Implementation of Fractional Fourier Transform in Digital Filter Design

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***Abstract*—**In literature, so many functions are available to process the signals. The quality of the window filters are mainly based on the following parameters like bandwidth (BW), Side Lobe Fall of Ratio (SLFOR) and Side Lobe Attenuation (SLA). Each window function is different and is not suitable for all applications. Every window has its own merits and demerits. Most of the time, the selections of a window function is made on trial and error basis. That is the reason, closed form fractional Fourier (FrFT) on spectral analysis of different window function [52] has been proposed. While going through the study, it is shown how windows functions break the traditional trade of between narrow band width and higher side lobe rejection. Also for the first time, we are presenting the FIR and IIR filters with four variables and also implement a differentiator using window based analysisthus,it'sa new beginning in the analysis of analog to discrete conversion. Here we presents the narrow band width and low computational cost of closed form FrFT for different window functions and also pointed out the demerit of this FrFT.

***Index Terms*—**FIR filters, IIRfilters, Window functions, differentiators

I. Introduction

Harris [1] proposed a concise review of spectral analysis on data window function using DFT. He presented all spectral characteristics like side lobe fall ratio (SLFOR), band width (BW) and side lobe attenuation (SLA) for conventional window functions. We normally have two types of windows like fixed and variable windows [1]. The fixed windows are having fixed window size (N) which can control the spectral parameters [2]-[5], whereas two or more parameters are necessary to control spectral characteristics in variable window functions [2]-[10], [51]. Most of the window functions have been developed with some optima criterion, unfortunately the trade-off is compromise between the conflicting requirements of a narrow main lobe width and small side lobe levels [11]-[18]. Filter is one of the most widely used operations in signal processing. In time domain, depending upon the transfer function the filters are classified into Finite Impulse Response (FIR)and Impulse Response (IIR)[17]. The FIR filters can be designed in various means Fourier series method, window functions method, frequency sampling method and optimal design methods. The variations in frequency response characteristics of FIR filters with different variable parameters are discussed in [19]–[26] and variations in IIR filters characteristics with variable parameters are discussed in [27]-[30]. From the above discussions we observed that the frequency characteristic variations of both FIR and IIR filter leads to different problems like high computation cost, implementation difficulties, replacement of filter coefficients every time and no sharp cut-off frequencies etc.,. Regarding differentiators, Al-Aloui [31] proposed a novel differentiator and integrator. The integrator is obtained by interpolation of the two most integrator techniques like trapezoidal and rectangular rules. The derived integrator performs better than individual rectangular and trapezoid integrator in frequency range and accuracy. The inverse of derived integrator leads to produce digital differentiator. The frequency range of this differentiator follows the ideal response is about 0.8 times of the Nyquist frequency, but it has no variations in its frequency characteristics. One of the most important tool to analyze signals on time –frequency scale is known as fast Fourier transform (FFT)[32]-[34], it has been used to compute the DFT using symmetry and periodicity properties in the twiddle factor and observed that the computation cost of FFT is lesser than DFT [32], [33]. It is used to decompose time signal into fundamental frequencies components. The disadvantage of FFT is that it cannot be used in local time frequency analysis and also in analyzing the non-stationary signals [32], [33]. Therefore a new analysis and synthesis method like fractional Fourier Transform (FrFT) has been proposed by V.Namias [35] and can be thought of as a generalization of FT. Thus FrFTis having an advantage of computation over than FFT [35]. The fractional FourierTransform (FrFT) is a family of linear transform and is generating the FT [36-44]. In recent years, the FrFT has attracted considerable amount of attention in many applications like optics and signal processing.. A comprehensive introduction to the FrFT and historical references found in[35]-[36]. The transfer function has become popular in optics, signal processing and communication follows the works of Ozaktus [36]-[39] and Almedia [34], [40]. Many definitions of FrFT are discussed in [41]-[46]. Some of the applications of FrFT explains include time-frequency analysis [47],

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communications [48], beam forming [49], [50] etc., up to now the FrFT has been digitally computed using variety of approaches [39]. However, these approaches are often far from exhibiting the internal consistency and analytical elegance. From above observations we present a novel closed form derivation for FrFT to analyze different window function and compared with discrete FrFT, proposed window function allows to provide breaks the conventional trade-\off, more number of variable parameters to tune the frequency characteristics of FIR ,IIR filters and differentiators. This paper contains four sections, section 2 describes the comparison between conventional and proposed FrFT, section 3 contains proposed window functionspectral characteristics, section 4 presents the variable FIR and IIR filters and followed by differentiators.

II. Comparison of Discrete and Proposed Closed Form FRFT

Here we present the comparison of our pastproposedFrFTderivation[52] for different window functions like Dirichlett eq.32 of [52], Bartlett under section 3.2 of [52], Hanning and Hamming windows eq.66 of [52] as an extension to compare with theexisting discrete FrFT[39,51]and whose spectral values are in shown in tables from tables 2.1 to 2.4, and frequency response graphs are shown in from fig.2.1 to fig. 2.4.As observed above figures from fig.2.1 to fig.2.4 and tables from 2.1 to 2.4, proposed derivation provides better narrow width (BW) almost 40% to 50% improvement and low computation speed than discrete FrFT of the above said windows and more or less approximately equal MSLL(or SLA) and SLFORvalues for observed windows with DFrFT.

A. The New Window Functions

A new window function is proposed as shown in the equation (2.1.1) in a closed form; it is a combination of Dirichlett [52] and Blackman-Harris window functions [53]. The features of this formulation is allowed to provide narrow band width with highest suppression side lobe levels and with more number of variable parameters like 'k' of the eqn.2.1.1 , '∝' of the FrFT, type of the window function and order of the window.

$\omega_{\alpha}(u)$ = k(FrFT of Dirichlett window)+(1-k)(FrFT of Blackmann-Harris window). (2.1.1)

III. Proposed FrFT Based FIR and IIR Filters

An analytical procedure for the tuning of (FIR) filters isintroduced. The tuning procedure adjusts a single frequency of the frequency response to the desired value while preserving the nature of the filter. In literature, many techniques are readily available to tune FIR filters [54]-[66]. After rigorous study on the tunable FIR filters here, a variable window based FIR low and high pass filters are presented in this paper with four variables.

A. Tuning Procedure of Firlowpass Filters Using Frft

Window based FIR digital filter operation (i.e.,H(n)) is described as a convolution of the finite duration of desired low pass impulse response (i.e., H_d(n)) with the window sequence W(n)[11]-[15] has shown in the equation (3.1.1.1) and whose pictorial operation is presented in fig.(3.1.1.1).

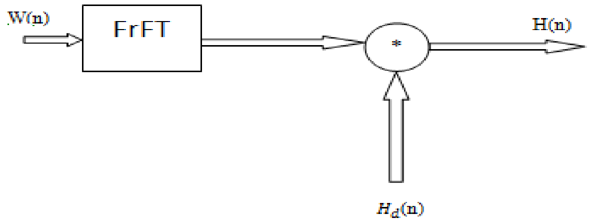

Fig. 3.1.1.1: Tuned LPF using FrFT

$$H(n)=H_d(n)*W(n) \qquad (3.1.1.1)$$

where, $H_d(n)$ is the desired or ideal low pass impulse response, W(n) type of window function and H(n) is tunable low pass filter. The equation (3.1.1.1) is valid for time domain and the corresponding frequency domain equation is presented in equation (3.1.1.2),

$$H(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} W(\lambda) H_d(\omega - \lambda) d\lambda, \qquad (3.1.1.2)$$

where, $H(\omega)$ is the frequency response of filter, $H_d(\omega)$, the desired or ideal frequency response of low pass filter), and $W(\omega)$, the frequency response of type of window. The frequency response of ideal low pass filter is shown in figure (3.1.1.2).

$$H_d(e^{jw}) = 1 \quad \text{for} \quad -\frac{\pi}{2} \le w \le \frac{\pi}{2}$$

$$= 0 \quad \text{for} \quad \frac{\pi}{2} \le |w| \le \pi$$

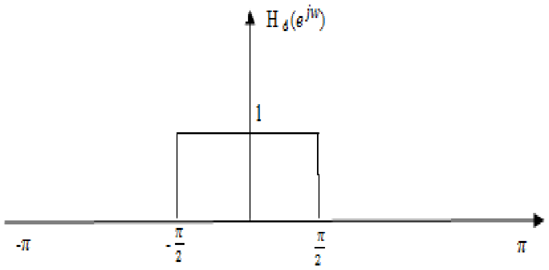

Fig. 3.1.1.2 Ideal low pass filter frequency response

The low pass filter coefficients can be obtained by using the formula

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{jw}) e^{jwn} dw$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{jwn} dw$$

$$= \frac{1}{2\pi jn} e^{jwn} \Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{\pi n(2j)} [e^{j\pi n/2} - e^{-j\pi n/2}]$$

$$= \frac{\sin\frac{\pi}{2}}{\pi n} \quad -\infty \le n \le \infty \quad (3.1.1.3)$$

The desired frequency response of high pass filter is shown in figure(3.1.1.3) below

$$H_d(e^{jw}) = 1 \text{ for } \frac{\pi}{4} \le w \le \pi$$

$$= 0 \text{ for } |w| \le \frac{\pi}{4}$$

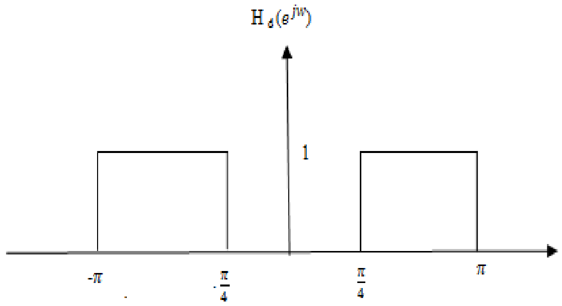

Fig. 3.1.1.3 Ideal high pass filter frequency response

The desired impulse response of low pass filter is calculated as,

$$H_d(n) = h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{jw}) e^{jwn} dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\pi/4} e^{jwn} dw + \int_{\pi/4}^{\pi} e^{jwn} dw$$

$$= \frac{1}{2\pi jn}\left[e^{jwn}\Big|_{-\pi}^{-\pi/4} + e^{jwn}\Big|_{\pi/4}^{\pi}\right]$$

$$= \frac{1}{\pi n(2j)}[e^{-j\pi n/4} - e^{-j\pi n} + e^{j\pi n} - e^{j\pi n/4}]$$

$$= \frac{1}{\pi n}[\sin \pi n - \sin(\pi/4)\, n] \text{ for } -\infty \le n \quad (3.1.1.4)$$

In this thesis, an algorithm is presented below to generate the FIR filter characteristics.

B. *Algorithm of Low Pass Tunable Fir Filter*

Step 1: The window is taken for the order 'N'.

Step 2: TheFrFTis used for an angle 'α',as α =aπ/2 as per the equation [52].

Step 3: The corresponding window function is computed as in equations 32,66 of [52].

Step 4: The step 3 isrepeated to calculate $W(n)$ by inserting negative value of 'a' as per step 2.

Step 5: The window function W(n) is convolved with H_d (n) (consider equation(3.1.1.3) for LPF and equation(3.1.1.4) for HPF.

Step 6: The filter coefficients are computed using eqn.3.1.1.1

Step 7: From step 2 to step 6 are repeated for different values of 'a'.

Thus, proposed low and high pass Bartlett and Blackman-Harris [53]windowsbased FIR filters are derived using above algorithm and whose spectral characteristic graphs and tables are shown in from fig.(3.3.1) to fig.(3.3.8) & table (3.3.1) to (3.3.2).respectively.

C. *Basic Concept of Proposed Tunable IIR Filters*

Here the proposed linear phase IIR filters have been developed by using FrFT high pass and low pass FIR filters under section 3.2 of this paper. Here, the formulation of high and low pass IIR filters are related to FIR filters as mentioned below[67],

$$H_{iirhighpass}(\omega) = \frac{H_{HP(FIR)N}(\omega)}{1 + H_{LP(FIR)D}(\omega)}$$

$$= \frac{|H_{HP(FIR)N}|\angle H_{HP(FIR)N}}{|H_{LP(FIR)D}|\angle H_{LP(FIR)D}}$$

$$= \frac{|H_{HP(FIR)N}|(\angle H_{HP(FIR)N} - H_{LP(FIR)D})}{|H_{LP(FIR)D}|} \quad (3.4.1)$$

where,

$H_{LP(FIR)N}(\omega)$is the FrFT of FIR low pass Filter (where N denotes numerator),

$H_{HP(FIR)D}(\omega)$ is the FrFT of FIR high pass Filter (where D denotes denominator) and

$H_{iirhighpass}(\omega)$is the proposed IIR high pass Filter.

$$H_{iirlowpass}(\omega) = \frac{H_{LP(FIR)N}(\omega)}{1 + H_{HP(FIR)D}(\omega)}$$

$$= \frac{|H_{LP(FIR)N}|\angle H_{LP(FIR)N}}{|H_{HP(FIR)D}|\angle H_{HP(FIR)D}}$$

$$= \frac{|H_{LP(FIR)N}|(\angle H_{LP(FIR)N} - H_{HP(FIR)D})}{|H_{HP(FIR)D}|} \quad (3.4.2)$$

where,

$H_{LP(FIR)N}(\omega)$ Is the FrFT of FIR low pass filter (where N denotes numerator),

$H_{HP(FIR)D}(\omega)$ Is the FrFT of FIR high pass filter (where D denotes denominator) and

$H_{iirlowpass}(\omega)$is the proposed IIR low pass filter.

The IIR(low & high pass) filters are designed from FIR filters(as in section 3.1).The frequency and stability responses of direct tunable low and high pass IIR filters are sketched in figures from fig.(3.5.1) to fig.(3.5.12).

IV. TUNABLEDIFFERENTIATOR

The frequency response of an ideal digital differentiator is linearly proportional to frequency. It is given by

$$H_d(e^{j\omega}) = j\omega \quad \textit{for} \quad (-\pi \le \omega \le \pi) \quad (4..1)$$

The ideal impulse response of a digital differentiator with linear phase is given by

$$h_{d(n)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n} d\omega$$
$$= \frac{\cos \pi(n-\beta)}{n-\beta} - \frac{\sin(n-\beta)\pi}{\pi(n-\beta)^2} \quad (4.2)$$

where,

$$\beta = \frac{N-1}{2}.$$

If 'N' is odd, 'β' is an integer and we have $\sin(n-\beta)\pi = 0$ for any integer n. If 'N' is even, then $\cos\left[\frac{2n-(N-1)}{2}\pi\right] = 0$ for any integer.

Thus we have, for 'N' odd

$$h_d(n) = \frac{\cos[(n-\beta)\pi]}{n-\beta} for n \neq \beta$$
$$= 0 \qquad for n = \beta \quad (4.3)$$

and for 'N' even

$h_d(n) = \frac{-\sin[(n-\beta)\pi]}{\pi(n-\beta)^2}$(4.4)

Both these equations 4.3 and 4.4have the property of symmetry (i.e. $h_d = -h_d(N-1-n)$). The coefficients are asymmetric and of infinite length. The finite impulse response can be obtained by truncating them by using window of length N. The proposed differentiator is obtained [14,16,18] as

$$h(\omega) = h_{diff}(\omega). F^a[\omega(n)] \quad (4.5)$$

where,$F^a\omega[(n)]$ he fractional Fourier domain window of different types here we calculated for combination Bartlett and boxcar window

$h_{diff}(\omega)$is desired frequency response of differentiator

One should follow the following steps to implement FrFT based differentiator

Step-1: The desired frequency response of differentiator is to be calculated as per equations 4.3 or 4.4 i.e. basing upon 'N' is even or odd).

Step-2: FrFT based rectangle window is multiplied with desired frequency response as per equation(4.5) denoted as $s_R(a)$.

Step-3: Step-2 is calculatedfor triangle window[$s_T(a)$].

Step-4: Nowproposeddifferentiator computed as

$$[s_D(a)]^P = P[s_R(a)] + (1-P)[s_T(a)] \text{ as in [31]} \quad (4.6)$$

it is variable for different values of 'P'.

where, $s_R(a)$ is FrFT of rectangle window, $s_T(a)$ is FrFT of triangular window and $s_D(a)$ is proposed differentiator . Finally the proposed differentiator in this thesis is selected for N=2,a=0.9 and P=0.9 which can be expressed as

$$\text{i.e.,} s_D(a) = 1.039112625\text{-}1.043Z^{-1} \quad (4.7)$$

The corresponding differentiator diagrams are shown in figures from 4.1 to 4.5.

A. Modified Proposed Differentiators

The proposed differentiator as in eqn.4.6 gives better results for magnitude response, when compared with rectangular and trapezoidal differentiators, but not suitable with Al-Aloui differentiator. So, we have proposed modified differentiator with shadow concept[59]. By substituting the backward difference formula in the derivative $\frac{dy(t)}{dt}$ at time t = nT. We get,

$$\left.\frac{dy(t)}{dt}\right|_{t=nT} = \frac{y(nT)-y(nT-T)}{T}$$
$$= \frac{y(n)-y(n-1)}{T} \quad (4.1.1)$$

where, T is sampling interval and y(n)=y(nT).The transfer function of analog differentiator is represented with H(s) = s and thedigital system, that produces the output $\frac{y(n)-y(n-1)}{T}$, which has the system function H(z)=$\frac{(1-z^{-1})}{T}$.These two cases can be compared to get the equivalence in frequency domain using the equation mentioned below,

$$s = \frac{(1-z^{-1})}{T} \quad (4.1.2)$$

So, modified proposed differentiator is given below based on shadow concept

$s^* = \frac{G(z)}{1-KH(z)}$ (eqn.3 as in[68])

We consider,

G(z) = $039112625 - 1.0433325\, Z^{-1}$ and H(z) = $\frac{(1-z^{-1})}{T}$

The above equation can be reduced to

$$s^* = \frac{1.039112625 - 1.0433325\, Z^{-1}}{1-K\frac{(1-z^{-1})}{T}} \quad (4.1.3)$$

where 'K' is any arbitrary feedback parameter, for different values of 'K' graphs are shown from fig. 4.1.1 to 4.1.2

We observed from the fig.4.2 proposed differentiator indicates lesser error than Al-Aloui[31] up to 1.6 bins on frequency scale from there on wards it is increased, whereas phase ismorelinear when compared to Al-Aloui differentiator. The demerit of this proposed differentiator is that, it has low cross-over frequency factor which is about 1.8 bins with 2.6 bins of Al-Alouidifferentiator. This draw back has been overcome by introduced modified proposed differentiator (eqn.4.1.3). From the fig.4.1.2 it is noticed that the cross over factor of modified proposed differentiator is about 2.8 bins.

B. Results

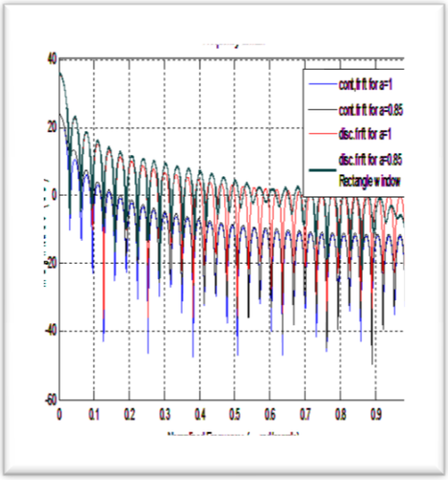

Fig. 2.1 Continuous and discrete FrFT responses for rectangle window at different values of 'a'

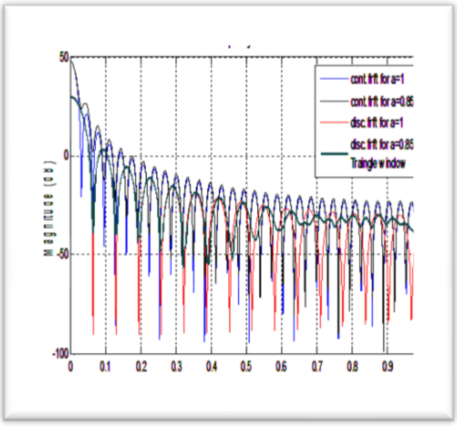

Fig. 2.2 Continuous andiscreteFrFT responses for Bartlett window at different values of ‘a’

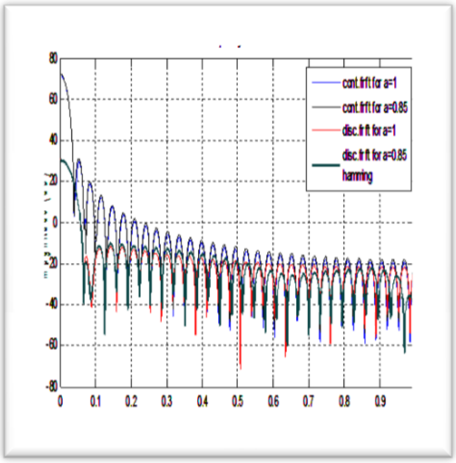

Fig. 2.3 Continuous and discrete FrFT responses for Hamming window at different values of ‘a’

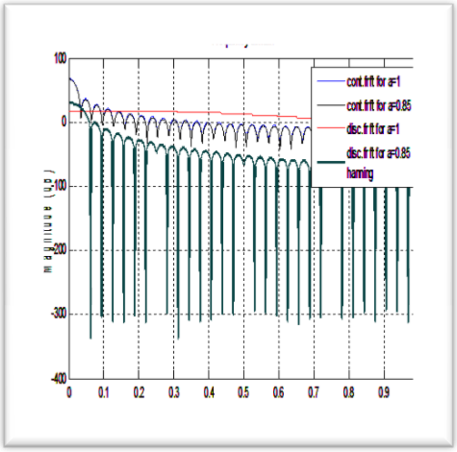

Fig. 2.4 Continuous and discrete FrFT responses for Hanning window at different values of ‘a’

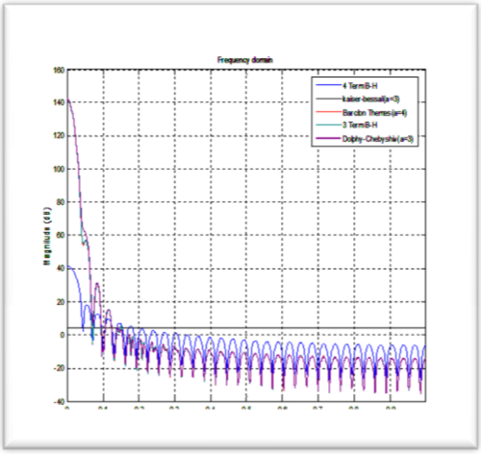

Fig. 2.1.1 spectral Responses of different windows based on ‘a’ value between 0.9218 to 1

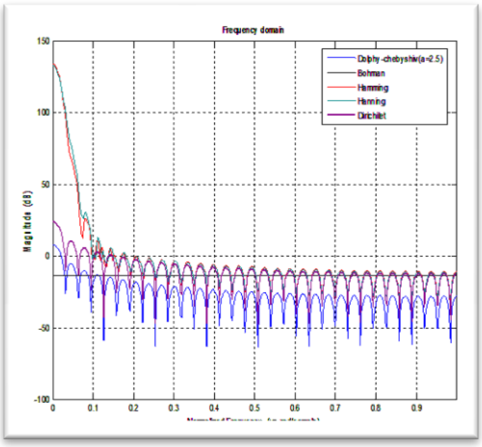

Fig. 2.1.2 spectral Responses of different windows based on ‘a’ value between 0.8650 to 0.9130

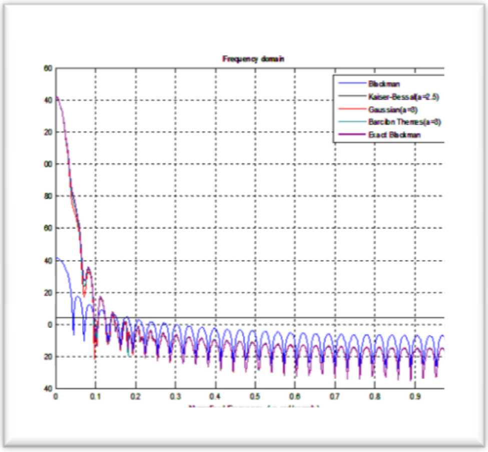

Fig. 2.1.3 spectral Responses of different windows based on ‘a’ value between 0.8620 to 1

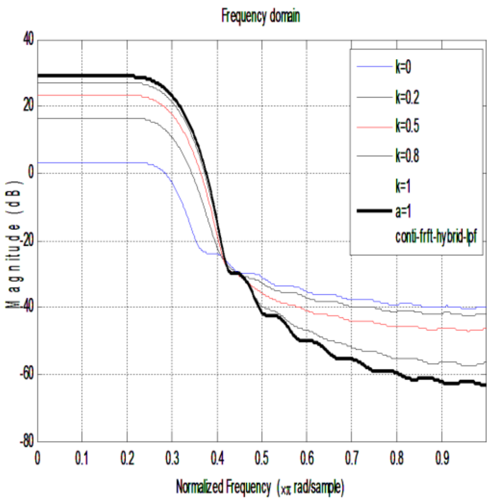

Fig. 3.3.1 Tuneable low pass filter with hybrid window function,a=1&k=0,0.2,0.5,0.8,1

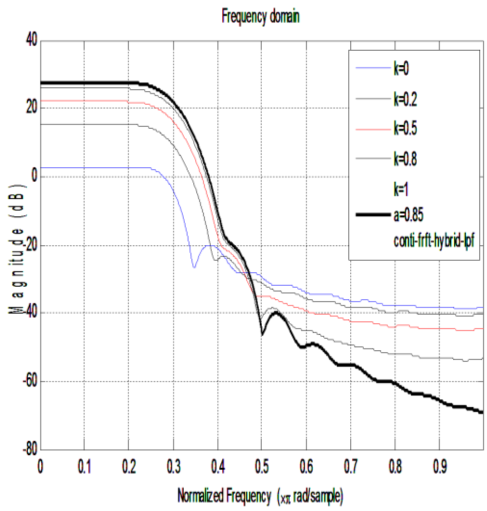

Fig. 3.3.4 Tuneable low pass filter with hybrid window function,a=0.85&k=0,0.2,0.5,0.8,1

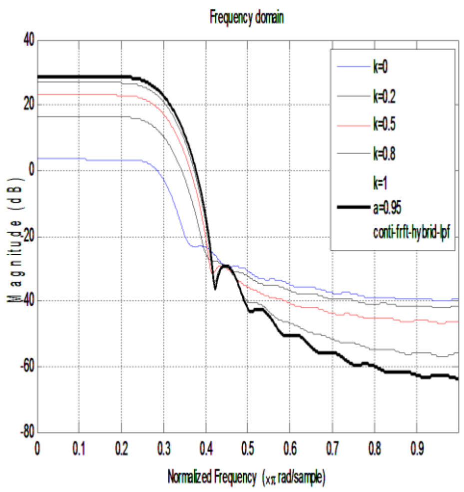

Fig. 3.3.2 Tuneable low pass filter with hybrid windowfunction,a=0.95&k=0,0.2,0.5,0.8,1

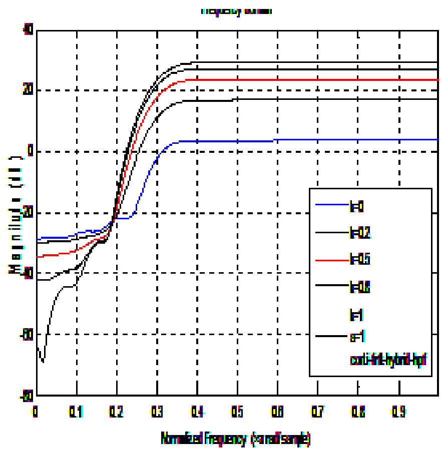

Fig. 3.3.5 Tuneable high pass filter with hybrid window function,a=1&k=0,0.2,0.5,0.8,1

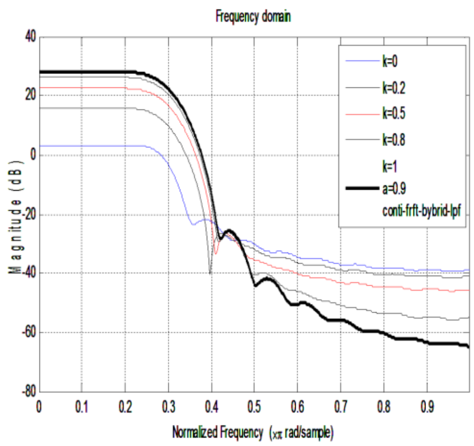

Fig. 3.3.3 Tuneable low pass filter with hybrid window function,a=.9&k=0,0.2,0.5,0.8,1

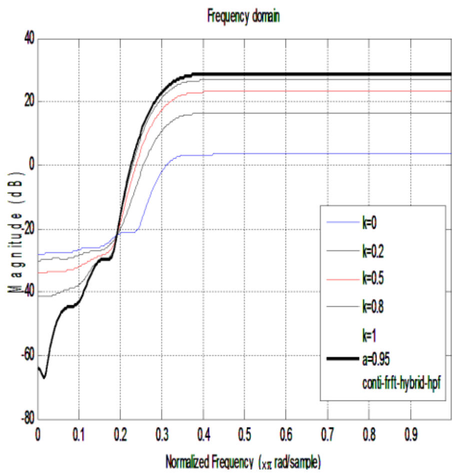

Fig. 3.3.6 Tuneable high pass filter with hybrid window function,a=.95&k=0,0.2,0.5,0.8,1

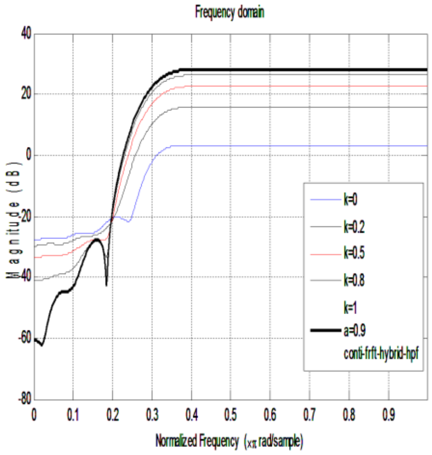

Fig. 3.3.7 Tuneable high pass filter with hybrid window function,a=.9&k=0,0.2,0.5,0.8,1

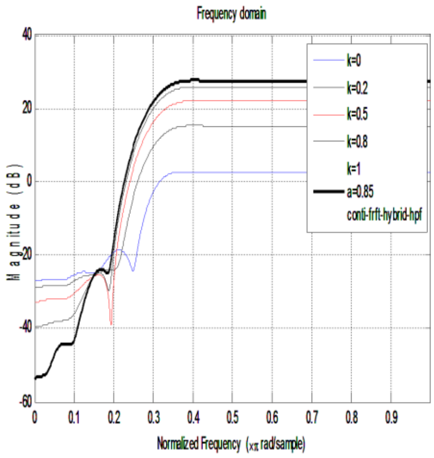

Fig. 3.3.8 Tuneable high pass filter with hybrid window function, a=0.851 & k=0,0.2,0.5,0.8,1

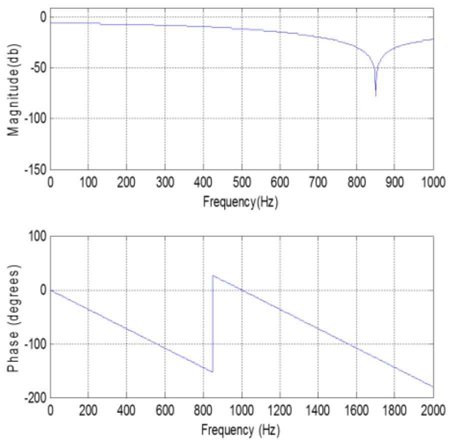

Fig. 3.5.1 Magnitude and phase response of proposed IIR Low pass Filter (a=0.85)

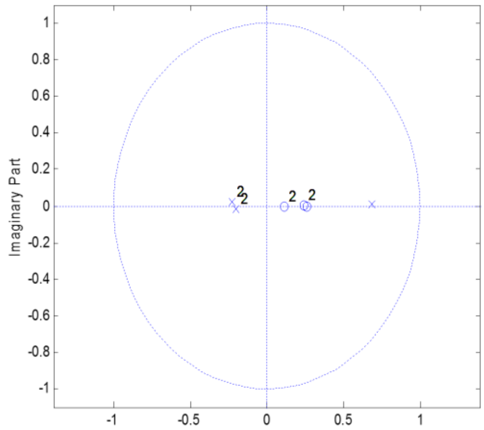

Fig. 3.5.2 Stability response of proposed IIR Low pass Filter (a=0.85)

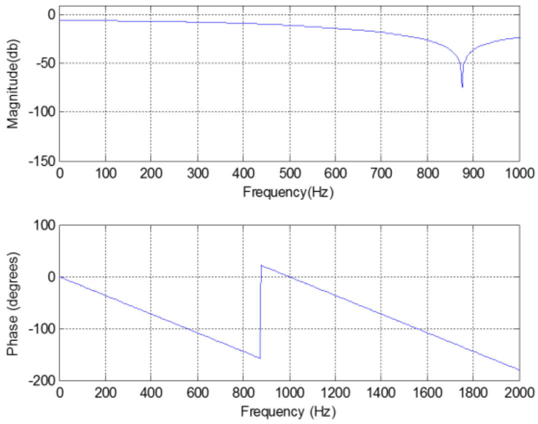

Fig. 3.5.3 Magnitude and phase response of Proposed IIR Low pass Filter (a=0.9)

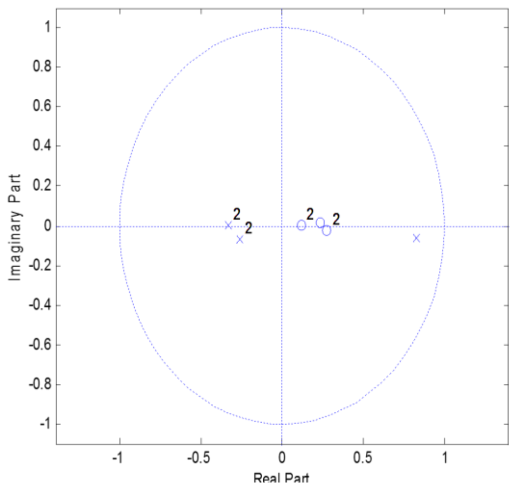

Fig. 3.5.4 Stability response of Proposed IIR Low pass Filter (a=0.9)

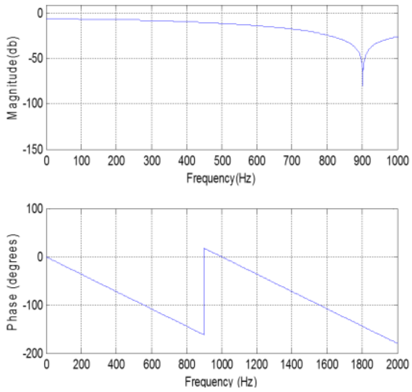

Fig. 3.5.5 Magnitude and phase response of proposed IIR Low pass Filter (a=0.95)

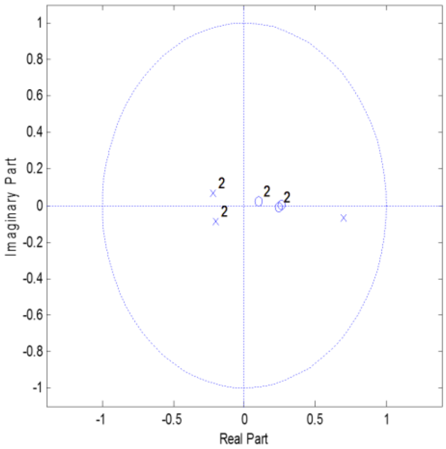

Fig. 3.5.6 Stability response of proposed IIR Low pass Filter (a=0.95)

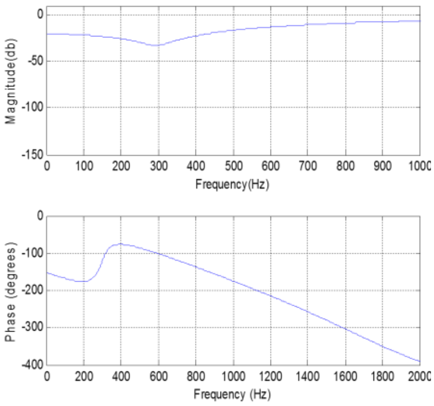

Figure 3.5.7 Magnitude and phase response of proposed IIR High pass Filter (a=0.85)

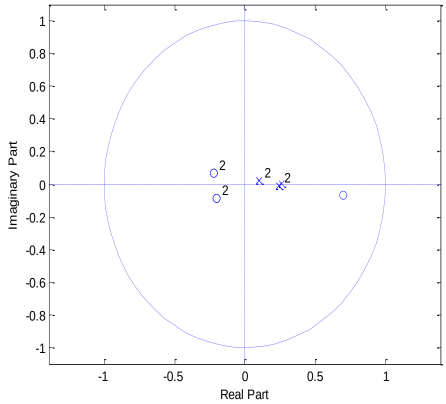

Figure 3.5.8 Stability response of proposed IIR High pass Filter (a=0.85)

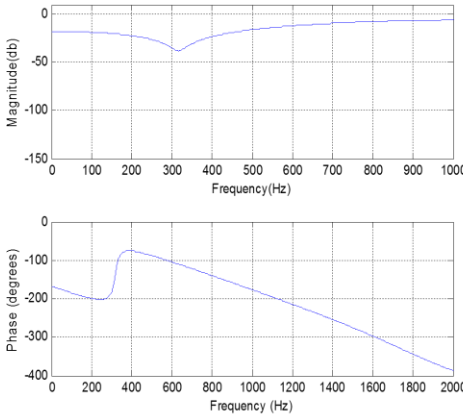

Figure 3.5.9 Magnitude response of proposed IIR High pass Filter (a=0.9)

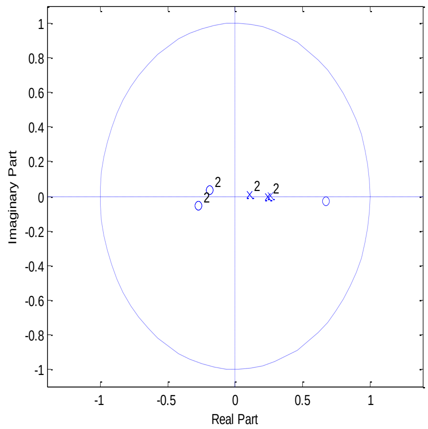

Figure 3.5.10 Stability response of proposed IIR High pass Filter (m=0.9)

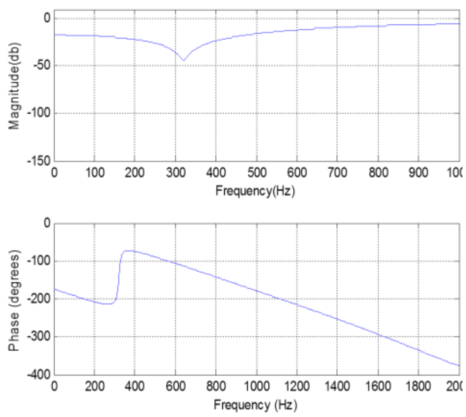

Figure 3.5.11 Magnitude and phase response of proposed IIR High pass Filter(a=0.95)

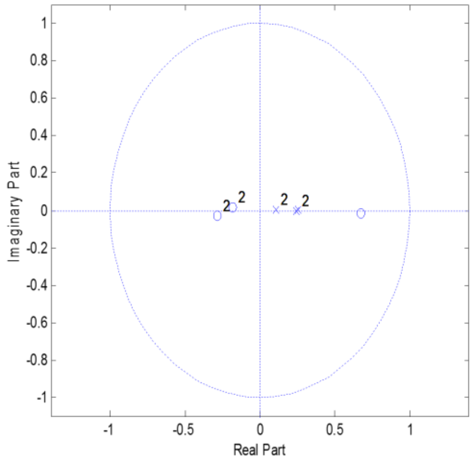

Figure 3.5.12 Stability response of proposed IIR High pass Filter (a=0.95)

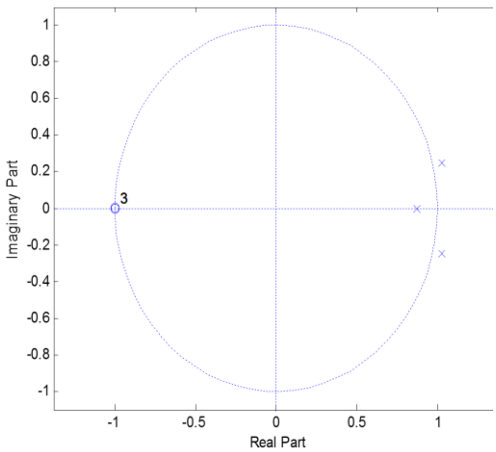

Figure 3.5.13 Stability responses of IIR Chebyshev Filter

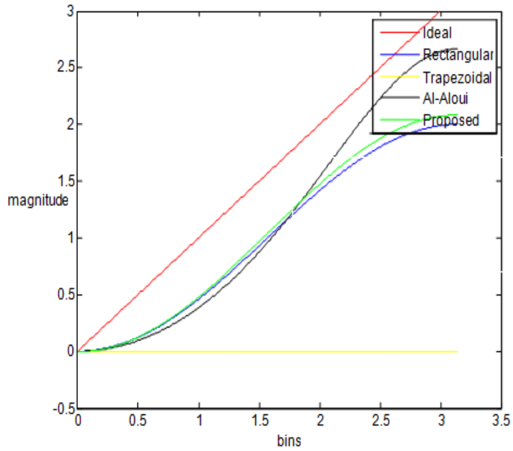

Figure 4.1 Comparison of proposed differentiator with Rectangle

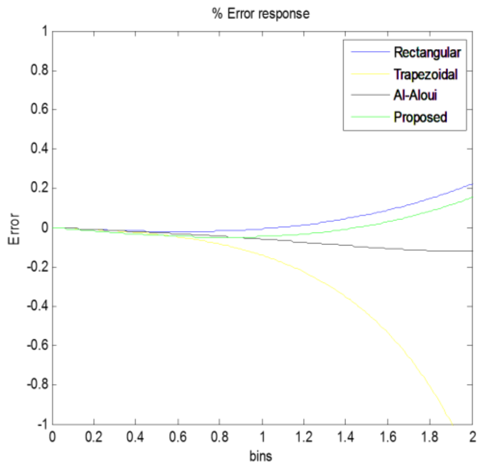

Figure 4.2 Error comparison of proposed differentiator with Rectangle, Trapezoidal, Al-Aloui differentiator

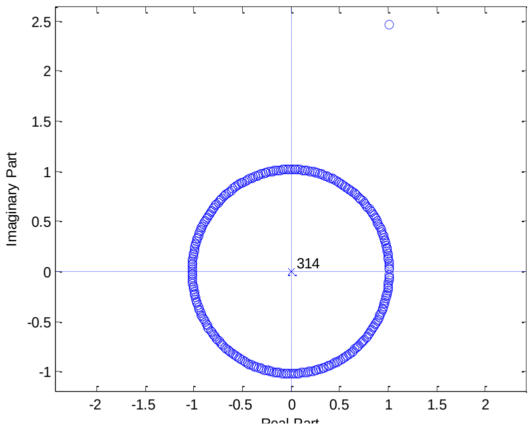

Figure 4.3 Pole-zero plot of proposed differentiator

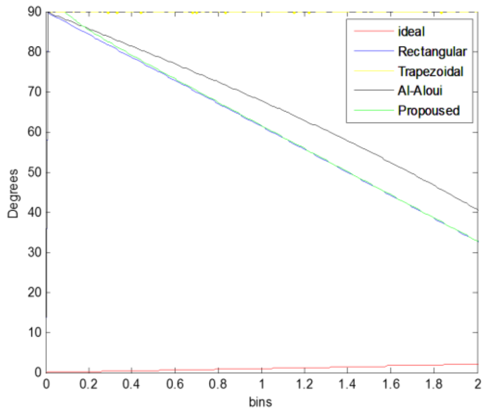

Figure 4.4 Phase response of proposed with Rectangle, Trapezoidal, Al-Aloui differentiator

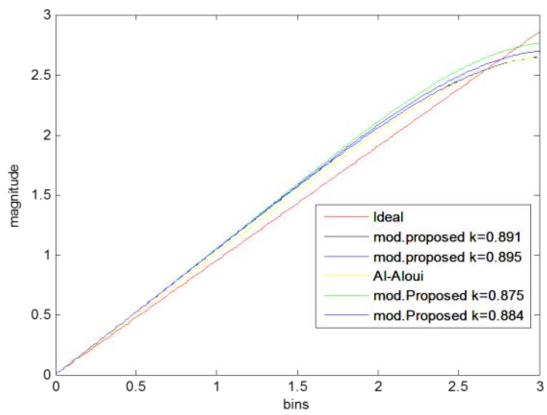

Figure 4.1.1:Magnitude response for modified proposed differentiator with Al-Aloui differentiator with various values of 'K'

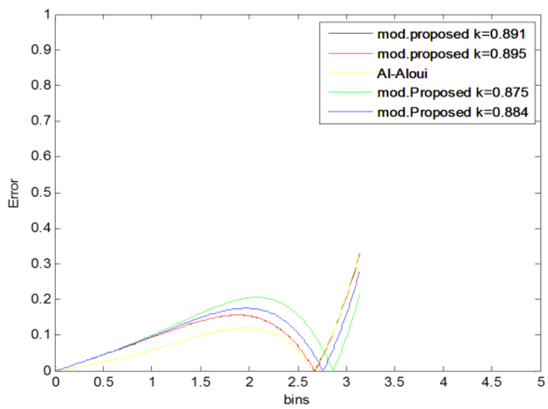

Figure 4.1.2:Error comparison of modified proposed differentiator with Al-Aloui differentiator with various values of 'K'

TABLE 2.1 RESPONSE OF CONTINUOUS AND DISCRETEFRFT OF BOXCAR WINDOW

Continuous FrFT					Discrete FrFT			
Tuning parameter (a)	*Band width in bins*	*Maximum Side lobelevel in(dB)(SLA)*	*Side lobe fall of ratio (dB)*	*Execution time in sec.*	*Band width in bins*	*Maximum Side lobe level in(dB)*	*Side lobe fall of ratio (d)*	*Execution time in sec.*
1	0.0273	-13.4	-22.87	0.387	0.0273	-13.3	-23.3	0.391
0.85	0.0273	-14.9	-25.18	0.399	0.0273	-13.1	-32.71	0.474

TABLE 2.2 RESPONSE OF CONTINUOUS AND DISCRETE FRFT OF BARTLETT WINDOW

Continuous FrFT					Discrete FrFT			
Tuning parameter(a)	*Band width in bins*	*Maximum Side lobelevel in(dB)(SLA)*	*Side lobe fall of ratio (dB)*	*Execution time in sec.*	*Band width in bins*	*Maximum Side lobe level in(dB)*	*Side lobe fall of ratio (dB)*	*Execution time in sec.*
1	0.0234	-42.5	-49.37	0.413	0.0392	-42.5	-5.29	0.557
0.85	0.0234	-43.3	-53.2	0.410	0.0392	-41.1	-16.68	0.419

TABLE 2.3 RESPONSE OF CONTINUOUS AND DISCRETEFRFT OF HAMMING WINDOW

Continuous FrFT					Discrete FrFT			
Tuning parameter(a)	*Band width in bins*	Maximum Side lobelevel in(dB)(SLA)	*Side lobe fall of ratio (dB)*	*Execution time in sec.*	*Band width in bins*	*Maximum Side lobe level in(dB)*	*Side lobe fall of ratio (dB)*	*Execution time in sec.*
1	0.0234	-42.5	-49.37	0.413	0.0392	-42.5	-5.29	0.557
0.85	0.0234	-43.3	-53.2	0.410	0.0392	-41.1	-16.68	0.419

TABLE 2.4 RESPONSE OF CONTINUOUS AND DISCRETEFRFT OFHANNING WINDOW

Continuous FrFT					Discrete FrFT			
Tuning parameter(a)	*Band width in bins*	*Maximum Side lobe level in(dB)(SLA)*	*Side lobe fall of ratio (dB)*	*Execution time in sec.*	*Band width in bins*	*Maximum Side lobe level in(dB)*	*Side lobe fall of ratio (dB)*	*Execution time in sec.*
1	0.0214	-32.1	-50.07	0.410	0.0429	-31.5	-87.10	0.395
0.85	0.0214	-32.5	-43.49	0.408	0.0429	-31.5	-59.9	0.411

TABLE 2.1.1 SPECTRAL PARAMETERS OF PROPOSED COMBINATION WINDOWS (EQN.2.1.1)

a	**K**	**MSLA in dB**	**HBW in dB**	**SLFOR in dB**
1	1	-13.1	0.0273	-22.89

0.8324	0.825	-31.5	0.0253	-39.67
0.8809	0.8	-42.1	0.0166	-41.6
0.85	0.5	-46.4	0.0205	-87.77
0.862	0.5	-50.1	0.0185	-85.8
0.865	0.5	-51.1	0.0185	-84.81
0.878	0.5	-53	0.0175	-85.1
0.8958	0.5	-55	0.0166	-82.45
0.908	0.5	-57	0.0166	-79.16
0.913	0.5	-58	0.0166	-80.03
0.9218	0.5	-60	0.0156	-79.09
0.942	0.5	-67	0.0156	-77.18
0.944	0.5	-68	0.0156	-75.89
0.946	0.5	-69	0.0156	-76.86
1	0	-92.1	0.0175	-99.25

TABLE 3.3.1 SLA,HBW AND SLFOR FOR LPF FOR DIFFERENT VALUES OF 'A' &'K'

S.NO	FrFT parameter (a)	K	SLA	HBW	SLFOR
1	1	0	-27.21	0.5625	15.9
2		0.2	-48.85	0.5546	9.75
3		0.5	-54.42	0.5546	15.5
4		0.8	-57.20	0.5546	26.52
5		1	-58.87	0.5546	33.38
6	0.95	0	-26.49	0.5625	16.11
7		0.2	-43.95	0.5546	13.91
8		0.5	-52.25	0.5546	16.82
9		0.8	-56.13	0.5546	29.52
10		1	-57.65	0.5546	34.45
11	0.9	0	-25.09	0.5625	17.11
12		0.2	-42.5	0.5546	14.48
13		0.5	-49.34	0.5546	18.71
14		0.8	-52.74	0.5546	28.65
15		1	-53.84	0.5546	38.71
16	0.85	0	-22.65	0.5625	18.41
17		0.2	-38.66	0.5546	17.01
18		0.5	-44.23	0.5546	22.46
19		0.8	-46.65	0.5546	32.55
20		1	-47.07	0.5546	49.64

TABLE 3.3.2 SLA,HBW AND SLFOR FOR HPF FOR DIFFERENT VALUES OF 'A' &'K'

S.NO	FrFT parameter (a)	K	SLA	HBW	SLFOR
1	1	0	-24.99	1.9961	6.66
2		0.2	-44.52	1.9961	2.63
3		0.5	-51.9	1.9961	5.98
4		0.8	-57.27	1.9961	11.94
5		1	-59.57	1.9961	33.36
6	0.95	0	-24.83	1.9961	6.84
7		0.2	-43.71	1.9961	2.69
8		0.5	-51.65	1.9961	5.47
9		0.8	-57.01	1.9961	11.4
10		1	-58.35	1.9961	35.15
11	0.9	0	-23.55	1.9961	7.44
12		0.2	-42.36	1.9961	3.09
13		0.5	-50.12	1.9961	5.95
14		0.8	-54.37	1.9961	12.84
15		1	-5.81	1.9961	32.89
16	0.85	0	-21.23	1.9961	8.41
17		0.2	-39.27	1.9961	4.85
18		0.5	-47.34	1.9961	7.25
19		0.8	-50.59	1.9961	14.61
20		1	-51.39	1.9961	-29.19

V. CONCLUSION

The comparison of frequency response continuous and discrete FrFT based windows like Dirichilett, Bartlett, Hamming and Hanning window functions are done presented in figures 2.4.1 to 2.4.4. The comparison of spectral values BW, MSLL and SLFOR (rejection side lobes ability) of Dirichilett, Bartlett, Hamming and Hanning window functions are presented in table 2.4.1 to 2.4.4. respectively. From the above observations our proposed FrFT of different window functions more or less variations in SLA and SLFOR values, but it provides very good improvement in B.W like, 50% (0.0164) for Hamming, 50% for Hanning(0.0215), 50% for Bartlett(0.0176) windows and with no change in Dirichlett window when compared to discrete FrFT.

Window based design filters mainly measures peak amplitude of side lobe, main lobe width and side lobe fall of ratio. As window length N, increases width parameters decreases, but side lobe attenuation remains more or less constant. Peak stop band level of the windowed spectrum is less than side lobe attenuation (i.e., side lobe fall of ratio) of the window itself. In other words side lobe fall ratio is high means stop band attenuation of the filter is typically greater. Ideally, the spectrum of a window should approximate an impulse. Most of the windows have been developed with some optimal criterion. Unfortunately the trade-off is compromising between the conflicting requirements of a narrow main lobe width (or a smallest transition width) and small side lobe levels.

As said compromising the trade-off between main lobe width and spectral leakage, the proposed combination window function is allowed to break the convention trade-off i.e., as 'k' decreases from 1 to 0 in the eqn.2.5.3 and table (2.5.1), the band width decreases from 0.0273 to 0.0175 and at same time the phase suppression of the leakage value increases from -22.89dB to -99.25dB. This is one of merit of our proposed window function. As we observed existing FrFT provides the improvement in suppression abilities with constant B.W[9,47,61]. Thus our proposed window provides highest rejection ability with lowest narrow band.

Proposed Differentiator provides better results than in [31] except cross over frequency (2.6 bins for Al-Aloui) and this problem is rectified by our modified proposed differentiator (eq.2.6.1.3) and its value is about 2.8 more value than Al-Aloui value shown in fig.(2.6.1.3) Thus, our modified differentiator can be treated s to z converter and also can be used this window based design will be provided a better s to z converter for further.

Implementation of variable FIR filters based on the above said window models are carried out. The proposed FIR filters are having more number variable parameters like type of the window, length of the window, 'a' value of FrFT and 'k' of the proposed eqn.2.5.3 as shown in figures from fig.(3.4.1) to fig.(3.4.12) to tune their frequency characteristics than any other FIR filters[47], [54], [56], [60], [65] and variable linear phase direct digital IIR filter. whose frequency characteristics are shown in figures from fig.(4.2.1) to fig.(4.2.12) are also provides more number variable parameters with linear phase than any other IIR filters in[27,28&60-68] and the proposed IIR filters do not require any mapping techniques [31]-[35] to convert from analog to digital as in case of conventional Butterworth and Chebyshev IIR filters[11]-[15].

The demerits of our proposal is only applicable for values of 'a' of FrFT between 0.8 to 1. Since in our derivation, we used kernel of exponential power multiplication and it is approximated to lower time values ('$t < 1$') values hence it is suitable for high frequency applications. Finally, we can conclude that these above windows, differentiator/integrator, FIR and IIR filters will be used for optimal designs to get better results in their characteristics responses.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

This research work was carried out as part of my Ph.D thesis, both S.K. Nayak and Trinadh Sahu are acted as guides of this thesis work.

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