Optimal Algorithm for Minimizing Interference with Two Power Levels in Wireless Sensor Networks

M. Prasanna Lakshmi and Pushparaj Shetty D.
Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal, 575025
Email: prasannasainitw@gmail.com, prajshetty@nitk.edu.in

Abstract—Interference is a major hindrance to the communication in wireless sensor networks which needs to be optimized in order to minimize the total power consumption of the network. A sensor node in a WSN is assigned certain transmission range for sensing and transmission of data. If the transmission between any two nodes is affected by a third node, then it leads to interference. Sender interference of a node in WSN is the number of nodes that lie within the transmission range of that vertex. The receiver interference of a node x is the number of other nodes which include x in their transmission range. In recent days WSNs are operated by a discrete set of power levels in which a limited number of power levels are available which can be assigned to a node. The problem of minimizing the maximum sender interference of a WSN using only two power levels is studied in this paper. An optimal algorithm is presented in this paper which assigns transmission power to the sensor nodes of a given network such that the maximum sender interference is minimized and it results in a connected topology. An algorithm for receiver interference is also proposed using a similar concept, and an extensive simulation is performed to compare the maximum sender and receiver interference for the same instances.

Index Terms—Dual power assignment, algorithm, range assignment, optimal solution, wireless sensor networks

I. Introduction

A Wireless Sensor Network (WSN) is composed of sensor nodes that have the capacity to sense and transmit the data within their range and the nodes communicate with the help of wireless radio. A WSN monitors a physical system in which the sensor nodes sense the environmental parameters such as pressure, humidity, temperature etc. and send the data to the control nodes through wireless communication. Research in WSNs has been increasing in recent years because of its variety of applications in the real world such as military, healthcare, biological detection, environmental monitoring etc. The sensors collaborate to perform a task required by the end user. Multi hop communication in which the nodes relay the data to the other nodes via intermediate nodes is suitable for an efficient energy utilization in a WSN. Nodes are assigned transmission range to perform the specified task. Fig. 1 shows the transmission range for different dimensional networks [1].

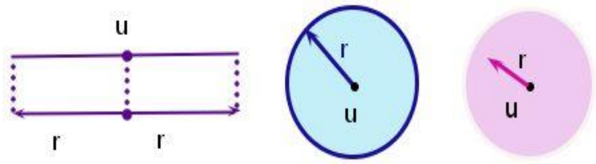

Fig. 1. Radio coverage in three different dimensional networks (a): one dimensional (b) two dimensional (c) three dimensional.

In this research, the nodes are deployed on a two dimensional Euclidean Plane and each node has a limited range. For any node v, the circle with center v and radius as its transmission range $R(v)$ is called transmission disk which is represented by $D(v, R(v))$ [2]. Each node broadcasts the data to all the nodes in its transmission disk and the communication occurs between two nodes if and only if transmission range of one node has the other node. Since the energy is a limiting factor, several mechanisms are developed for the efficient utilization of energy consumption [3]. Topology control problem [4], [5] in a WSN deals with an assignment of transmission power to the nodes of the network such that at least one bidirectional path exists between any two sensor nodes and the total energy consumption is minimized, this problem is also termed as range assignment problem [6]. Energy minimization problem is significant as the sensor nodes are equipped with a small battery of limited capacity [7].

A WSN is modeled as an undirected graph in which each vertex represents a node and an edge exists between two nodes if they can communicate directly with each other [3]. The nodes are deployed on a two dimensional Euclidean plane and the distance between any pair of two nodes is computed using the distance formula. The weight function is defined as $w: v \times v \rightarrow R^+$ where $w(xy)$ indicates the Euclidean distance between x and y. In this paper, the terms node and vertex are used interchangeably. Once a WSN is modeled as an undirected graph, the range assigned to a vertex in a graph G is the maximum of all its adjacent edge weights in G and is mathematically given as follows:

$$\mathrm{R(v)} = \max\{\mathrm{uv} | \mathrm{uv} \in \mathrm{E(G)}\}$$

Manuscript received March 10, 2019; revised November 8, 2019.
Corresponding author email: prasannasainitw@gmail.com.
doi:10.12720/jcm.14.12.1198-1204

Fig. 2 shows a reduced topology of nine vertices and the range values assigned to the vertices of this subgraph are reported in Table I.

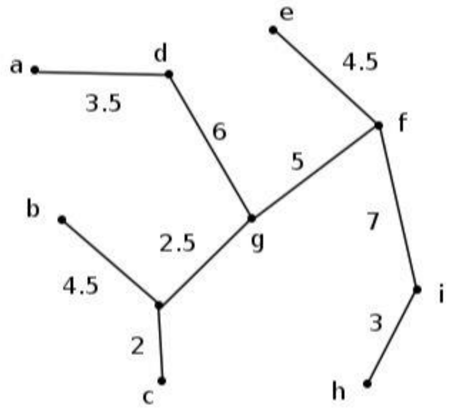

Fig. 2. Sample topology of nine vertices.

TABLE I: ILLUSTRATION OF RANGE ASSIGNMENT

Vertex	Range
a	3.5
b	4.5
c	2
d	6
e	4.5
f	7
g	6
h	3
i	3

The objective of minimum range assignment problem is to allocate transmission range to the nodes of a network such that the obtained reduced network satisfies specified connectivity constraints and its total energy is minimized. Strong Minimum Energy Topology (SMET) problem is studied by Cheng *et al*. [8] in which the connectivity constraint is simple connectivity. Authors proposed two heuristics named: Minimum spanning tree based heuristic and Incremental Heuristic for SMET problem and presented simulation results. Authors also proved that SMET problem is NP-complete by using a polynomial time reduction from 3 planar vertex cover problem and proved that SMET is of 2 approximation ratio.

Discrete power level assignment problem has gained the attention of several researchers in recent days as the sensors in the market are operated by a discrete set of power levels. A node in the network can be assigned with one of the power levels only from the given set. In this research, two power levels (high and low) are considered for assignment and the problem of assigning nodes of the network with any of the two power levels is termed as Dual power management problem and it is proved that this problem is NP-complete [9]. A 2-approximation algorithm was proposed by Rong *et al*. [9] where the lower bound is the number of components obtained when all the nodes are assigned low power. The formation of components after assigning low power to all the nodes initially for a given set of nodes is demonstrated in Fig. 3. In this figure, there are four components and each node can communicate with all the other nodes of that component which it belongs to. The number of components obtained at this stage is the lower bound for the number of high power nodes for dual power assignment problem since at least one node from each component must be assigned high power to ensure the connectivity of the resultant subgraph.

An improved algorithm with the same approximation ratio was proposed by Shetty and Lakshmi [10]. The ratio is improved to 11/6 by Carmi and Katz [11] and further improved to 1.61 by Calinescu [12]. Affash *et al*. [13] proposed an algorithm for this problem and improved the ratio to 1.57. Hoffmann *et al*. [14] studied dual power vertex degree problem where the degree of each vertex in the resultant graph is at least Δ, and proposed an algorithm of approximation ratio $1 + \log 5\Delta$, where Δ is the maximum degree.

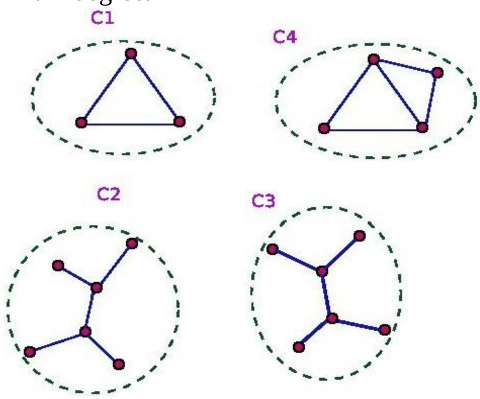

Fig. 3. Formation of components after assigning low power to all the nodes.

Interference is one of the primary issues in WSNs which is a major hindrance to the communication in WSNs. Interference of a node in a wireless network is the number of nodes affecting its communication. Minimizing the interference is equivalent to minimizing the total energy as interference leads to the loss of packets and hence retransmission of data. So, it is requisite to keep the interference low at each node of a network. There are two types of interference namely sender interference and receiver interference which are formally defined in terms of graph theoretic model as follows:

Let $G = (V, E, w)$ be the given complete graph:

Definition I.1. Sender interference of a vertex v is the number of vertices in its transmission disk and denoted by $I_S(v) = |\{u \in V \setminus \{v\} | u \in D(v, R(v))\}|$. Maximum sender interference is defined as $I(S) = \max_{v \in V} I_S(v)$.

Definition I.2. Receiver interference of a vertex v is the number of transmission disks which consist of v and is denoted by $I_R(v) = |\{u \in V \setminus \{v\} | v \in D(u, R(u))\}|$. Maximum receiver interference is defined as $I(R) = \max_{v \in V} I_{SR}(v)$

Theoretically, the sender interference has its own significance but in practice, the receiver interference plays a significant role in a WSN.

Fig. 4 shows a sample topology which has six vertices with their respective transmission disks (or ranges). The sender and receiver interference numbers of all the vertices are reported in Table II.

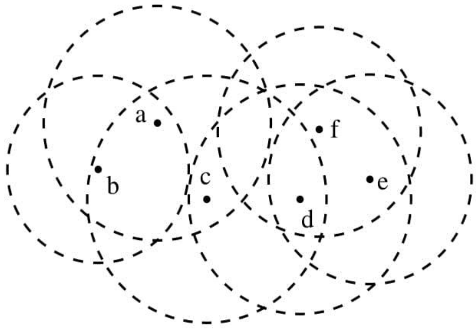

Fig. 4. Sample topology to illustrate interference.

TABLE II: INTERFERENCE OF THE TOPOLOGY IN FIG. 1.

Vertex	$I_S(v)$	$I_R(v)$
a	2	2
b	1	2
c	3	2
d	3	3
e	2	2
f	2	2

An optimal algorithm for minimizing the maximum node interference is proposed by Panda and Shetty [15] Authors also proposed a 2-approximation algorithm for minimizing the average node interference of a given network. Tan *et al*. [16] proposed exact algorithms for minimizing the average and maximum interference for the highway model where the nodes are distributed on a line. Authors proposed an $\mathrm{O}(\mathrm{n}^3\Delta)$ exact algorithm to minimize the average interference where n is the number of nodes and Δ is the maximum degree and an exact sub exponential time algorithm to minimize the maximum interference. Agrawal and Das [17] studied the problem of minimizing the sender interference and gave an optimal algorithm of running time $O((P_n + n^2)\log n)$ for minimizing the maximum sender interference for any connectivity predicate P, where $O(P_n)$ is the polynomial running time required to check the connectivity constraint. Some of the connectivity predicates are strong connectivity, simple connectivity, broadcast connectivity, k-vertex connectivity etc. [18]. Panda and Shetty [19] proposed two new models SUM and MAX to estimate the coverage based and transmission based interferences and proposed algorithms for minimizing the average node interference and maximum node interference. Authors have proved that under the MAX model, the algorithm gives an optimum solution for sender interference minimization problem and gives a 2 approximation algorithm for the average node interference problem.

Buchin [20] proved the NP-hardness of the receiver interference problem using a polynomial time reduction from the problem of determining a Hamiltonian path of a grid graph with maximum degree 3. Sharma *et al*. [21] proposed a polynomial heuristic for the problem of minimizing the maximum receiver interference of given network and analyzed the performance through the simulation. Shetty and Lakshmi [2] proposed two algorithms for receiver interference problem whose objective is to find a spanning tree that minimizes the maximum receiver interference and presented simulation to show the stability of the proposed algorithms. Authors also proposed an optimal algorithm for minimizing the maximum and total receiver interference in broadcast networks. Bilo and Proietti [22] gave an optimal algorithm for the interference problem for any connectivity predicate π . Authors also proved that minimizing the maximum receiver interference problem cannot be approximated within a factor of $\left(\frac{1}{2} - \epsilon\right) \ln n$ unless $NP \subseteq \mathrm{DTIME}\, n^{O(\log\log n)}$ for any $\epsilon > 0$. Rickenbach *et al*. [23] investigated the interference problem for the highway model and obtained an algorithm whose approximation ratio is $O(4\sqrt{\Delta})$, where Δ is the maximum node degree. Shetty and Lakshmi [24] studied the dual power receiver interference problem whose objective is to assign range to the nodes from a set of two power levels such that the obtained subgraph is connected and maximum receiver interference is minimized. NP-completeness is proved by reducing from degree constrained minimum spanning tree problem. Authors also gave an approximation algorithm using an arbitrary approximation algorithm of dual power management problem and presented simulation results to support the proposed algorithm.

In this paper, we consider the problem of assigning power levels to the nodes of a given network from a specified set of two power levels such that the maximum interference is minimized and the network connectivity is preserved. An optimal algorithm which runs in polynomial time is proposed for sender interference and a similar algorithm is examined for the case of receiver interference.

TABLE III: NOTATIONS USED IN THE ALGORITHMS

Notation	Explanation
$R(v)$	Range of vertex v
$I_S(v)$	Sender interference of vertex v
$I(S)$	Maximum Sender interference
$I_R(v)$	Receiver interference of vertex v
$I(R)$	Maximum Receiver interference
H_{no}	Number of nodes with high power
$I(S)_L$	Maximum sender interference when all the nodes are assigned low power
H	Resultant subgraph
$R(H)$	Total range of resultant subgraph H
$I_{OPT}(S)$	Maximum sender interference of optimal solution

There are different methods for interference minimization like signal-to-interference-plus-noise (SINR) ratio which is a real physical representation of interference [25] and topology control etc. The focus of this paper is to minimize the interference by controlling the topology of a given network using the centralized algorithms. Total power and maximum interference values of the resultant subgraphs are computed and compared for the same instances through simulation. Notations used in this paper are listed in Table III. Organization of the rest of the paper is as follows: section II formulates the problem and proposes the algorithms. Results are exhibited in section III and finally, section IV concludes the paper.

II. Problem Statement

The objective of the problem of minimizing the maximum sender interference with two power levels is to assign each node a power level from the set of two power levels to such that the reduced topology satisfies the property of strong bidirectional connectivity and the maximum sender interference is minimized. Let $G = (V, E, w)$ be the given complete graph which represents the network where V is the set of vertices and $E = \{uv | u \neq v,\ u, v\ \in V\}$ is the set of edges. The set of edges i.e., E is partitioned into two sets i.e., set of low power edges and set of high power edges denoted by $E_l = \{uv \in E | w(uv) \leq L_p\}$ and $E_h = \{uv \in E | L_p < w(uv) \leq H_p\}$, respectively. It is to be noted that $E_l \cap E_h = \emptyset$ and $E_l \cup E_h$ need not equal E. The mathematical formulation of the problem is as follows:

Problem: Minimizing the maximum sender interference with two power levels.

Input: Complete graph $G = (V, E, w)$, $w: v \times v \rightarrow R^+$, two power levels, H_P and L_P.

Question: A spanning subgraph H of G such that $\cup_{v \in V} R(v) = \{L_P, H_P\}$ and maximum sender interference of H is minimized.

The problem of minimizing the maximum sender interference with two power levels (MSI-TPL) takes a complete graph as input with Euclidean distances as weights and returns a subgraph whose maximum sender interference is minimized and only two power levels are used for assignment.

Initially, the low power edges are added to the resultant subgraph. It is to be noted that the sender interference of this subgraph is the lower bound for the maximum sender interference of the optimal solution. Now, the sender interference is computed for each vertex v if it were assigned high power. Next, the vertices are sorted in the non decreasing order of their sender interference denoted by $I_S(v)$ if they were assigned high power. Let $u_1, u_2, \dots, u_n$ be the vertices in the non decreasing order of their sender interference i.e., $I_S(u_i) \leq I_S(u_{i+1})$, for $1 \leq i \leq n - 1$. First, we assign high power to vertex u_i (initially $i = 1$) and verify whether the resultant subgraph H is connected, if yes then the subgraph H is the optimal solution, else high power is assigned to the subsequent vertex u_{i+1} and the connectivity is verified. Each time a new vertex u_i is assigned high power, the resultant graph gets updated by $H = H \cup \{u_i u | u_i u \in E_h, R(u) = H_P\}$ and this process terminates when the resultant subgraph becomes connected. Above explained method is presented in Algorithm 1.

Algorithm 1: MSI-TPL

Input: A complete graph $= (V, E, w)$, $w: v \times v \rightarrow R^+$, H_P and L_P.

Output: A spanning subgraph of G such that $\cup_{v \in V} R(v) = \{L_P, H_P\}$ and maximum sender interference is minimized.

```
Step1:   H = E_l = E_h = ∅.
Step 2:  For each edge xy ∈ E do
           If (w(xy) ≤ L_P) then
            E_l = E_l ∪ {xy}
            End If
           If (L_p < w(xy) ≤ H_P) then
           E_h = E_h ∪ {xy}
           End If
           End For
Step 3:  For each v ∈ V do
            R(v) = L_P
            End For
Step 4:  For each edge e ∈ E_l do
            H = H ∪ {e}
            End For
Step 5: Let u_1, u_2, ..., u_n be the vertices in the non
decreasing order of their sender interference with high
power
Step 6:  do
            R(v) = H_P
            i + +
            For each vertex v ≠ u_i ∈ V do
            H = H ∪ {u_i v | u_i v ∈ E_h && R(v) = H_P}
           End For
           While (DFS(H) < n)
Step 7: Return H, I_S(u_i).
```

Note II.1. In Algorithm 1, the procedure $DFS(H)$ performs the depth first search traversal on H and returns the number of vertices visited [26]. If it returns the value n, then it is sure that the subgraph H is connected.

Theorem II.2. Algorithm MSI-TPL always returns a connected subgraph.

Proof. Since the algorithm performs DFS and checks the connectivity each time when a vertex is assigned high power, connectivity of the resultant subgraph is achieved.

Theorem II.3. Algorithm MSI-TPL always results in an optimal solution.

Proof. The algorithm considers a complete graph as input and computes the sets of low power and high power edges i.e., E_l and E_h and assigns low power to all the vertices. Next, the vertices are sorted in the non decreasing order of their sender interference if they were assigned high power. Starting from the vertex u_i $(i = 1)$ with the least value of maximum sender interference, it checks if the subgraph becomes connected if that vertex were assigned high power. If yes then the subgraph is the optimal solution, else it assigns high power to the next vertex u_{i+1} and repeats the same procedure until the subgraph becomes connected.

Let $u_1, u_2, \ldots, u_n$ be the vertices in the non decreasing order of their sender interference i.e., $I_S(u_i) \leq I_S(u_{i+1})$, for $1 \leq i \leq n-1$. Let us partition the set of vertices of V into subsets S_i such that $\cup_{i=1}^{k} S_i = V$ and $\cap_{i=1}^{k} S_i = \emptyset$ and vertices of the same set have the same maximum sender interference i.e., no two sets have the same maximum sender interference. Let $S_1, S_2, \ldots, S_k$ be the subsets of vertices such that $I(S_1) < I(S_2) \ldots < I(S_k)$ where $I(S_i)$ represents the interference of the vertices of the set S_i. Fig. 5 shows the partition of the vertices into sets $S_1, S_2, \ldots, S_k$ such that all the vertices of a particular set have the same sender interference and no two sets have the same sender interference. In the algorithm, when the connectivity constraint is satisfied after high power is assigned to a particular vertex $v \in S_i$, it indicates that even if all the vertices of the set S_{i-1} are assigned high power, the connectivity is not satisfied. So, there cannot be a connected topology with lesser interference than $I(S_i)$.

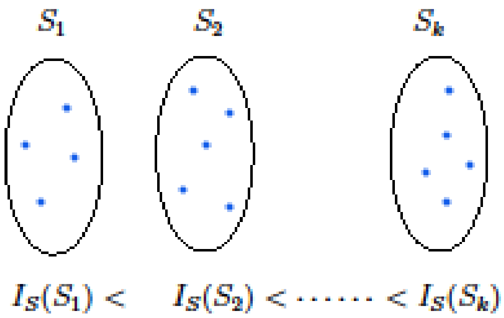

Fig. 5. Illustration of Theorem II. 3

Theorem II.4. Algorithm MSI-TPL runs in $O(n^3)$ time.

Proof. Computing E_l and E_h, the set of low power and high power edges respectively takes $O(n^2)$ running time. Assigning high power and computing the sender interference of each vertex takes $O(n^2)$ running time. Sorting the sets according to their maximum sender interference takes $O(n\,log\,n)$ using the best technique merge sort [27]. Next, traversing through the subgraph to check the connectivity constraint using DFS [19] after assigning high power to each vertex takes $O(n.n^2)$ time. So, the algorithm takes $O(n^3)$ running time.

Remark II.5. Using divide and conquer [28] method which takes $O(log\,n)$ running time while assigning high power to the vertices will result in a better computational complexity. High power is assigned to the vertex at the middle position and if connectivity is achieved then we withdraw the assignment and proceed with the left subset else we proceed with the assignment to the vertices from the right subset. This procedure is repeated until we remain with a single element in the array. In such a case, the complexity would be $O(n^2)$.

A similar algorithm named MRI-TPL for minimizing the maximum receiver interference with two power levels is presented using the same technique which is presented in Algorithm 2.

Algorithm 2: MRI-TPL
Input: A complete graph $G = (V, E, w)$, $w: v \times v \rightarrow R^+$, H_P and L_P.
Output: A spanning subgraph of G such that $\cup_{v \in V} R(v) = \{L_P, H_P\}$ and maximum receiver interference is minimized.
Step 1: Repeat steps 1 to 4 of Algorithm 1
Step 2: Let $u_1, u_2, \ldots, u_n$ be the vertices in the non decreasing order of their sender interference with high power
Step 3: Repeat step 6 of Algorithm 1
Step 4: Return $H, I(R)$.

Theorem II.6. Algorithm MRI-TPL runs in $O(n^3)$ time.

Proof. The proof of this theorem is similar to the that of Theorem II.4.

III. Results

The simulation results are presented in this section for which a 1000 × 1000, 2D plane is considered to deploy the nodes using a random function that follows the uniform distribution. A positive real number is associated with each pair of nodes which is the distance between those two nodes. Transmission power assigned to each node is a function of distance, transmission threshold value t and path loss coefficient α which depends on various environmental factors [29]. The total transmission power of the reduced topology is given by $\sum_{i=1}^{n} t.R(v_i)^{\alpha}$. The values $t = 1$ and $\alpha = 2$ are used in this experiment and the two power levels H_P and L_P are given as input to the problem.

TABLE IV: Optimal Sender Interference by MSI-TPL

N	H_{no}	$I(S)_L$	$I_{OPT}(S)$	$R(H)$
10	4	4	4	2397875.75
20	11	5	5	2303819.50
30	18	5	7	2263196.00
40	26	6	7	2224561.75
50	40	6	8	2163434.75
60	13	8	9	2143374.75
70	37	8	9	2122247.00
80	28	7	10	2111000.00
90	34	7	9	2100421.00
100	51	6	8	2034881.12
200	61	9	10	2028162.00
300	111	7	9	1962488.50
400	272	6	8	1914522.75
500	260	7	9	1972800.12
600	246	7	8	1772758.00
700	333	7	8	1713035.37
800	312	7	8	1668968.25
900	564	6	7	1583267.87
1000	444	7	8	1740944.00

The headers in Table IV denoted by N, H_{no}, $I(S)_L$, $I_{OPT}(S)$ and $R(H)$ represent the number of nodes, number of nodes with high power, maximum sender interference when all the nodes are assigned low power, maximum sender interference of optimal solution and total power of the resultant subgraph, respectively. The total power values obtained by both the algorithms MSI-TPL and MRI-TPL for the same instances are plotted in Fig. 6.

Maximum sender interference of the subgraph obtained when all the nodes are assigned low power initially and optimal sender interference of the resultant subgraph by MSI-TPL are illustrated in Fig. 7. Similarly, Fig. 8 shows the above explained parameters for maximum receiver interference obtained by the MRI-TPL algorithm are illustrated in Fig. 8.

TABLE V: COMPARISON OF ALGORITHMS MSI-TPL, MRI-TPL

N	MSI-TPL		MRI-TPL		$R(H)$	
	H_{no}	$I_{OPT}(S)$	H_{no}	$I(R)$	MSI-TPL	MRI-TPL
10	7	5	2	7	3045313.50	1873336.12
20	17	5	9	9	2766613.2	1958861.50
30	24	7	9	10	2506536.50	1518501.75
40	31	6	21	10	2098797.50	1663817.00
50	38	8	29	10	1998524.62	1700380.62
60	43	7	34	9	1878793.00	1617694.00
70	64	7	54	8	1764775.37	1567151.12
80	55	7	45	7	1861279.62	1640671.50
90	82	6	67	8	1506592.75	1311504.50
100	63	8	47	8	1770359.62	1494061.12

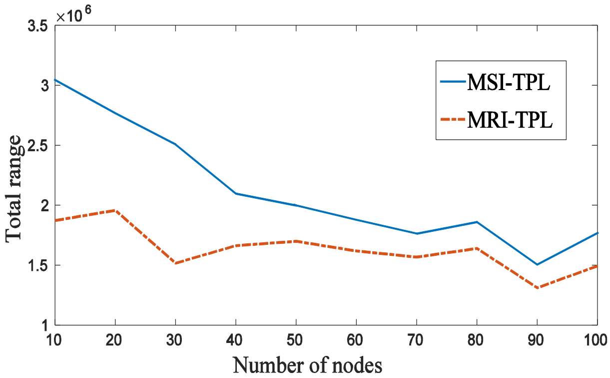

Fig. 6. Comparison of total range by MSI-TPL and MRI-TPL.

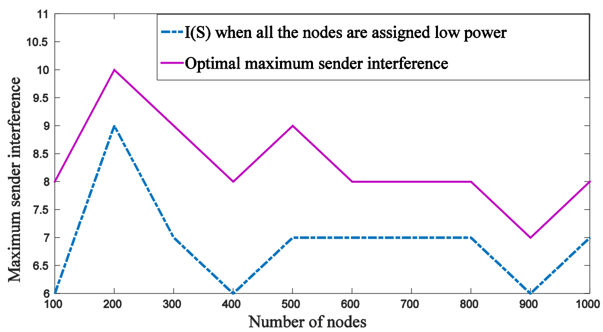

Fig. 7. Maximum sender interference by MSI-TPL.

Algorithm MRI-TPL is also executed to analyze the receiver interference of the given network. Algorithms MSI-TPL and MRI-TPL are compared for the same instances and the obtained numerical values are reported in Table V. In this table, the number of high power nodes, total power consumption, and interference (respective) by both the algorithms are compared for the same instances by varying the number of nodes from 10 to 100 in steps of 10.

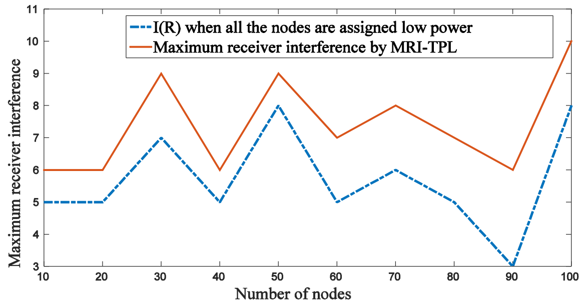

Fig. 8. Maximum receiver interference by MSI-TPL.

IV. CONCLUSIONS

In this research, an optimal algorithm is proposed for the problem of determining a spanning subgraph which is connected and uses only two power levels for the assignment such that the maximum sender interference of the subgraph is minimized. Simulation results are also presented to analyze the proposed algorithms for various numbers of nodes. Using a similar method, an algorithm for minimizing the maximum receiver interference is also proposed and both the algorithms are compared for the same network with same power levels. Minimizing the maximum the interference can be studied for any number of power levels for both sender and receiver interference.

ACKNOWLEDGMENT

The authors would like to extend thanks to the National Institute of Technology Karnataka, Surathkal for the support in this research work.

REFERENCES

[1] P. Santi, *Topology Control in Wireless Ad Hoc and Sensor Networks*, John Wiley & Sons, 2005.

[2] D. P. Shetty and M. P. Lakshmi, "Algorithms for minimizing the receiver interference in a wireless sensor network," in *Proc. Distributed Computing, VLSI, Electrical Circuits and Robotics*, pp. 113–118, 2016.

[3] M. M. Halld´orsson and T. Tokuyama, "Minimizing interference of a wireless ad-hoc network in a plane," *Theor. Comput. Sci*., vol. 402, no. 1, pp. 29–42, 2008.

[4] E. L. Lloyd, R. Liu, M. V. Marathe, R. Ramanathan, and S. S.Ravi, "Algorithmic aspects of topology control problems for ad hoc networks," *MONET*, vol. 10, no. 1-2, pp. 19–34, 2005.

[5] G. Calinescu, S. Kapoor, and M. Sarwat, "Bounded-hops power assignment in ad hoc wireless networks," *Discrete Applied Mathematics*, vol. 154, no. 9, pp. 1358-1371, 2006.

[6] L. M. Kirousis, E. Kranakis, D. Krizanc, and A. Pelc, "Power consumption in packet radio networks," *Theor. Comput. Sci*., vol. 243, no. 1-2, pp. 289–305, 2000.

[7] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "Wireless sensor networks: A survey," *Computer Networks*, vol. 38, no. 4, pp. 393–422, 2002.

[8] X. Cheng, B. Narahari, R. Simha, M. X. Cheng, and D. Liu, "Strong minimum energy topology in wireless sensor networks: NP-completeness and heuristics," *IEEE Transactions on Mobile Computing*, vol. 2, no. 3, pp. 248-256, 2003.

[9] Y. Rong, H. Choi, and C. Hyeong-Ah, "Dual power management for network connectivity in wireless sensor networks," in *Proc. 18th International Parallel and Distributed Processing Symposium (IPDPS 2004), CD-ROM / Abstracts Proceedings*, Santa Fe, New Mexico, USA, 2004.

[10] D. P. Shetty and M. P. Lakshmi, "Minimizing the total range with two power levels in wireless sensor networks," in *Advanced Computing and Communication Technologies*, J. K. Mandal, D. Bhattacharyya, and N. Auluck, Eds., Singapore, 2019, pp. 183–191.

[11] P. Carmi and M. J. Katz, "Power assignment in radio networks with two power levels," *Algorithmica*, vol. 47, no. 2, pp. 183–201, 2007.

[12] G. C˘alinescu, "1.61-approximation for min-power strong connectivity with two power levels," *J. Comb. Optim.*, vol. 31, no. 1, pp. 239–259, 2016.

[13] A. K. Abu-Affash, P. Carmi, and A. P. Tzur, "Dual power assignment via second hamiltonian cycle," *J. Comput. Syst. Sci.*, vol. 93, pp. 41–53, 2018.

[14] S. Hoffmann, T. Kampermann, and E. Wanke, "Minimizing the number of max-power users in ad-hoc wireless networks with minimum node degree requirements," *Inf. Process. Lett.*, vol. 136, pp. 25–29, 2018.

[15] B. S. Panda and D. P. Shetty, "Strong minimum interference topology for wireless sensor networks," in *Advanced Computing, Networking and Security. ADCONS 2011. Lecture Notes in Computer Science*, P. S. Thilagam, A. R. Pais, K. Chandrasekaran, and N. Balakrishnan, Eds., Springer, Berlin, Heidelberg, 2011, vol. 7135.

[16] H. Tan, T. Lou, Y. Wang, Q. S. Hua, and F. C. Lau, "Exact algorithms to minimize interference in wireless sensor networks," *Theoretical Computer Science*, vol. 412, no. 50, pp. 6913-6925, 2011.

[17] P. Agrawal and G. K. Das, "Improved interference in wireless sensor networks," CoRR, abs/1207.5708, 2012.

[18] D. B. West, *et al*., "Introduction to graph theory," in *Prentice Hall Upper Saddle River*, 2001, vol. 2.

[19] B. S. Panda and D. P. Shetty, "Minimum interference strong bidirectional topology for wireless sensor networks," *IJAHUC*, vol. 13, no. 3-4, pp. 243–253, 2013.

[20] K. Buchin, "Minimizing the maximum interference is hard," CoRR, abs/0802.2134, 2008.

[21] A. K. Sharma, N. Thakral, S. K. Udgata, and A. K. Pujari, "Heuristics for minimizing interference in sensor networks," in *Distributed Computing and Networking. ICDCN 2009. Lecture Notes in Computer Science*, V. Garg, R. Wattenhofer, and K. Kothapalli Eds., Springer, Berlin, Heidelberg, 2009, vol. 5408.

[22] D. Bil`o and G. Proietti, "On the complexity of minimizing interference in ad-hoc and sensor networks,' *Theor. Comput. Sci.*, vol. 402, no. 1, pp. 43–55, 2008.

[23] P. von Rickenbach, S. Schmid, R. Wattenhofer, and A. Zollinger, "A robust interference model for wireless ad-hoc networks," in *Proc. 19th International Parallel and Distributed Processing Symposium*, Denver, CO, USA, 2005.

[24] D. P. Shetty and M. P. J. Lakshmi. Appl. Math. Comput. (2019). [Online]. Available: https://doi.org/10.1007/s12190-019-01242-5

[25] L. Xiang-Yang, *et al*., "Interference-aware topology control for wireless sensor networks," in *Proc. Second Annual IEEE Communications Society Conference on Sensor and Ad Hoc Communications and Networks*, 2005.

[26] J. Kleinberg and E. Tardos, "Algorithm design," Pearson Education India, 2006.

[27] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms*, 3rd Edition. MIT Press, 2009.

[28] K. H. Rosen, *Discrete Mathematics and Its Applications*, New York: McGraw-Hill, 2011.

[29] P. Carmi and L. Chaitman-Yerushalmi, "On the minimum cost range assignment problem," in *Proc. Algorithms and Computation - 26th International Symposium*, Nagoya, Japan, 2015, pp. 95–105.

M. Prasanna Lakshmi did her B.Sc. in M.S.Cs. (Mathematics, Statistics and Computer science) and M.Sc. in Mathematics with scientific computing. She is pursuing her Ph.D. from National Institute of Technology, Karnataka, Surathkal, India.

Pushparaj Shetty D. did his BE, ME in Computer Science and Engineering and Ph.D from Indian Institute of Technology Delhi, India. He is working as an Assistant Professor in the Department of Mathematical and Computational Sciences at National Institute of Technology, Karnataka, Surathkal, India. His research interests are Graph algorithms, algorithmic aspects of wireless sensor networks etc.