

Performance Analysis of Wireless Dynamic Cooperative Relay Networks Using Fountain Codes

Weijia Lei¹, Xianzhong Xie² and Guangjun Li¹

¹School of Comm. and Info. Engineering, University of Electronic Science and Technology of China, Chengdu, China

²Institute of Personal Communications, Chongqing University of Posts and Telecommunications, Chongqing, China

Email: leiweijia@gmail.com, xiexzh@cqupt.edu.cn, gjli@uestc.edu.cn

Abstract—The use of fountain codes in wireless cooperative relay networks can improve the system performance in aspects such as transmission time, energy consumption, transmission efficiency and outage probability etc. This has been proved when the number of the relay nodes and their relaying capabilities are stationary. But in practical networks the relay nodes are variable and dynamic. This paper proposes a simple relay scheme adopting fountain codes, and its performance in the case when the number of the relay nodes and their capabilities to relay data are randomly changing is analyzed. The performance of a normal relay scheme in the same network condition is also analyzed for comparison. The average number of transmitted data packets and transmission time required to transfer a certain number of original data packets from the source node to the destination node are derived. We carry out numerical calculation and simulation for the two schemes. The numerical results and simulation results match very well, and they show it clearly that the fountain transmission scheme can cut down the transmission time greatly.

Index Terms—cooperative relay, wireless dynamic relay networks, fountain codes, transmission time

I. INTRODUCTION

The basic idea of cooperative communication lies in the cooperation among the terminals (or nodes) of the wireless network. Cooperation can improve the system performance in aspects such as power consumption, bit error rate, outage probability, cover region, etc [1]-[4]. In the decentralized wireless network such as Ad hoc network, data transmission from the source node to the destination node is achieved via relay nodes located between. It is the cooperation that makes the communication between any two nodes possible. In such centralized network as cellular network, the technique of relay transmission can also be used to extend the coverage of the center station.

Fountain codes [5]-[7], also called rateless code, have the capabilities of generating limitless encoding symbols, and the source information can be recovered from any subset of the encoding symbols as long as the information conveyed by them is enough. The digital fountain code has the properties similar to those of a fountain of water: anyone wanting water from the fountain only needs to fill his own bucket without caring what drops of water have

fallen in and how others fill theirs. With the digital fountain code, the transmitter is just like a fountain, which encodes the original information to get potentially infinite encoded symbols and sends them out; for the receivers, only if they have received enough encoded symbols, they can reconstruct the original information. An idealized digital fountain code should have two properties [6]: (1) infinite encoded data can be generated from finite source data. (2) if the length of the source data is M , the receiver can reconstruct the source data from any M encoded data, and the decoding process should be fast enough. It is difficult for any digital fountain code in use to satisfy both requirements, with either a limited ability to generate infinite encoded data or a requirement of data greater than M to decode the encoded data. A practical digital fountain code can work well if it can generate enough encoded data and the required data are not too larger.

The use of fountain codes in the wireless relay system has attracted a lot of attention in recent years. Molisch et al. studied the use of fountain codes in the cooperative relay network [8][9], proposed quasi-synchronous and asynchronous transmission schemes, and analyzed their performances based on the mutual information. The authors proposed a cooperative relay scheme based on the information accumulation by employing fountain codes [10], analyzed its performance based on the block error rate, and scaled the performance by the transmission time and energy consumption. Liu introduced several single relay node schemes based on acknowledgment signals and derived their information theoretic achievable rates [11]. Castura and Mao introduced a relaying protocol in which a relay successful in decoding the source's message collaborates with the source by forming a distributed space-time coding scheme [12]. The schemes of Liu and Castura studied the scenario in which there is only one relay node and a direct link exists between the source node the destination node. Nikjah and Beaulieu proposed two other protocols built upon previous works of Liu and Castura, which select the relay nodes with the best channels to the destination node to relay the information [13], and their schemes have better energy efficiency. The results of these papers show that rateless codes can help to improve the performance of the wireless relay system. But in all of them the statuses of relay nodes are supposed to remain constant, including the number of relay nodes and their relay capabilities. In

a practical relay network, because of the mobility of relay nodes, their number can not remain the same. Moreover, a relay node also has its own data to transmit while relaying data for other nodes, and the number of the source nodes which require it to relay information is also variable. So a relay node's capability to relay information for a source node may change at any time. Its capability is also constrained by the life of its battery. So the statuses of the relay nodes are dynamic.

In this paper, a transmission scheme is proposed in a dynamic relay network employing fountain codes (called fountain transmission in this paper), and its performance is analyzed by using the theory of probability and stochastic process. To compare, we also analyze the performance of a normal relay network (called normal transmission in this paper) in the same condition. The average number of the transmitted data packets and transmission time required to transfer a certain number of original data packets from the source node to the destination node are derived. We also simulate the two networks on erasure channels. The results of the analysis and simulation show that the employment of fountain codes can help to improve the capability of the network to adapt to the dynamic change of relay nodes, and shorten the transmission time greatly.

The rest of the paper is organized as follows. We describe the system model of the dynamic relay network and the transmission scheme employing fountain codes in section II, and a normal transmission scheme is also introduced; we analyze the performances of the two transmission schemes with fixed number of relay nodes in section III, and their performances with variable number of relay nodes in section IV; in section V, we give the numerical results and the simulation results of the two relay schemes on erasure channels; section VI is the conclusion of the paper.

II. THE SYSTEM MODEL AND TRANSMISSION SCHEMES

The system model of a cooperative relay system is shown in Fig. 1. We assume that there is no direct link between the source node and the destination node. Data transferred from the source node to the destination node are relayed by L relay nodes. The data are transmitted in two phases: firstly, the source node broadcasts the data, and the relay nodes receive; then, the relay nodes transmit the data to the destination node. In a dynamic relay network, the number of the relay nodes and their relay capabilities are variable. We assume that data

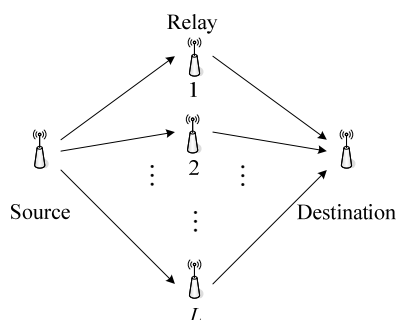


Fig. 1. The system model of a cooperative relay system

transmission and the change of the relay nodes abide by the following rules:

1. Data are transmitted in packets.
2. A relay node might or might not relay data at any time. If it relays data, it must relay a full packet. The relay capabilities of the relay nodes may vary after each one data packet is transmitted.
3. The number of the relay nodes is fixed while M original information packets are being transferred from the source node to the destination node. It may change after M original information packets have been transferred. Here, M is the number of the original data packets to be grouped into a block for fountain codes to encode.

A. Fountain Transmission Scheme

In the first phase, the source node encodes the original information using fountain codes, and broadcasts the encoded data to the relay nodes. The relay nodes accumulate the encoded data until they have got enough data to recover the original information. The source node terminates the transmission when all the relay nodes have finished receiving. In the second phase, the relay nodes re-encode the information using fountain codes. Then they transmit the encoded data to the destination node when they can relay data. The number of the data packets transmitted by any relay node is not fixed. Due to the property of fountain codes that the original information can be recovered from any unordered subset of the encoded symbols, the encoded data packets can be distributed to all relay nodes to transmit, and synchronization among the relay nodes is not required. The destination node accumulates the data received from the relay nodes, and recovers the original data.

B. Normal Transmission Scheme

To compare the performance of fountain transmission, a normal relay network which does not adopt fountain code is analyzed too. To avoid synchronization among the relay nodes, we use the following transmission scheme. In the first phase, if at least one relay node can relay data, the source node sends data packets. Only the relay nodes which can relay data receive data. In the second phase, only the relay node which first completes the data receiving process in the first phase relays data packets. So the source node sends the same packet continuously until one relay node has correctly received it. If no relay node can relay data in the first phase, the source node suspends the transmission until there is at least one relay node that can relay data coming out. The suspension can help to decrease the energy consumption. In the second phase, the relay node which relays data sends the packet repeatedly until it has been correctly received by the destination node.

In the next two sections we will analyze the performances of the fountain relay network and the normal relay network. First, we analyze the performances when the relay capabilities of the relay nodes are variable and the number of the relay nodes is fixed. Then we analyze the performances when both of them are variable.

III. PERFORMANCE ANALYSIS – WHEN THE NUMBER OF RELAY NODES IS FIXED

In this section we fix the number of the relay nodes to L . In the analysis of the whole paper, we assume all channels in the networks are erasure channel, and all channels share the same erasure probability P_{eB} ; the number of the original data packets to be grouped into a block and encoded by fountain codes is M , and the receivers need any M encoded data packets to recover the original data; the probability for any relay node to relay data at any moment is P_r ; the time that the source node and the relay nodes need to transmit a data packet is normalized to 1. The meanings of the subscripts in the formulas of this paper are: n – normal transmission, f – fountain transmission, (M) – the transference of M original data packets, s – source node, r – relay node, d – destination node. We use the number of the transmitted data packets and the transmission time to scale the performances of the transmission schemes.

A. Fountain Transmission

1) From the Source Node to the Relay Nodes

In the first phase of transmission, the source node groups M original data packets into a block, and encodes them using fountain codes. The source node sends the encoded data continuously until all L relay nodes have received M packets correctly. Assume the number of the data packets sent by the source node to make sure that all relay nodes can recover the original data is $N_{fs(M)}$. The condition for $N_{fs(M)} = k$ is: at least one relay node does not succeed in receiving M data packets until the source node has transmitted k data packets, while other relay nodes do after the source node has transmitted M to $k-1$ encoded data packets. The probability for $N_{fs(M)} = k$ is

$$\begin{cases}
 P(N_{fs(M)}=M)=(1-P_{eB})^{M \times L} \\
 P(N_{fs(M)}=k)=\binom{L}{1}P_{(M)k} \left(\sum_{i=M}^{k-1} P_{(M)i} \right)^{L-1} + \binom{L}{2} \left(P_{(M)k} \right)^2 \left(\sum_{i=M}^{k-1} P_{(M)i} \right)^{L-2} \\
 \dots + \binom{L}{L-1} \left(P_{(M)k} \right)^{L-1} \sum_{i=M}^{k-1} P_{(M)i} + \left(P_{(M)k} \right)^L \\
 = \sum_{j=1}^L \left[\binom{L}{j} \left(P_{(M)k} \right)^j \left(\sum_{i=M}^{k-1} P_{(M)i} \right)^{L-j} \right] \\
 = \left(\sum_{i=M}^k P_{(M)i} \right)^L - \left(\sum_{i=M}^{k-1} P_{(M)i} \right)^L, \quad k \geq M+1
 \end{cases} \tag{1}$$

where $\binom{n}{m} = \frac{n!}{m!(n-m)!}$ is the binomial coefficient. $P_{(M)i}$

in (1) is the probability that a relay node correctly receives M packets among the i ($i \geq M$) packets sent by the source node, and the M -th correctly received packet is the i -th packet sent by the source node:

$$P_{(M)i} = \binom{i-1}{M-1} P_{eB}^{i-M} (1-P_{eB})^M, \quad i \geq M. \tag{2}$$

To enable all relay nodes to recover M original data packets, the average number of the encoded data packets sent by the source node $\bar{N}_{fs(M)}$ is

$$\begin{aligned}
 \bar{N}_{fs(M)} &= \sum_{k=M}^{\infty} k \cdot P(N_{fs(M)}=k) \\
 &= M \cdot (1-P_{eB})^{M \times L} + \sum_{k=M+1}^{\infty} k \cdot \left[\left(\sum_{i=M}^k P_{(M)i} \right)^L - \left(\sum_{i=M}^{k-1} P_{(M)i} \right)^L \right]. \tag{3}
 \end{aligned}$$

The average time that the source node needs to transmit the data packets is

$$\bar{T}_{fs(M)} = \bar{N}_{fs(M)} \times 1 = \bar{N}_{fs(M)}. \tag{4}$$

2) From the Relay Nodes to the Destination Node

Because data packets might be erased in the transmission process, so the number of the data packets sent by the relay nodes may be greater than that the destination node needs. The condition for the relay nodes to send at least $K = k$ data packets is: the destination correctly receives M data packets among the k data packets, and the M -th correctly received packet is the k -th packet sent by the relay nodes. Its probability is

$$P(K=k) = \binom{k-1}{M-1} P_{eB}^{k-M} (1-P_{eB})^M, \quad k \geq M. \tag{5}$$

The K data packets are relayed by all L relay nodes, but at some time, there may be some relay nodes that can not relay data. Denote the number of the relay nodes that can relay data as L_{ri} at time i , and L_{ri} is a random variable with binomial distribution:

$$P(L_{ri}=l) = \binom{L}{l} P_r^l (1-P_r)^{L-l}, \quad l=0,1,\dots,L \tag{6}$$

where P_r is the probability that one relay node can relay data.

In the duration of a unit time of a data packet to be transmitted (it is normalized to 1), the L_{ri} relay nodes will transmit L_{ri} data packets. Assume the required time for relaying the K data packets is T_{frK} , which is a random variable. The conditions for $T_{frK} = t$ are: the number of the data packets transmitted by the relay nodes from time 1 to $t-1$ is $\sum_{i=1}^{t-1} N_{fri}$, $K-L \leq \sum_{i=1}^{t-1} N_{fri} \leq \min(K-1, L \times (t-1))$, and it is $N_{frt} \geq K - \sum_{i=1}^{t-1} N_{fri}$ at time t . Here, N_{fri} is the number of the data packets transmitted by all relay nodes at time i , which equals to the number of the relay nodes which can relay data at that time, i.e. $N_{fri} = L_{ri}$. So the probability for $T_{frK} = t$ is

$$\begin{aligned}
 P(T_{frK}=t) &= P\left(K-L \leq \sum_{i=1}^{t-1} N_{fri} \leq \min(K-1, L \times (t-1)), N_{frt} \geq K - \sum_{i=1}^{t-1} N_{fri} \right) \\
 &= P\left(K-L \leq \sum_{i=1}^{t-1} L_{ri} \leq \min(K-1, L \times (t-1)), L_{rt} \geq K - \sum_{i=1}^{t-1} L_{ri} \right), \quad t \geq \left\lceil \frac{K}{L} \right\rceil \tag{7}
 \end{aligned}$$

Here, the $\lceil x \rceil$ rounds the elements of x to the nearest integer greater than or equal to x . For the convenience of writing, we denote $\sum_{i=1}^u L_{ri}$ as L_{rau} , which is the sum of u random variables. We develop its probability distribution from the transformation of its characteristic function. We have known that L_{ri} ($i = 1, 2, 3, \dots$) are a serial of binomial distribution random variables independent of

each other. All random variables have the same probability mass functions (6). The characteristic function of L_{ri} is

$$\Phi_{L_{ri}}(\omega) = \Phi_{L_r}(\omega) = [P_r e^{j\omega} + (1-P_r)]^L. \quad (8)$$

Since the characteristic function of the sum of u statistically independent random variables is equal to the product of characteristic functions of the individual random variables [14], we get the characteristic function of L_{rau} :

$$\Phi_{L_{rau}}(\omega) = \prod_{i=1}^u \Phi_{L_{ri}}(\omega) = [\Phi_{L_r}(\omega)]^u = [P_r e^{j\omega} + (1-P_r)]^{L \times u}. \quad (9)$$

From the unique relationship between the distribution function and the characteristic function, we know that L_{rau} is also a binomial distribution random variable, and its probability mass function is

$$P(L_{rau} = l_a) = \binom{L \times u}{l_a} P_r^{l_a} (1-P_r)^{L \times u - l_a}, \quad l_a = 0, 1, \dots, L \times u. \quad (10)$$

So we get

$$\begin{aligned} P(T_{frk} = t) &= P(K-L \leq L_{ra(t-1)} \leq \min(K-1, L \times (t-1)), L_{rt} \geq K - L_{ra(t-1)}) \\ &= \sum_{l_a=K-L}^{\min(K-1, L \times (t-1))} P(L_{ra(t-1)} = l_a, L_{rt} \geq K - l_a) \\ &= \sum_{l_a=K-L}^{\min(K-1, L \times (t-1))} \left[P(L_{ra(t-1)} = l_a) \cdot P(L_{rt} \geq K - l_a) \right] \quad (11) \\ &= \sum_{l_a=K-L}^{\min(K-1, L \times (t-1))} \left[P(L_{ra(t-1)} = l_a) \cdot \sum_{l=K-l_a}^L P(L_{rt} = l) \right] \\ &= \sum_{l_a=K-L}^{\min(K-1, L \times (t-1))} \left[\binom{L \times (t-1)}{l_a} P_r^{l_a} (1-P_r)^{L \times (t-1) - l_a} \right. \\ &\quad \left. \cdot \sum_{l=K-l_a}^L \binom{L}{l} P_r^l (1-P_r)^{L-l} \right], \quad t \geq \left\lceil \frac{K}{L} \right\rceil \end{aligned}$$

The mean of the time to relay K data packets T_{frk} is

$$\begin{aligned} \bar{T}_{frk} &= \sum_{t=\lceil K/L \rceil}^{\infty} t \cdot P(T_{frk} = t) \\ &= \sum_{t=\lceil K/L \rceil}^{\infty} \left\{ t \cdot \sum_{l_a=K-L}^{\min(K-1, L \times (t-1))} \left[\binom{L \times (t-1)}{l_a} P_r^{l_a} (1-P_r)^{L \times (t-1) - l_a} \right. \right. \\ &\quad \left. \left. \cdot \sum_{l=K-l_a}^L \binom{L}{l} P_r^l (1-P_r)^{L-l} \right] \right\} \quad (12) \end{aligned}$$

The number of the data packets sent by the relay nodes K ($\geq M$) is a random variable. So is \bar{T}_{frk} . The average transmission time $\bar{T}_{fr(M)}$ of the relay nodes is the expectation of \bar{T}_{frk} :

$$\begin{aligned} \bar{T}_{fr(M)} &= E[\bar{T}_{frk}] = \sum_{k=M}^{\infty} P(K=k) \bar{T}_{frk} \\ &= \sum_{k=M}^{\infty} \left\{ \binom{k-1}{M-1} P_{eB}^{k-M} (1-P_{eB})^M \cdot \sum_{t=\lceil k/L \rceil}^{\infty} \left\{ t \cdot \sum_{l_a=K-L}^{\min(k-1, L \times (t-1))} \right. \right. \\ &\quad \left. \left[\binom{L \times (t-1)}{l_a} P_r^{l_a} (1-P_r)^{L \times (t-1) - l_a} \sum_{l=k-l_a}^L \binom{L}{l} P_r^l (1-P_r)^{L-l} \right] \right\} \quad (13) \end{aligned}$$

During the time from 1 to $\bar{T}_{fr(M)}$, the number of the packets relayed by the relay nodes is $N_{fr(M)} = \sum_{i=1}^{\bar{T}_{fr(M)}} L_{ri} = L_{ra \bar{T}_{fr(M)}}$. We have known it is a binomial distribution random variable:

$$\begin{aligned} P(N_{fr(M)} = l_a) &= P(L_{ra \bar{T}_{fr(M)}} = l_a) \\ &= \binom{L \times \bar{T}_{fr(M)}}{l_a} P_r^{l_a} (1-P_r)^{L \times \bar{T}_{fr(M)} - l_a}, \quad l_a = 0, 1, \dots, L \times \bar{T}_{fr(M)}. \quad (14) \end{aligned}$$

Its mean is

$$\bar{N}_{fr(M)} = \sum_{l_a=0}^{L \times \bar{T}_{fr(M)}} l_a \cdot P(N_{fr(M)} = l_a) = L \times \bar{T}_{fr(M)} \times P_r. \quad (15)$$

It should be noted that the number of the data packets sent by the relay nodes may be greater than that the destination node needs. This is brought about by the last data transmission process of these data packets from the relay nodes to the destination node. In the last transmission process, if the number of the remaining data packets is smaller than that of the relay nodes which can relay data, there will be more data packets than the destination needs to be transmitted.¹

Thus, the overall number of the transmitted data packets and the transmission time needed for the destination node to correctly obtain the M original data packets are:

$$\begin{aligned} \bar{N}_{f(M)} &= \bar{N}_{fs(M)} + \bar{N}_{fr(M)} \\ \bar{T}_{f(M)} &= \bar{T}_{fs(M)} + \bar{T}_{fr(M)} \quad (16) \end{aligned}$$

B. Normal Transmission

1) From the Source Node to the Relay Nodes

In the normal relay system, the data are dealt with and transmitted in packets. According to the scheme introduced afore, the source node sends a data packet continuously until at least one relay node has correctly received the packet. If there is no relay node that can relay data, the source node suspends the transmission. The number of the relay nodes which can relay data L_r is a binomial distribution random variable:

$$P(L_r = l) = \binom{L}{l} P_r^l (1-P_r)^{L-l}, \quad l = 0, 1, \dots, L. \quad (17)$$

Assume the times that the source node needs to send a packet to make at least one relay node which can relay data correctly receive it is N_{ns} . The condition for $N_{ns} = k$ is: at least one relay node correctly receives the packet after the source node has sent it k -th times, while the other relay nodes do not receive it correctly. Its probability is

$$\begin{aligned} P(N_{ns} = k | L_r = l) &= \binom{l}{1} P_k P_{ek}^{l-1} + \binom{l}{2} P_k^2 P_{ek}^{l-2} + \dots + \binom{l}{l-1} P_k^{l-1} P_{ek} + P_k^l \end{aligned}$$

¹ Actually, K in (5) is the minimum of the number of data packets sent by the relay node. Its mean is

$$\bar{K} = E[K] = \sum_{k=M}^{\infty} k \cdot P(K=k) = \sum_{k=M}^{\infty} \left[k \cdot \binom{k-1}{M-1} P_{eB}^{k-M} (1-P_{eB})^M \right] = \frac{M}{1-P_{eB}}$$

\bar{K} is not affected by L or P_r , and is not greater than $\bar{N}_{fr(M)}$. The more the relay nodes are and the higher is the probability P_r , the more redundant data packets are sent by the relay nodes.

$$= \sum_{j=1}^l \binom{l}{j} P_k^j P_{ek}^{l-j} = (P_k + P_{ek})^l - P_{ek}^l, \quad k \geq 1. \quad (18)$$

Here, P_k is the probability that one relay node correctly receives the packet when the source node sends it k -th times, and P_{ek} is the probability that one relay node still has not received the packet correctly after the source node has sent it k times. They are

$$\begin{aligned} P_k &= P_{eB}^{k-1} (1 - P_{eB}) \\ P_{ek} &= P_{eB}^k \end{aligned} \quad (19)$$

Substitute (19) into (18), we can simplify (18) to

$$\begin{aligned} P(N_{ns} = k | L_r = l) &= [P_{eB}^{k-1} (1 - P_{eB}) + P_{eB}^k]^l - P_{eB}^{k \times l} \\ &= P_{eB}^{(k-1) \times l} - P_{eB}^{k \times l} = P_{eB}^{(k-1) \times l} (1 - P_{eB}^l), \quad k \geq 1 \end{aligned} \quad (20)$$

According to the total probability theorem we get

$$P(N_{ns} = k) = \sum_{l=0}^L [P(N_{ns} = k | L_r = l) \cdot P(L_r = l)], \quad k \geq 1. \quad (21)$$

We notice that the source node suspends transmission if $L_r = 0$, so $L_r = 0$ should be excluded from (21), and (17) needs to be mended. There are

$$\tilde{P}(L_r = l) = \frac{\binom{L}{l} P_r^l (1 - P_r)^{L-l}}{1 - P(L_r = 0)} = \frac{\binom{L}{l} P_r^l (1 - P_r)^{L-l}}{1 - (1 - P_r)^L}, \quad l = 1, \dots, L, \quad (22)$$

and

$$\begin{aligned} P(N_{ns} = k) &= \sum_{l=1}^L P(N_{ns} = k | L_r = l) \cdot \tilde{P}(L_r = l) \\ &= \frac{1}{1 - (1 - P_r)^L} \cdot \sum_{l=1}^L P_{eB}^{(k-1) \times l} (1 - P_{eB}^l) \binom{L}{l} P_r^l (1 - P_r)^{L-l}, \quad k \geq 1 \end{aligned} \quad (23)$$

The average number of the data packets sent by the source node to ensure at least one relay node correctly receives a data packet is the expectation of N_{ns} :

$$\begin{aligned} \bar{N}_{ns} &= \sum_{k=1}^{\infty} k \cdot P(N_{ns} = k) \\ &= \frac{1}{1 - (1 - P_r)^L} \cdot \sum_{k=1}^{\infty} k \cdot \sum_{l=1}^L [P_{eB}^{(k-1) \times l} (1 - P_{eB}^l) \binom{L}{l} P_r^l (1 - P_r)^{L-l}] \\ &= \frac{1}{1 - (1 - P_r)^L} \cdot \sum_{l=1}^L (1 - P_{eB}^l) \binom{L}{l} P_r^l (1 - P_r)^{L-l} \cdot \sum_{k=1}^{\infty} [k \cdot P_{eB}^{(k-1) \times l}] \\ &= \frac{1}{1 - (1 - P_r)^L} \cdot \sum_{l=1}^L \frac{\binom{L}{l} P_r^l (1 - P_r)^{L-l}}{1 - P_{eB}^l} \end{aligned} \quad (24)$$

Now we calculate the time required for the first phase. Since the source node suspends the transmission if $L_r = 0$, the total time of first phase is the sum of the time to transmit the packets and the suspension time. The former simply equals to the average number of the transmitted packets. Now we analyze the suspension time. Denote the time of $L_r = 0$ as L_{r0} , and $L_{r0} = k$ means that the stretch of time during which there is no relay node can relay data is k . So the suspension time equals to the value of L_{r0} . The condition for $L_{r0} = k$ is: $L_r = 0$ occurs consecutively k times before $L_r \neq 0$. The probability of $L_{r0} = k$ is

$$\begin{aligned} P(L_{r0} = k) &= [(1 - P_r)^L]^k \cdot [1 - (1 - P_r)^L] \\ &= (1 - P_r)^{k \times L} - (1 - P_r)^{(k+1) \times L}, \quad k \geq 0 \end{aligned} \quad (25)$$

The mean of L_{r0} is

$$\begin{aligned} \bar{L}_{r0} &= \sum_{k=0}^{\infty} k \cdot P(L_{r0} = k) = \sum_{k=0}^{\infty} k \cdot [(1 - P_r)^L]^k \cdot [1 - (1 - P_r)^L] \\ &= \frac{(1 - P_r)^L}{1 - (1 - P_r)^L} \end{aligned} \quad (26)$$

The average time for accomplishing the first phase is

$$\bar{T}_{ns} = (\bar{N}_{ns} + \bar{L}_{r0}) \times 1 = \bar{N}_{ns} + \bar{L}_{r0}. \quad (27)$$

2) From the Relay Nodes to the Destination Node

There is only one relay node that transmits data to the destination node. Assume the times that the relay node transmits a data packet to make the destination node correctly receive it is N_{nr} . The probability that $N_{nr} = k$ is

$$P(N_{nr} = k) = P_{eB}^{k-1} \cdot (1 - P_{eB}), \quad k \geq 1. \quad (28)$$

The mean of N_{nr} is

$$\bar{N}_{nr} = \sum_{k=1}^{\infty} k \cdot P(N_{nr} = k) = \sum_{k=1}^{\infty} k \cdot P_{eB}^{k-1} (1 - P_{eB}) = \frac{1}{1 - P_{eB}}. \quad (29)$$

The average time for the relay node to transmit the data packets is

$$\bar{T}_{nr} = \bar{N}_{nr} \times 1 = \bar{N}_{nr}. \quad (30)$$

Therefore, the overall number of the transmitted data packets and the transmission time needed for the destination to correctly obtain one original data packet are:

$$\begin{aligned} \bar{N}_n &= \bar{N}_{ns} + \bar{N}_{nr} \\ \bar{T}_n &= \bar{T}_{ns} + \bar{T}_{nr} \end{aligned} \quad (31)$$

IV. PERFORMANCE ANALYSIS – WHEN THE NUMBER OF RELAY NODES IS VARIABLE

The composition of the relay nodes may change after an information block has been transferred from the source node to the destination node. Assume that the number of the relay nodes L is limited in the range of $[L_{\min}, L_{\max}]$, the change process of the number of the relay nodes is a Markov chain, and its state space is $E = \{L_{\min}, L_{\min} + 1, L_{\min} + 2, \dots, L_{\max}\}$. Its transition probability matrix is

$$\mathbf{P} = \begin{bmatrix} P_{L_{\min}, L_{\min}} & P_{L_{\min}, L_{\min}+1} & P_{L_{\min}, L_{\min}+2} & \cdots & P_{L_{\min}, L_{\max}} \\ P_{L_{\min}+1, L_{\min}} & P_{L_{\min}+1, L_{\min}+1} & P_{L_{\min}+1, L_{\min}+2} & \cdots & P_{L_{\min}+1, L_{\max}} \\ P_{L_{\min}+2, L_{\min}} & P_{L_{\min}+2, L_{\min}+1} & P_{L_{\min}+2, L_{\min}+2} & \cdots & P_{L_{\min}+2, L_{\max}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{L_{\max}, L_{\min}} & P_{L_{\max}, L_{\min}+1} & P_{L_{\max}, L_{\min}+2} & \cdots & P_{L_{\max}, L_{\max}} \end{bmatrix}. \quad (32)$$

Here p_{ij} is the probability that the number of the relay nodes changes from i to j , and it satisfies

$$p_{ij} \geq 0, \quad \sum_{j=L_{\min}}^{L_{\max}} p_{ij} = 1, \quad i, j = L_{\min}, L_{\min} + 1, \dots, L_{\max}. \quad (33)$$

It is easy to know that this Markov chain is ergodic, so we can get its stationary distribution:

$$p_q = P(L = q), \quad q = L_{\min}, L_{\min} + 1, \dots, L_{\max}. \quad (34)$$

Our ultimate purpose is to get the mean values of the number of the transmitted data packets and the required time, so it is not necessary to derive their distribution functions. We can derive the values from the results of

section III. In the formulas of this section, the subscript of “v” means that the number of the relay nodes is variable.

A. Fountain Transmission

1) From the Source Node to the Relay Nodes

We have got the average number of the data packets sent by the source node in (3) when the number of the relay nodes is fixed to L . L is a random variable when the number of the relay nodes is variable. Thus, $\bar{N}_{fs(M)}$ is the function of L . Using the fundamental theorem of expectation [14], we can get the average number of the data packets sent by the source node when the number of the relay nodes is variable:

$$\begin{aligned} \bar{N}_{fs(M)} &= \sum_{q=L_{\min}}^{L_{\max}} [P(L=q) \cdot \bar{N}_{fs(M)}] \\ &= \sum_{q=L_{\min}}^{L_{\max}} p_q \cdot \left\{ M(1-P_{eB})^{M \times q} + \sum_{k=M+1}^{\infty} k \cdot \left[\left(\sum_{i=M}^k P_{(M)i} \right)^q - \left(\sum_{i=M}^{k-1} P_{(M)i} \right)^q \right] \right\} \end{aligned} \quad (35)$$

The average time to transmit the data packets is

$$\bar{T}_{fs(M)} = \bar{N}_{fs(M)} \times 1 = \bar{N}_{fs(M)}. \quad (36)$$

2) From the Relay Nodes to the Destination Node

Similarly, when the number of the relay nodes is variable, the average number of the data packets transmitted by the relay nodes and the average transmission time can be calculated on the basis of (15) and (13):

$$\begin{aligned} \bar{N}_{fr(M)} &= \sum_{q=L_{\min}}^{L_{\max}} P(L=q) \cdot \bar{N}_{fr(M)} = \sum_{q=L_{\min}}^{L_{\max}} (p_q \times q \times \bar{T}_{fr(M)} \times P_r) \\ &= \sum_{q=L_{\min}}^{L_{\max}} \left\{ p_q \cdot q \cdot P_r \cdot \sum_{k=M}^{\infty} \binom{k-1}{M-1} P_{eB}^{k-M} (1-P_{eB})^M \cdot \sum_{t=\lceil k/q \rceil}^{\infty} \left\{ t \cdot \sum_{l_a=k-q}^{\min(k-1, q(t-1))} \right. \right. \\ &\quad \left. \left. \left[\binom{q(t-1)}{l_a} P_r^{l_a} (1-P_r)^{q(t-1)-l_a} \cdot \sum_{l=k-l_a}^q \binom{q}{l} P_r^l (1-P_r)^{q-l} \right] \right\} \right\} \end{aligned} \quad (37)$$

$$\begin{aligned} \bar{T}_{fr(M)} &= \sum_{q=L_{\min}}^{L_{\max}} P(L=q) \cdot \bar{T}_{fr(M)} \\ &= \sum_{q=L_{\min}}^{L_{\max}} \left\{ p_q \cdot \sum_{k=M}^{\infty} \binom{k-1}{M-1} P_{eB}^{k-M} (1-P_{eB})^M \cdot \sum_{t=\lceil k/q \rceil}^{\infty} \left\{ t \cdot \sum_{l_a=k-q}^{\min(k-1, q(t-1))} \right. \right. \\ &\quad \left. \left. \left[\binom{q(t-1)}{l_a} P_r^{l_a} (1-P_r)^{q(t-1)-l_a} \cdot \sum_{l=k-l_a}^q \binom{q}{l} P_r^l (1-P_r)^{q-l} \right] \right\} \right\} \end{aligned} \quad (38)$$

B. Normal Transmission

1) From the Source Node to the Relay Nodes

Similarly, we obtain the average number of the data packets transmitted and the suspension time in the first phase based on (24) and (26):

$$\begin{aligned} \bar{N}_{nsv} &= \sum_{q=L_{\min}}^{L_{\max}} P(L=q) \cdot \bar{N}_{ns} \\ &= \sum_{q=L_{\min}}^{L_{\max}} \left\{ p_q \cdot \frac{1}{1-(1-P_r)^q} \cdot \sum_{l=1}^q \frac{\binom{q}{l} P_r^l (1-P_r)^{q-l}}{1-P_{eB}^l} \right\}, \end{aligned} \quad (39)$$

$$\bar{L}_{r0v} = \sum_{q=L_{\min}}^{L_{\max}} [P(L=q) \cdot \bar{L}_{r0}] = \sum_{q=L_{\min}}^{L_{\max}} p_q \cdot \frac{(1-P_r)^q}{1-(1-P_r)^q}. \quad (40)$$

The average time required in the first phase is

$$\bar{T}_{nsv} = (\bar{N}_{nsv} + \bar{L}_{r0v}) \times 1 = \bar{N}_{nsv} + \bar{L}_{r0v}. \quad (41)$$

2) From the Relay Nodes to the Destination Node

The change of the number of the relay nodes does not affect the number of the data packets sent by the relay nodes and the time required, and they are identical to these in (31):

$$\begin{aligned} \bar{N}_{nrV} &= \frac{1}{1-P_{eB}} \\ \bar{T}_{nrV} &= \bar{N}_{nrV} \end{aligned} \quad (42)$$

V. NUMERICAL AND SIMULATION RESULTS

To clearly show the performances of the networks, we carry out numerical calculations based on the results of sections III and IV. In the mean time, to verify the theoretic results, we also simulate the performances of the networks. In the calculation and simulation, the channels are erasure channel, and all the channels share the same erasure probability.

Because the numerical and simulation results of the fountain relay network are based on the transference of M original data packets, and those in the normal relay network are based on the transference of 1 original data packet, so to compare more directly the performances of the two schemes, the results of the fountain relay network are divided by M when they are illustrated or listed in the figures and table in this section. The parameter M is 100 in the numerical calculations and simulations.

A. When the Number of the Relay Nodes Is Fixed

Figs. 2 and 3 are the average number of the data packets sent by the source node and the transmission time required to make the relay nodes correctly receive one original data packet. Numerical results are indicated by solid lines and the simulation results by dashed lines. In the fountain network, the transmission from the source node to the relay nodes is a broadcast transmission. It is required that all the relay nodes correctly receive the data, so the number of the transmitted data packets and transmission time are increasing along with the increase of the number of the relay nodes.² The probability P_r that the relay nodes can transmit data does not affect the results. In the normal network, only one relay node which can relay data at that time is required to correctly receive the data packet. The number of the transmitted packets decreases with the increase of the number of the relay nodes and P_r . The transmission time is the time that the source node needs to send data plus the suspension time.

² Because of the use of fountain codes, this increase is very slight. For example, when the erasure probability P_{eB} is 0.1, and there is 1 relay node, the average number of the data packets sent by the source node is 1.111, the use of fountain codes does not affect the number in this case. But if the number of the relay nodes increases to 8, it is only 1.164 if fountain codes are employed – increased by 4.73% and it is 1.656 if fountain codes are not used – increased by 49% [10].

The value of the former is identical to the number of the transmitted packets. The latter will have a very large value when the number of the relay nodes and P_r are low.

Figs. 4 and 5 are performances of the second phase (in Fig. 5, because the results of numerical calculation and simulation are very close, so it is difficult to distinguish the two categories of curves.). In the normal network, only the relay node that first accomplishes data receiving in the first phase transmits data, so the change of the number of the relay nodes and the probability P_r do not affect the performance of the second phase. In the fountain network, the data packets are transmitted by all the relay nodes. For any one of the relay nodes that can relay data, if there are data that need to be transmitted, it relays them. So the more the relay nodes are and the higher is P_r , the shorter is the transmission time. This is shown in Fig. 5. Fig. 4 shows that the number of the data packets sent by the relay nodes in the fountain network increases slightly along with the increase of the number of the relay nodes and P_r .

Figs. 6 and 7 are the overall number of the data packets sent by the source node and the relay nodes and the time required to make one original data packet correctly transferred from the source node to the destination node. In the fountain network, the transmission time is shortened greatly with only the slight increase of the number of the transmitted data packets. For example, when the number of the relay nodes is 10, the probability P_r is 0.5, P_{eB} is 0.1, for a data packet to be correctly transferred from the source node to the destination node,

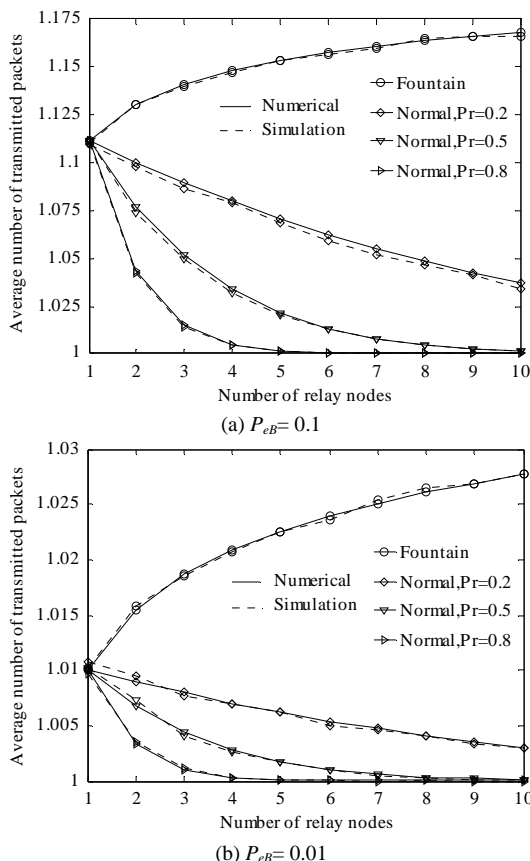


Fig. 2. The average number of transmitted packets from the source node to the relay nodes. The number of the relay nodes is fixed.

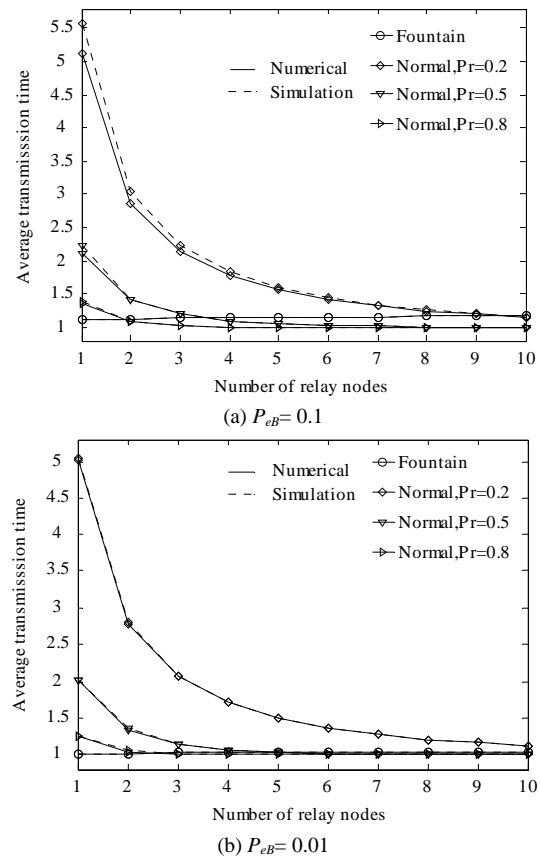


Fig. 3. The average transmission time from the source node to the relay nodes. The number of the relay nodes is fixed.

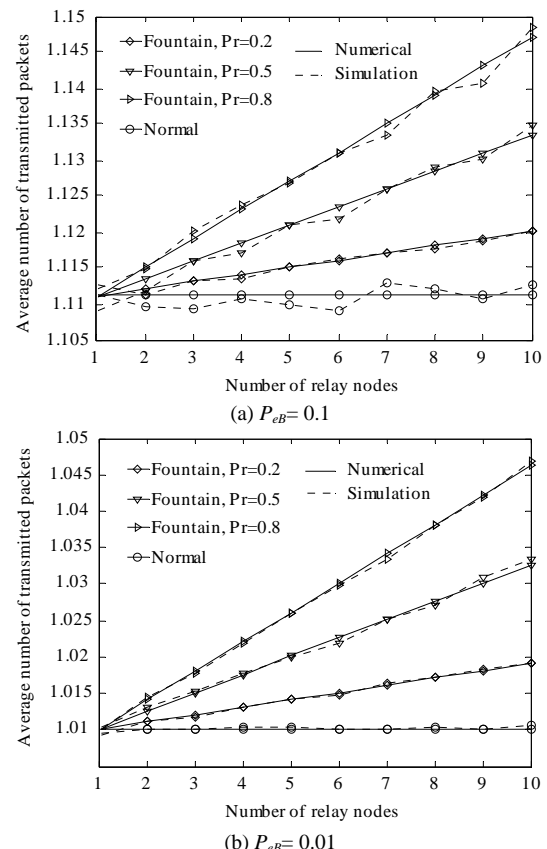


Fig. 4. The average number of the transmitted packets from the relay nodes to the destination node. The number of the relay nodes is fixed.

the average number of the transmitted data packets and the transmission time are 2.113 and 2.114 respectively in the normal network, and they are 2.302 and 1.395 in the fountain network. The number of the transmitted data packets increases by 8.94%, but the transmission time is cut down by 34.01%.

B. When the Number of the Relay Nodes Is Variable

In the numerical calculation and simulation for both networks with a variable number of the relay nodes, we assume that the number of the relay nodes L ranges from 1 to 10. The transition probability matrix of the number of the relay nodes is

$$\mathbf{P} = \begin{bmatrix} 2/3 & 1/3 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1/3 & 1/3 & 1/3 & \dots & 0 & 0 & 0 \\ & & & & \vdots & & & \\ 0 & 0 & 0 & 0 & \dots & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1/3 & 2/3 \end{bmatrix} \quad (43)$$

It is easy to get its stationary distribution, which is:

$$p_q = P(L=q) = \frac{1}{10}, \quad q=1,2,\dots,10. \quad (44)$$

Figs. 8 to 13 are the results of numerical calculation and simulation of the two networks. In the figures, the numerical results are shown by the solid bars, and the simulation results are shown by the hollow bars. The performances are calculated and simulated when the erasure probabilities of the channels are 0.1 and 0.01. Figs. 12 and 13 are the average number of the overall

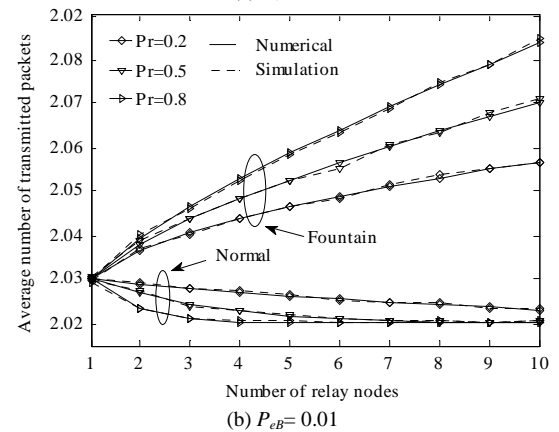
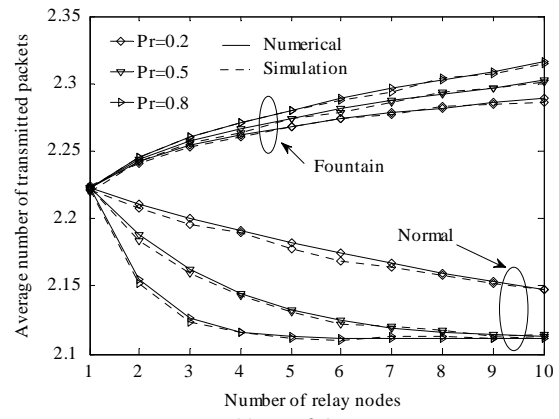


Fig. 6. The average overall number of transmitted packets. The number of the relay nodes is fixed.

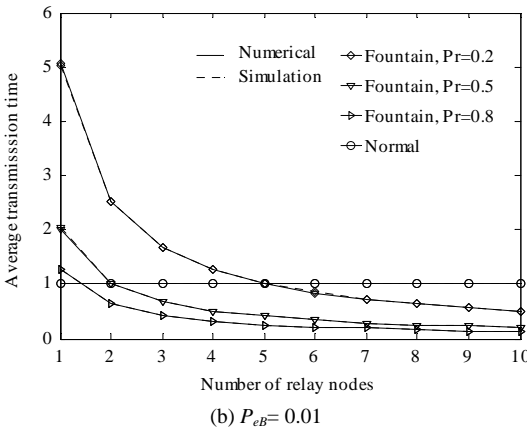
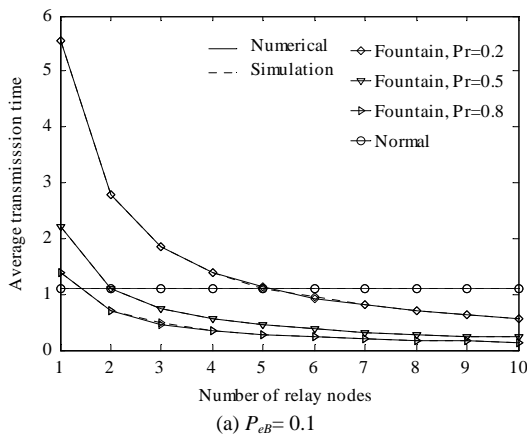


Fig. 5. The average transmission time from the relay nodes to the destination node. The number of the relay nodes is fixed.

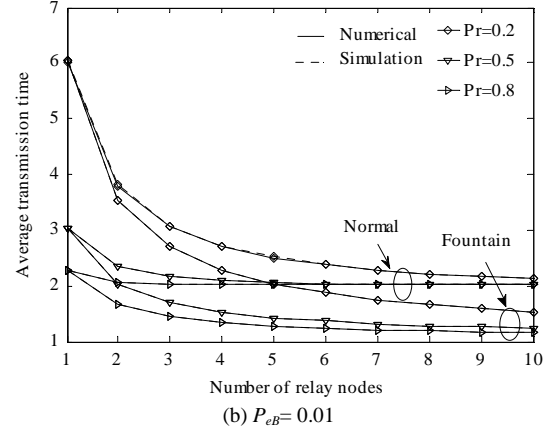
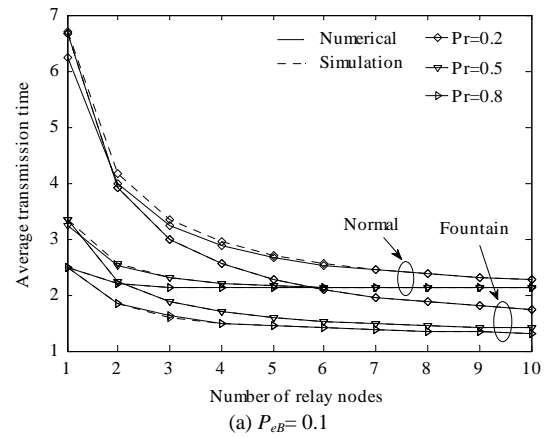


Fig. 7. The average overall transmission time. The number of the relay nodes is fixed.

transmitted packets and the overall transmission time to transfer a data packet from the source node to the destination node. Because the performances on the whole from the source node to the destination node are more significant, we also list the numerical results of Figs. 12 and 13 in Table I. From Table I, we can find that the average number of the transmitted data packets in the fountain network is slightly higher than that in the normal network, but the transmission time is much shorter. The system performance improvement brought by the use of fountain code is very attractive.

VI. CONCLUSION

We have analyzed the performance of a wireless relay network employing fountain codes. For comparison, we have also analyzed the performance of the normal relay network. The analyses are done in the condition that the relay nodes are dynamic, including their number and their capabilities to relay data. Due to the mobility, power constraint, and the payload change of the relay nodes, the statuses of the relay nodes are constantly changing in a practical network. That the numerical results and the simulation results match very well proves that our analyses are correct. These results show that the use of fountain codes can help to cut down the transmission time dramatically with only the tiny increase of the number of the transmitted data packets. Another merit of the fountain relay scheme is that in the second phase of the transmission no synchronization among the relay nodes is required. This is valuable because synchronization is always a difficult problem in a practical system. In the analyses, we have used the theory of probability and stochastic process as a chief mathematical tool. The dynamic property of the relay nodes is modeled by binominal distribution and Markov chain, which simplifies our analyses. To more accurately analyze the performance of the dynamic wireless relay networks employing fountain codes, a more accurate statistical model is required to describe the dynamic characteristic of the relay nodes. To find the models will be valuable work in the future.

TABLE I.

THE AVERAGE NUMBER OF THE OVERALL TRANSMITTED PACKETS AND THE OVERALL TRANSMISSION TIME. THE NUMBER OF THE RELAY NODES IS VARIABLE.

	$P_{eB} = 0.1$					
	$P_r = 0.2$		$P_r = 0.5$		$P_r = 0.8$	
	Packets	Time	Packets	Time	Packets	Time
Fountain	2.266	2.781	2.273	1.805	2.279	1.5604
Normal	2.181	3.091	2.144	2.304	2.129	2.1590
Difference	0.085 3.89%	-0.311 -1005%	0.129 6.00%	-0.499 -21.69%	0.150 7.07%	-0.599 -27.73%
	$P_{eB} = 0.01$					
	$P_r = 0.2$		$P_r = 0.5$		$P_r = 0.8$	
	Packets	Time	Packets	Time	Packets	Time
Fountain	2.036	2.505	2.043	1.617	2.050	1.3951
Normal	2.016	2.927	2.013	2.174	2.0116	2.0418
Difference	0.020 1.00%	-0.422 -14.43%	0.030 1.50%	-0.557 -25.60%	0.038 1.90%	-0.647 -31.67%

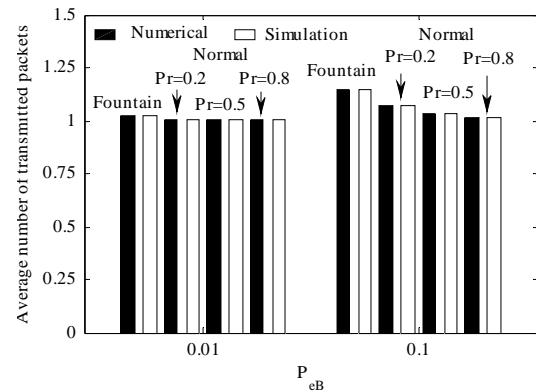


Fig. 8. The average number of the transmitted packets from the source node to the relay nodes. The number of the relay nodes is variable.

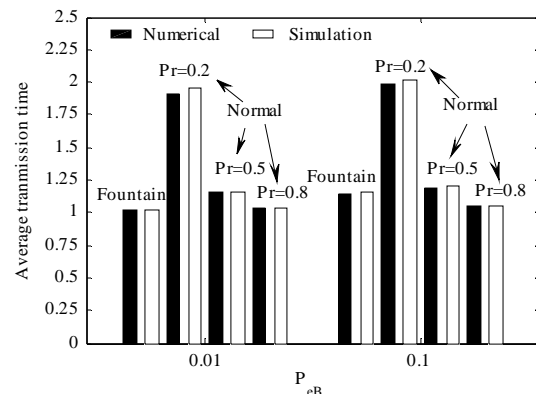


Fig. 9. The average transmission time from the source node to the relay nodes. The number of the relay nodes is variable.

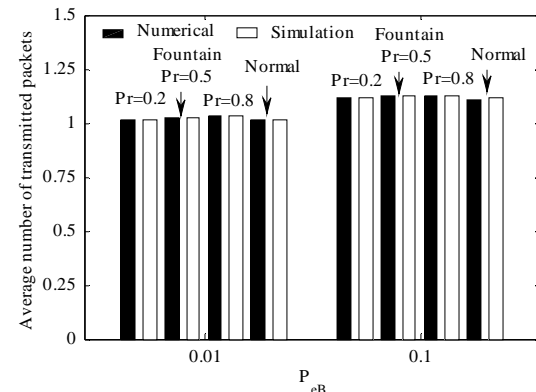


Fig. 10. The average number of the transmitted packets from the relay nodes to the destination node. The number of the relay nodes is variable.

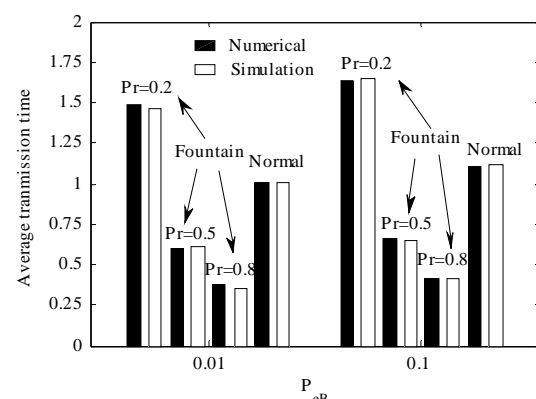


Fig. 11. The average transmission time from the relay nodes to the destination node. The number of the relay nodes is variable.

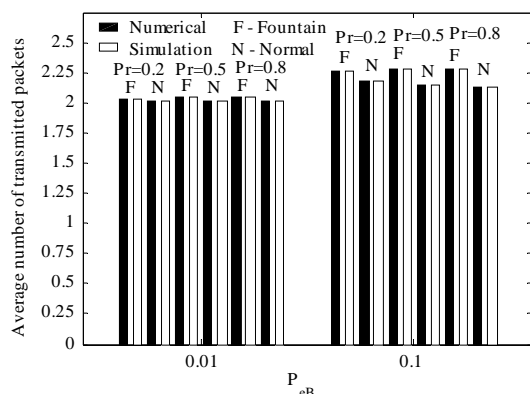


Fig. 12. The average overall number of transmitted packets. The number of the relay nodes is variable.

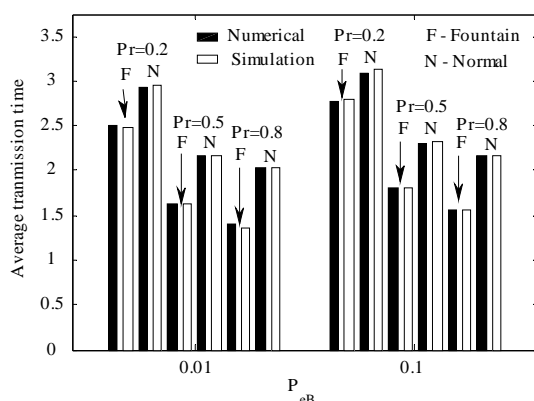


Fig. 13. The average overall transmission time. The number of the relay nodes is variable.

ACKNOWLEDGMENT

This work was supported by NNSF of China (60872037), CQ CSTC (2008BB2411) and Open Research Fund of KLMC, CQUPT.

REFERENCES

- [1] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity Part I: System Description," *IEEE Transaction on Communications*, vol. 51, no. 11, pp. 1927-1938, Nov. 2003.
- [2] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity Part II: Implementation Aspects and Performance Analysis," *IEEE Transaction on Communications*, vol. 51, no. 11, pp. 1939-1948, Nov. 2003.
- [3] A. Nosratinia and T. E. Hunter, A. Hedayat, "Cooperative Communication in Wireless Networks," *IEEE Communications Magazine*, vol. 42, no. 10, pp. 74-80, Oct. 2004.
- [4] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.
- [5] J. Byers, M. Luby, and M. Mitzenmacher, "A Digital Fountain Approach to Asynchronous Reliable Multicast," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 8, pp. 1528-1540, Oct. 2002.
- [6] M. Mitzenmacher, "Digital Fountains: A Survey and Look Forward," in *Proc. IEEE Information Theory Workshop*, 24-29, Oct. 2004, pp. 271-276.
- [7] D. J. C. Mackay, "Fountain Codes," *IEE Communications Proceedings*, vol. 152, no. 6, pp. 1062-1068, Dec. 2005.
- [8] A. F. Molisch, N. B. Mehta, J. S. Yedidia, and J. Zhang, "Cooperative Relay Networks Using Fountain Codes". In *Proc. IEEE GLOBECOM 2006*, Nov. 27 - Dec. 1, 2006, pp. 1-6.
- [9] A. F. Molisch, N. B. Mehta, J. S. Yedidia, and J. Zhang, "Performance of Fountain Codes in Collaborative Relay Networks". *IEEE Transactions on Wireless Communications*, vol. 6, no. 11, pp. 4108-4119, Nov. 2007.
- [10] W. Lei, X. Li, X. Xie, and G. Li, "Wireless Cooperative Relay System Using Digital Fountain Codes," In *Proc. IEEE ICCAS 2009*, Jul. 23-25, 2009, pp. 123-127.
- [11] Y. Liu, "A Low Complexity Protocol for Relay Channels Employing Rateless Codes and Acknowledgement," In *Proc. IEEE ISIT 2006*, Jul. 9-14, 2006, pp. 1244-1248.
- [12] J. Castura and Y. Mao, "Rateless Coding for Wireless Relay Channels", *IEEE Transactions on Wireless Communications*, vol. 6, no. 5, pp. 11638-1642, May 2007.
- [13] R. Nikjah and N. C. Beaulieu, "Novel Rateless Coded Selection Cooperation in Dual-Hop Relaying Systems", In *Proc. IEEE GLOBECOM 2008*, Nov. 30-Dec. 4, 2008, pp. 1-6.
- [14] H. P. Hsu, *Probability, Random Variables, and Random Processes*. New York: McGraw-Hill Companies, Inc., 1997.



Weijia Lei received his M.S. degree in communications and information system from Beijing University of Posts and Telecommunications, Beijing, China in 1999 and B.S. degree in communication engineering from Chongqing University of Posts and Telecommunications (CQUPT), Chongqing, China in 1992. He is an associate professor in CQUPT, and currently is studying for a Ph.D.

degree in University of Electronic Science and Technology of China (UESTC). His main research field is in wireless communications technology.



Xianzhong Xie received the Ph.D. degree in communication and information system from Xidian University, Xi'an, China in 2000. He is currently with School of Computer Science and Technology at CQUPT, where he is a professor and director of Institute of Personal Communications. His research interests include cooperative diversity, cognitive radio

networks, MIMO and precoding, and signal processing for wireless communications. He is the principal author of four books on 3G, MIMO, Cognitive Radio, and TDD Technology. He has published more than 80 papers in journals and 30 papers in international conferences.



Guangjun Li received his M.S. degree from University of Electronic Science and Technology of China, Chengdu, China in 1985. From 1991 to 1992, he was a visiting scholar at RWTH Aachen University, Germany. He is currently a professor and vice dean of School of Communication and Information Engineering in UESTC. His main research interests include wireless and mobile communication system and embedded system.