Evolutionary Programming: A Population-Based Optimization Algorithm for Coded Multiuser Systems

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Abstract—We consider iterative channel detection and estimation for coded multiuser systems. The conventional A Posteriori Probability (APP) channel detector has a computational complexity growing exponentially with the number of users. In this paper, we study the channel detection problem from a combinatorial optimization viewpoint and derive a low-complexity soft-output channel detector based on the Evolutionary Programming (EP) optimization algorithm. An iterative channel estimator based on tentative soft estimates fed back from channel decoders is used to provide refined channel parameters to the channel detector. It is shown that the proposed iterative receiver can significantly reduce the computational complexity with slight performance degradation compared to the conventional receiver based on APP detection.

Index Terms—Evolutionary programming, combinatorial optimization, channel detection, coded multiuser systems

I. INTRODUCTION

A number of modern communication and data storage systems can be formulated by linearly correlated equations, e.g., multiuser underwater acoustic networks [1], multiple-input multiple-output (MIMO) systems [2], two-dimensional (2-D) bit-patterned media recording [3], filter-bank multi-carrier (FBMC) systems [4], solid-state non-volatile memory (NVM) devices [5], to name a few.

In the scenario of coded multiuser systems [1] [6]-[8], it is well-known that a synchronous channel can be viewed as a block code, while an asynchronous channel is equivalent to a time-varying convolutional code. This observation has significantly stimulated the research on iterative decoding techniques for coded multiuser systems. In [9], iterative receivers for coded multiuser systems were developed. The proposed receiver consists of a soft-input/soft-output (SISO) channel detector based on the a posteriori probability (APP) algorithm and a bank of channel decoders. Simulation results have shown that with iterative decoding schemes, the performance of coded multiuser systems can approach that of the channel code over additive white Gaussian noise (AWGN) channels for moderate-to-high signal-to-noise ratios (SNR). The computational complexity of the APP detector, however, is prohibitive for medium-to-large systems.

In this paper, we consider the channel detection problem from a combinatorial optimization viewpoint. To make optimal decisions for channel detection, we need to solve a binary-constrained optimization problem that is known as a binary quadratic program (BQP) in the area of optimization. It is well known that optimal multiuser detection is, in general, a non-deterministic polynomial-time (NP)-hard BQP. Various optimization algorithms have been proposed to solve the BQP approximately. Evolutionary programming (EP), which is a type of evolutionary algorithms inspired by biological evolution and natural selection, is known for its high efficiency as a global optimization procedure. EP was first proposed by Fogel in [10] as a method to generate artificial intelligence. Since then, it has been extended to process arbitrary data structures, such as applications involving continuous parameter optimization and combinatorial optimization. EP optimization incorporating self-adaptive mutations was introduced in [11]. Comprehensive investigations into various applications [12] [13] [14] suggest that evolutionary algorithms have the potentials of being applied to solve the channel detection problem over multiuser channels as well. The challenge, however, is that they converge slowly to a near-optimal solution and have high computational complexities as a large number of generations or population size (in the order of tens or hundreds) is typically used. In addition, the schemes produce only hard decisions of transmitted bits and hence are not suitable for iterative detection over coded multiuser channels. In this paper, we propose a low-complexity soft-output channel detector based on a computationally efficient implementation of the EP optimization algorithm. The proposed detector uses tentative hard estimates fed back from channel decoders to form the initial population. This new scheme gives a complexity of $O(G^{2Q+1}K)$ per bit, where $G$ is the number of generations, and $Q$ ($0 \leq Q \leq K$) is an arbitrary integer that controls the population size.

The performance of the proposed channel detector is investigated over low-density parity-check (LDPC) [15] and convolutional coded multiuser channels, respectively. We also assume that channel parameters remain constant over the frame duration. No a priori knowledge of channel statistical properties or channel noise variance is known to the receiver. Hence, we consider the joint channel detection and estimation problem in that the receiver needs to estimate channel coefficients on a block
basis. In this paper, we develop an iterative channel estimator based on tentative soft estimates fed back from decoders to provide refined parameters to the detector at each iteration. It is noted that the channel estimator itself can be viewed as an optimization procedure that searches for a better solution over one dimension at a time. Numerical results show that the proposed receiver can significantly reduce the complexity with slight performance degradation compared to the APP algorithm. Moreover, the proposed approach can be readily extended to more complex MIMO channels, which is generally viewed as a dominant solution for next-generation wireless communication systems due to their potential to achieve higher capacity, diversity gain and superior interference suppression capabilities.

This paper is organized as follows. In Section II, the system model is introduced. In Section III, a soft-output channel detector based on EP optimization is proposed. An iterative channel estimator is also developed. In Section IV, performance results over coded multiuser channels are presented. Lastly, the conclusion is drawn in Section V.

II. SYSTEM MODEL

We focus on synchronous coded multiuser channels as shown in Fig. 1. The binary data bits \( \{d_k\} \) for user \( k \), \( k=1,\cdots, K \), are encoded using a channel encoder. We assume that the same channel code is used for all users. The code bits \( \{b_k\} \) are binary-phase shift-keying (BPSK) modulated and multiplied by a normalized signature sequence \( s_i(t) \) with duration \( N \) chips. The signature sequence employed by the \( k \)th user is supported over one bit interval \([0,T_c)\) and consists of \( N \) chips, i.e.,

\[
s_i(t) = \sum_{j=0}^{N-1} s_{ij} c(t-jT_c)
\]

where \( s_{ij} \in [-1/\sqrt{N},1/\sqrt{N}] \) is uniformly distributed over each chip duration \( T_c \) with \( T_c=T_o/N \). The continuous-time waveform observed at the \( i \)th bit interval is given by

\[
r_i(t) = \sum_{j=1}^{L} \sum_{k=1}^{K} b_{ik} w_{ik} s_{ij} c(t-jT_c-iT_s)+n_i(t)
\]

where \( n_i(t) \) is additive white Gaussian noise (AWGN) with zero mean and variance \( \sigma^2=N_o/2 \). \( N_o \) is the one-sided power spectral density level, and \( w_{ik} \) is the \( k \)th user’s signal amplitude. At the receiver, the waveform is converted to a discrete-time signal by passing through a filter matched to the chip waveform followed by being sampled at the chip rate \( 1/T_c \). The output of the chip matched filter (MF) can be expressed in a matrix notation as

\[
r = S b + n
\]

where \( S_i \) is a diagonal matrix, i.e., \( S_i=\text{diag}(w_1,\ldots,w_k) \). \( S_i \) is a \( N\times K \) matrix defined in terms of the signature sequence employed by each user at the \( i \)th bit interval, i.e., \( S_i=[s_{i1},\ldots,s_{iK}] \), \( b_i=[b_{i1},\ldots,b_{iK}]^T \) is a \( K \times 1 \) vector of code bits, \( n_i=[n_{i1},\ldots,n_{iK}]^T \) is a \( K \times 1 \) AWGN vector with zero mean and auto-correlation \( E[n_i n_i^T] = \sigma^2 I_k \), and \( I_k \) is an identity matrix of rank \( K \).

Please note that (4) is a generalized formulation in the literature to model received signals over correlated channels. As described in Section IV, the proposed approach can be readily extended to more complex MIMO channels and has the potentials as well to be applied to systems in [1]-[8].

III. ITERATIVE RECEIVER

A. Full-Complexity Receiver

For channel detection over synchronous multiuser channels, it is sufficient to consider the chip matched filter output in one bit interval as shown in (3). The full-complexity multiuser detector based on the APP algorithm produces the \textit{a posteriori} log-likelihood ratio (LLR) for the \( k \)th code bit at the \( i \)th bit interval, which is given by

\[
\Lambda_{i,k}^{(m)} = \log \frac{P(b_{ik}=+1|r_i)}{P(b_{ik}=-1|r_i)}
\]

where the superscript \( m \) denotes the channel detector. Using Bayes’ rule, we can write (5) as

\[
\Lambda_{i,k}^{(m)} = \sum_{b_{ik}=-1}^{+1} P(r_i|b_{ik})P(b_{ik}) - \sum_{b_{ik}=-1}^{+1} P(r_i|b_{ik}^c)P(b_{ik}^c)
\]

where the \textit{a priori} probability of code bit vector \( b_i \) is related to its \textit{a priori} LLR \( \Lambda_{i} \) as,
It is assumed that different user’s bits are independent of each other. The first term at the right-hand side (RHS) of (7) does not depend on \( b_i \) and thus can be omitted in the calculation. The conditional probability density function (pdf) of the chip matched filter output \( r \), is given by

\[
P(r|b_i) = C \cdot \exp\left[ \frac{1}{2\sigma^2} \left( 2[S_i^T r]^T W_i b_i - b_i^T W_i S_i^T W_i b_i \right) \right] \tag{8}
\]

where \( C \) is a constant. Based on (7) and (8), we can rewrite the LLR expression as:

\[
\Lambda_{k,i} = \log \left( \frac{\sum b_{n\rightarrow i} \cdot \exp(\Omega(b_i))}{\sum b_{n\rightarrow i} \cdot \exp(\Omega(b_i))} \right) \tag{9}
\]

where the metric of binary vector \( b_i \) is given by

\[
\Omega(b_i) = \frac{1}{2\sigma^2} \left( 2[S_i^T r]^T W_i b_i - b_i^T W_i S_i^T W_i b_i \right) + \frac{1}{2} \left( r_i - b_i \right) \tag{10}
\]

The summations in the numerator and denominator of (9) are over all \( 2^{K-1} \) binary vectors with \( b_{j_1}=1 \) and \( b_{j_2}=1 \), respectively. Hence, the computational complexity of the APP detector is \( O(2^K) \) per bit, which is prohibitive for medium-to-large systems.

**B. Soft-Output EP Detector**

The computational complexity of the APP algorithm can be effectively reduced by decoupling the soft-output channel detection problem into two separate feasible stages. In the first stage, EP optimization is used to search for a near-optimal hard estimate of transmitted code bits; while in the second stage, we produce the \textit{a posteriori} information based on trial vectors searched in the EP procedure as well as those in a local neighborhood of the EP hard estimate. It can be seen that EP optimization starts with a population of potential solutions, where the population size \( N_{EP} \) is generally fixed for all generations. At each generation, each member in the population generates an offspring via mutation, and better individuals among parents and offspring are selected as parents of the next generation based on their fitness values. The cycle (mutation, selection, and replacement) as shown in Fig. 2 can be repeated until an optimization criterion is met.

![Flowchart of the proposed EP-based detector for coded multiuser channels.](image)

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1) **Initialization of the Population**

In many EP applications, the initial population of potential solutions is randomly selected from the search space. In this study, a different approach is adopted. In particular, we form tentative hard estimates \( \hat{b}_i \) of transmitted bits based on the \textit{a posteriori} LLR \( \Lambda_{i,k} \) at the output of channel decoders as

\[
\hat{b}_{i,j} = \begin{cases} +1 & \Lambda_{i,j} > 0 \\ -1 & \Lambda_{i,j} < 0 \end{cases} \tag{11}
\]

where \( k=1,\ldots,K \). The initial population is generated by identifying and perturbing the \( Q \) least reliable bits \( \hat{b}_{i_1}, \hat{b}_{i_2}, \ldots, \hat{b}_{i_q} \) in the vector \( \hat{b}_i \). Hence, the population consists of \( N_{EP}=2^Q \) members and can be expressed as a set of binary vectors \( e_l = \{b^{(l)}_i\} \), \( l = 0,\ldots,2^Q-1 \), as

\[
e_l = \begin{cases} \hat{b}_{i_{l_1}}, \ldots, +\hat{b}_{i_{l_q}}, \ldots, +\hat{b}_{i_{l_{K+1}}}, \ldots, +\hat{b}_{i_{l_{K+Q}}}, \ldots, \hat{b}_{i_{l_{K+2Q}}}, \ldots, \hat{b}_{i_{l_{K+3Q}}} \\
-\hat{b}_{i_{l_1}}, \ldots, -\hat{b}_{i_{l_q}}, \ldots, -\hat{b}_{i_{l_{K+1}}}, \ldots, -\hat{b}_{i_{l_{K+1}}}, \ldots, -\hat{b}_{i_{l_{K+2Q}}}, \ldots, -\hat{b}_{i_{l_{K+3Q}}} \\
\end{cases} \tag{12}
\]

where we define the reliability of bit \( b_{i,j} \) as the absolute value of its \textit{a posteriori} LLR \( |\Lambda_{i,j}| \). At the first iteration (i.e., iteration between the channel detector and decoders), no \textit{a priori} information is available. A conventional channel detector has to be employed to deliver the input vector \( \hat{b}_i \) to EP, for example, a minimum-mean-square-error (MMSE) multiuser detector. Compared with a solution randomly selected from the search space \{−1,+1\}^K, \( \hat{b}_i \) benefits from channel decoding efficiency and is a better estimate of transmitted bits, thus resulting in faster convergence of the algorithm.

2) **Efficient fitness evaluation of parents**

The goal of the fitness function is to evaluate the status of each member in the population. In the channel detection problem, the fitness level is evaluated based on the objective function (10) derived from the APP algorithm. The direct implementation of (10) for all \( 2^K \) members in the initial population involves \( 2^K (K+3K) \) floating-point operations, which could be computationally intensive for large values of \( K \) and \( Q \). Based on the inherent structure of the set \( e_l \), we can develop a recursive approach to compute fitness values efficiently. Note that the direct calculation is still required for the first vector in \( e_l \), that is, we have \( \Omega^{(0)} = \Omega^{(0)} \).

After some manipulation, the fitness values associated with the other \( 2^K-1 \) vectors in the initial population can be computed based on a recursive approach as

\[
\Omega^{(n)} - \Omega^{(0)} = \frac{1}{2\sigma^2} \left[ -4b_{i,j}^{(n)} w_{i,j}(s^T, r) - \sum_{k=0}^{K+Q} b_{i,j}^{(0)} \right] + \Lambda_{i,j}^{(n)} \tag{13}
\]

where \( n=2^k, \ h=0,\ldots,\ Q-1, \ l=0,\ldots,2^h-1, \ n=1,\ldots,2^Q-1 \). The recursive relation in (13) is based on the observation that \( b^{(n)}_i \) and \( b^{(0)}_i \) differ only in bit position \( j \). With this
efficient approach, the number of operations required for calculating $2^g$ fitness values is reduced to $K^2+2^K$. For example, for a system with $K=15$ and $Q=4$, the computational effort can be saved by 89%.

3) Mutation operator

There are three major operations in an evolutionary algorithm: crossover, selection, and mutation. EP places emphasis on the behavioral linkage between parents and offspring rather than emulating specific genetic operators. Hence, mutation is the main operation in EP and is introduced to maintain a bias between exploiting local neighborhood and exploring large search space. From a performance point of view, Cauchy (or Lévy) mutation might be preferable when search points are far away from the global optimum due to its high probability of making long jumps; whereas Gaussian mutation is usually adopted when search points are in the neighborhood of the global optimum due to its better fine-tuning ability. For the channel detection problem considered here, the input vector $\mathbf{b}$, to EP is obtained from channel decoders and is likely to be estimated well. Hence, we use Gaussian mutation to generate offspring for each member in the population. The EP with self-adaptive Gaussian mutation works as follows.

Each vector $\hat{\mathbf{b}}^{(i)}$, $l=0,\cdots,2^Q-1$, in the set $\mathcal{E}_l$ is assigned a real-valued $K$-tuple vector $\mathbf{\eta}^{(i)} = [\eta_k^{(i)}]$, $k=1,\cdots,K$, where $\mathbf{\eta}^{(i)}$ are standard deviations for Gaussian mutation. For each parent $\mathbf{b}^{(i)}$, we create an offspring $\hat{\mathbf{b}}^{(i)}$ according to the updating rule as

$$\hat{\eta}_k^{(i)} = \eta_k^{(i)} \exp\left[\tau N^0(0,1) + \varepsilon N_i(0,1)\right]$$

and

$$\hat{b}_k^{(i)} = \text{sign}(\hat{\eta}_k^{(i)} + \eta_k^{(i)} N_i(0,1))$$

where $N^0(0,1)$ stands for a standard Gaussian random variable and is fixed for a given member index $l$, and $N_i(0,1)$ is a Gaussian random variable generated anew for each bit position $k$. The parameters $\tau$ and $\tau'$ are commonly set to $\tau = 1/\sqrt{2N_{\text{EP}}}$ and $\tau' = 1/\sqrt{2N_{\text{EP}}}$, where $N_{\text{EP}}=2^Q$ is the population size and is assumed to be the same for all generations.

4) Selection and replacement

Selection is an operator that emulates the survival-of-the-fittest mechanism in nature. Selection in EP is usually carried out by using the tournament scheme, that is, by performing pairwise comparison over the union of parents $\mathcal{E}_l = \mathbf{b}^{(i)}$, and offspring $\mathcal{E}_l = \mathbf{b}^{(i)}$, $l=0,\cdots,2^Q-1$. For each individual member, opponents are chosen uniformly at random from all the parents and offspring. If the individual’s fitness is no smaller than that of the opponent, the individual receives a “win”. Based on comparison results, the $2^g$ members out of $\mathcal{E}_l$ and $\mathcal{E}_l$ that have the most wins are selected as parents of the next generation. Based on the above principle, we rank the parents and offspring in the descending order of their fitness values and select the first $2^g$ members to form the basis of the next generation. The fitness values of the selected members are also passed along to avoid repeated computation.

Steps 3) and 4) described above can be repeated until the maximum number of generations $G$ is reached. In the end, we have a set $\mathcal{E}$ that consists of all trial binary vectors searched in EP with cardinality $|\mathcal{E}|=2G^Q+1$, as well as the best individual member $b^{(EP)}$ in $\mathcal{E}$ that corresponds to the largest fitness value. We consider the solution $b^{(EP)}$ as a suboptimal hard estimate of transmitted bits.

5) Soft information

To be incorporated into the iterative receiver, the EP detector must produce the a posteriori LLR of code bits as the input for channel decoders. One approach is to perform the LLR calculation over the set of trial vectors $\mathcal{E}$. From (9), it can be seen that a simple restriction on $\mathcal{E}$ is that for each bit position $k=1,\cdots,K$, there must exist at least two vectors in $\mathcal{E}$ whose $k$th element are $-1$ and $+1$, respectively. Otherwise, either the numerator or the denominator of (9) becomes zero, thus resulting in an infinite LLR. To ensure that this requirement is met as we are mainly interested in using a few generations in EP optimization for the sake of lower computational complexity, we propose to include another set of $K$ binary vectors in the LLR calculation. A heuristic scheme to form these vectors is based on the 1-opt local search neighborhood $\xi$ of the EP hard estimate $b^{(EP)}$,

$$\xi = \{ b_l \in [-1,1]^K \| b_l - b^{(EP)} \|_r \leq 1 \}$$

where $\|b\|_r$ denotes the Hamming weight of its vector argument. The set $\xi$ can be expressed as

$$\xi = \{ (-b_{k,1}^{(EP)}, b_{k,2}^{(EP)}, \cdots, b_{k,|K|}^{(EP)}), \} \begin{array}{c}
\begin{array}{c}
(b_{k,1}^{(EP)}, b_{k,2}^{(EP)}, \cdots, b_{k,|K|}^{(EP)})
\end{array}
\end{array}.$$
C. Channel Estimation

In the development of the EP channel detector, we assume that the receiver has the knowledge of users’ signal amplitudes. In this section, we develop an iterative channel estimator based on soft estimates fed back from channel decoders to provide channel parameters to the detector.

1) Unbiased EM channel estimator

Let us define a $K \times 1$ vector $\mathbf{w} = [w_1, \ldots, w_K]^T$ as $K$-users’ signal amplitudes to be estimated for the current frame. It is well known that the joint maximum-likelihood (ML) detection and estimation is computationally prohibitive; while the accuracy of the data-aided estimate based on short training sequences is rather low for practical applications. On the other hand, the expectation-maximization (EM) algorithm [16], which can be viewed as a unidimensional optimizer, provides us with a feasible approximate solution to the ML problem by iteratively updating channel estimates.

After some manipulation, we obtain the unbiased channel estimates as

$$\hat{\mathbf{w}}^{(t)} = \left[ \mathbf{R}^{(t)} \right]^{-1} \mathbf{Y}^{(t)}$$

(19)

where the $K \times K$ correlation matrix $\mathbf{R}^{(t)}$ and the $K \times 1$ vector $\mathbf{Y}^{(0)}$ are given by, respectively,

$$\mathbf{R}^{(t)} = \frac{1}{L} \sum_{\tau = 0}^{\tau_{\text{max}}} \text{diag} \left( \mathbf{b}_{\tau}^T \mathbf{s}_{\tau} \mathbf{s}_{\tau}^T \text{diag} \left( \mathbf{b}_{\tau} \right) \right)$$

(20)

and

$$\mathbf{Y}^{(t)} = \frac{1}{L} \sum_{\tau = 0}^{\tau_{\text{max}}} \text{diag} \left( \mathbf{b}_{\tau}^T \mathbf{s}_{\tau} \mathbf{r}_{\tau} \right)$$

(21)

2) Steepest descent implementation

A direct implementation of the unbiased channel estimate in (19) involves the computation of the matrix inverse at each iteration. To reduce the computational complexity, we could simply approximate the matrix $\mathbf{R}^{(t)}$ as a diagonal matrix. This approximation, however, is based on the assumption that mutually orthogonal signature sequences are used in the system. Simulation shows that this approximation results in significant performance degradation.

On the other hand, note that the product $\hat{\mathbf{w}}^{(t)} = \left[ \mathbf{R}^{(t)} \right]^{-1} \mathbf{Y}^{(t)}$ can be approximated by solving the linear equation $\mathbf{R}^{(t)} \hat{\mathbf{w}}^{(t)} = \mathbf{Y}^{(t)}$ using iterative methods like the steepest-descent (SD) algorithm. At the $r$th iteration, the algorithm performs the following computation

$$\hat{\mathbf{w}}^{(t,r+1)} = \hat{\mathbf{w}}^{(t,r)} - 2\mu \left( \mathbf{R}^{(t,r)} \hat{\mathbf{w}}^{(t,r)} - \mathbf{Y}^{(t)} \right)$$

(22)

where the parameter $\mu$ is the step size chosen to ensure convergence of the algorithm. Since channel estimation is performed only once per iteration, we use the same superscript $t$ to denote both EM iteration and channel iteration in the receiver. The superscript $(t,r)$ denotes the quantities obtained at the $r$th steepest-descent search step for the $t$th channel iteration. The updating step in (22) can be repeated as many times as allowed by the computational resources. In this paper, the maximum number of search steps is set to $\tau_{\text{max}} = 2$.

We can improve the convergence speed of the steepest-descent channel estimator by choosing the optimal step size $\mu_{\text{opt}}$ as [17]

$$\mu_{\text{opt}} = \frac{1}{\rho_{\text{max}} + \rho_{\text{min}}}$$

(23)

where $\rho_{\text{max}}$ and $\rho_{\text{min}}$ are the maximum and minimum eigenvalues of the symmetrical matrix $\mathbf{R}^{(t)}$, respectively.

To find the optimal step size, we need to solve an eigenvalue problem that is usually as costly as solving the linear equation itself. To circumvent this problem, we can use the Gershgorin circle theorem [18] to obtain approximate extreme eigenvalues of $\mathbf{R}^{(t)}$. The theorem delivers an interval $[(\hat{\rho}_{\text{min}}, \hat{\rho}_{\text{max}})]$ such that $\hat{\rho}_{\text{min}} \leq \rho_{\text{min}}$ and $\hat{\rho}_{\text{max}} \geq \rho_{\text{max}}$.

$$\hat{\rho}_{\text{min}} = \max \left( 0, \min_{j} \left( R_{j,j}^{(t)} - \sum_{j \neq j} |R_{j,j}^{(t)}| \right) \right)$$

(24)

$$\hat{\rho}_{\text{max}} = \max \left( R_{j,j}^{(t)} + \sum_{j \neq j} |R_{j,j}^{(t)}| \right)$$

(25)

The optimal step size $\mu_{\text{opt}}$ is thus approximated by using $\hat{\rho}_{\text{min}}$ and $\hat{\rho}_{\text{max}}$.

The steepest-descent algorithm requires an initial estimate $\hat{\mathbf{w}}^{(t,0)}$ as an input vector. Since each channel iteration provides refined estimates, we use the estimate from the most recent channel iteration $\hat{\mathbf{w}}^{(t+1,\tau_{\text{max}})}$ as the initial estimate for the current $t$th iteration, i.e., by setting $\hat{\mathbf{w}}^{(t,0)} = \hat{\mathbf{w}}^{(t+1,\tau_{\text{max}})}$. At the first iteration, however, no a priori information is available. In this case, the initial estimate can be evaluated in a data-aided fashion by inserting training sequences into each user’s transmitted frame.

D. Receiver Structure

The proposed receiver consists of three main building blocks: a soft-output EP channel detector, an iterative channel estimator, and a bank of decoders. The proposed receiver involves three iterative processes.

For each overall iteration between the channel detector and decoders, the EP detector operates on $G$ generations of potential solutions each with population size $2^Q$; while the channel estimator performs $\tau_{\text{max}}$-step steepest-descent search to update channel parameters. When LDPC codes are used in the system, the LDPC decoder itself involves the maximum of $P$ decoding iterations to produce hard decisions of data bits as well as the a posteriori LLR of LDPC code bits $\mathbf{X}$. The details of the sum-product algorithm (SPA) for LDPC decoding may be referred to [15]. In addition to the a priori information $\mathbf{X}$, the channel detector, the receiver also feeds back soft estimates of coded bits $\mathbf{\hat{b}}$ to the channel estimator as well as tentative hard estimates $\mathbf{b}$ to the EP multiuser detector as an input vector. Hence, the proposed receiver benefits from channel decoding efficiency and time
diversity brought by random interleaving. Simulation results in Section IV will show that the proposed receiver converges with a few generations and significantly reduces the computational complexity with slight performance degradation compared to the APP algorithm.

IV. PERFORMANCE RESULTS

A. Full-Complexity Receiver

In this section, we present simulation results of the proposed iterative receiver for synchronous LDPC coded multiuser systems. All the users in the system employ the same rate-1/2 (504,252) random LDPC code with column weight 3 and are assumed to have the same received power. Each user uses a different random interleave that is updated for every frame transmission. The MMSE detector is used as the first stage to deliver initial hard estimates to the EP detector.

First, we consider the iterative receiver based on the full-complexity APP algorithm (i.e., full-complexity iterative receiver) over a 5-user channel with random signature sequences of length 7, i.e., \( K=5 \) and \( N=7 \), where we assume that the receiver has the perfect knowledge of channel amplitudes and channel noise variance. Fig. 3 shows the bit-error-rate (BER) performance of the receiver at the 8th iteration. In the simulation, the maximum number of local iterations inside LDPC decoders varies from \( P=5 \) to \( P=30 \) per overall iteration. For comparison purposes, we also present the performance of the receiver over a 16-state (31, 33) convolutional coded channel (curve labeled “Conv”) as well as the performance of single-user (SU) LDPC decoding with 30 decoding iterations \( (P=30) \). It is shown that the number of local LDPC iterations \( P \) has an effect on BER performance. Increasing \( P \) from \( P=5 \) to \( P=7 \) results in a performance gain of about 0.5 dB at BER of \( 10^{-5} \), while increasing \( P \) further to \( P=10 \) brings an additional gain of 0.2 dB. Compared with the convolutional coded system, the performance of the LDPC coded system is much better for moderate-to-high SNR. At BER of \( 10^{-5} \), the performance gain with \( P=30 \) is about 1.5 dB. Fig. 3 shows that with 30 local iterations, the system can approach closely the performance of the LDPC code.

![Fig. 3. BER performance of the full-complexity iterative receiver for a \( K=5, N=7 \) LDPC coded multiuser system with perfect channel knowledge.](image-url)

Next, we consider the performance of the iterative receiver based on the EP optimization. Fig. 5 shows the performance of a LDPC coded system with \( K=5 \) and \( N=7 \), where perfect channel knowledge is assumed at the receiver. In the following simulations, we set the maximum number of local iterations within LDPC decoders to \( P=30 \). A set of mutually orthogonal training sequences of length 12 is used to provide the data-aided channel estimate at the first overall iteration. Fig. 4 shows that approximating the correlation matrix \( R^{(o)} \) in (20) as a diagonal matrix (curve labeled “Diag. Approx.”) results in the worst performance among several channel estimation methods. On the other hand, the channel estimator provides a gain of 2 dB at BER of \( 10^{-5} \) at the expense of the matrix inverse computation. However, due to the bias of channel estimates, there is still a performance loss of 1 dB compared to the system with perfect channel knowledge (curve labeled “Perfect Est.”). Fig. 4 shows that we can further improve BER performance by using the steepest-descent (SD) algorithm in (22) to realize unbiased channel estimation. It is shown that the receiver with 2-step channel estimation has negligible performance loss at BER of \( 10^{-5} \).

In Fig. 4, we present the performance of the receiver for the same system considered in Fig. 3. However, no knowledge of channel parameters is assumed here at the receiver. For comparison purposes, we fix the number of local iterations within LDPC decoders to \( P=30 \). A set of mutually orthogonal training sequences of length 12 is used to provide the data-aided channel estimate at the first overall iteration. Fig. 4 shows that approximating the correlation matrix \( R^{(o)} \) in (20) as a diagonal matrix (curve labeled “Diag. Approx.”) results in the worst performance among several channel estimation methods. On the other hand, the channel estimator provides a gain of 2 dB at BER of \( 10^{-5} \) at the expense of the matrix inverse computation. However, due to the bias of channel estimates, there is still a performance loss of 1 dB compared to the system with perfect channel knowledge (curve labeled “Perfect Est.”). Fig. 4 shows that we can further improve BER performance by using the steepest-descent (SD) algorithm in (22) to realize unbiased channel estimation. It is shown that the receiver with 2-step channel estimation has negligible performance loss at BER of \( 10^{-5} \).

Next, we consider the performance of the iterative receiver based on the EP optimization. Fig. 5 shows the performance of a LDPC coded system with \( K=5 \) and \( N=7 \), where perfect channel knowledge is assumed at the receiver. In the following simulations, we set the maximum number of local iterations within LDPC decoders to \( P=30 \). Three soft-output EP detectors with \( G=1 \) and \( Q=1, 2, 3 \) are considered in the simulation, respectively, where \( G \) denotes the number of generations and \( Q \) determines the population size (i.e., \( N_{EP}=2^Q \)). For this system, EP converges with 1 generation and increasing the value of \( G \) results in similar performance.
The proposed EP detector with $G=1$ and $Q=1$, 2, 3 searches 9, 13, 21 binary vectors in the LLR calculation, respectively. For comparison purposes, the performance of the full-complexity (FC) iterative APP receiver at the 8th iteration is also presented. Compared with the full-complexity receiver, the EP receiver with $Q=1$ has a performance loss of 0.5 dB at BER of $10^{-2}$. However, the performance loss becomes negligible for moderate SNR and the receiver with $Q=3$ approaches the LDPC code performance within 0.2 dB at BER of $10^{-3}$. Fig. 5 shows that the performance can be improved by increasing the population size. The receiver with $Q=3$ performs close to the LDPC code performance within 0.1 dB at BER of $10^{-3}$ and has a negligible loss at BER of $10^{-5}$. In Fig. 6, we present the performance of the EP receiver for the same 5-user system, where a 2-step steepest-descent (SD) algorithm is used to estimate channel parameters. Compared with the receiver with perfect channel knowledge (curve labeled “Perfect Est.”), it can be seen that the receiver can almost fully compensate the effect of imperfect channel estimates. At BER of $10^{-5}$, the receiver with $Q=1$ and $Q=3$ can approach SU performance within 0.3 dB and 0.1 dB, respectively.

Fig. 5. BER performance of the iterative EP receiver for a $K=5$, $N=7$ LDPC coded multiuser system with perfect channel knowledge.

Fig. 6. BER performance of the iterative EP receiver for a $K=5$, $N=7$ LDPC coded multiuser system with estimated channel parameters.

In Fig. 7, we consider a larger LDPC coded system with $K=10$ and $N=15$, where perfect channel knowledge is assumed at the receiver. For a 10-user system, the full-complexity iterative APP receiver is infeasible for practical implementation, since it searches through $2^{10}$ binary vectors per iteration. The EP receiver, however, has a reasonable and adjustable computational complexity, and can be easily implemented for large systems. The receiver with $Q=2$ (corresponding to population size of 4) and various number of generations $G=1$, 2, 4 searches 18, 26, and 42 binary vectors per iteration, respectively; while the receiver with $G=1$, $Q=1$, and $G=2$, $Q=5$ searches 14 and 138 vectors in EP optimization, respectively. Fig. 7 shows that increasing the values of $G$ and $Q$ results in better performance as more trial vectors are used in the LLR calculation. In Fig. 8, we present the performance of the receiver with estimated channel parameters for the same 10-user system. In the simulation, the number of EP generations is set to $G=3$, while the value of $Q$ varies from 1 to 4 (corresponding to population size $2^Q$). Hence, the EP receiver searches 22, 34, 58, and 106 binary vectors out of 1024 vectors in EP optimization, respectively. Fig. 8
shows that the receiver with $G=3$ and $Q=2$ can approach SU performance within 0.5 dB at BER of $10^{-5}$, while increasing the value of $Q$ to 3 reduces the gap to 0.2 dB.

In Fig. 9, we present the performance of a LDPC coded system with $K=20$ and $N=31$, where a 2-step steepest-descent (SD) channel estimator is used in the system. It is shown that the receiver with $G=3$ and $Q=2$ has a performance loss of 1 dB from that of the single-user system at BER of $10^{-5}$, while increasing the value of $Q$ to 3 brings a gain of 0.3 dB. At BER of $10^{-5}$, the receiver with $G=3$ and $Q=4$ approaches single-user performance within 0.5 dB. For this system, the reduction in the computational complexity is significant. The receiver with $G=3$ and $Q=1, 2, 3, 4$ searches 32, 44, 68, and 116 binary vectors in EP optimization, respectively, compared with $2^{20}=1,048,576$ vectors for the full-complexity APP algorithm.

![Fig. 9. BER performance of the iterative EP receiver for a $K=20$, $N=31$ LDPC coded multiuser system with estimated channel parameters.](image)

**B. EP Receiver for MIMO Channels**

In this section, we present numerical results to demonstrate the performance of the proposed receiver over coded multiuser MIMO channels. It is assumed that the channel is quasi-static. All users employ the same rate-1/2 convolutional code with generator polynomial $[23_8, 25_8]$ in octal notation. The code bit frame size is 1008, which corresponds to 504 QPSK symbols. In addition, we assume that the number of receive antennas is the same as the number of $K$ users’ transmit antennas. Furthermore, all users transmit their symbols with equal power, a scenario that causes severe multiple-access interference from the interference suppression viewpoint. At the first iteration, the MMSE multiuser detector is used as a front stage to deliver initial estimates of transmitted code bits. The probability of mutation in the EP optimization is set to $p_m=0.1$. We vary the population size $P$ and the number of generations $G$ to study their effect on BER performance.

First, we consider a multiuser MIMO system transmitted over unknown channels where the number of users and the number of transmit antennas per user are $K=2$ and $N=2$, respectively, and the total number of receive antennas is $M_r=4$. The simulation is tested for 8 iterations. For clarity, only performance curves at the 8th iteration are presented in Fig. 10. The curve labeled “Pilot only” corresponds to the receiver that performs channel estimation only once based on pilot symbols. The other solid curves (labeled “*est.”) correspond to the receiver that estimates the channel at each iteration based on both pilot symbols and soft-estimated symbols fed back from channel decoders. For comparison purposes, we also include in Fig. 10 the performance curve of a single-user iteratively MAP-decoded system that is equipped with the same number of receive antennas as that in the multiuser system (i.e., $N_r=2$ and $M_r=8$) over perfectly known channels (curve labeled “SU MAP, known”). As in the context of multiuser systems [16], it is reasonable to view the performance of the iteratively MAP-decoded single-user system as a lower bound of coded multiuser MIMO systems. Fig. 10 shows that with an increasing population size of $P=2^P$, the proposed receiver gradually approaches the performance of the single-user system. The receiver with $Q=5$, $G=2$, and $N=1$ searches over 112 candidate bit vectors and performs close to the single-user system within 0.5 dB at BER of $10^-5$.

![Fig. 10. Performance of the EP-based iterative receiver for a rate-1/2 coded multiuser MIMO system with $K=4$, $N=2$, and $M_r=8$ over unknown channels.](image)

In Fig. 11, we consider an 8-user coded MIMO system with $N_r=2$ transmit antennas per user and a total of $M_r=16$ receive antennas. The performance of the iteratively MAP-decoded single-user (SU) system with $N_r=2$ and $M_r=16$ over perfectly known channels is shown in the figure. Fig. 11 shows that the proposed receiver performs much better than the scheme that uses only pilot symbols in channel estimation. Furthermore, even with estimated channel knowledge, the receiver with $Q=3$ (corresponding to population size $P=8$) approaches the single-user (SU) performance over perfectly known channels within 0.5 dB at BER of $10^{-5}$ by searching only 56 candidate vectors, which is a tiny fraction of $2^{32} \approx 4.3 \times 10^9$, the total number of hypotheses required for
the MAP multiuser detector. In Fig. 12, the averaged channel estimation error up to the 15th iteration is presented at SNR=−2.0, −0.5, and 2.0 dB, respectively. It is shown that with the proposed receiver, the channel estimation error is reduced significantly with increasing iterations.

<table>
<thead>
<tr>
<th>Q</th>
<th>G</th>
<th>N</th>
<th>Known</th>
<th>Est.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>SU MAP, known</td>
<td>Q=3,G=2,N=1, est.</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>Q=2,G=2,N=1, est.</td>
<td>Q=3,G=2,N=1, est.</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>Q=4,G=2,N=1, est.</td>
<td>Q=4,G=2,N=1, Pilot only</td>
</tr>
</tbody>
</table>

3.0, 1.5, and 2.0 dB, respectively. It is shown that with the proposed receiver, the channel estimation error is reduced significantly with increasing generations and can significantly reduce the computational complexity with slight performance degradation compared to the APP algorithm. The proposed approach is shown to be readily generalized to more complex systems, e.g., coded multiuser MIMO channels and achieve an impressive performance/complexity tradeoff.

**CONFLICT OF INTEREST**

The authors declare no conflict of interest.

**AUTHOR CONTRIBUTIONS**

Yu Qin and Zhiliang Qin conducted the research and experiments on combinatorial optimization and prepared the paper; Zhongkai Zhang, Yingying Li, and Qidong Lu contributed to the simulation studies with respect to the full-complexity receiver. All authors had approved the final version.

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