Power and Energy Efficiency of Multilevel Baseband Transmission Systems: Analysis, Optimization and Improvements

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***Abstract*—**Power and energy consumption of communication networks contribute to the resource demand and hence they impact the sustainability of the society. Transmission links form basic parts of communication networks and thus they add to the overall communicatio nnetworks' resourcedemand. Therefore, it is important to optimize transmission links in order to support the energy-efficient design of communication networks. In transmission systems the constellation size in general is a degree of freedom when designing or adapting a transmission link. In this article, multilevel baseband transmission via additive white Gaussian noise (AWGN) channels and over copper cables is optimized with respect to minimum power and energy demand, respectively – for the relevant case of given throughput and fixed transmission quality of a transmission link. The results are two-fold: In the AWGN case the minimum power and energy is utilized at the smallest possible constellation size. In case of band-limited baseband transmission over twisted-pair copper wires with linear equalization the optimum constellation size increases as the band limitation becomes stronger, i. e., as the cable gets longer at fixed bit rate or as the bit rate is increased at fixed link length. The optimization of the constellation size enables significant power and energy savings per link in the range of 25 % to approximately 90 % compared to a conventional two-level baseband system.

***Index Terms*—**Communication, transmission, transmit power, energy efficiency, cable, pulse amplitude modulation.

I. Introduction

A. Motivation and Topical Background

As communication networks are significant consumers of electrical energy in developed economies [1], [2], the sustainability and the energy demand of such networks have been of considerable interest during the past few decades. This stimulated research and development activities of the related sciences leading to a better understanding of the energy distribution in communication networks (e. g. [1], [3]–[5]) and to proposals for improvement (e. g. [6]–[17] and many more).

On the one hand, communication networks are large-Coften regional or nation-wide – infrastructures exhibiting a large cumulated energy demand when considered as a whole system, e. g., from the viewpoint of an organization operating the network. Typically, communication operator networks exhibit a share of roughly a percentage point of the national electricity consumption [2]—adding up to a worldwide total amount of several hundreds of terawatt hours per year [1]. In this regard, measures for reducing the communication networks' energy demand are applicable on operational and organizational levels,e.g. [2]. On the other hand, such networks consist of avery large multitude of individual elements and in particular transmission systems forming their overall energy consumption. When looking from this viewpoint it is important to minimize the energy demand of each network element and transmission system in order to limit the aggregated electricity demand of a communication network, e. g. [8], [18]. The power or energy needed for transferring signals via transmission links can be seen as minimum electrical power or energy that is needed for operating transmission links – as at least the information has to be transferred from a transmitter to a receiver [19]. Other network functions, e. g. switching, routing and computing, require power and energy, too, that adds to the power and energy needed for the pure information and signal transfer.

The work in this article focusses on the electrical power and energy consumed by individual communication links operating via given transmission channels. On the one hand, this forms a classical problem of transmission engineering in principle targeting at information transmission at high bit rate, low bit-error probability – and low power and energy demand. On the other hand, it contributes to limiting power and energy usage in communication systems. When minimizing the energy demand of transmission links for given network usage patterns the overall electricity demand of networks can be limited or even reduced – as the energy demands of large-sized communication networks are formed by the demands of the components they consist of.

B. Problem Statement and Related Work

The task considered in this article is to analyse a transmission link between a transmitter and a receiver at a given bit rate and a fixed transmission quality via an existing transmission channel, as depicted generally in Fig. 1-targeting minimized power and energy demand of this link.[1] This is a common problem when designing or extending a communication network energy efficiently. Often, the bit rate

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[1] This article is an extended version of a conference paper published in [20].

requirement is explicitely stated and the required bit error probability is implicitely supposed to be met in order to achieve a sufficiently good transmission quality.

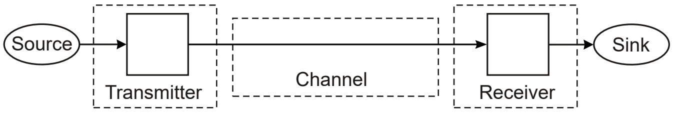

Fig. 1. Model of a general transmission system

In this article, for a multilevel baseband transmission system it is studied whether there is a possibility to control the amount of transmit power or electrical energy, respectively, used by a transmission link via adjusting transmission parameters appropriately. This type of transmission is chosen in order to enable a concise analytical formulation and calculation. This, in turn, allows for insights on interdependencies and options for governing the power and energy demand. In particular, the number of signalling levels – also designated as constellation size of the transmission system – is a degree of freedom to adjust and optimize in such systems using pulse amplitude modulation (PAM). In the first part a baseband PAM transmission system operating over an additive white Gaussian noise (AWGN) channel is studied to introduce the basic concept. In the second part, the transmission over a twisted-pair copper cable – as a practically important example of a linear distorting transmission channel – is analyzed in conjunction with pulse amplitude modulation: A baseband PAM transmission system operating on such a copper cable with linear equalization in the receiver is investigated. Twisted-pair copper cables are widely used for broadband data transmission in telecommunication access networks. They were originally installed for the telephone system and since several decades they are also used to provide apartment buildings and individual homes with broadband data services [21] – even in an era where optical fibers are installed in access networks [22], [23]. Currently, prevalently multicarrier systems, e. g. [24]–[28], are utilized for broadband data transmission over copper access lines: On the one hand, the results obtained in this article for the baseband system are in principle transferrable to other – bandpass and multicarrier – modulation formats and on the other hand the simpler implementations of baseband transmission systems could lead to a regained practical interest in baseband transmission for copper-based access networks, at least for cost-sensitive special applications, e. g. sensor networks. Especially since the telephone service in the recent past moved to Voice over IP (Internet Protocol) technology and thus also uses the underlying broadband data transmission to the user it is no longer necessary to keep the lower frequency bands free for telephone services and therefore also baseband systems are viable in this field of application.

The novelty of the investigations in this article is based on a concise derivation of the transmit power and energy per bit for multilevel baseband transmission systems as a function of the constellation size starting from AWGN systems and extending this consideration to twisted-pair copper cable transmission systems with linear equalization in the receiver. The analysis in this article explicitly focuses on the power and energy efficiency of AWGN and cable baseband systems whereas related work, e. g. in [13], [14], [26], [28], concentrates on wired multicarrier systems – in parts also treating aspects of their power and energy demand. A degree of freedom is identified that allows for minimizing the power and energy demand when designing or adapting baseband transmission schemes: The constellation size – or the number of signalling levels – is optimized with respect to minimum transmit power or energy per bit, respectively. The results show that by optimal choice of this transmission parameter significant improvements of power and energy efficiency can be achieved with respect to conventional two-level twisted-pair baseband cable transmission systems.

C. Structure of the Article

The remaining part of this paper is organized as follows: In Section II, the transmission model for a multilevel baseband transmission via an AWGN channel is introduced and the transmit power is derived for a given bit rate and a fixed bit error probability. Based on this, the energy per transmitted bit as an indicator for energy efficiency is calculated and the results are presented. In Section III, the transmission model for the cable transmission is specified and both, the transmit power and energy per bit, respectively, are derived for this extended transmission setup encompassing a distorting transmission channel and a linear equalization. Results are presented and discussed. Section IV provides major findings and concluding remarks.

II. Power and Energy Efficiency of AWGN Transmission

A. AWGN Baseband Transmission Model

Usually digital data are binary data – as the source produces a sequence of bits (i. e. 0s and 1s). In general, for the transmission over a given channel a signal is needed that represents the digital data stream and matches the essential characteristics of the transmission channel [29]. Therefore, according to the transmission model shown in Fig. 2 binary data with bit rate f_{B} are converted to symbols with s amplitude signalling levels in a multilevel coder at the transmitter side that maps bits to s-ary symbols, i. e., the baseband signal has a constellation size of s.[2] After filtering with the transmit filter transfer function $G_{\mathrm{s}}(f)$ a pulse amplitude modulated transmit signal with s signalling levels is transmitted over the channel. The multilevel coding allows for controlling the bandwidth of the transmit signal and it can be adapted to the channel characteristics. In the AWGN channel white Gaussian noise with power spectral density Ψ_0 is added to the transmit signal. At the receiver side after filtering with the receive filter transfer function $G_{\mathrm{e}}(f)$ and subsequent symbol rate sampling the detector decides on the received signal amplitude. The multilevel decoder maps the detected symbols to the received bits and the sink is provided with a binary data sequence.

The transmit and receive filters $G_{\mathrm{s}}(f)$ and $G_{\mathrm{e}}(f)$ are square-root raised cosine filters with roll-off factor r, respectively:

[2]The bit rate f_{B} can be interpreted as the bit repetition frequency, given in the unit $1/\mathrm{s} = \mathrm{Hz}$.

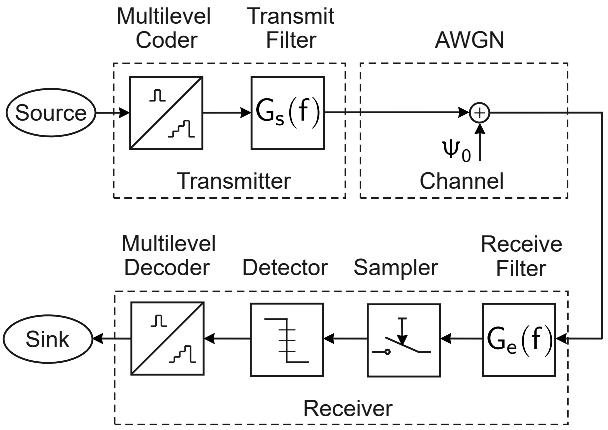

Fig. 2. Model of the AWGN baseband transmission system

Over the cascade of transmit and receive filter the first Nyquist criterion is satisfied and the receive signal at the detector is free from intersymbol interference (ISI) [29]–[31].

The s-ary transmit signal exhibits a symbol rate of

$$f_{\mathrm{T}} = \frac{f_{\mathrm{B}}}{\mathrm{ld}(s)} \tag{1}$$

and each symbol carries $\mathrm{ld}(s)$ bits [29].[3] At a fixed bit rate f_{B} the symbol rate f_{T} is different for each value of s. The half-level amplitude is denoted as U_{s}, i. e. the distance between neighbouring signal amplitude levels or signal points in the constellation, respectively, is $2\,U_{\mathrm{s}}$. Then, the average transmit power is obtained as

$$P_{\mathrm{s}} = \frac{U_{\mathrm{s}}^2}{3}\left(s^2 - 1\right) \tag{2}$$

for an equally distributed s-ary random baseband signal, a redundancy-free source and the given square-root raised cosine transmit filter, e. g. [30].[4]

For assessing the transmission quality, the signal-to-noise ratio

$$\varrho = \frac{(\text{Half Vertical Eye Opening})^2}{\text{Noise Power}} = \frac{U_{\mathrm{A}}^2}{U_{\mathrm{R}}^2} \tag{3}$$

at the detector input is used – based on [32]. As the first Nyquist criterion is met over the cascade of transmit and receive filter, the half vertical eye opening U_{A} equals the half-level transmit signal amplitude U_{s}: $U_{\mathrm{A}} = U_{\mathrm{s}}$.

For the given square-root raised cosine receive filter $G_{\mathrm{e}}(f)$ together with the power spectral density Ψ_0 of the additive white Gaussian noise the noise power at the detector input is calculated as (e. g. [29], [30], [33])

$$U_{\mathrm{R}}^2 = \Psi_0 \int\limits_{-\infty}^{+\infty} \left|G_{\mathrm{e}}(f)\right|^2 \,\mathrm{d}f = \Psi_0 f_{\mathrm{T}} = \Psi_0 \cdot \frac{f_{\mathrm{B}}}{\mathrm{ld}(s)} \quad . \tag{4}$$

[3]The notation $\mathrm{ld}(x)$ describes the *dyadic logarithm*: $\mathrm{ld}(x) = \log_2(x)$.

[4]In this paper a system-theoretic power with the dimension of a $(\mathrm{voltage})^2$ (with the unit V^2) is used. At a real, constant resistance R this system-theoretic power is proportional to the physical power and it is translated into a physical power (with the unit Watt) by dividing the system-theoretic power by R.

Assuming Gray coding [29], [30], the bit error probability of the s-ary baseband transmission yields [29]

$$P_{\mathrm{b}} = \frac{s-1}{s\,\mathrm{ld}(s)}\left[1 - \mathrm{erf}\left(\sqrt{\frac{\varrho}{2}}\right)\right] \tag{5}$$

As a sequence of bits is transmitted the bit error probability is the decisive quality criterion in binary data transmission.[5]

B. Calculation of Power and Energy Efficiency Indicators

In order to assess the power and energy efficiency of the AWGN baseband transmission system appropriate indicators have to be chosen or to be defined. An obvious approach is to calculate the transmit power P_{s} taking the constraints of given bit rate f_{B} and given bit error probability P_{b} into account. The transmit power represents a key physical quantity for the power and energy demand of transmission systems. From this result different energy efficiency measures can be derived. Here, the energy per bit as an important measure for energy efficiency will be calculated [6], [7].

1) Transmit Power: As the first Nyquist criterion is met over the cascade of transmit and receive filter the relation $U_{\mathrm{A}} = U_{\mathrm{s}}$ holds. Then, the signal-to-noise ratio (3) in the ISI-free case can be expressed as

$$\varrho = \frac{U_{\mathrm{s}}^2}{U_{\mathrm{R}}^2} \tag{6}$$

By solving (6) for U_{s}^2 and combining the result with (2) the transmit power becomes

$$P_{\mathrm{s}} = \frac{U_{\mathrm{R}}^2 \cdot \varrho}{3}\left(s^2 - 1\right) \tag{7}$$

and with (4) it further results in

$$P_{\mathrm{s}} = \frac{\left(s^2 - 1\right)}{3} \cdot \Psi_0 \cdot \frac{f_{\mathrm{B}}}{\mathrm{ld}(s)} \cdot \varrho \quad . \tag{8}$$

Solving (5) for the signal-to-noise ratio ϱ yields

$$\varrho = 2\left[\mathrm{erf}^{-1}\left(1 - \frac{s\,\mathrm{ld}(s)}{s-1}P_{\mathrm{b}}\right)\right]^2 \quad . \tag{9}$$

Finally, by combining (8) and (9) the transmit power P_{s} in an ISI-free AWGN channel for given bit rate f_{B} and given bit error probability P_{b} is obtained:

$$P_{\mathrm{s}}(s) = \frac{\left(s^2 - 1\right)}{3} \cdot \frac{\Psi_0 \cdot f_{\mathrm{B}}}{\mathrm{ld}(s)} \cdot 2\left[\mathrm{erf}^{-1}\left(1 - \frac{s\,\mathrm{ld}(s)}{s-1}P_{\mathrm{b}}\right)\right]^2 \quad . \tag{10}$$

Equation (10) shows that the transmit power only depends on the given quantities (f_{B} and P_{b}) and the noise power spectral density Ψ_0 of inevitable noise disturbances – as well as on the number of transmission levels s. This number of transmission levels s is a degree of freedom in the design or adaptation process of the respective transmission system: Targeting minimum power and energy demand it is subject to optimization and a constellation size can be derived and used for transmission that leads to minimum transmit power for given constraints (bit rate, bit error probability, noise characteristic). For the transmit power function the notation $P_{\mathrm{s}}(s)$ is used, to emphasize its dependency on the constellation size s.

[5]The function $\mathrm{erf}(x)$ denotes the Gaussian error function (e. g. [34], [35]).

2) Energy per Bit: The energy per bit – or equivalently the power per bit rate – is an important performance indicator when it comes to power and energy efficiency of digital information transmission systems and links [6], [7]. It takes into account the power $P_\mathrm{s}(s)$ as well as the bit rate f_B as a measure for the benefit this power demand provides for the information transfer. The energy per bit is obtained according to

$$E_\mathrm{b}(s) = \frac{P_\mathrm{s}(s)}{f_\mathrm{B}} \ , \tag{11}$$

and with (10) it results in

$$E_\mathrm{b}(s) = \frac{2\cdot(s^2-1)}{3}\cdot\frac{\Psi_0}{\mathrm{ld}(s)}\cdot\left[\mathrm{erf}^{-1}\left(1-\frac{s\,\mathrm{ld}(s)}{s-1}P_\mathrm{b}\right)\right]^2 \tag{12}$$

for the AWGN baseband transmission link. Again, the denotation $E_\mathrm{b}(s)$ is used to underline the dependency of the energy-per-bit result on the constellation size s. The energy per bit (only) depends on the required transmission quality (bit error probability P_b), the noise characteristic (noise power spectral density Ψ_0) and the constellation size s. It is interesting to observe that the energy per transmitted bit does not depend on the bit rate f_B in the AWGN channel.

C. Power and Energy Efficiency Results

The numerical results for the calculated power and energy efficiency indicators presented in the following are exemplarily based on the given values for the bit error probability $P_\mathrm{b} = 1.5\cdot 10^{-9}$ and the noise power spectral density $\Psi_0 = 10^{-12}\,\mathrm{V}^2/\mathrm{Hz}$. These numerical assumptions are based on practically relevant values for real baseband transmission systems. In case of modified numerical values of P_b and Ψ_0 the absolute results will change, however, the optimum values and the achievable savings will hold in tendency and in the order of magnitude. The roll-off factors of the square-root raised cosine transmit and receive filters are chosen to be $r = 0.5$. As the constellation size s is a degree of freedom the power and energy efficiency results are depicted as a function of this parameter s to obtain hints on the power and energy efficient design of baseband transmission links. At the axis of abscissas the values of $\mathrm{ld}(s)$ are shown – representing the number of transmitted bits per symbol and allowing implicitly for a logarithmic division of the axis with respect to the constellation size s. In order to accommodate the results in the diagrams conveniently the axis of ordinates is divided logarithmically. The transmit powers and energies per bit are calculated and plotted in a system-theoretic way and thus the results are given in the units V^2 and V^2/Hz, respectively. In doing so, the results are independent of concrete transmission technologies. Real physical powers and energies – in W or $\mathrm{W}/\mathrm{Hz} = \mathrm{J}$, respectively – can be obtained by dividing the results shown in the diagrams by the resistance R that is valid for a given transmission technology.

The transmit power and energy-per-bit results presented in the following are obtained by numerical calculations with the transmission parameters given above.

1) Transmit Power: Numerical transmit power results for the AWGN baseband transmission system are shown in Figure 3: When considering a particular fixed bit rate f_B in the analysis, the transmit power $P_\mathrm{s}(s)$ increases with rising constellation size s at a fixed bit error probability. The minimum transmit power is observed at the minimum constellation size of $s = 2$, i. e., for a basic pulse amplitude transmission system with two signalling levels. With rising order of the constellation size the transmit power increases since a higher signal power is necessary to reliably distinguish between the constellation signal points – at the given transmission quality (P_b). Implicitly it is assumed that the bandwidth for the considered setup is guaranteed for each constellation size.

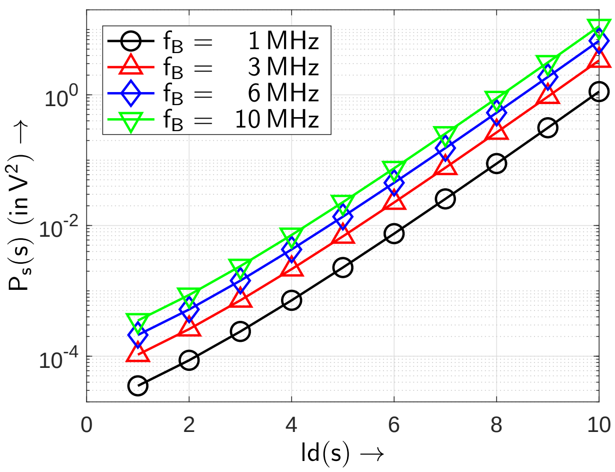

Fig. 3. Transmit power $P_\mathrm{s}(s)$ as a function of the constellation size s at several fixed bit rates f_B (parameters: $P_\mathrm{b} = 1.5\cdot 10^{-9}$ and $\Psi_0 = 10^{-12}\,\mathrm{V}^2/\mathrm{Hz}$)

In order to obtain more results and hence insights, the analysis has been performed for several (fixed) bit rates: In case the bit rate f_B is increased and hence the requirements regarding the throughput and thus the performance capability of the transmission system are enhanced (see Fig. 3), the transmit power $P_\mathrm{s}(s)$ rises accordingly – as the transmit power $P_\mathrm{s}(s)$ is directly proportional to the bit rate f_B according to (10): In the diagram in Fig. 3 this behaviour is expressed by the different transmit power functions for distinct bit rates.

2) Energy per Bit: In Fig. 4 exemplary numerical results for the energy $E_\mathrm{b}(s)$ per transmitted bit are shown: This energy per bit also rises with increasing constellation size s – as it is essentially the transmit power $P_\mathrm{s}(s)$ divided by a fixed (given) value for the bit rate f_B. Since the energy per bit according to (12) does not depend on the bit rate f_B only one single graph represents the result for all possible bit rates f_B.

The interesting result of (12) and Figure 4 of the same energy demand *per bit* arises since the transmit power $P_\mathrm{s}(s)$ is multiplied by the bit duration $T_\mathrm{b} = 1/f_\mathrm{B}$. Thus, the time T_b is adjusted to each bit rate f_B – leading to the observed result.

The situation with regard to the energy demand changes, when a fixed time T is considered – that is not connected to the bit rate f_B – for determining the energy (instead of the bit duration T_b), i. e. the transmission system is operated over a

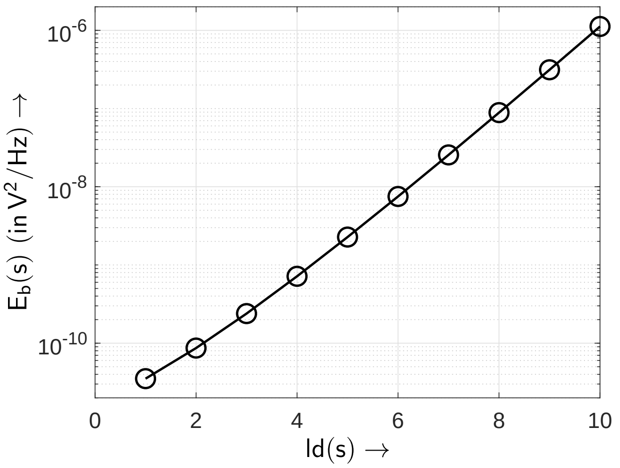

Fig. 4. Energy per transmitted bit $E_{\mathrm{b}}(s)$ as a function of the constellation size s (parameters: $P_{\mathrm{b}} = 1.5 \cdot 10^{-9}$ and $\Psi_0 = 10^{-12}\ \mathrm{V}^2/\mathrm{Hz}$)

given duration T of, e. g., an hour, a day, a week or a year: Then the transmit power $P_{\mathrm{s}}(s)$ – which is different for the various bit rates – will be multiplied by a fixed time T and the energy demand will depend on the bit rate f_{B}. The resulting energy demand for given time T is proportional to the transmit power $P_{\mathrm{s}}(s)$ and results will be scaled versions of the transmit power results shown in Fig. 3 (scaled by T). Then higher bit rates f_{B} require higher energies for operating the AWGN baseband transmission link over a fixed time duration T.

3) Summary of the Transmit Power and Energy-per-Bit Results: Fig. 5 and Fig. 6 illustrate the results obtained for the transmit power and the energy per bit in the AWGN channel: Both, the transmit power and the energy per bit are each shown in a three-dimensional plot as a function of the constellation size s and the bit rate f_{B}, respectively.

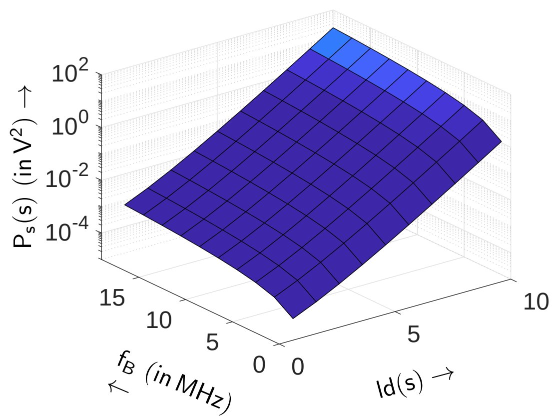

Fig. 5. Transmit power $P_{\mathrm{s}}(s)$ as a function of the constellation size s and of the bit rate f_{B} (parameters: $P_{\mathrm{b}} = 1.5 \cdot 10^{-9}$ and $\Psi_0 = 10^{-12}\ \mathrm{V}^2/\mathrm{Hz}$)

The behaviour of the functions (10) and (12) especially with respect to the dependency on the bit rate f_{B} is recognizable: The transmit power $P_{\mathrm{s}}(s)$ ascends with both, rising constellation size s and increasing bit rate f_{B}, whereas the energy per bit $E_{\mathrm{b}}(s)$ in the AWGN channel increases with rising constellation size s—but shows no dependency on the bit rate f_{B}.

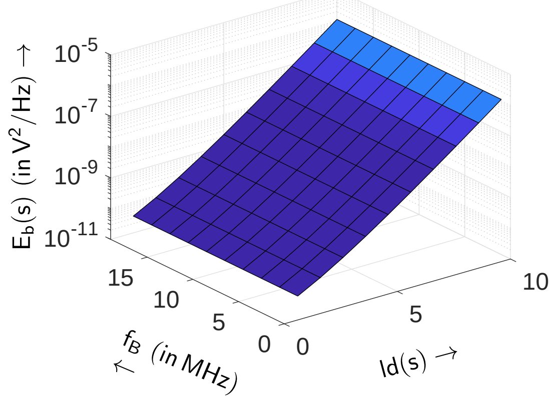

Fig. 6. Energy per bit $E_{\mathrm{b}}(s)$ as a function of the constellation size s and of the bit rate f_{B} (parameters: $P_{\mathrm{b}} = 1.5 \cdot 10^{-9}$ and $\Psi_0 = 10^{-12}\ \mathrm{V}^2/\mathrm{Hz}$)

III. Power and Energy Efficiency of Cable Transmission

A. Cable Transmission Model

Baseband techniques are practically applied for digital information transmission via twisted-pair copper cables. The model of the baseband cable transmission system is shown in Fig. 7.

As a channel model the cable transfer function

$$G_{\mathrm{k}}(f) = \mathrm{e}^{-\ell\sqrt{\mathrm{j}\frac{f}{f_0}}} \tag{13}$$

is introduced, where ℓ denotes the cable length (in km) and f_0 represents the characteristic cable frequency (in $\mathrm{MHz} \cdot \mathrm{km}^2$). The characteristic cable frequency f_0 is a cable-specific constant that depends, e. g., on the wire diameter and on the insulation material [36]. The transfer function (13) can be derived via transmission line theory for the RC range – leading to a strong low-pass behaviour of the channel. Noise disturbances are modelled by an additive white Gaussian noise of power spectral density Ψ_0.

The distorting impact of the cable on the wanted signal is completely equalized by the inverse of the cable transfer function $(1/G_{\mathrm{k}}(f))$ at the receiver side. There are a lot of equalization techniques established – e. g. [29]: Assuming an equalizer like mentioned above allows to obtain analytical results and insights – which is the main aim of this contribution.

The components cable transfer function $G_{\mathrm{k}}(f)$ and linear equalizer $1/G_{\mathrm{k}}(f)$ are the only elements that are newly introduced to the transmission system. All other components of the transmission link are maintained from the AWGN system (Fig. 2): The transmit and receive filters $G_{\mathrm{s}}(f)$ and $G_{\mathrm{e}}(f)$ are again square-root raised cosine filters with roll-off factor r, respectively. This way, in conjunction with the complete linear equalization of the cable transfer function's impact on the wanted signal the first Nyquist criterion holds from the transmit filter's input to the receive filter's output – resulting (again) in an ISI-free signal. The multilevel coder maps the binary bit sequence into an s-ary symbol sequence. In the receiver, the symbol rate sampler and detector perform the detection of the symbols before the s-ary symbol sequence is demapped into a binary bit sequence by the multilevel decoder.

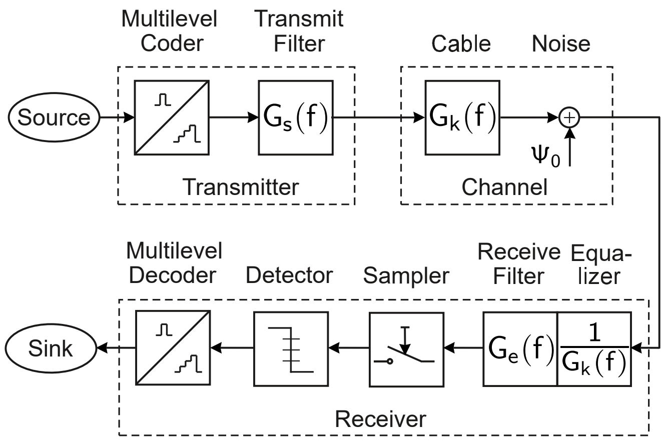

Fig. 7. Model of the cable baseband transmission system

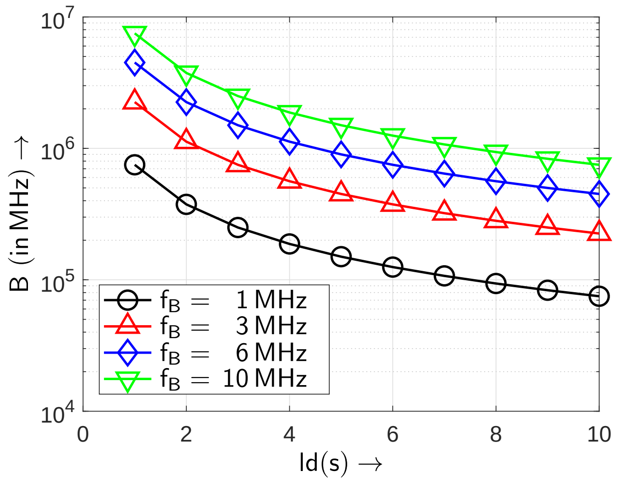

Fig. 8. Filter bandwidth B as a function of the constellation size s for several fixed bit rates f_B (roll-off factor: $r = 0.5$)

Within the cable transmission system the useful signal at the detector input is essentially identical to that in the AWGN transmission link. The only modification concerns the noise filtering in the receiver. Therefore almost all equations derived for the AWGN system still hold—with one exception: The noise power at the detector input now yields

$$U_\mathrm{R}^2 = \Psi_0 \int_{-\infty}^{+\infty} \left| \frac{G_\mathrm{e}(f)}{G_\mathrm{k}(f)} \right|^2 \mathrm{d}f \ . \tag{14}$$

B. Calculation of Power and Energy Efficiency Indicators

The cable transmission is assessed with respect to power and energy efficiency with the methodology used for the AWGN case. As stated above most of the equations obtained for the AWGN system are still valid and can be re-used.

1) Transmit Power: With (7) and (9) the transmit power for an ISI-free cable transmission system according to Fig. 7 for given bit rate f_B and bit error probability P_b yields

$$P_\mathrm{s}(s) = \frac{(s^2-1)}{3} \cdot U_\mathrm{R}^2 \cdot 2 \left[\mathrm{erf}^{-1} \left(1 - \frac{s\,\mathrm{ld}(s)}{s-1} P_\mathrm{b} \right) \right]^2 \ . \tag{15}$$

The noise power U_R^2 is calculated according to (14) and contains the dependency on the bit rate f_B – since the (one-sided) bandwidth of the square-root raised cosine receive filter $G_\mathrm{e}(f)$

$$B = \frac{f_\mathrm{T}}{2} \cdot (1 + r) = \frac{f_\mathrm{B}}{2 \cdot \mathrm{ld}(s)} \cdot (1 + r) \ , \tag{16}$$

depends on the bit rate f_B [33]. Fig. 9 shows the receive filter bandwidth (16) – which equals the transmit filter bandwidth and thus exhibits the (absolute one-sided) signal bandwidth – for several fixed bit rates f_B: Higher bit rates require higher bandwidth at the same constellation size and with rising constellation size s the filter (and signal) bandwidth decreases.

The receive filter bandwidth (16) in conjunction with the equalizer's transfer function governs the effective noise power at the detector input. The transmit power (15) further depends – again – on the constellation size s as a degree of freedom.

2) Energy per Bit: With (15) and (11) the energy per transmitted bit results in

$$E_\mathrm{b}(s) = \frac{(s^2-1)}{3} \cdot \frac{U_\mathrm{R}^2}{f_\mathrm{B}} \cdot 2 \left[\mathrm{erf}^{-1} \left(1 - \frac{s\,\mathrm{ld}(s)}{s-1} P_\mathrm{b} \right) \right]^2 \ . \tag{17}$$

C. Power and Energy Efficiency Results

Numerical power and energy efficiency results for the cable transmission system are exemplarily based on the bit error probability $P_\mathrm{b} = 1.5 \cdot 10^{-9}$ and the noise power spectral density $\Psi_0 = 10^{-12}\,\mathrm{V}^2/\mathrm{Hz}$ (the same values as in the AWGN case). For the cable a characteristic cable frequency of $f_0 = 0.178\,\mathrm{MHz \cdot km}^2$ is assumed describing a twisted-pair copper cable with a wire diameter of $0.6\,\mathrm{mm}$. The cable's length is chosen to be $\ell = 2\,\mathrm{km}$: This is a typical average length of copper cables in access networks that connect apartment buildings or individual homes to access nodes residing in a local exchange.

1) Transmit Power: Numerical transmit power results for the baseband transmission via a twisted-pair copper cable with given characteristics are shown in Fig. 9: The transmit power $P_\mathrm{s}(s)$ shows a minimum at an optimum constellation size s_opt that depends on the bit rate f_B. When starting from s_opt, with rising constellation size s the impact of higher-order constellation sizes known from the AWGN transmission is effective: The higher number of transmission levels leads to a higher density of signal points and an enhanced transmit power is required to achieve a fixed transmission quality. Towards smaller constellation sizes s another mechanism takes effect: The noise power is enhanced since the signal bandwidth – and hence the receive filter bandwidth according to (16) – is increased. This, in conjunction with the noise amplification induced by the equalizer, requires an increased transmit power when targeting a constant-quality transmission, i. e., maintaining a fixed bit error probability.

As expected, furthermore it is recognizable that higher bit rates f_B demand a higher transmit power $P_\mathrm{s}(s)$ when

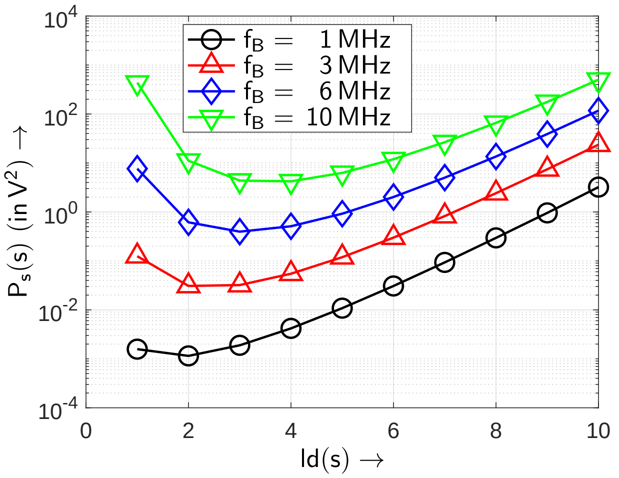

Fig. 9. Transmit power $P_{\mathrm{s}}(s)$ as a function of the constellation size s for several fixed bit rates f_{B} (parameters: $f_0 = 0.178\,\mathrm{MHz} \cdot \mathrm{km}^2$, $\ell = 2\,\mathrm{km}$, $P_{\mathrm{b}} = 1.5 \cdot 10^{-9}$, $\Psi_0 = 10^{-12}\,\mathrm{V}^2/\mathrm{Hz}$)

the transmission link operates at a fixed transmission quality (P_{b}) and distance ℓ. Also, it is noticeable that at higher bit rates f_{B} the optimum constellation size s_{opt} tends to shift to higher values of s: The low-pass characteristic of the cable becomes more severe to signals with higher bit rates (and hence higher bandwidths) requiring the equalization of a broader frequency range. This, in turn leads to a more intense noise amplification and so the effective noise power at the detector input is increased. In essence, the higher bandwidth at smaller constellation sizes s results in a stronger noise power enhancement – leading to a higher transmit power to maintain a given transmission quality.

2) Energy per Bit: Numerical results for the energy per transmitted bit $E_{\mathrm{b}}(s)$ are shown in Fig. 10 assuming the same preconditions as in Fig. 9.

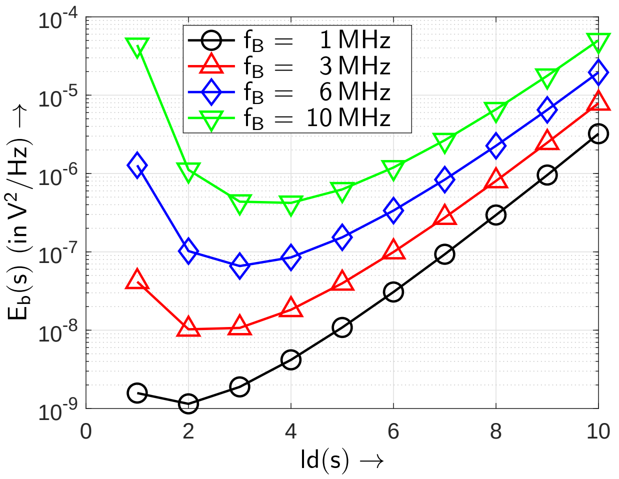

Fig. 10. Energy per transmitted bit $E_{\mathrm{b}}(s)$ as a function of the constellation size s for several fixed bit rates f_{B} (parameters: $f_0 = 0.178\,\mathrm{MHz} \cdot \mathrm{km}^2$, $\ell = 2\,\mathrm{km}$, $P_{\mathrm{b}} = 1.5 \cdot 10^{-9}$, $\Psi_0 = 10^{-12}\,\mathrm{V}^2/\mathrm{Hz}$)

The curves of the energies per bit show a very similar behaviour with respect to the constellation size s as the transmit power functions in Fig. 9. The reason is found in the fact that each of the transmit power curves is divided (i. e., scaled) by a constant – but for each curve different – value of the bit rate f_{B}.

In case of cable baseband transmission, i. e., the transmission via a band-limited low-pass channel, unlike in the AWGN system the energies per bit increase with rising bit rate f_{B} since the energy per bit according to (17) depends on the constellation size s via the signal bandwidth and the noise power U_{R}^2 for each bit rate f_{B} differently.

As both, the transmit power and the energy per bit show essentially very similar shapes of trajectories with respect to the constellation size s, especially leading to the same optimum constellation sizes s_{opt}, it makes no important difference whether an optimization is performed with respect to minimum energy per bit $E_{\mathrm{b}}(s)$ or minimum transmit power $P_{\mathrm{s}}(s)$. This may become of practical relevance in transmission system engineering and for energy-aware load-adaptive link operation, e. g. [2].

3) Summary of the Transmit Power and Energy-per-Bit Results: Fig. 11 and Fig. 12 illustrate the results obtained for the transmit power and the energy per bit in the linearly equalized cable transmission channel: Both, the transmit power and the energy per bit are each shown in a three-dimensional plot as a function of the constellation size s and the bit rate f_{B}, respectively.

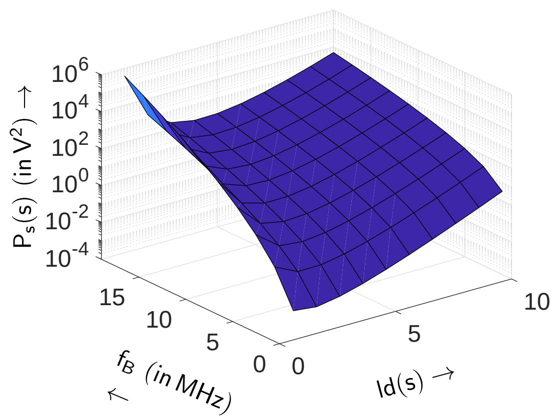

Fig. 11. Transmit power $P_{\mathrm{s}}(s)$ as a function of the constellation size s and of the bit rate f_{B} (with: $f_0 = 0.178\,\mathrm{MHz} \cdot \mathrm{km}^2$, $\ell = 2\,\mathrm{km}$, $P_{\mathrm{b}} = 1.5 \cdot 10^{-9}$, $\Psi_0 = 10^{-12}\,\mathrm{V}^2/\mathrm{Hz}$)

The behaviour of the functions (15) and (17) especially with respect to the dependency on the bit rate f_{B} and the constellation size s is recognizable: Both, the transmit power $P_{\mathrm{s}}(s)$ and the energy per bit $E_{\mathrm{b}}(s)$ ascend with increasing bit rate f_{B} (at fixed s), whereas concerning the constellation size s both quantities show a minimum at an optimum constellation size $s = s_{\mathrm{opt}}$ (at a fixed bit rate f_{B}).

4) Transmit Power and Energy-per-Bit Savings: The diagrams in Fig. 9 and Fig. 10 (and also in Fig. 11 and Fig. 12) show the results for transmit power and energy per bit in a logarithmic scale as functions of the constellation size s: Improvements are recognizable, but it is hard to estimate the magnitude of the transmit power or energy-per-bit savings, respectively.

In order to make those savings easier accessible, a relative saving in terms of transmit power or energy per bit is defined.

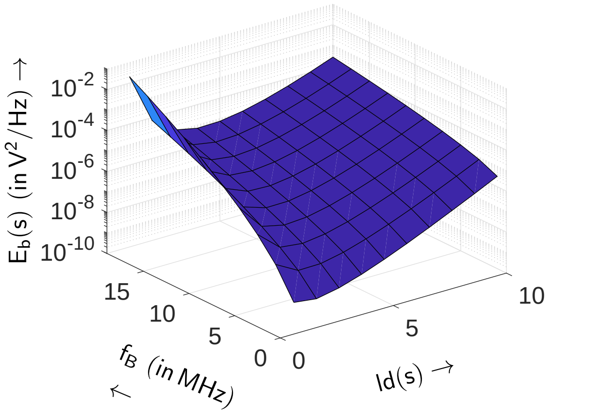

Fig. 12. Energy per bit $E_{\rm b}(s)$ as a function of the constellation size s and of the bit rate $f_{\rm B}$ (with: $f_0 = 0.178\,\text{MHz}\cdot\text{km}^2$, $\ell = 2\,\text{km}$, $P_{\rm b} = 1.5\cdot 10^{-9}$, $\Psi_0 = 10^{-12}\,\text{V}^2/\text{Hz}$)

Thereby, as a reference system the twisted-pair cable transmission system with two signalling levels ($s = 2$) and linear equalization in the receiver is used—as this is a conventional system with a simple and convenient implementation.

The relative transmit power or energy-per-bit savings are defined as

$$\varepsilon(s) = 1 - \frac{P_{\rm s}(s)}{P_{\rm s}(2)} = 1 - \frac{E_{\rm b}(s)}{E_{\rm b}(2)} \quad . \tag{18}$$

The transmit power $P_{\rm s}(2) = P_{\rm s}(s = 2)$ and the energy $E_{\rm b}(2) = E_{\rm b}(s = 2)$ describe the respective power or energy-per-bit values obtained for the reference system (with $s = 2$). For the transmit power and energy per bit functions also here the notation $P_{\rm s}(s)$ and $E_{\rm b}(s)$ is used, respectively, to emphasize their dependency on the constellation size s as compared to the values $P_{\rm s}(2)$ and $E_{\rm b}(2)$ for a single constellation size ($s = 2$) each. With (11) both terms of (18) can be transferred into each other and therefore it is irrelevant whether the savings are calculated by using the transmit power or the energy per bit according to (18)—as both are linked by the – for each curve fixed – bit rate $f_{\rm B}$.

The parameter $\varepsilon(s)$ makes the achievable savings visible. In Fig. 13 numerical results are presented: The power or energy-per-bit savings $\varepsilon(s)$ are depcited as functions of the constellation size s. Only positive values of $\varepsilon(s)$ are shown in Fig. 13 since only those are actual *savings* by definition according to (18). Negative values of $\varepsilon(s)$ arise when the required power or energy per bit, respectively, for the considered constellation size s is higher than in the reference case $s = 2$: These are not shown in Fig. 13.

It becomes obvious that at the bit rate of $f_{\rm B} = 1\,\text{MHz}$ approximately $25\,\%$ savings can be achieved by using the optimum constellation size $s_{\rm opt} = 2^2 = 4$. At higher bit rates the achievable savings are much more significant, e. g. $\approx 75\,\%$ for a bit rate of $f_{\rm B} = 3\,\text{MHz}$ – also at optimum constellation size $s_{\rm opt} = 4$ – and $> 90\,\%$ for $f_{\rm B} = 6\,\text{MHz}$ at $s_{\rm opt} = 2^3 = 8$, respectively. At the highest investigated bit rate of $f_{\rm B} = 10\,\text{MHz}$ for the given constraints transmit power or energy-per-bit savings as high as $> 95\,\%$ are registered. Here the optimum number of signalling levels is $s_{\rm opt} = 2^4 = 16$. However, also in the cases of $f_{\rm B} = 6\,\text{MHz}$ and $f_{\rm B} = 10\,\text{MHz}$ switching to the suboptimal constellation size $s = 4$ enables the largest part of possible savings—at a moderately increased implementation complexity as compared to the reference system.

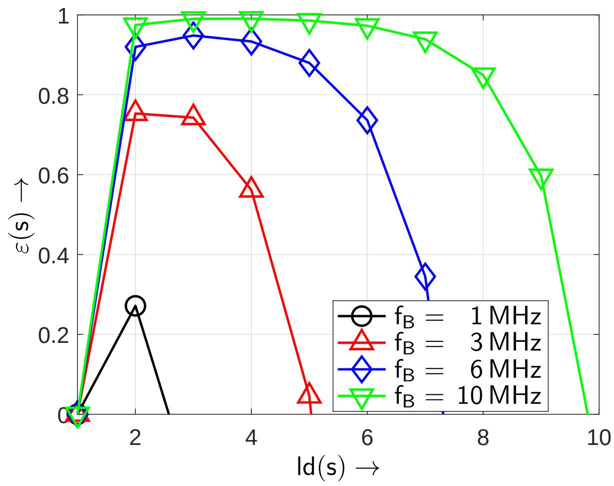

Fig. 13. Relative transmit power or energy-per-bit savings $\varepsilon(s)$, respectively, as a function of the constellation size s for several fixed bit rates $f_{\rm B}$ (parameters: $f_0 = 0.178\,\text{MHz}\cdot\text{km}^2$, $\ell = 2\,\text{km}$, $P_{\rm b} = 1.5\cdot 10^{-9}$, $\Psi_0 = 10^{-12}\,\text{V}^2/\text{Hz}$).

When considering the conventional transmission with two signalling levels ($s = 2$) as reference, the savings of constellations-size optimized transmission – with $s_{\rm opt}$ – can be already significant in a range of $25\ldots75\,\%$ at the lower bit rates and they can become as high as $> 90\,\%$ for the higher bit rates in the considered application.

Fig. 14 shows the relative savings in a three-dimensional plot as a function of both, the constellation size s and the bit rate $f_{\rm B}$, respectively. It is recognizable that there is a region especially at high bit rates (in relation to the channel's bandwidth) where significant to high savings in terms of transmit power or energy per bit, respectively, can be achieved by switching to optimum constellation sizes $s_{\rm opt} > 2$. The relative savings at a fixed constellation size s increase with rising bit rate $f_{\rm B}$. At lower bit rates only small to moderate savings are recognizable. For very low bit rates the cable transmission system approaches the behaviour of the AWGN system, where no savings are possible by switching to higher constellation sizes, i. e., the optimum constellation size is $s_{\rm opt} = 2$ leading to the lowest transmit power and energy per bit under the given constraints.

The achievable relative savings in terms of transmit power and energy per bit depend on the concrete values of the given parameters, in particular cable length ℓ (and characteristic cable frequency f_0) and bit rate $f_{\rm B}$, i. e., on the interrelationship of signal bandwidth and low-pass characteristic of the cable. The improvements by using optimum constellation sizes in terms of transmit power and energy per bit are especially high in cases when the transmission system operates at high bit rates in relation to the band limitation induced by the cable's low pass characteristic.

5) Power and Energy per Bit at Variable Cable Length: In typical telecommunication access networks twisted-pair

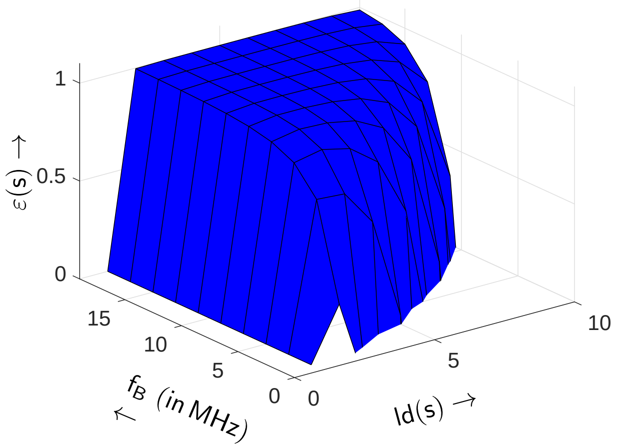

Fig. 14. Relative transmit power or energy-per-bit savings $\varepsilon(s)$, respectively, as a function of the constellation size s and of the bit rate f_{B} (parameters: $f_0 = 0.178\,\mathrm{MHz}\cdot\mathrm{km}^2$, $\ell = 2\,\mathrm{km}$, $P_{\mathrm{b}} = 1.5\cdot10^{-9}$, $\Psi_0 = 10^{-12}\,\mathrm{V}^2/\mathrm{Hz}$).

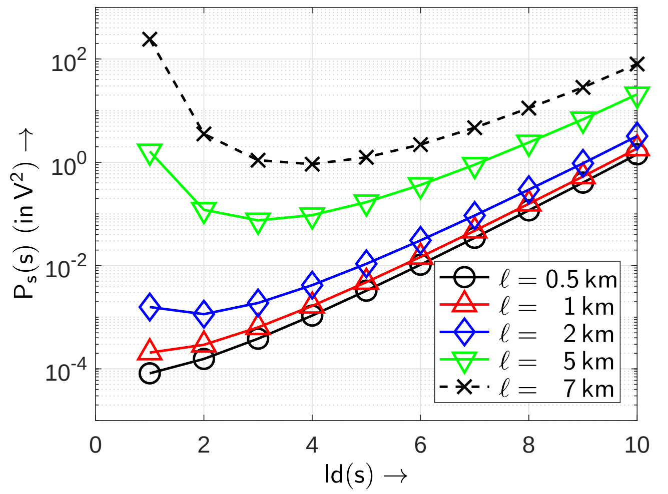

Fig. 15. Transmit power $P_{\mathrm{s}}(s)$ as a function of the constellation size s at several cable lengths ℓ (parameters: $f_0 = 0.178\,\mathrm{MHz}\cdot\mathrm{km}^2$, $f_{\mathrm{B}} = 1\,\mathrm{MHz}$, $P_{\mathrm{b}} = 1.5\cdot10^{-9}$, $\Psi_0 = 10^{-12}\,\mathrm{V}^2/\mathrm{Hz}$)

copper cables of very different length are common: On the one hand the customers connected to an access node are randomly distributed in service areas and so the distances to be bridged range from a few hundred meters to several kilometers (in Germany, for example, mostly up to a maximum of approximately $\ell = 5\ldots7\,\mathrm{km}$). On the other hand there are different access network architectures: When the customer premises are directly connected via copper lines to the access node residing in a local exchange the above mentioned distances are completely bridged by the respective copper cables and in case of, e. g., fiber-to-the-curb or fiber-to-the-building architectures remote nodes are connected by optical fibers [22], [37] and the twisted-pair copper cable usually spans a few hundred meters from the remote node to the customers' homes.

As copper cables of very different lengths are utilized in practical environments, it is of interest, what impact the cable length has on the required transmit power or energy per bit, respectively. In principle, for longer cables the low-pass effect of the channel becomes stronger and on shorter cables its low-pass impact on the signal is relaxed. Thus, the cable length ℓ is an important parameter for the transmit power and energy per bit results. In order to study the impact of the cable length, in Figure 15 and Figure 16 results are depicted for the transmit power $P_{\mathrm{s}}(s)$ and the energy per bit $E_{\mathrm{b}}(s)$ at an exemplary fixed bit rate $f_{\mathrm{B}} = 1\,\mathrm{MHz}$ as functions of the constellation size s, respectively, for different cable lengths ℓ.

At short cables – e. g. $\ell = 0.5\,\mathrm{km}$ – the transmission properties essentially resemble the principle transmission behaviour of the AWGN transmission system where the minimum transmit power and energy per bit is required at the minimum constellation size $s = 2$ — mainly caused by the fact that the exemplarily chosen bit rate is rather low for the distance of a few hundreds of meters. With increasing constellation size s the transmit power or energy per bit rises. When the cable length ℓ increases two effects are observed: The transmit power (or energy per bit) increases and the optimum constellation size s_{opt} is shifted towards higher values of s. This becomes especially obvious for the longest cable considered with a length of $\ell = 7\,\mathrm{km}$. Both of the effects are due to the stronger low-pass effect of the cable at higher cable length ℓ: The lower cable (i. e. channel) bandwidth at a fixed signal bandwidth makes a higher equalizer gain necessary in the equalized frequency range to maintain ISI-free transmission and this in turn causes a higher noise power at the detector input leading to a higher transmit power (and energy per bit) for assuring a given transmission quality.

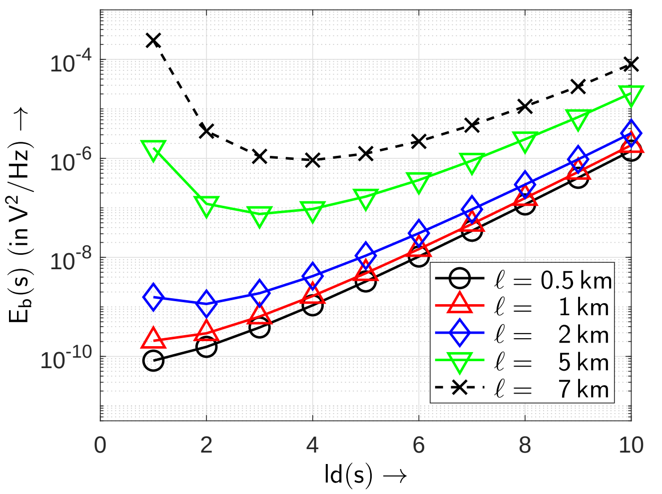

Fig. 16. Energy per transmitted bit $E_{\mathrm{b}}(s)$ as a function of the constellation size s at several cable lengths ℓ (parameters: $f_0 = 0.178\,\mathrm{MHz}\cdot\mathrm{km}^2$, $f_{\mathrm{B}} = 1\,\mathrm{MHz}$, $P_{\mathrm{b}} = 1.5\cdot10^{-9}$, $\Psi_0 = 10^{-12}\,\mathrm{V}^2/\mathrm{Hz}$)

The trajectories of the energy per bit in Fig. 16 resemble the trajectories of the transmit power in Fig. 15 in a rescaled way since the energy per bit function $E_{\mathrm{b}}(s)$ is obtained by dividing each individual transmit power curve $P_{\mathrm{s}}(s)$ by the same – exemplary – bit rate of $f_{\mathrm{B}} = 1\,\mathrm{MHz}$.

IV. Conclusions

Power and energy efficiency of baseband transmission in AWGN and twisted-pair cable channels have been studied for pulse amplitude modulated transmission systems with s

signalling levels—focussing on the dependency of the power and energy efficiency indicators transmit power $P_{\mathrm{s}}(s)$ and energy per bit $E_{\mathrm{b}}(s)$ on the constellation size s, i. e. the number of transmit signal levels or equivalently the number of transmitted bits per symbol $\mathrm{ld} s$. The transmission quality (bit error probability P_{b}), the performance capability (bit rate f_{B}) and the noise characteristic (Ψ_0) have been considered given constraints.

In the AWGN baseband transmission system the transmit power $P_{\mathrm{s}}(s)$ increases with rising constellation size s for a fixed bit rate f_{B}. Also in case higher bit rates f_{B} are required the transmit power $P_{\mathrm{s}}(s)$ increases. The energy per bit $E_{\mathrm{b}}(s)$ also rises with increasing constellation size s – but it is independent of the bit rate f_{B}.

In linearly equalized copper cable baseband transmission the transmit power $P_{\mathrm{s}}(s)$ and the energy per bit $E_{\mathrm{b}}(s)$ depend on the constellation size s, too. There is an optimum constellation size s_{opt} where the transmit power or energy per bit show a minimum. The optimum constellation size s_{opt} depends on the interrelationship between signal bandwidth and cable low-pass characteristic, i. e., the interdependency between f_{B}, ℓ (and f_0) plays a major role for the determination of s_{opt}. The results show that for exemplary but typical numerical constraints significant savings in terms of transmit power or energy per bit, respectively, can be registered when applying optimized constellation sizes in linearly equalized twisted-pair copper cable transmission systems – as compared to a basic cable baseband reference system with two signalling levels.

After discussing the basics by using an AWGN channel, the considerations in this work focussed on the transmission over a twisted-pair copper cable as a practically relevant linearly distorting transmission channel – in conjunction with pulse amplitude modulation. For future work it will be interesting to extend the investigations to other relevant transmission channels, e. g. wireless and mobile radio channels, in conjunction with suitable modulation formats, such as quadrature amplitude modulation (QAM), multicarrier modulation and MIMO systems (multiple input multiple output).

The optimization of the constellation size in transmission systems is an important pre-requisite for energy-efficient transmission of information and in turn for the operation of energy-optimized and sustainable communication networks.

Conflict of Interests

The authors declare no conflict of interests.

Author Contributions

The contributions of the authors to the work presented in this article are as follows: Conceptualization, C. L.; methodology, C. L. and A. A.; formal analysis, C. L.; investigation, C. L.; validation, C. L. and A. A.; writing—original draft preparation, C. L.; writing—review and editing, C. L. and A. A.; visualization, C. L. and A. A.; supervision, C. L. and A. A. All authors have read and agreed to the published version of the manuscript.

References

[1] S. Lambert, W. Van Heddeghem, W. Vereecken, B. Lannoo, D. Colle, and M. Pickavet, "Worldwide Electricity Consumption of Communication Networks," *Optics Express*, vol. 20, no. 26, pp. B513–B524, December 2012.

[2] C. Lange, D. Kosiankowski, A. Betker, H. Simon, N. Bayer, D. von Hugo, H. Lehmann, and A. Gladisch, "Energy Efficiency of Load-Adaptively Operated Telecommunication Networks," *Journal of Lightwave Technology*, vol. 32, no. 4, pp. 571–590, 2014.

[3] C. Lange, D. Kosiankowski, R. Weidmann, and A. Gladisch, "Energy Consumption of Telecommunication Networks and Related Improvement Options," *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 17, no. 2, pp. 285–295, 2011.

[4] R. Bolla, R. Bruschi, F. Davoli, and F. Cucchietti, "Energy Efficiency in the Future Internet: A Survey of Existing Approaches and Trends in Energy-Aware Fixed Network Infrastructures," *IEEE Communications Surveys & Tutorials*, vol. 13, no. 2, pp. 223–244, 2011.

[5] J. M. H. Elmirghani, T. Klein, K. Hinton, L. Nonde, A. Q. Lawey, T. E. H. El-Gorashi, M. O. I. Musa, and X. Dong, "Greentouch GreenMeter Core Network Energy-Efficiency Improvement Measures and Optimization," *Journal of Optical Communications and Networking*, vol. 10, no. 2, pp. A250–A269, 2018.

[6] R. S. Tucker, "Green Optical Communications Part I: Energy Limitations in Transport," *IEEE Journal of Selected Topics In Quantum Electronics*, vol. 17, no. 2, pp. 245–260, March/April 2011.

[7] ——, "Green Optical Communications Part II: Energy Limitations in Networks," *IEEE Journal of Selected Topics In Quantum Electronics*, vol. 17, no. 2, pp. 261–274, March/April 2011.

[8] H. Khodakarami and W. Shieh, "On the Energy Efficiency of Modulation Formats for Optical Communications," *IEEE Photonics Technology Letters*, vol. 25, no. 3, pp. 275–278, Februar 2013.

[9] E. Uysal-Biyikoglu, B. Prabhakar, and A. El Gamal, "Energy-Efficient Packet Transmission Over a Wireless Link," *IEEE/ACM Transactions on Networking*, vol. 10, no. 4, pp. 487–499, 2002.

[10] S. N. Roy, "Energy Logic: A Road Map to Reducing Energy Consumption in Telecommunications Networks," in *International Telecommunications Energy Conference (INTELEC)*, September 2008.

[11] B. Puype, W. Vereecken, D. Colle, M. Pickavet, and P. Demeester, "Multilayer Traffic Engineering for Energy Efficiency," *Photonic Network Communications*, vol. 21, no. 2, pp. 127–140, April 2011.

[12] F. Idzikowski, S. Orlowski, C. Raack, H. Woesner, and A. Wolisz, "Dynamic Routing at Different Layers in IP-over-WDM Networks — Maximizing Energy Savings," *Optical Switching and Networking*, vol. 8, no. 3, pp. 181–200, 2011.

[13] J. Maes and C. Nuzman, "Energy Efficient Discontinuous Operation in Vectored G.fast," in *2014 IEEE International Conference on Communications (ICC)*, 2014, pp. 3854–3858.

[14] M. Guenach, C. Nuzman, P. Tsiaflakis, and J. Maes, "Power Optimization in Vectored and Non-Vectored G.fast Transmission," in *2014 IEEE Global Communications Conference*, 2014, pp. 2229–2233.

[15] S. Lambert, P. Ananth, P. Vetter, K.-l. Lee, J. Li, X. Yin, H. Chow, J.-P. Gelas, L. Lefevre, D. Chiaroni, B. Lannoo, and M. Pickavet, "Road to Energy-Efficient Optical Access: Greentouch Final Results," *Journal of Optical Communications and Networking*, vol. 8, no. 11, pp. 878–892, 2016.

[16] A. Deylamsalehi, D. A. P. Davis, P. Afsharlar, M. Bahrami, W.-P. Chen, and V. M. Vokkarane, "Using Machine Learning to Balance Energy Cost and Emissions in Optical Networks," *Journal of Optical Communications and Networking*, vol. 10, no. 10, pp. D72–D83, Oct 2018.

[17] M. Zhu, J. Gu, and G. Li, "PWC-PON: An Energy-Efficient Low-Latency DBA Scheme for Time Division Multiplexed Passive Optical Networks," *IEEE Access*, vol. 8, pp. 206 848–206 865, 2020.

[18] E. Agrell and M. Karlsson, "Power-Efficient Modulation Formats in Coherent Transmission Systems," *Journal on Lightwave Technology*, vol. 27, no. 22, pp. 5115–5126, November 2009.

[19] D. C. Kilper, D. Neilson, D. Stiliadis, D. Suvakovic, and S. Goyal, "Fundamental Limits on Energy Use in Optical Networks," in *36th European Conference and Exhibition on Optical Communication*, no. Tu.3.D.1, 2010, pp. 1–6.

[20] C. Lange and A. Ahrens, "Power and Energy Efficiency Optimization of Baseband Transmission Links," in *Signal Processing Symposium (SPSympo)*, Lodz (Poland), September 2021, pp. 147–152.

[21] J. Maes and C. J. Nuzman, "The Past, Present, and Future of Copper Access," *Bell Labs Technical Journal*, vol. 20, pp. 1–10, 2015.

[22] P. W. Shumate, “Fiber-to-the-Home: 1977–2007,” *Journal of Lightwave Technology*, vol. 26, no. 5, pp. 1093–1103, May 2008.
[23] T. Pfeiffer, P. Dom, S. Bidkar, F. Fredricx, K. Christodoulopoulos, and R. Bonk, “PON going beyond FTTH,” *Journal of Optical Communications and Networking*, vol. 14, no. 1, pp. A31–A40, 2022.
[24] T. Starr, J. M. Cioffi, and P. J. Silverman, *Understanding Digital Subscriber Line Technology*. Upper Saddle River: Prentice Hall, 1999.
[25] J. A. C. Bingham, *ADSL, VDSL, and Multicarrier Modulation*. New York: Wiley, 2000.
[26] M. Timmers, M. Guenach, C. Nuzman, and J. Maes, “G.fast: Evolving the Copper Access Network,” *IEEE Communications Magazine*, vol. 51, no. 8, pp. 74–79, 2013.
[27] W. Coomans, R. B. Moraes, K. Hooghe, A. Duque, J. Galaro, M. Timmers, A. J. van Wijngaarden, M. Guenach, and J. Maes, “XG-fast: The 5th Generation Broadband,” *IEEE Communications Magazine*, vol. 53, no. 12, pp. 83–88, 2015.
[28] J. M. Cioffi, K. J. Kerpez, C. S. Hwang, and I. Kanellakopoulos, “Terabit DSLs,” *IEEE Communications Magazine*, vol. 56, no. 11, pp. 152–159, 2018.
[29] J. G. Proakis and M. Salehi, *Digital Communications*, 5th ed. Boston; New York; San Francisco: McGraw-Hill, 2008.
[30] A. Goldsmith, *Wireless Communications*. Cambridge: Cambridge University Press, 2005.
[31] R. F. H. Fischer, *Precoding and Signal Shaping for Digital Transmission*. New York: IEEE Press; John Wiley & Sons, 2002.
[32] D. Kreß, *Theoretische Grundlagen der Übertragung digitaler Signale [in German]*. Berlin: Akademie-Verlag, 1979.
[33] J. B. Anderson, *Digital Transmission Engineering*, 2nd ed. Piscataway, NJ: IEEE Press; John Wiley & Sons, 2005.
[34] J. Spanier and K. B. Oldham, *An Atlas of Functions*. Washington, New York, London: Hemisphere Publishing, 1987.
[35] G. B. Arfken and H. J. Weber, *Mathematical Methods for Physicists*, 5th ed. San Diego, San Francisco, New York: Harcourt/Academic Press, 2001.
[36] D. Kreß and M. Krieghoff, “Elementare Approximation und Entzerrung bei der Übertragung von PCM-Signalen über Koaxialkabel [in german],” *Nachrichtentechnik Elektronik*, vol. 23, no. 6, pp. 225–227, 1973.
[37] G. Keiser, *FTTX Concepts and Applications*. Hoboken, NJ: John Wiley & Sons, 2006.