Power Allocation Under Transmitter Channel Uncertainty and QoS Constraints: Volumetric Water-Filling Solution

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Abstract— **We study optimal power allocation for a set of parallel channels when the transmitter has limited knowledge of the Channel State Information (CSI). This case is typical for mobile OFDMA systems like LTE and LTE-A where the transmitter relies on CSI feedback from a receiver that is subject to measurement errors, sparse sampling, quantization errors (especially noticeable in MIMO systems), processing and transmission delays. Based upon per-channel targeted Packet Error Rates (PER) we construct a power allocation scheme that maximizes the sum capacity. The optimal algorithm can be cast into a modified 3-D (Volumetric) version of the Water-Filling algorithm. Additionally, the solution obtained here improves performance of real-time delay sensitive traffic such as voice and real-time video by favoring the traffic with lower target PER. Simulations show that the gain in sum capacity over traditional Water-Filling solution can reach as high 90% in some cases.**

Index Terms— **Wireless Cellular Networks, 3G, 4G, LTE, WCDMA, HSPA, OFDMA, Radio Resource Management, RRM, Waterfilling, CSI.**

I. Introduction & Problem Statement

A. Dropped calls, QoS, Channel Adaptation

In a modern Mobile Broadband Wireless Network a RF channel between Base Station (Access Point) and User Terminal undergoes rapid and unpredictable fluctuations in time and across the spectrum. Accordingly, the radio resources used (transmit power, sub-channel allocations, coding & modulation) have to be adapted in real time to track RF channel variations. Unfortunately, due to measurement errors, feedback processing delay and quantization, and because of fundamental unpredictability of the time-varying mobile RF channel, a transceiver has to operate under an assumption of some channel uncertainty [1], [2]. The situation is worsened when MIMO is deployed, as the amount of feedback provided in such a case is limited to a discrete codebook coarsely quantized in spacial (antennae) dimensions [2], [3].

Quality of Service (QoS) requirements of various types of traffic could be quite different. Real time traffic such as voice has low tolerance for packet errors and cannot tolerate delays caused by retransmissions (no H-ARQ for voice). On the other hand, delay tolerant traffic such as FTP, HTTP, and email exchanges are delay tolerant and can withstand higher packet error rates provided retransmissions (e.g. Hybrid ARQ) take care of delivering failed packets [3], [4]. Coexistence of both types of traffic within the same spectrum at the same time and for the same pair of transceivers can detrimentally affect real-time traffic such as voice and can lead to excessive dropped calls. Proper link adaptation and power management can equalize the radio resource utilization and balance out diverging QoS requirements.

B. OFDM Based LTE, Wi-Fi, Wi-MAX systems

Common among all most recent broadband wireless technologies is the fact that they are based on the *Orthogonal Frequency Division Multiplex* (OFDM) channelization technique. By using Fast Fourier Transform (FFT) in the Time Domain sin/cos based signaling forms are generated that are invariant with respect to a multipath distortion within the duration of the cyclic prefix [5]. Thus, a set of transmitted parallel non-interfering (no cross-tack) OFDM sub-channels (aka OFDM tones) remain *Inter Channel Interference* (ICI) free after passing through time-dispersive multipath channel. Of course, it comes with a price. Multipath nature of the wireless channel manifests itself by channel frequency selectivity in the frequency domain [6]. As a result, the entire broadband channel becomes a collection of parallel non-interfering channels with widely varying channel gains. Adaptive modulation, bit and power loading become important tool in optimizing channel capacity and reducing communication latency [7]. When the information about a channel is fully known to both receiver (via measurement) and transmitter (via feedback) such an optimization problem has an exact solution known as the *Water Filling* algorithm [7]–[9].

C. Waterfilling for Parallel Channels

For a set of parallel non-interfering channels with Additive Gaussian Noise (AWGN) the problem of determining

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the maximum sum-capacity under the joint power constraint was solved at the dawn of Information Theory [7]. The solution known as *Waterfilling* provides a closed form direct allocation algorithm that assigns powers and, via the Shannon capacity formula, data rates to each individual sub-channel. Waterfilling always had theoretical significance [8] but it has been gaining practical importance in wireless communications since adoption of *Orthogonal Frequency Division Multiplex* (OFDM) that partitions the frequency domain radio resources into set of parallel linear channels. Original AWGN version of Waterfilling makes some assumptions that are not quite applicable to mobile wireless communications. These shortcomings caused renewed interest in the Waterfilling-like methods in recent years [9].

For example, classic Waterfilling is based on the assumption that a transmitter has the exact knowledge of all the channel gains and other relevant *Channel State Information* (CSI) for all of the parallel sub-channels. This knowledge is used in the optimization problem to choose sub-channel achievable rates in such a way to maximize the overall sum-capacity. In a real-life mobile broadband communication channel one has to deal with time-varying frequency selective channel with incomplete channel state information at the transmitter. In this case naive use of existing Waterfilling solution can cause radio resource management mismatch leading to transmission outage (when the transmitter overestimates channel quality) or waste of radio resources (when the transmitter underestimates channel quality). We chose to address this problem based on the concept of outage capacity [10] discussed in next section.

D. Outage Capacity Approach

In this paper we consider a Wireless Communication System where transmitter and receiver operate over a set of parallel communication channels such as *Orthogonal Frequency Division Multiplex* (`OFDM`) tones or Frequency bands (Frequency Division Multiplex (`FDM`) or Multi-Carrier signaling) or spreading codes `CDMA`. Current 3G (WCDMA, HSPA) as well 4G systems (LTE, LTE-A) fall under this description. We also assume a frequency selective (multipath) and time varying channels so that different OFDM tones or spreading codes could have different channel quality. In addition, interference levels on each sub-channel could be different as well. We assume AWGN noise and model interference by means of a Gaussian process. We choose Gaussian signaling [7] to approximate modern communication systems such as 3GPP LTE that employ adaptive coding-modulation techniques and are within the reach of the Shannon limit [3], [4].

Under the conditions outlined above a sub-channel capacity is determined by the famous Shannon's capacity formula [7] which we parameterize in the following way

$$C_n = B_n \ln(1 + g_n u_n) \tag{1}$$

where:

B_n is sub-channel's bandwidth

$g_n = \dfrac{|h_n|^2 P_{\text{tot}}}{N_0 + I_0}$ sub-channel quality, where h_n – sub-channel gain; P_{tot} – total power for all channels.

N_0, I_0 – Thermal Noise and Interference respectively.

$u_n = P_n / P_{\text{tot}}$ fraction of power allocated to the sub-channel n, $u_n \in [0, 1]$

Unfortunately, as described in preceding sections, in a real mobile wireless communication system, the value of per-channel channel gain g_n is known to the transmitter only approximately. And such channel-state-information (CSI) uncertainties can cause substantial performance degradation. The likelihood of channel estimation error that causes an outage depends on many factors and can be described by a statistical distribution. This distribution can be measured from the trace of channel and traffic measurements when running receivers in debugging mode or/and performing drive tests.

II. Water-filling for channels with imperfect feedback

A. Optimal solution for Mean Outage Capacity

We start with a set of N parallel orthogonal scalar channels with channel quality gains g_n (1) coming from some random distribution. A transmitter does not know exact values of g_n. Instead, it is provided with imperfect channel state information $\mathbf{\Gamma}$ from which the gain estimates γ_n may be derived. Thus, a transmitter bases its transmission policy (power assignments, rates) upon γ_n which can bias the true gains g_n in some clever way.

We consider outage capacity per transmission for Gaussian signaling over this set of parallel channels subject to an exact power constraint. It means that if the actual channel quality is less than the perceived by the transmitter $g_n < \gamma_n$ then the transmission will fail due to insufficient actual SNR for the chosen data rate (over-shoot). On the other hand, if $g_n > \gamma_n$ then it will succeed to deliver the bits at the rate that transmitter determined based on a γ_n which in this case is more conservative choice (under-shoot). The expected value of the sum-rate capacity now becomes

$$\mathrm{E}_{\boldsymbol{g}}[C(\boldsymbol{\gamma},\boldsymbol{\Gamma})] = \int \mathrm{d}^N \boldsymbol{g} p(\boldsymbol{g}|\boldsymbol{\Gamma}) \tag{2a}$$

$$\sum_{n=1}^{N} B_n \ln(1+\gamma_n u_n)\Theta(g_n-\gamma_n)$$

$$\text{s.t.:} \quad \sum_{n=1}^{N} u_n = 1; \quad u_n \geq 0 \tag{2b}$$

$$\text{where:} \quad \Theta(x) \equiv \begin{cases} 0 & ; \text{ if } \ x<0 \\ 1/2 & ; \text{ if } \ x=0 \\ 1 & ; \text{ if } \ x>0 \end{cases}$$

Indicator function $\theta(g_n - \gamma_n)$ counts outages that occur when real channel quality gain g_n is less than the perceived value γ_n estimated by the transmitter. For a continuous random fading process the probability measure of it exactly hitting a threshold $g=\gamma$ is zero $\Pr\{g=\gamma\}=0$. Therefore, the indicator step function $\theta(x)$ can be defined at $x=0$ by any real number between 0 and 1 without affecting the results. We chose $1/2$. Other popular choice are 1 and 0. Eq.(2) can be rewritten in a more compact form as follows:

$$\mathrm{E}_{\boldsymbol{g}}[C(\boldsymbol{\gamma},\boldsymbol{\Gamma})] = \sum_{n=1}^{N} s_n(\boldsymbol{\gamma}) \ln(1+\gamma_n u_n) \tag{3a}$$

$$\text{s.t.:} \quad \sum_{n=1}^{N} u_n = 1; \quad u_n \geq 0 \tag{3b}$$

$$\begin{aligned} s_n(\boldsymbol{\gamma}) \equiv\ & B_n \int \mathrm{d}^N \boldsymbol{g} p(\boldsymbol{g}|\boldsymbol{\Gamma})\Theta(g_n-\gamma_n) \\ =\ & B_n \Pr\{g_n \geq \gamma_n|\boldsymbol{\Gamma}\} = B_n w_n \end{aligned} \tag{3c}$$

$$\text{where:} \quad w_n \equiv \Pr\{g_n \geq \gamma_n|\boldsymbol{\Gamma}\} \tag{3d}$$

Constraints $u_n \geq 0$ can be easily eliminated if new variables are introduced

$$u_n \equiv x_n^2; \quad \text{for} \ \ n=1,\ldots,N \tag{4}$$

The remaining total power constraint can be addressed by using Lagrange multipliers method [11]. The resulting unconstrained cost function appears as

$$J(\boldsymbol{x},\lambda|\boldsymbol{\gamma}) = \ \lambda + \sum_{n=1}^{N} \left[s_n(\boldsymbol{\gamma})\ln(1+\gamma_n x_n^2) - \lambda x_n^2\right] \tag{5}$$

Its extreme points can be easily found

$$\frac{\delta J}{\delta x_k} = 2x_k \left[\frac{s_k(\boldsymbol{\gamma})\gamma_k}{(1+\gamma_k x_k^2)} - \lambda\right] \stackrel{!}{=} 0 \tag{6a}$$

$$\frac{\delta J}{\delta \lambda} = 1 - \sum_{n=1}^{N} x_n^2 \stackrel{!}{=} 0 \tag{6b}$$

(6a) always admits trivial solution $x_n = u_n = 0$. Its non-trivial solution can be written in a quite simple form

$$\frac{u_k}{s_k(\boldsymbol{\gamma})} \equiv \frac{x_k^2}{s_k(\boldsymbol{\gamma})} = \lambda^{-1} - \left[\gamma_k s_k(\boldsymbol{\gamma})\right]^{-1} \tag{7}$$

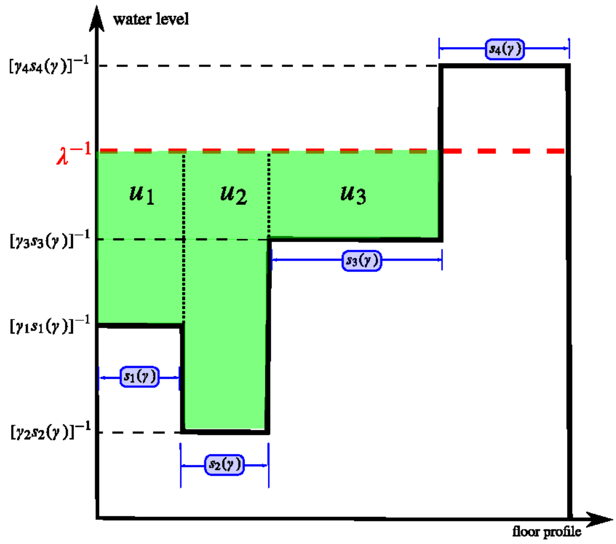

Figure 1. Water–filling interpretation of the optimum power allocation for imperfectly known parallel channels. Sub-channel power allocation fractions $u_k = P_k/P_t$ equals to the **volume** of water that occupies the imaginary parallelepiped with the base area $s_k(\boldsymbol{\gamma})$. The depth of this parallelepiped is counted from the floor level $\left(\gamma_k s_k(\boldsymbol{\gamma})\right)^{-1}$ and to the water level.

Eq.(7) has a ***Volumetric (3-D) Water-Filling*** interpretation. Let us take a vessel with a floor profile shown on Fig. **II-A**. The height of the floor element is $h_k \equiv \left(\gamma_k s_k(\boldsymbol{\gamma})\right)^{-1}$ while the area of its rectangular base is $A_k \equiv s_k(\boldsymbol{\gamma})$. We pour incompressible liquid (water) into that vessel until the common level becomes λ^{-1}. We also identify transmit power P_k with the **volume** of water inside an imaginary parallelepiped sitting on top of the corresponding floor element and stretching to the surface. The area of its base is $s_k(\boldsymbol{\gamma})$ by construction and the water depth can be expressed through its volume as

$$\frac{V_k}{s_k(\boldsymbol{\gamma})} = \lambda^{-1} - \left(\gamma_k s_k(\boldsymbol{\gamma})\right)^{-1}$$

which is precisely (7) if we declare $u_k \equiv V_k$. The value of common water level λ can be found from the total power constrain $\sum u_n = 1$ leading to a nonlinear equation for λ

$$\sum_{k=1}^{N} \max\left(0, \ s_k(\boldsymbol{\gamma})\lambda^{-1} - \gamma_k^{-1}\right) = 1$$

which can be easily solved numerically.

On the other hand, in a text-book version of the Water-Filling procedure [7], [8] there is only one degree of freedom that matters – water level height. It leaves no room for extra quantities such as Packet Error Rate to be accounted for.

B. Simulations and Analysis

Simulations can be used to study achievable capacity gains. So far we were able to observe capacity gains

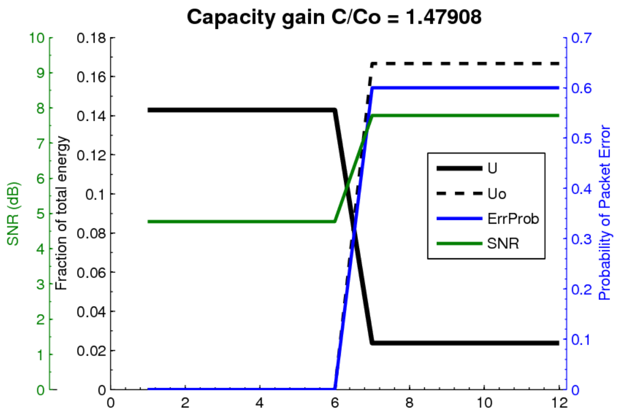

Figure 2. This scenario demonstrates 47% capacity gain over conventional water-filling that does not take into account for packet errors. As a side effect of new scheme, real-time traffic (low error probability) is boosted compared to old scheme in-line with typical network operations goals. U is a vector of channel allocations from (7) while U_0 represents conventional water-filling (without accounting for PER).

ranging from 10% all way to 90%. For illustration purposes we consider a case when voice traffic with a target Packet Error Rate (PER) of 1% coexists with a delay tolerant data traffic that, thanks to H-ARQ, can tolerate much larger error rates on individual transmissions. Let us say, due a deep fade an instantaneous PER of the data traffic on the first H-ARQ round is degraded to 60%. For such case Fig. **II-B** shows side-by-side the channel allocations obtained by means of our algorithm (7) as well as the allocations produced by conventional water-filling procedure [7], [8]. As one can clearly see, channel allocation U_0 that comes from conventional water-filling solution does not account for excessive PER in the data sub-channels and allocates excessive fraction of total power to those "high SNR" but poor PER links. Such power allocation policy results in waste of power of the transmitter, loss of goodput at the intended receiver and excessive interference for the unintended receivers. On the other hand, new power allocation U that follows (7) balances out excessive PER of the data sub-channels by allocating more power to more resilient voice channels improving the sum capacity by almost 50%. We believe that such "self-healing" properties of the new water-filling algorithm are direct reflections of the SNR versus PER optimization trade-off.

C. Summary and Future Work

In this paper we studied o problem of maximizing sum-rate of the set of parallel AWGN channels under imperfect channel information at the transmitter. We obtained the optimal solution under total power constraint and provided an elegant interpretation of the solution in the form of the 3D water-filling procedure. Numerical simulations show that in addition to the gain in sum-capacity of the link the new algorithm is trying to allocate some extra power to low PER (real-time) traffic at the expense of high PER data traffic (deep fade or aggressive link adaptation). For mobile wireless operators this is very useful side effect, for it helps prioritize real-time traffic such as voice.

For future work we would like to combine this type of power allocation with MIMO and STC to provide joint radio resource management optimization framework which explicitly takes into account CSI uncertainty and different target QoS for different types of traffic.

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REFERENCES

[1] S. T. Rickard, R. V. Balan, H. V. Poor, and S. Verdu, "Canonical time-frequency, time-scale, and frequency-scale representations of time-varying channels," *Communications in Information and Systems*, vol. 5, no. 1, pp. 1–30, 2005.
[2] 3GPP, "Evolved Universal Terrestrial Radio Access (E-UTRA): Physical Layer Procedure," 3GPP TS 36.213, 2011.
[3] A. Ghosh, J. Zhang, J. G. Andrews, and R. Muhamed, *Fundamentals of LTE*. Prentice Hall, 2011.
[4] H. Holma and A. Toskala, *WCDMA for UMTS HSPA Evolution and LTE*, 4th ed. John Wiley & Sons, 2007.
[5] L. Hanzo and T. Keller, *OFDM and MC-CDMA: A Primer*. John Wiley & Sons, 2006.
[6] O. Edfors, M. Sandell, J.-J. van de Beek, D. Landstrm, and F. Sjberg, "An Introduction to Orthogonal Frequency-Division Multiplexing," Lulea University of Technology, Tech. Rep., 1996, http://epubl.luth.se/avslutade/0347-0881/96-16/esb96rc.pdf.
[7] R. G. Gallager, *Information Theory and Reliable Communication*. John Wiley & Sons, 1968.
[8] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. John Wiley & Sons, 2006.
[9] A. Lozano, A. Tulino, and S. Verdu, "Optimum Power Allocation for Parallel Gaussian Channels with Arbitrary Input Distributions," *IEEE Trans. Inf. Theor.*, vol. 52, no. 7, pp. 3033–3051, July 2006.
[10] G. Caire, G. Taricco, and E. Biglieri, "Optimum Power Control Over Fading Channels," *IEEE Trans. Inf. Theor.*, vol. 45, no. 5, pp. 1468–1489, 1999.
[11] D. P. Bertsekas, *Constrained Optimization and Lagrange Multiplier Methods*. Athena Scientific, 1996.