numbered time epochs too.[5] Nevertheless, we use the term CCC in the sense that it is a cyclic version of SCC.

For CCC-OFDM, we have the following expression for the DFT output, for $0 \le n < \frac{N}{2}$ and for $\xi = n, n + \frac{N}{2}$,

$$R_\xi = \tilde{A}^{\mathrm{C}}(\xi) a'_n + \tilde{I}^{\mathrm{C}}(\xi) + W_\xi,$$

where we let

$$\tilde{A}^{\mathrm{C}}(\xi) \triangleq \frac{1}{\sqrt{2}} \left(A_{n,\xi} - A_{n+\frac{N}{2},\xi} \right) \quad (31)$$

$$\tilde{I}^{\mathrm{C}}(\xi) \triangleq \sum_{m=0,\, m\neq n}^{\frac{N}{2}-1} a'_m \left(A_{m,\xi} - A_{m+\frac{N}{2},\xi} \right) \quad (32)$$

[5]Since the signals at odd-numbered time epochs and at even-numbered time epochs interfere to each other, however, constellation rotation is required to attain full diversity as shown in [38].

The BER expression is obtained as in the same manner as SCC-OFDM and is omitted.

2) Relationships to V-OFDM: Let us insert another $\frac{N}{2}$-IDFT outputs to the place of zero-valued samples in (30) and modify the result expression a little bit as

$$\begin{cases} x_{2k} = \sum_{m=0}^{\frac{N}{2}-1} a_{2m} e^{j\frac{2\pi mk}{N/2}}, \\ x_{2k+1} = \sum_{m=0}^{\frac{N}{2}-1} a_{2m+1} e^{j\frac{2\pi mk}{N/2}}, \end{cases}$$

for given N symbols a_n, $n = 0, 1, \cdots, N-1$. We can show that $\{x_k\}$ is an ordinary OFDM signal for a set of subcarrier values $\{b_n\}$ with a special structure. In fact, applying DFT, we have

$$\begin{cases} b_n = \frac{1}{2}\left(a_{2n} + a_{2n+1} e^{-j\frac{2\pi n}{N}} \right) \\ b_{n+\frac{N}{2}} = \frac{1}{2}\left(a_{2n} - a_{2n+1} e^{-j\frac{2\pi n}{N}} \right) \end{cases}$$

for $n = 0, 1, \cdots, \frac{N}{2}-1$, or, letting $a'_{2n} = a_{2n}$ and $a'_{2n+1} = a_{2n+1} e^{-j\frac{2\pi n}{N}}$, we have

$$\begin{cases} b_n = \frac{a'_{2n} + a'_{an+1}}{2} \\ b_{n+\frac{N}{2}} = \frac{a'_{2n} - a'_{an+1}}{2} \end{cases}$$

This signaling method is nothing but the (single-carrier) vector OFDM (V-OFDM) [37] with vector length 2. Although, contrary to the the case of SCC-OFDM and DCT-OFDM, the relationship between CCC-OFDM and V-OFDM (of vector length 2) is not straightforward since V-OFDM yields artificial ICI between the nth and $n+\frac{N}{2}$th subcarriers, we may expect the same robustness against the Doppler spread and CFO in both schemes.

VI. Conclusion

We have reviewed PCC-OFDM, SCC-OFDM, and windowing for uncoded OFDM systems and showed that PCC is nothing but a matched windowing scheme which uses windowing both at the transmitter and receiver and that SCC-OFDM is DCT-OFDM with double subcarrier space. We considered PAPR properties of PCC-OFDM and SCC-OFDM signals and showed that the PCC-OFDM signal has a relatively high PAPR due to its characteristic as a matched windowing scheme. We next discussed SCC-OFDM and CCC-OFDM as diversity schemes and discussed relationships between SCC-OFDM and DCT-OFDM and between CCC-OFDM and (CR)V-OFDM. In the second part of this two-part paper, we theoretically analyze SINR and BER of these schemes based on the Gaussian ICI assumption and compare these scheme in terms of SINR and BER.

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APPENDIX I
THE FLAWS IN [24]

A general time-domain correlation function $r(f_D T_s x)$ in [24], we only consider the specific correlation function $r(f_D T_s x) = J(f_D T_s x)$. Then, the authors' claim below Expression (6) in [24] is written as

$$J(f_D T_s x) \sum_{\substack{n=-\infty \\ n=\text{even} \\ n\neq 0}}^{\infty} e^{-j2\pi nx} = 1 - J\left(\frac{f_D T_s x}{2}\right),$$

but this equality is not correct.

To show it, we first note the identity

$$\sum_{\ell=-\infty}^{\infty} e^{-j4\pi\ell x} = \frac{1}{2}\sum_{m=-\infty}^{\infty} \delta\left(x - \frac{m}{2}\right). \qquad (33)$$

The identity can be derived in the following manner. We first note, for an arbitrary continuous function $f(x)$, the formal integration

$$\int_{-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} e^{-j4\pi\ell x} f(x)\,dx = \sum_{\ell=-\infty}^{\infty} F\left(j\frac{2\pi\ell}{(1/2)}\right),$$

where $F(j\omega)$ is the Fourier transform of $f(x)$. Since the last summation is the aliased sum of $F(j\omega)$ at $\omega = 0$ and since the aliased sum represents the discrete-time Fourier transform of samples of $f(x)$, that is, for $s > 0$, [42]

$$\frac{1}{2}\sum_{\ell=-\infty}^{\infty} F\left(j\left[\omega + \frac{2\pi\ell}{s}\right]\right) = \sum_{n=-\infty}^{\infty} f(ns)e^{-j\omega ns},$$

we have, for an arbitrary continuous function $f(x)$,

$$\int_{-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} e^{-j4\pi\ell x} f(x)\,dx = \frac{1}{2}\sum_{n=-\infty}^{\infty} f\left(\frac{n}{2}\right).$$

This proves the desired identity.

Thus, from the identity (33), the correct expression for $x \in (-1, 1)$ is

$$\begin{aligned} & J(f_D T_s x) \sum_{\substack{n=-\infty \\ n=\text{even} \\ n\neq 0}}^{\infty} e^{-j2\pi nx} \\ = \; & \frac{1}{2}J(f_D T_s x) \sum_{n=-1,0,1} \delta\left(x + \frac{n}{2}\right) - J\left(\frac{f_D T_s x}{2}\right) \end{aligned} \qquad (34)$$

Thus the expression (7) in [24] must be

$$\begin{aligned} & \sum_{\substack{n=-\infty \\ n=\text{even} \\ n\neq 0}}^{\infty} \mathrm{E}\left[|h_n|^2\right] \\ = \; & \frac{1}{2} + \frac{1}{2}J\left(\frac{f_D T_s}{2}\right) - \int_{-1}^{1} (1-|x|)J(f_D T_s x)\,dx. \end{aligned}$$

According to the same reasoning, the equalities below Expression (8) in [24] are incorrect and hence Expression (9) in [24] is.