On Golden Code Performances in Impulsive Noise MIMO Channels

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Abstract —Wireless technologies are the ones who provide fast communications on long distances. MIMO (Multiple-Input-Multiple-Output) Systems become an important factor in communication standards. At the beginning, this technology involved several antennas for both transmission and reception. Nowadays, in order to improve the performance of wireless transmissions, in addition to multi-channel propagation, error correction codes are used. Golden Code has some advantages, such as: maximum rate and diversity or coding gain. In this paper it is proposed an approach of using this code to mitigate the impulsive noise effects in a MIMO communication system. The Middleton Class-A noise was considered. The simulation was done for different values of the impulsive noise model parameters and showed that the probability density function depends on index impulse and on gaussian factor and the number of noise sources has no influence.

Index Terms—MIMO system; Golden Code; impulsive noise; Middleton Class-A

I. Introduction

The conventional transmission systems using single antenna on transmitter and receiver use techniques that operate either in time domain, in frequency domain or in both. In this case, in order to overcome the effects of fading due to the multipath propagation different channel coding methods are used.

Because the number of wireless devices and services users has increased, the channel capacity demands increased, so the MIMO (Multiple–Input–Multiple-Output) systems have gained significant importance. The use of communications systems with multiple antennas has been shown a successful and effective approach for improving the communication performance.

A MIMO system refers to a wireless system containing one or multiple links. Wireless MIMO communications systems use spatial multiplexing in order to increase spectrum efficiency and antenna diversity for improving link reliability [1]-[3]. This is one of the essential benefits of the system.

In order to improve the security of data transmissions in MIMO systems, especially those with high speeds, by increasing spatial diversity and minimizing the possibility of errors, space-time codes have been proposed [4].

One of the best choices for space-time codes is the Golden Code [5] which is a full rate with full diversity 2x2 block code with linear dispersion. This code was built using cyclic division algebras. Thanks to its algebraic structure the Golden Code is provided with a sum of exceptional properties such as: non-vanishing minimum determinant and cubic shaping.

This code has been proposed as an optimum 2x2 Space-Time Block Code (STBC) with linear dispersion for a 2 x 2 MIMO system. [6]

Although the Golden Code is better than the Alamouti Code on flat fading channels, in the case of an environment with frequency selective channels the Golden Code loses its properties, because of the inter-symbol interference [7]. In order to cut down this disadvantage and increase performance of the code, OFDM (orthogonal frequency division multiplexing) modulation is used [8].

In [9] the performance of MIMO-OFDM systems using golden codes is analyzed after using an ICI (inter carrier interference) SC (self-cancelation) technique. Here, the BER performance of the MIMO-OFDM system using Golden Code is shown in the case of using and not using ISI-SC. The result shows that the performance is improved significantly due to the reduction of ICI coefficients.

Also, in [10] is presented a solution for modifying the Golden Code in order to use a less complex sphere decoder without compromising the error rates.

The most used model for representing noise in a communication system is the AWGN (Additive White Gaussian Noise) model, which represents the thermal noise arriving at the receiver. This kind of representation ignores the impulsive appearance of electromagnetic interferences, atmospheric noise or noises caused by humans using other machines.

When discussing of indoor wireless communications, impulsive noise could appear due to devices that use electromechanical switches such as refrigerators, electrical motors of elevators or due to other devices that could produce impulsive noise on the same frequency bands as the operating frequencies of the wireless transceivers. Impulsive noise is thus one of the important factors that impact the performance of a wireless transmission system [11], [12]. Space time block codes improve the performance systems on channel affected by this type of noise [13].

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Many models have been used for representing the impulsive noise. In [14] are discussed two models without memory (Middleton Class A and Bernoulli-Gaussian) and two models with memory (Markov-Middleton and Markov-Gaussian).

All the above tests for Golden Code was done for channels affected by Additive White Gaussian Noise (AWGN). In this paper, the abovementioned code was used to mitigate the impulsive noise effects in MIMO systems. We use a Belfiore-Rekaya-Viterbo Golden Code [6] and the Middleton Class-A Impulsive noise model. The simulation results for impulsive noise are compared with those for gaussian one.

The rest of the paper is organized as follow: Section II presents the model for the non-gaussian noise, in Section III is the system design. The simulation results are highlighted in Section IV and Section V concludes the paper.

II. Middleton Class-A Noise

One of the important issues that appears in wireless communication systems is the perturbations that affect the transmitted information. There are several causes of signal degradation, such us: the multipath propagation or the noise [15]. The multipath waves are reflected replies of the transmitted signal who hit the surrounding objects and the phenomenon is called fading. In this paper, we used Rayleigh model to describe it.

The noise who appears on the communication channel can be [15]:

- External - radiations, industrial noise or made by the spark plugs etc. This type of noise is called impulsive or non-gaussian noise. There are several models to describe it: Middleton Class-A, Class-B or Class-C; Bivariate Class-A; Alpha-stable or Mixture Gaussian [16].
- Internal – noise produced by the electronic devices used in the receiver.

From the above models we used in this paper the first one: Middleton Class-A. This type of noise is additive to the transmitted information, so we named the channel affected by it AWCN (Additive White Class-A Noise). The noise sources are multiple, the model includes the white noise from the receiver and the interferences between antennas are not correlated. The probability density function (pdf) it is [17]:

$$p(n)=\sum_{m=0}^{\infty}\frac{A^m e^{-A}}{\sqrt{2\pi}m!\sigma_m}\exp\left(-\frac{n^2}{2\sigma_m^2}\right) \quad (1)$$

From (1) it can be observed that it is a sum of independent interferences. This type of noise has two components: a gaussian one and an impulsive one. The parameters have the following significations:

- m – is the number of noise sources;
- A ϵ [10-2, 1] is the average number of impulses;
- $\sigma_m^2=\sigma^2\cdot\dfrac{\frac{m}{A}+T}{1+T}$, $\sigma^2=\sigma_g^2+\sigma_i^2$, σ_g^2 - is the gaussian variance and σ_i^2 is the impulsive variance,
- $T=\dfrac{\sigma_g^2}{\sigma_i^2}$ ϵ [10-6, 1] - is the gaussian factor.

A and *T* show how impulsive is the noise. When *A* and/or *T* have low values, the noise is highly impulsive and when *A*=*T*=1, the noise is additive white gaussian.

III. System Design

A N_T x N_R MIMO system has N_T antennas at transmitter and N_R antennas at receiver. The relation that describe it is:

$$Y = H\ X + N \quad (2)$$

where *Y* is the received signal, *H* is the channel matrix, *X* represents the transmitted codeword and N is the noise vector. The *H* matrix contains the Rayleigh fading coefficients considered random complex Gaussian variables and *N* has noise samples (gaussian or impulsive).

Antenna *j* receive at moment *t* the signal:

$$y_j^t=\sum_{i=1}^{N_T}h_{ji}^t\cdot x_i^t+\eta_j^t \quad (3)$$

In this paper we considered a 2x2 MIMO system like in Fig. 1:

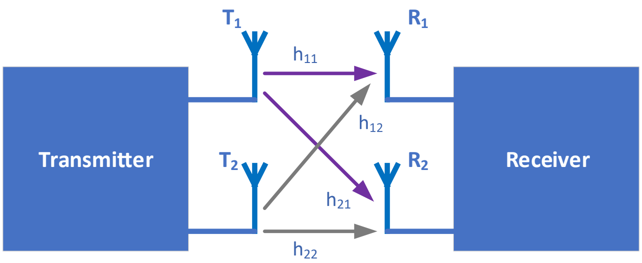


Fig. 1. 2x2 MIMO system.

In order to improve the data transmissions, by increasing spatial diversity and minimizing the error probability, space-time codes have been used in MIMO systems. One of these are Golden codes [6], that achieve the diversity-multiplexing gain [18]. Due to their advantages: maximum rate and diversity, spatial-temporal coding gain, they have been called perfect codes [19].

The Golden codes are based on the gold number $\theta=\frac{1+\sqrt{5}}{2}\cong 1.618$, one of the roots of the equation: $\theta^2-\theta-1=0$. The codeword matrix *X* in this case is defined as below [6]:

$$X=\frac{1}{\sqrt{5}}\begin{bmatrix}\alpha[a+b\theta] & \alpha[c+d\theta]\\ i\sigma(\alpha)[c+d\sigma(\theta)] & \sigma(\alpha)[a+b\sigma(\theta)]\end{bmatrix} \quad (4)$$

where *a*, *b*, *c*, *d* are the information symbols from QAM constellation, $i=\sqrt{-1}$, $\sigma(\theta)=\frac{1-\sqrt{5}}{2}=1-\theta$, $\sigma\in Q[i]$, $\alpha=1+i-i\theta$.

Replacing the variables defined above in relation (4), the *X* matrix becomes:

$$X=\frac{1}{\sqrt{5}}\begin{bmatrix}[1+i\sigma(\theta)]a+[\theta-i]b & [1+i\sigma(\theta)]c+[\theta-i]d \\ [i-\theta]c+[1+i\sigma(\theta)]d & [1+i\theta]a+[\sigma(\theta)-i]b\end{bmatrix} \quad (5)$$

At transmitter, the symbols will be elements from a column matrix. Thus, the matrix will be written:

$$X=\frac{1}{\sqrt{5}}\begin{bmatrix}[1+i\sigma(\theta)]a+[\theta-i]b \\ [i-\theta]c+[1+i\sigma(\theta)]d \\ [1+i\sigma(\theta)]c+[\theta-i]d \\ [1+i\theta]a+[\sigma(\theta)-i]b\end{bmatrix} \quad (6)$$

At receiver, a maximum likelihood (ML) algorithm is used. The decoder estimates the transmission matrix:

$$\hat{X}=\arg\min_{X}\|Y-HX\|^2 \quad (7)$$

IV. Simulation Results

The simulations were done for a 2x2 MIMO system with Golden code and 4-QAM modulation. We considered a channel affected by Rayleigh fading and two types of noise: additive gaussian white noise and impulsive noise described with Middleton Class-A model.

First, we have analyzed the impulsive noise in terms of the parameters influence on its pdf. In the second subsection, the Golden code performances were evaluated for different values of the noise model parameters. All the simulations were compared with the AWGN channel.

A. *Middleton Class-A Noise*

In this subsection we generated with Matlab Toolbox from [16] some impulsive noise samples and pdf families for different values of *A* and *T*, compared with AWGN. These are represented in Fig. 2. If *A* or *T* are 1, the impulsive noise is almost gaussian. If *A* or *T* have low values, the impulses are more frequent, so the noise is more impulsive. For *A*=*T*=0.01 the impulses are rarer but with greater amplitude.

The influence of the number of noise sources *m* on the probability density function for AWCN channel is analyzed and represented in Fig. 3a). For 2 < *m* < 10, a family of 9 pdfs was estimated and illustrated for *A*=*T*=0.1. It can be observed that the pdfs almost overlap, the differences are very small, and practically the distributions depend very little on the *m* value.

In Fig. 3b) are the 3D representations of the pdfs. These allow distinct observation of each distribution. In this figure it can be observed very clear that the distributions are practically the same for all *m* values.

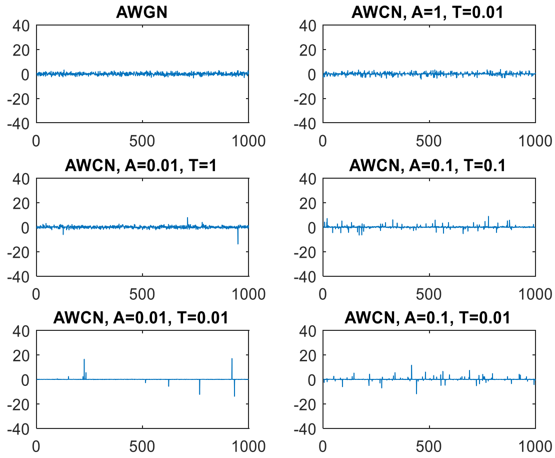


Fig. 2. Noise samples.

In Fig. 4 and Fig. 5 are 2D and 3D representations for AWCN probability density function families for *T*=0.1 and *A* = 0.01:1.01 and *A* = 0.1 and *T* = 0.01:1.01 respectively. Both parameters have an important influence on the probability density function; as their values decrease, the pdf becomes sharper and increases in amplitude.

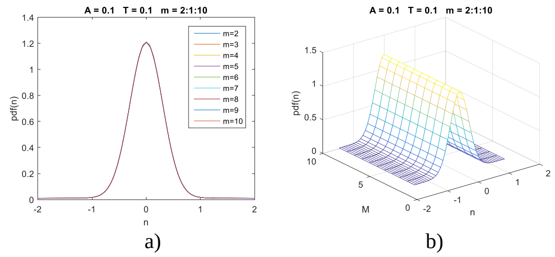


Fig. 3. The *m* influence on AWCN pdf.

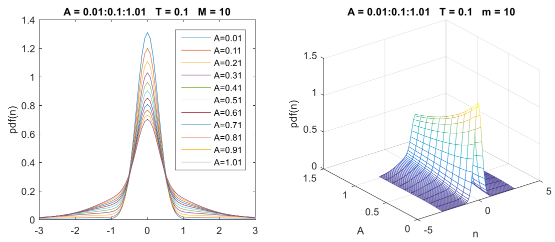


Fig. 4. AWCN pdfs for different values of *A*

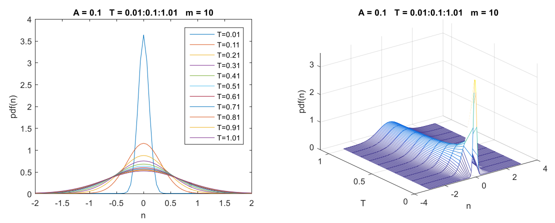
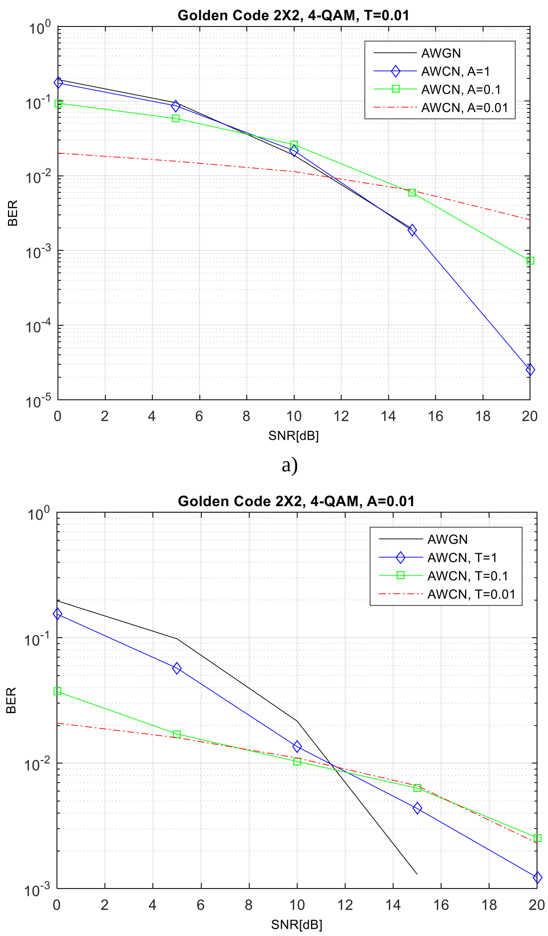


Fig. 5. AWCN pdfs for different values of *T*.

B. *Golden Code Performance on AWCN Channel*

In Fig. 6 are illustrated the Golden code performances for 2 transmitting and 2 receiving antennas, 4-QAM modulation and a channel affected by Rayleigh fading and impulsive noise described by Middleton Class-A model. The simulations were done for different values of *A* and *T*, compared with AWGN channel. If *T*=0.01 and *A* varies, up to a SNR value of 8dB (except for *A*=0.01, where SNR=12dB), the Golden code leads to better performance in AWCN channel and after 9dB, the AWGN channel has a low Bit Error Rate (BER). If *A*=1, the BER is closer to AWGN. In Fig. 6b), *A*=0.01 and *T* varies. The conclusions are similar to previous situations, after 12dB of SNR, the more impulsive is the noise, the BER increases.



a)

b)

Fig. 6. Golden code performance on AWCN channel

V. Conclusions

In this paper we have used the Middleton Class-A model to describe the impulsive noise. We generated some noise samples (gaussian and non-gaussian) and as *A* and *T* decrease, the noise becomes more impulsive. At their lowest values, the impulses number is smaller, but the amplitude is higher. The analysis of the parameters influences on pdf showed that the number of noise sources did not affect the distribution, but as *A* and *T* values decrease, the pdf becomes sharper and increases in amplitude. Also, the simulations results showed that the number of impulsive noise sources has no influence on probability density function.

The Golden code performances on AWCN channel was investigated on 2x2 MIMO system, with Rayleigh fading and 4-QAM modulation. The simulations were done for different values of *A* and *T*, and showed that for SNR values above 8dB, the BER increases compared to AWGN channel, as the impulsive model parameters get lower. The performances are better for impulsive noise, for low SNR values. If *A*=1, on AWCN channel, the Golden Code has almost the same error rate as in the case of gaussian noise.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

George Petrea has made the state of the art and he will be the presenter, Viorel Nicolau took care of the part with impulsive noise pdfs and Mihaela Andrei on Golden code performances and she was the coordinator. Andrei and Petrea wrote the paper.

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