Mitigation of Nonlinear Effects with Volterra and Wiener Hammerstein Electrical Equalizer in LDPC Coded Coherent Optical OFDM System

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Abstract —Coherent Optical Orthogonal Frequency Division Multiplexing (CO-OFDM) finds a number of applications in the Telecommunication world. Though CO-OFDM is immune to the dispersive effect of the fiber channel it is subjective to the channel induced nonlinear effects. In our paper, we discuss the techniques to mitigate the intensity dependent nonlinear self – phase modulation (SPM) effects. Its highly important to reduce the phase shift introduced by the SPM since it results in added pulse broadening effect which remains uncompensated with the addition of cyclic prefix-suffix and the dispersion compensation schmes. A combination of Low Density –Parity Check(LDPC) codes with Volterra and Wiener-Hammerstein Electrical Equalizers are proposed to mitigate the SPM effect.

Index Terms—CO-OFDM, self-phase modulation, low density –parity check codes, fiber channel

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a revolutionary scalable modulation scheme used in wireless fidelity LAN (IEEE 802.11 a/g), Digital Audio and Video and Audio Broadcasting (DAV/DAB), and Digital Subscriber Line (DSL) for internet access, video conferencing, Voice over IP (VOIP), high-speed cellular modem and streaming multimedia. The two challenging Fourth-Generation (4G) mobile communications Worldwide Interoperability for Microwave Access (WiMAX, or IEEE802.16) from the computing community and Long-Term Evolution (LTE) from the telecommunication community have tailored OFDM for high-speed data transmission to meet the ever-increasing demand for services. OFDM being a multicarrier modulation scheme is a supremo compared to the already existing single carrier modulation schemes despite the complexity involved in the signal processing. Efficient computation of the Fast Fourier Transform (FFT) ensures the multiplexing of low data streams onto subcarriers which are orthogonal to each other. The cyclic prefix added with OFDM signal makes it reluctant to the dispersive channel effect. Also, the scalability of the spectrum among the subcarriers into multiple sub-bands of the OFDM spectrum provides intense flexibility in contrast to single carrier spread over the entire spectrum.

CO-OFDM is recommended to encounter chromatic dispersion (1-5). It is expanded towards polarization-diversity detection and presented to be robust to fiber Polarization Mode Dispersion (PMD). The CO-OFDM transmission is testified for 1000 km SSMF fiber distance at 8 Gb/s, and reported for a maximum fiber distance of 4160 km at 20 Gb/s(2). The first nonlinearity reduction is reported for CO-OFDM systems (3) emphasising the coherent optical OFDM and the direct detection optical OFDM in multimode fiber, short and long distance SMF transmissions.

Optical OFDM is found to be similar and different from RF OFDM. The inherent problem of OFDM is peak-to-average power ratio (PAPR) and sensitivity to phase and frequency noise. The fiber channel introduces linear effect like dispersion and other nonlinear effects like Self-Phase Modulation (SPM), Four-Wave Mixing (FWM) and Cross-Phase Modulation (CPM) effects.

We propose the Volterra nonlinear equalizer (VNLE) and Wiener –Hammerstein nonlinear Equalizer for mitigating the nonlinear SPM effect. A comparison of VNLE and LDPC with VLNE and WHNLE and LDPC with WHNLE are compared.

II. Simulation Set-up

The proposed block schematic of CO-OFDM system is pictured in Fig. 1.

A 100 Gbps data signal is coded with Low-Density Parity Check codes. The coded bits are mapped onto 512 subcarriers into a complex-valued function with real and imaginary parts with 16-Quadrature Amplitude Modulation. The signal points from subcarriers are complex- valued and are deliberated as inputs for 1024 FFT of a multicarrier OFDM signal. To reduce ISI, a large number of subcarriers is selected such that the duration of the OFDM symbol is considerably higher than the spread pulse duration. Overlapping of adjacent OFDM symbols due spreading leads to loss of orthogonality of subcarriers. The addition of cyclic prefix prevents this effect. The Guard interval by adding cyclic

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prefix/suffix should be prolonged than the delay spread induced by dispersion in order to avoid overlapping of adjacent OFDM symbols. A section of the OFDM symbol is appended at the start of the symbol as prefix and at the end of the symbol as suffix. The OFDM symbols are passed through digital to analog converter and then upconverted to Radio Frequencies. Coherent modulation is done using two Mach-Zehnder modulators and a CW laser.

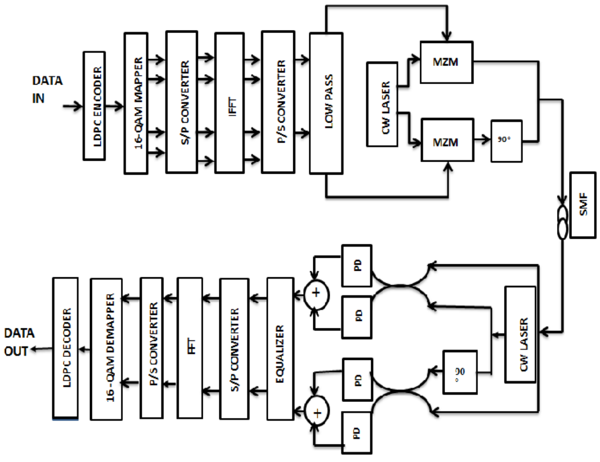

Fig. 1. Schematic of CO-OFDM System with Equalizer

The optical OFDM signal is transmitted through single mode fiber G.652 of coefficient of attenuation 0.2 dB/km, coefficient of polarization mode dispersion 0.10 ps $km^{-1/2}$ and coefficient of chromatic dispersion 16.75 $psnm^{-1}km^{-1}$ and, non-linear index of refraction is $2.6x10^{-20}$ m^2/W and effective area 80 μm^2. An amplifier of 4dB gain is introduced in the link to compensate attenuation loss. An optical filter is employed at the receiver to reduce the effect of Amplified spontaneous emission (ASE) noise developed at the EDFA. The wavelength of the laser source at the receiver which acts as a local oscillator signal is matched with the wavelength used for modulation. The In- Phase component and the Quadrature – Phase component of the OFDM signal is recovered by a 2×4 90° hybrid pair of photo-detectors. The effect of noises at the Photo-detector, such as shot noise, thermal noise, dark current and ASE noise is considered in the simulation studies. Equalization is done in Mat lab, after the optical receiver to reduce the effect on nonlinear self-phase modulation. A training sequence of length 16384 bits is chosen to determine the initial filter coefficients. The filter coefficients are updated with joint normalized least mean squared algorithm. The output of the equalizer is now passed into the 1024 FFT block and then into 16-QAM demodulator. BER and OSNR are analyzed.

TABLE I: SYSTEM PARAMETERS SET IN THE SIMULATION SET-UP

S.No	Parameters	Values
1	Bit Rate	100 Gbps
2	SMF Fiber Attenuation	0.2 $dBkm^{-1}$
3	Chromatic Dispersion Coefficient	16.75 $psnm^{-1}km^{-1}$
4	Non-linear index of refraction	$2.6x10^{-20}$ m^2/W
5	Effective area	80 μm^2
6	Number of subcarriers	512
7	FFT points	1024
8	Mapping	16-QAM

III. VOLTERRA EQUALIZER

For a continuous-time Volterra model (5-7), its output signal *y(t)* can be represented by the sum of linear, quadratic, cubic and higher-order convolution integrals of input signal *x(t)*.

The input-output relation for the Volterra series is given by

$$y(n) = \sum_{i=0}^{\infty} g_1(i)x(n-i) + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} g_2(i,j)\, x(n-i)x(n-j) + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} g_3(i,j,k)\, x(n-i)x(n-j)x(n-k)+.... \quad (1)$$

where the input signal is *x(.)*, the output signal is *y(.)*, and th-order kernels of the Volterra model is $g_i(.)$. The communication channel is modelled as band pass filter and the input-output signals are represented as complex envelopes. The even order Volterra kernels are ignored in a higher order Volterra series because they do not generate signal that lies in band. Ignoring the second order kernel in the third order kernel of the Ignoring the second order kernel in the third order kernel of the Volterra model with finite memory can be represented by

$$y(n) = \sum_{i=0}^{N} g_1(i)x(n-i) + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=0}^{N} g_3(i,j,k)\, x(n-i)x(n-j)x^*(n-k) + e(n) \quad (2)$$

where N is the memory length, $()^*$ denotes the complex conjugate, x(n) and y(n) are the input and output signal complex envelopes, e(n) is modeling error, $g_1(i)$ and $g_3(i,j,k)$ are the linear and cubic Volterra kernels respectively.

The normalized mean square error (NMSE) is represented as:

$$\frac{E^T E}{Y^T Y} = \frac{(Y-\sum_{i=1}^{\infty} v_i q_i)^T (Y-\sum_{i=1}^{\infty} v_i q_i)}{YY^T} = 1-\sum_{i=1}^{L} D_i \quad (3)$$

$$D_i = \frac{v_i^2 q_i^T q_i}{YY^T} \quad (4)$$

The most significant v_i are identified out of the coefficients *L*. The number of coefficients *L* required is $(N+1) + (N+1)^3$. Volterra series is a combination of linear Volterra kernels. As a result Volterra kernels can be updated using LMS, RLS, and CMA algorithms.

IV. Wiener Hammerstein Nonlinear Equalizer

The major drawback of Volterra based nonlinear equalizer is the computational complexity. Wiener–Hammerstein model(7) consists of two linear filters and a memoryless nonlinearity. It has a simpler structure and requires less calculation. A wiener- Hammerstein consists of two linear filters and one memoryless nonlinearity. Finite Impulse Response (FIR) filters are used as linear filters and only odd-order terms of the polynomial are chosen as memoryless nonlinearity.

The input to the first FIR filter is represented in the vector form as

$$S'(n) = [S(1), S(2), \dots \dots, S(n - M_1 + 1)]^T \quad (5)$$

The input to the nonlinear filter is represented

$$Y(n) = [y(1), y(2), \dots \dots, y(n - M_1 + 1)]^T \quad (6)$$

in the vector form as and

The input to the second FIR filter is represented in the vector form as

$$Q(n) = [q(1), q(2), \dots \dots, q(n - M_1 + 1)]^T \quad (7)$$

Let S*'(n)* be the input to the equalizer, the output of first FIR filter, Y(n) can be given by

$$Y(n) = \sum_{i=0}^{P_1} Z(i) S(n - i) \quad (8)$$

When Z(i) is the linear filter coefficient and the length of the filter memory is P_1.

The input y(n) and output Q(n) of the nonlinear channel are related by

$$Q(n) = v(0)Y(n) + v(1)Y^2(n)Y^*(n) \quad (9)$$

The output signal B(n) and the input signal Q(n) of the second filter are related by

$$B(n) = \sum_{i=0}^{P_2} C(i) Q(n - i) \quad (10)$$

where P_2 is the length of memory length of the second FIR filter and C(i) is the second FIR filter coefficient.

V. Results and Discussion

The variation of BER as a function of OSNR in Volterra Equalizer updated with LMS, RLS and CMA in LDPC coded OFDM is displayed in Fig. 2. With LDPC codes PAPR is reduced. With Volterra based nonlinear equalizer the effect of SPM is reduced. With the combination of LDPC and Volterra based nonlinear equalizers, the effect nonlinearities are totally reduced and it can be observed.

On determining the filter coefficients with LMS algorithm, BER is below 10^{-3} at 8.8dB OSNR, with CMA algorithm BER is 10^{-3} ^{a}t 8.8 dB OSNR and with RLS algorithm BER is below $10^{-4.5}$ at OSNR 8 dB. Determining the Volterra equalizer with RLS algorithms outperforms LMS and RLS. The PAPR effect is reduced by LDPC codes and RLS quickly converges quickly so that the effect of SPM is greatly reduced and thereby the BER is very low.

Wiener – Hammerstein nonlinear equalizer is less complex compared to the computational complexity of Volterra nonlinear equalizers. BER vs OSNR plot of Wiener Hammerstein based electrical equalizer for OFDM with LDPC is pictured in Fig.3. The equalizer is updated with LMS, RLS and CMA algorithms one at a time. It is observed that LMS resulted in BER less than 10^{-4} at 9.6dB OSNR. RLS resulted in BER below $10^{-4.5}$ at 8.2dB OSNR and CMA of BER less than $10^{-3.5}$ at OSNR 8.4 dB.

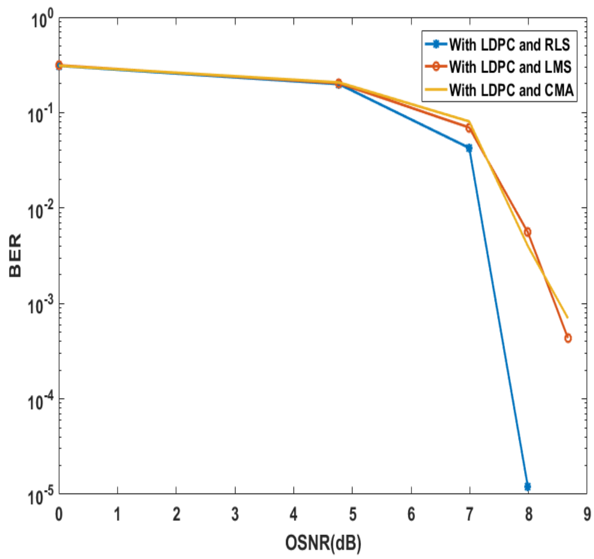

Fig. 2. Variation of BER as a Function of OSNR with Volterra with LMS, RLS, CMA Equalizers in LDPC coded CO-OFDM

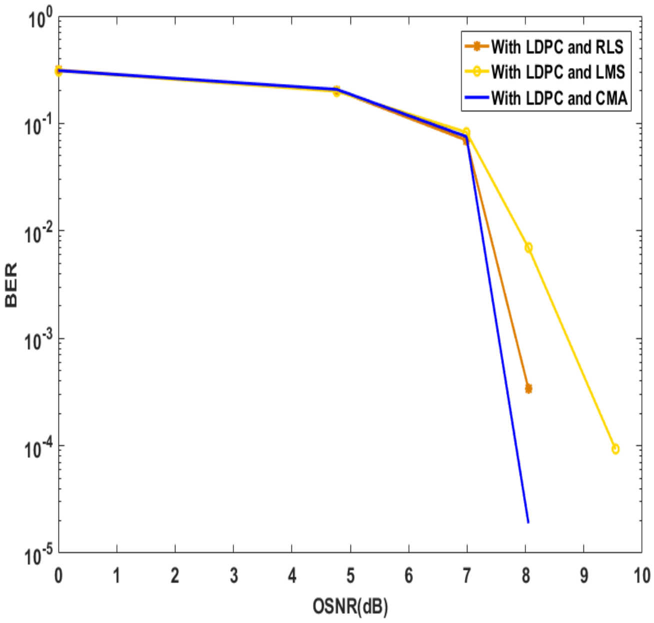

Fig. 3. Variation of BER as a Function of OSNR with LDPC Coded Wiener Hammerstein with LMS, RLS, CMA Equalizers

The variation of BER in response to launch power is pictured in Fig. 4. The optical launch power is varied from -5dBm to 5dBm and the BER is observed with and without LDPC codes in a CO-OFDM system with Volterra Nonlinear Electrical Equalizer. Results reveal that error rate is reaching $10^{-3.5}$ with Volterra Nonlinear Electrical Equalizer and without LDPC codes. With the LDPC codes, the error rate is less than $10^{-4.5}$ at 0dBm. As the launched optical power is increased, the effect of phase modulation is increased and hence the effect can be seen as an increase in BER.

SPM effect is reduced by Volterra Nonlinear Electrical Equalizer. Similar results as shown in Fig. 5 can also be observed in a LDPC coded CO-OFDM system with Wiener- Hammerstein Electrical Equalizer. Without PAPR compensation, BER is $10^{-3.5}$ and with LDPC codes it is less than $10^{-4.5}$ in a CO-OFDM system with Wiener-Hammerstein Electrical Equalizer.

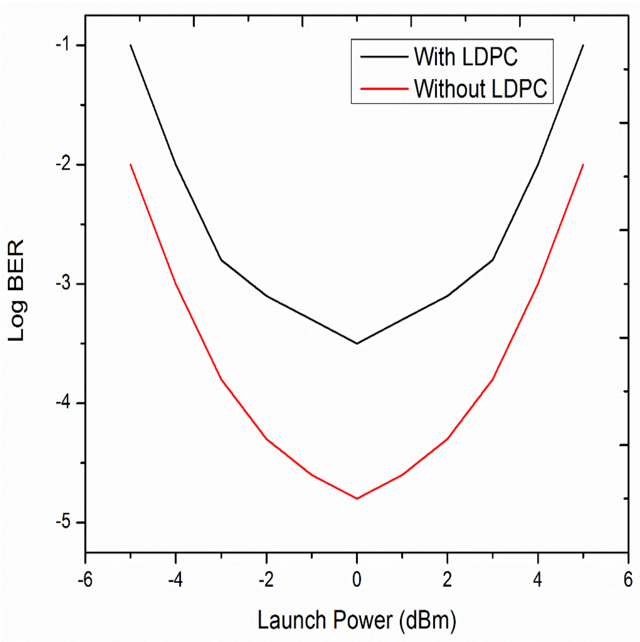

Fig. 4. Variation of BER as a function varying launch powers in CO-OFDM with volterra nonlinear electrical equalizer

The maximum transmission distance supported for various scenarios is shown in Fig. 5. A CO-OFDM system supports a maximum fiber length less than 500 km. Without PAPR reduction with LDPC codes, it is nearing 900 km with Volterra Nonlinear Electrical Equalizer. Similar fiber length is supported with Wiener Hammerstein Nonlinear Electrical Equalizer without LDPC codes. Both the Nonlinear Electrical Equalizers on reducing SPM effect allow a maximum reach distance of 1000 km with PAPR reduction using LDPC codes.

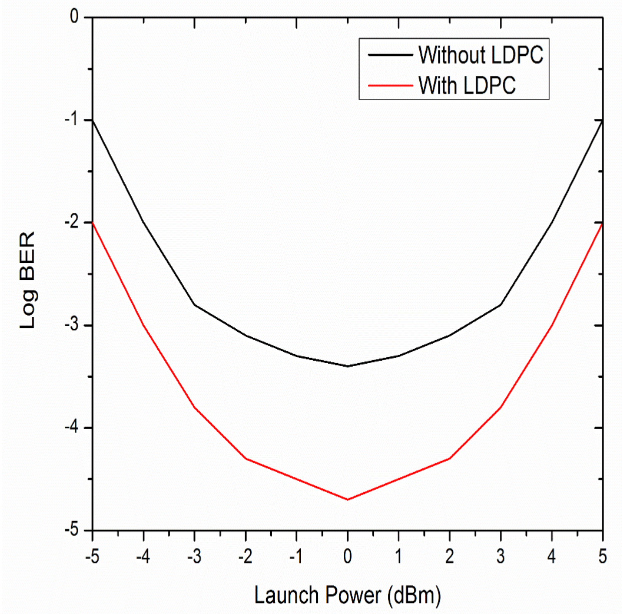

Fig. 5. Variation of BER as a Function varying Launch powers in CO-OFDM with Wiener – Hammerstein Nonlinear Electrical Equalizer

The variation of maximum transmission length as a function of different launch power is presented in Fig. 6. The maximum transmission distance obtained is 1000 km at 0 dBm launch for a LDPC coded CO-OFDM system with Volterra and Wiener –Hammerstein Electrical Equalizers. It is because of simultaneous PAPR and SPM reduction.

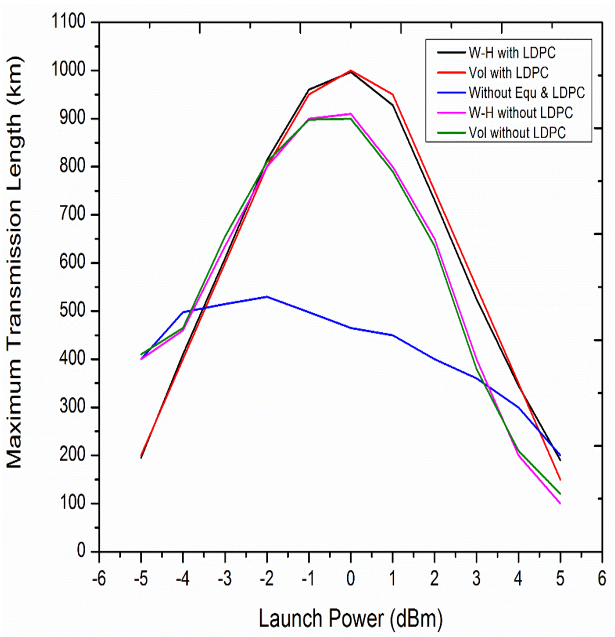

Fig. 6. Maximum transmission distance at varying Launch Power

VI. Conclusion

A 4 dB improvement in OSNR can be obtained with Volterra and Wiener - Hammerstein equalizer for BER of $10^{-4.8}$ at 8 dB OSNR for a distance of 1000 km compared with Adaptive decision feedback equalizer in LDPC coded OFDM system. LDPC codes are not only used for error detection and correction A tenfold reduction in BER is clearly shown in the results with LDPC coded in a CO-OFDM system with nonlinear electrical equalizers compared to the results published by Jie Pan and Chi-Hao Cheng (2011) with Volterra and Wiener – Hammerstein Nonlinear Electrical Equalizer for a maximum transmission fiber length of 800 km.

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