Downlink Cooperative Transmission by Superposition Modulation: Performance Analysis and Power Allocation Strategy

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Abstract— **In this paper, we proposed 2 new downlink cooperative transmission strategies based on superposition modulations: Scheme (a), QPSK superimposed modulation and scheme (b), orthogonal 4-PAM superimposed modulation. In both schemes, Rayleigh fading and path loss effect are introduced into channel model. We analyzed the symbol error rate (SER) performances and derived their theoretical expressions. The analytical SER expression of scheme (b) is achieved. Furthermore, we investigated the optimal power allocation problem through both analytical and numerical approach. Finally, the derived analytical expressions were verified by simulation results. Our proposed superposition scheme (b) can outperform the classical downlink cooperative scheme by 2 dB, and the performance advantage of Scheme (a) achieves by 5 dB.**

Index Terms— **cooperative transmission, superposition modulation, power allocation, SER**

I. Introduction

Cooperative transmission [1] has attracted a lot of interest in recent years because it can provide a simple and effective way to achieve diversity gain for portable devices in wireless communication environment. In terms of the signal processing method employed, existing cooperative protocols can roughly be divided into two categories: amplify-and-forward (AF) mode and decode-and-forward (DF) mode [2], [3]. In AF mode, the relay or cooperative node just amplify and retransmit the signal it receives, while in the DF mode, the received signals are decoded first, then re-encoded and retransmitted by the cooperative partners.

Recently, cooperative diversity transmission strategies based on superposition modulation were proposed and investigated in [4], [5], and [6]. Among those works mentioned above, uplink user cooperative transmission model were adopted, they all use soft demodulation method and LLRs (log-likelihood ratio) were calculated to detect cooperative users' information bits. Then frame error rate analysis were presented to prove that performance improvements of cooperation transmission can be achieved owing to the superposition modulation technique. But to the best of our knowledge, downlink user cooperative transmission strategy was not considered yet, and hard decision demodulation was not employed in previous work. Because hard demodulation is much less complicated than soft demodulation, it is more desirable to apply hard demodulation in downlink cooperative users. And theoretical analysis of symbol error rate (SER) performance of cooperation transmission via superposition modulation would reveal the key characteristics of superposition modulated cooperation, such as diversity gain and coding gain.

In this paper, we first propose two kinds of *Superposition Downlink Cooperative transmission* (SDC) schemes: QPSK superimposed modulation (we call it scheme (a)) and Orthogonal 4-PAM superimposed modulation (we call it scheme (b)). We verify them by both theoretical analysis and simulations. The contributions of this paper can be summarized into following two aspects.

- SER performance analysis of downlink cooperative transmission with superposition modulation over combined Rayleigh fading and path loss channels are presented for the first time. Analytical SER expressions are presented for the system, in which the cyclic redundancy check (CRC) codes at relay is utilized to avoid error propagation.
- We presented the method to compute the optimal power allocation ratio for two-user cooperation at Base Station. All the analysis are verified by Monte-Carlo simulation results.

The rest of this paper is organized as follows: Section II describes the cooperative communication system model with superposition coding. SER expressions and optimal performances are derived in Section III. In Section IV, numerical results of both simulation and theoretical results are provided. Finally, section V draws a conclusion on this work .

II. Downlink Cooperative System With Superposition Modulation

We consider a downlink wireless transmission system, as shown in Fig. 1, which comprises 3 nodes: Base station (BS), User 1 (U1) and User 2 (U2). All the terminals are equipped with single antenna and work in half-duplex

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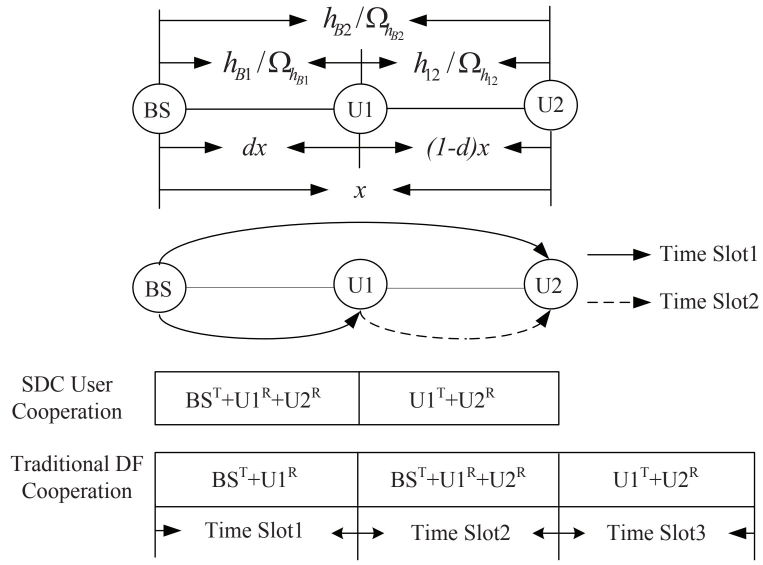

Figure 1. SDC Cooperative communication system model with path loss where $\Omega_{h_{B2}} \propto d^{-\beta}$, $\Omega_{h_{B1}} = d^{-\beta}\Omega_{h_{B2}}$ and $\Omega_{h_{12}} = (1-d)^{-\beta}\Omega_{h_{B2}}$ with $\beta = 4$ for urban environment [10], where $\Omega_{h_{B2}} = 1$.

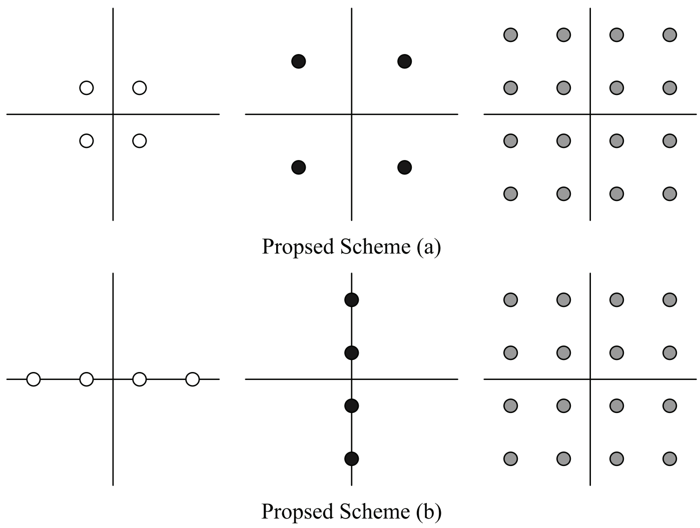

Figure 2. Optimal constellations for proposed SDC scheme (a) and (b). The white , black and gray circles denote the constellation of s_1, s_2 and superimposed constellation respectively. The power of s_1 is $\sqrt{P}\alpha$, and the power of s_2 is $\sqrt{P(1-\alpha^2)}$ for both scheme (a) and (b).

mode. BS intends to broadcast information to U1 and U2. Fig. 1 illustrates the channel model of the SDC system. It is assumed that the channels between each terminal are independent and with Rayleigh flat fading. We also assume both U1 and U2 could know perfect channel state information by channel estimation. The communications scenario considered in this paper could be regarded as a special case of the hybrid three-node network with privacy message in [11], in which achievable rate regions for both Decode-and-Forward and Compress-and-Forward protocols are given.

As Fig. 1 depicted, our SDC schemes and traditional DF cooperation scheme are compared, where the superscripts $^{\mathrm{T}}$ and $^{\mathrm{R}}$ denote transmit and receive, respectively. Whole SDC cooperative transmission occurs over two time slots. We assume that cyclic redundancy check (CRC) codes are used for transmitted signal frame by frame.

We proposed two kinds of SDC superposition modulation, which are described in Fig. 2.

Time Slot 1: In this time slot, BS broadcasts a superimposed signals to U1 and U2. For SDC scheme (a), the signal is QPSK superimposed, and for SDC scheme (b), the signal is orthogonal 4-PAM superimposed. The received signals for the User i can be expressed as

$$y_{Bi}^{1} = \sqrt{P}h_{Bi}\left(\alpha s_1 + \sqrt{1-\alpha^2}s_2\right) + n_i, (i = 1, 2) \quad (1)$$

where s_1 and s_2 are the modulated signals for U1 and U2 respectively, h_{Bi} denotes the channel coefficient between BS and the ith user, which is with zero-mean complex Gaussian distribution, i.e., $h_{B_i} \sim \mathcal{CN}(0, \Omega_{h_{Bi}})$, and α is a weight which can adjust the power allocated to U1 and U2.

Time Slot 2: In this time slot, U1 decodes differently

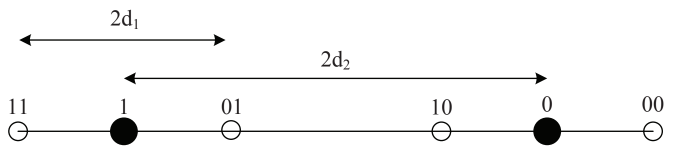

Figure 3. SDC-BPSK constellation where $2d_1$ is the distance between s_1 and $2d_2$ is the distance between s_2 .

in scheme (a) and scheme (b). For scheme (a), U1 first decodes message s_2 with taking its own message s_1 as noise. Then U1 subtracts s_2 from its received signal and decode s_1. This decoding process is also referred to as serial interference cancelation (SIC). If the decoding of s_2 is successful, which is detected by CRC, U1 would retransmits s_2 to U2 with power P. Otherwise, it retransmit nothing. For scheme (b), U1 decodes s_2 and decides whether to retransmit s_2 depending on the CRC results. Then, it decodes U1 s_1 from the received signal. In both cases, if U1 retransmits s_2, the received signal at U2 is given by

$$y_{12}^2 = \sqrt{P}h_{12}s_2 + n_3. \quad (2)$$

Otherwise, it will not take part in relaying. Consequently, U2 will receive nothing in time slot 2.

If U1 can relay s_2 , U2 will combine the signals received in time slot 1 and time slot 2 using maximum ratio combiner (MRC). Then it decodes message s_2 by treating s_1 as noise for scheme (a) and decodes s_2 directly for scheme (b). Otherwise, since U2 only receives signal in time slot 1, it will decode s_2 by treating s_1 as noise for scheme (a) and decode s_2 directly for scheme (b).

In our simulations, each SDC scheme (a) and (b) frame contain 100 bits of message and 24 CRC bits. The generator polynominal of CRC is $x^{24}+x^{23}+x^{14}+x^{12}+x^8+1$. And we use deterministic analysis method for simplicity of expression.

III. SER Analysis and Optimal Performance

A. Scheme (a)

Since the SER expression of proposed SDC scheme (a) with QPSK modulation can be derived from the BER expression of the scheme with BPSK modulation, we first study the performance of SDC-BPSK modulation over AWGN channel. The signal of SDC-BPSK can be expressed as

$$y_{\text{AWGN}}^{\text{BPSK}} = \sqrt{P}\left(\alpha s_1 + \sqrt{1-\alpha^2}s_2\right) + N. \quad (3)$$

We assume the power of noise is N_0, $\gamma = \frac{P}{N_0}$, and without loss of generality, $0 < \alpha < \frac{\sqrt{2}}{2}$ is assumed (when $\frac{\sqrt{2}}{2} < \alpha < 1$, the analysis process is vice versa).

Fig. 3 shows the SDC-BPSK constellation. The black symbols represent only fictitious symbols, the actual transmitted symbols are the white cycles. $2d_1$ is constellation distance between s_1 and $2d_2$ is constellation distance between s_2 . From (3), we get $d_1 = \sqrt{P}\alpha$ and $d_2 = \sqrt{P\left(1-\alpha^2\right)}$.

As derived in [8], the average bit error probability for s_2 is given by (4) on next page. where $\operatorname{erfc}(x) = \frac{2}{\sqrt{x}}\int_x^\infty e^{-t^2}dt$ and $Q(x) = \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)$.

And the average bit error probability for s_1 is given by (5).

Since SDC-QPSK can be thought of as a superposition of two orthogonal SDC-BPSK constellations, its SER expression can be directly derived from BER of SDC-BPSK as

$$SER_{\text{AWGN}}^{\text{QPSK}}(s_2) = 1 - \left[1 - \tfrac{1}{2}Q\left(\sqrt{\gamma}g_2\right) - \tfrac{1}{2}Q\left(\sqrt{\gamma}g_3\right)\right]^2, \quad (6)$$

and

$$\begin{aligned} SER_{\text{AWGN}}^{\text{QPSK}}(s_1) = 1 - [1 - Q(\sqrt{\gamma}g_1) + \tfrac{1}{2}Q(\sqrt{\gamma}g_2)) \\ -\tfrac{1}{2}Q(\sqrt{\gamma}g_3) - \tfrac{1}{2}Q(\sqrt{\gamma}g_4) + \tfrac{1}{2}Q(\sqrt{\gamma}g_5)]^2, \end{aligned} \quad (7)$$

where $g_1 = \alpha$, $g_2 = \alpha + \sqrt{1-\alpha^2}$, $g_3 = \sqrt{1-\alpha^2} - \alpha$, $g_4 = 2\sqrt{1-\alpha^2} + \alpha$ and $g_5 = 2\sqrt{1-\alpha^2} - \alpha$.

Moreover, the received signal for SDC-QPSK scheme over fading channel can be written as

$$y_{\text{fading}}^{\text{QPSK}} = \sqrt{P}h\left(\alpha s_1 + \sqrt{1-\alpha^2}s_2\right) + N. \quad (8)$$

We assume the channel fading coefficient h is with Rayleigh distribution, its variance is σ^2, and the power of noise is 1. Thus the probability density function of SNR, $\hat{\gamma} = \left(\sqrt{P}\,|h|\right)^2$, is given by

$$p_{\hat{\gamma}}(\hat{\gamma}) = \frac{1}{P\sigma^2}\exp\left(-\frac{\hat{\gamma}}{P\sigma^2}\right). \quad (9)$$

Averaging the SER expression of AWGN channel over this probability density, the SER expression of s_1 and s_2 can be written as

$$SER_{\text{fading}}^{\text{QPSK}}(s_1) = \int_0^\infty SER_{\text{AWGN}}^{\text{QPSK}}(s_1)\, p_{\hat{\gamma}}(\hat{\gamma})\, d\hat{\gamma} \quad (10)$$

$$SER_{\text{fading}}^{\text{QPSK}}(s_2) = \int_0^\infty SER_{\text{AWGN}}^{\text{QPSK}}(s_2)\, p_{\hat{\gamma}}(\hat{\gamma})\, d\hat{\gamma} \quad (11)$$

According to derivation in [7, Equation (5), (6) and (7)], the analytical expression of (10) and (11) can be rewritten as equation (12) and (13) on the top of next page, where

$$J_1(x) = \tfrac{1}{2}\left(1 - \sqrt{\tfrac{x^2P\sigma^2}{2+x^2P\sigma^2}}\right), \quad (14)$$

$$J_2(x) = \tfrac{1}{4} - \tfrac{1}{\pi}\sqrt{\tfrac{x^2P\sigma^2}{2+x^2P\sigma^2}}\tan^{-1}\left(\sqrt{\tfrac{2+x^2P\sigma^2}{x^2P\sigma^2}}\right), \quad (15)$$

and

$$\begin{aligned} J_3(x_1,x_2) = \tfrac{1}{4} - \tfrac{1}{2\pi}\Bigg[\sqrt{\tfrac{x_1^2P\sigma^2}{2+x_1^2P\sigma^2}}\tan^{-1}\left(\tfrac{x_1}{x_2}\sqrt{\tfrac{2+x_1^2P\sigma^2}{x_1^2P\sigma^2}}\right) \\ +\sqrt{\tfrac{x_2^2P\sigma^2}{2+x_2^2P\sigma^2}}\tan^{-1}\left(\tfrac{x_2}{x_1}\sqrt{\tfrac{2+x_2^2P\sigma^2}{x_2^2P\sigma^2}}\right)\Bigg], \end{aligned} \quad (16)$$

$$\begin{aligned} P_b(s_2) &= \tfrac{1}{4}\left[P_b(s_2|00_{\text{sent}}) + P_b(s_2|01_{\text{sent}}) + P_b(s_2|10_{\text{sent}}) + P_b(s_2|11_{\text{sent}})\right] \\ &= \tfrac{1}{2}\left[\tfrac{1}{2}\text{erfc}\left(\tfrac{\mathrm{d_1+d_2}}{\sqrt{\mathrm{N_0}}}\right) + \tfrac{1}{2}\text{erfc}\left(\tfrac{\mathrm{d_2-d_1}}{\sqrt{\mathrm{N_0}}}\right)\right] \\ &= \tfrac{1}{2}Q\left(\sqrt{2\gamma}\left(\alpha+\sqrt{1-\alpha^2}\right)\right) + \tfrac{1}{2}Q\left(\sqrt{2\gamma}\left(\sqrt{1-\alpha^2}-\alpha\right)\right), \end{aligned} \tag{4}$$

$$\begin{aligned} &P_b(s_1) = \tfrac{1}{4}\left[P_b(s_1|00_{\text{sent}}) + P_b(s_1|01_{\text{sent}}) + P_b(s_1|10_{\text{sent}}) + P_b(s_1|11_{\text{sent}})\right] \\ &= \tfrac{1}{4}\left[\text{erfc}\left(\tfrac{\mathrm{d_1}}{\sqrt{\mathrm{N_0}}}\right) - \text{erfc}\left(\tfrac{\mathrm{d_1+d_2}}{\sqrt{\mathrm{N_0}}}\right) + \text{erfc}\left(\tfrac{\mathrm{2d_2+d_1}}{\sqrt{\mathrm{N_0}}}\right) + \text{erfc}\left(\tfrac{\mathrm{d_1}}{\sqrt{\mathrm{N_0}}}\right) + \text{erfc}\left(\tfrac{\mathrm{d_2-d_1}}{\sqrt{\mathrm{N_0}}}\right) - \text{erfc}\left(\tfrac{\mathrm{2d_2-d_1}}{\sqrt{\mathrm{N_0}}}\right)\right] \\ &= Q\left(\sqrt{2\gamma}\alpha\right) - \tfrac{1}{2}Q\left(\sqrt{2\gamma}\left(\sqrt{1-\alpha^2}+\alpha\right)\right) + \tfrac{1}{2}Q\left(\sqrt{2\gamma}\left(2\sqrt{1-\alpha^2}+\alpha\right)\right) + \tfrac{1}{2}Q\left(\sqrt{2\gamma}\left(\sqrt{1-\alpha^2}-\alpha\right)\right) \\ &-\tfrac{1}{2}Q\left(\sqrt{2\gamma}\left(2\sqrt{1-\alpha^2}-\alpha\right)\right). \end{aligned} \tag{5}$$

$$\begin{aligned} SER_{\text{fading}}^{\text{QPSK}}(s_1) &= 2J_1(g_1) - J_1(g_2) + J_1(g_3) + J_1(g_4) - J_1(g_5) - J_2(g_1) - \tfrac{1}{4}J_2(g_2) - \tfrac{1}{4}J_2(g_3) - \tfrac{1}{4}J_2(g_4) \\ &-\tfrac{1}{4}J_2(g_5) + J_3(g_1,g_2) - J_3(g_1,g_3) - J_3(g_1,g_4) + J_3(g_1,g_5) + \tfrac{1}{2}J_3(g_2,g_3) + \tfrac{1}{2}J_3(g_2,g_4) - \tfrac{1}{2}J_3(g_2,g_5) \\ &-\tfrac{1}{2}J_3(g_3,g_4) + \tfrac{1}{2}J_3(g_3,g_5) + \tfrac{1}{2}J_3(g_4,g_5), \end{aligned} \tag{12}$$

$$SER_{\text{fading}}^{\text{QPSK}}(s_2) = J_1(g_2) + J_1(g_3) - \tfrac{1}{4}J_2(g_2) - \tfrac{1}{4}J_2(g_3) - \tfrac{1}{2}J_3(g_2,g_3), \tag{13}$$

1) SER Analysis of User 1: For U1, since we assume the path loss factor $\beta = 4$, the variance of channel coefficient between BS and U1 is $\frac{1}{d^4}$. By substituting this variance into (13), we can derive the analytical SER expression of s_2 , which we denote as $SER_{\text{U1}}^{a}(s_2)$. The superscript $^{\text{a}}$ means it is the SER performance of SDC scheme (a).

By substituting $\sigma^2 = \frac{1}{d^4}$ into (12), we can also derive the expression of SER_{U1}^{a} .

2) SER Analysis of User 2: The decoding of U2 comprises 2 cases. In the first case, U1 will not take part in relaying. And in the second case, U1 would relay the signal of U2 for cooperation.

The probability of the occurrence of first case is $1 - (1 - SER_{\text{U1}}^{a}(s_2))^M$, where M is the number of information symbol in one frame. Since we use deterministic analysis for the packet with 100 bits of message and QPSK modulation is considered, M equals 50 here. The SER analysis of this case is similar to U1, by substituting $\sigma^2 = \frac{1}{(1-d)^4}$ into (13), we can get the performance of $SER_{\text{U2}}^{a-\text{case1}}$.

The probability of the occurrence of the second case is $(1 - SER_{\text{U1}}^{a}(s_2))^M$, for the same reason as case 1, M is also 50 here. The signals U2 receives in this case are (1) and (2). They are combined by MRC at U2, then

$$\begin{aligned} \overline{y} &= P\left|h_{B2}\right|^2 \alpha s_1 + P\left(\left|h_{12}\right|^2 + \left|h_{B2}\right|^2\sqrt{1-\alpha^2}\right)s_2 \\ &+\overline{N}, \end{aligned} \tag{17}$$

where the variance of $\overline{N}$ is $P\left(\left|h_{B2}\right|^2 + \left|h_{12}\right|^2\right)$. The performance of case 2 can be expressed as (18) on the top of next page, where $x_1 = \left|h_{B2}\right|^2$, $x_2 = \left|h_{12}\right|^2$, $m_1 = P\alpha x_1$, $m_2 = P\left(x_2 + x_1\sqrt{1-\alpha^2}\right)$, $m_3 = \sqrt{P(x_1+x_2)}$. Although this integral has no analytical expression, to evaluate it through numerical method is still feasible. In Section IV, we will give numerical results based on this expression.

Combing the two cases above, we can derive the SER expression of U2 as

$$\begin{aligned} SER_{\text{U2}}^{a} &= \left[1 - \left(1 - SER_{\text{U1}}^{a}(s_2)\right)^{50}\right] SER_{\text{U2}}^{a-\text{case1}} \\ &+ \left(1 - SER_{\text{U1}}^{a}(s_2)\right)^{50} SER_{\text{U2}}^{a-\text{case2}} \end{aligned} \tag{19}$$

3) Optimal Performance Analysis: With the analytical results of previous two subsections, the final expression for optimization is given by

$$SER^{a} = SER_{\text{U1}}^{a} + SER_{\text{U2}}^{a}. \tag{20}$$

The full expression was not given due to the limit of space. Our goal is to find power allocation factor α to minimize (20). Because the integral in (18) has no analytical expression, we only use numerical brute-force method to find optimal factor.

B. Scheme (b)

For superimposed modulation scheme (b), the signals of U1 and U2 are orthogonal 4-PAM superimposed, there is no interference between their signals, thus we can decode their signals as if no other signal is superimposed.

1) SER Analysis of User 1: For U1, the average receiving signal to noise ratio of s_1 is $\overline{\gamma_{\text{U1}}^{b}}(s_1) = P\alpha^2\left|h_{B1}\right|^2$. The superscript $^{\text{b}}$ denotes it is the SDC scheme (b). Since $\Omega_{h_{B1}} = d^{-4}$, $\overline{\gamma_{\text{U1}}^{b}}(s_1) = P\alpha^2 d^{-4}$.

According to [9],We can get the SER expression of U1 decoding s_1 as

$$\begin{aligned} SER_{\text{U1}}^{b}(s_1) &= \tfrac{3}{4}\left(1 - \sqrt{\tfrac{\overline{\gamma_{\text{U1}}^{b}}(s_1)}{5+\overline{\gamma_{\text{U1}}^{b}}(s_1)}}\right) \\ &= \tfrac{3}{4}\left(1 - \sqrt{\tfrac{P\alpha^2}{5d^4+P\alpha^2}}\right) \end{aligned} \tag{21}$$

The average SNR of s_2 is $\overline{\gamma_{\text{U1}}^{b}}(s_2) = P\left(1-\alpha^2\right)d^{-4}$, and its SER expression is given by

$$SER_{\text{U1}}^{b}(s_2) = \tfrac{3}{4}\left(1 - \sqrt{\tfrac{P(1-\alpha^2)}{5d^4+P(1-\alpha^2)}}\right) \tag{22}$$

$$SER_{\mathrm{U2}}^{\mathrm{case2}} = (1-d)^4 \int_0^\infty \int_0^\infty \left[1 - \left(1 - \frac{1}{2}Q\left(\frac{m_1+m_2}{m_3}\right) - \frac{1}{2}Q\left(\frac{m_2-m_1}{m_3}\right)\right)^2\right] \exp[-x_1 - x_2(1-d)^4] dx_1 dx_2 \tag{18}$$

2) SER Analysis of USER 2: As analysed in scheme (a), There are 2 cases in U2's decoding. The probability of U1 retransmit nothing is $1 - \left(1 - SER_{\mathrm{U1}}^b(s_2)\right)^M$, M equals 50 for each frame contains 100 4-PAM bits. In this case, U2 only receives message in time slot 1. For the same approach as analyzed in U1, Its SER expression is

$$SER_{\mathrm{U2}}^{b-\mathrm{case1}} = \frac{3}{4}\left(1 - \sqrt{\frac{P(1-\alpha^2)}{5+P(1-\alpha^2)}}\right). \tag{23}$$

The other case is U1 takes part in message retransmission. The probability of this case is $\left(1 - SER_{\mathrm{U1}}^b(s_2)\right)^{50}$.

The average receiving SNR of s_2 in U1-U2 link is $P\left|h_{12}\right|^2 = P(1-d)^{-4}$, for $\Omega_{h_{12}} = (1-d)^{-4}$. And the average SNR of s_2 in BS-U2 link is $P\left(1-\alpha^2\right)\left|h_{12}\right|^2 = P\left(1-\alpha^2\right)$, for $\Omega_{h_{B2}} = 1$. We can evaluate its SER expression using MGF approach. For Rayleigh fading, $M_\gamma(s) = (1 - s\gamma)^{-1}$, then, according to equation 9.19 in [9], we can get the SER expression of case 2 as

$$\begin{aligned} SER_{\mathrm{U2}}^{b-\mathrm{case2}} &= \frac{3}{2\pi}\int_0^{\pi/2} \prod_{l=1}^{2} M_{\gamma_l}\left(-\frac{g_{AM}}{\sin^2\phi}\right) d\phi \\ &= \frac{3}{2\pi}\int_0^{\pi/2} \frac{1}{1+\frac{0.2P}{\sin^2\phi(1-d)^4}} \frac{1}{1+\frac{0.2P(1-\alpha^2)}{\sin^2\phi}} d\phi \end{aligned} \tag{24}$$

where $g_{AM} = 0.2$ for 4-PAM modulation.

For $m_1 = m_2 = 1$, according to equation 5A.58 in [9], the close-form expression can be given by

$$SER_{\mathrm{U2}}^{b-\mathrm{case2}} = \frac{3}{4}\left(1 - \frac{C_1}{C_1-C_2}\sqrt{\frac{C_1}{1+C_1}} + \frac{C_2}{C_1-C_2}\sqrt{\frac{C_2}{1+C_2}}\right) \tag{25}$$

where $C_1 = \frac{0.2P}{(1-d)^4}$, $C_2 = 0.2P\left(1-\alpha^2\right)$.

Finally, we can derive SER expression of U2 as

$$\begin{aligned} SER_{\mathrm{U2}}^b &= \left[1 - \left(1 - SER_{\mathrm{U1}}^b(s_2)\right)^{50}\right] SER_{\mathrm{U2}}^{b-\mathrm{case1}} \\ &+ \left(1 - SER_{\mathrm{U1}}^b(s_2)\right)^{50} SER_{\mathrm{U2}}^{b-\mathrm{case2}} \end{aligned} \tag{26}$$

3) Optimal Performance Analysis: With the analytical results of previous 2 subsections, we can derive the final analytical SER expression for optimization as

$$SER^b = SER_{\mathrm{U1}}^b + SER_{\mathrm{U2}}^b \tag{27}$$

The full expression is not given here due to the limit of space. Let $\frac{dSER^b}{d\alpha} = 0$, we can get the optimal α which could result best performance of scheme (b).

In practical systems that may employ some form of channel coding, the analytical framework derived in this paper should be modified to analyze the pairwise error probability of coded bits for frame-based codewords. The possible analytical approach is to follow the methods provided by Simon and Alouini in [9].

IV. Numerical and Simulation Results

In this section, we provide simulation results to verify the proposed SDC schemes. We also perform numerical computation with theoretical SER expressions described in the Section III.

Since the distance d is a slow variable and can be estimated in advance, the derived expressions in our paper provide a quantitative power allocation method for SDC schemes.

In order to compare fairly, the conventional DF cooperation scheme without superimposition coding uses 8-PSK modulation, and each frame has 186 bits totally, consisting of 162 bits of message and 24 bits of CRC respectively. Our proposed scheme (a) and (b) use one frame of 124 bits, where 100 bits is for message and 24 bits is for CRC. Since the frame length of proposed schemes is different from that of conventional scheme, it is not fair to compare those schemes for the same form channel coding with different error-correction capabilities due to different coding rate. Thus we only consider uncoded situation in all the schemes. For convenience, we use “SDCa” to denote our proposed SDC scheme (a), “SDCb” to denote our proposed SDC scheme (b), and “TDF” to denote traditional DF cooperation scheme.

For TDF scheme, as shown in Fig. 1, the BS station transmits s_1 in time slot 1 with power $P\alpha^2$. In time slot 2, BS transmits s_2 with power $P(1-\alpha^2)$, and in time slot 3, if U1 demodulates s_2 successfully, it relays s_2 with power P. Otherwise, it remains silence.

We use numerical brute-force method to find optimal factor of scheme (a) for each SNR and distance d , and the results of $\frac{dSER^b}{d\alpha} = 0$ to find the optimal factor of scheme (b). It turns out the optimal α are unique in all of the 3 schemes. Then we put these optimal α in the simulation to verify our numerical results. These factors are supposed to be known by both users.

Figure. 4 shows the optimal performance for all the setups when U1 locates $0.1x$ away from BS. All of the 3 lines correspond to the average symbol error probability of U1 and U2 by numerical computation using derived expression. It is observed from Fig. 4 that when SER = 10^{-2}, our proposed scheme (b) outperforms TDF scheme by 2 dB , scheme (a) outperforms TDF scheme by 5 dB and scheme (b) by 3 dB . We concluded the performance gain of our proposed scheme (a) and (b) over TDF comes from their higher spectral efficiency, in which the whole transmissions take 2 time slots while the TDF scheme takes 3 time slots. The advantage of proposed scheme (a) over scheme (b) lies in the fact that under the same power constraint, QPSK modulation performs better than 4-PAM modulation. On the other hand, the derived analytical SER

TABLE I.
OPTIMAL POWER ALLOCATION FACTOR (α) FOR FIG . 6

	Distance (d) between BS and U1								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10 dB SDCa	0.14	0.26	0.32	0.32	0.34	0.35	0.37	0.38	0.4
10 dB SDCb	0.18	0.35	0.44	0.47	0.5	0.54	0.6	0.66	0.69
10 dB TDF	0.17	0.34	0.41	0.44	0.5	0.58	0.64	0.68	0.7
15 dB SDCa	0.16	0.3	0.35	0.36	0.37	0.37	0.38	0.39	0.4
15 dB SDCb	0.2	0.4	0.5	0.53	0.54	0.56	0.58	0.62	0.67
15 dB TDF	0.19	0.38	0.46	0.48	0.5	0.54	0.58	0.64	0.68

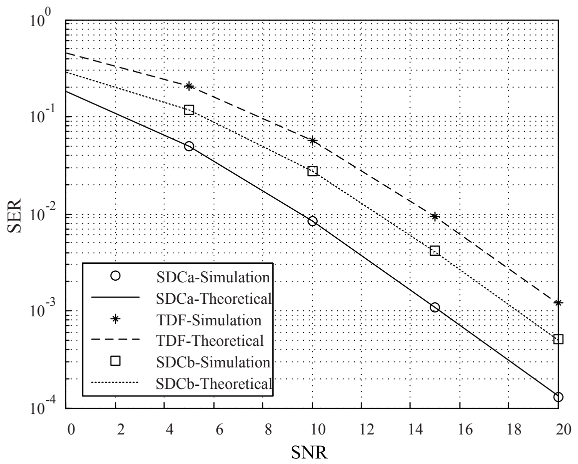

Figure 4. Optimal performance comparison when BS and U1 distance is $0.1x$, α=[0.16 0.13 0.14 0.16 0.2] for SDC scheme (a), α=[0.22 0.18 0.18 0.2 0.25] for SDC scheme (b) and α=[0.24 0.19 0.17 0.19 0.23] for TDF scheme.

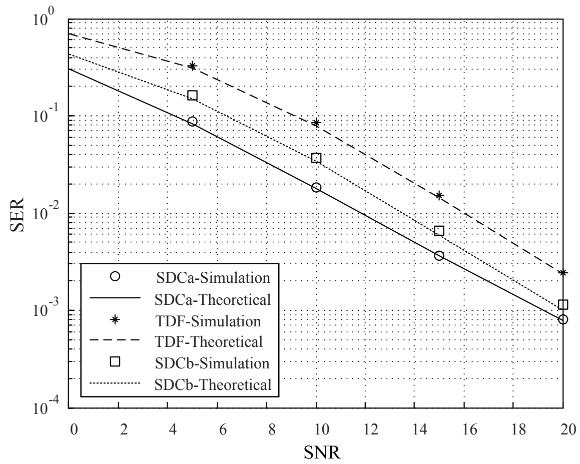

Figure 5. Optimal performance comparison when BS and U1 distance is $0.3x$, α=[0.3 0.29 0.32 0.35 0.4] for SDC scheme (a), α=[0.47 0.42 0.44 0.5 0.59] for SDC scheme (b) and α=[0.52 0.42 0.41 0.46 0.54] for TDF scheme.

expressions match the simulation results very well.

When the distance between BS and U1 increases to $0.3x$, Fig. 5 shows that the performance gap between the proposed scheme (b) and the conventional scheme keeps at about 2 dB. The plot of proposed scheme (a) is different. It is observed from Fig. 5 that it outperforms TDF scheme by 5 dB when SER equals to 10^{-1}, but its advantage diminishes to about 3 dB when SER equals to 10^{-3}. It is concluded that the performance advantage degradation of proposed scheme (a) is because the signal interference between U1 and U2. As the distance between BS and U1 grows larger, more power are allocated to U1, the power difference between U1 and U2 decreased, thus the interference of U1 and U2 becomes innegligible, which degrade its performance gain. While in both TDF scheme and proposed scheme (b), the signals of U1 and U2 are orthogonal modulated (time orthogonal for TDF and constellation orthogonal for scheme (b)), there are no inter-user interferences, 2 diversity order are achieved in both cases.

The optimal α for Fig. 6 is numerated and given by table I. In Fig. 6, the SER performances for SNR equal to 10 dB and 15 dB are simulated and computed based on analytical expression with different distances between BS and U1. It is observed from Fig. 6 that the advantages of the proposed scheme (a) and (b) are achieved in both two cases. The performance gap between scheme (b) keeps the same as BS and U1 distance increases, but for scheme (a),

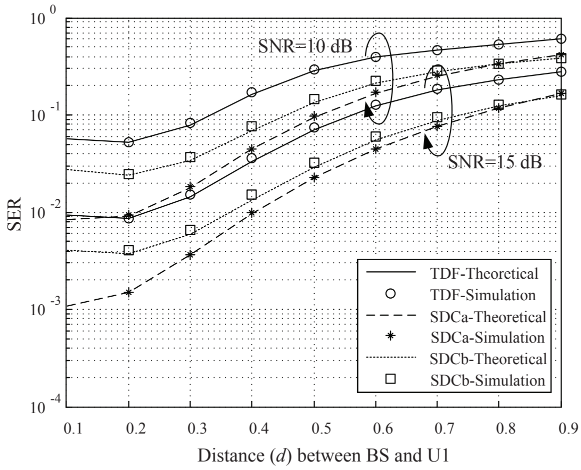

Figure 6. Optimal performance comparison when SNR=10 dB and SNR=15 dB.

the gap decrease as the distance increases. It could also be explained by the fact that as increasing distance between U1 and BS, more power should be allocated to U1. In that case, the interference between U1 and U2 become more severe.

Remark: As we can see from the simulations, no matter what the distance between BS and U1 is, the performance gain of scheme (b) over TDF is always the same. And since the signal of both users are orthogonal superimposed, no inter-user interference occurs, the decoding of both users are the same as TDF scheme, no additional complexities are introduced. Thus scheme (b) can always deployed to get performance gain without any decoding complexity increasement. While in scheme (a), SIC decoding is needed, which increases the complexity of users. Moreover, when both U1 and U2 are very close and far away from BS, its performance gain is negligible. So scheme (a) is only favorable when U1 locates near the BS and U2 locates far away.

V. CONCLUSION

We have proposed a new downlink cooperative transmission based on two type of superposition modulation schemes and derived their SER expressions for two-user cases over combined path-loss and Rayleigh fading channel. All the numerical computations and simulations of SER performances are presented to show the superiority of proposed schemes over conventional cooperative transmission scheme. We also investigated the impact of distance between Base station and users on SER performance. The proposed scheme (b) can outperform the classical downlink cooperative transmission by 2 dB, and scheme (a) outperforms it by almost 5 dB when SER equals to 10^{-2} as U1 is very close to the Base Station. Of course, there are still some open problems, such as the superposition cooperative scheme for multiple-user and channel estimation issue.

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