Design and Implementation of an Improved Error Correcting Code for 5G Communication System

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Abstract—Channel coding, which means adding a redundancy to information in order to minimize a bit error rate is one of key technologies in development of each generation of mobile systems. Recently, Low-Density Parity-Check (LDPC) Code has been selected to provide channel coding for 5G communication systems due to its high performance that approaches closely Shannon's limit, and its low complexity. However, there is still some work that is related to throughput, flexibility, and complexity of LDPC codes needs to be considered to meet all the 5G requirements. In this paper, an improved code of high throughput and flexibility was proposed. Here, a code is flexible if it supports a large number of coding rates and block lengths. In addition, the proposed code can be encoded and decoded with low computational and implementation complexities. The encoder and decoder can be implemented based on IEEE WiMAX standards, so the generated codes can be compared to the standard WiMAX LDPC codes. Theoretical calculations show that the proposed codes achieve significant improvements in throughput, latency, and computational and implementation complexity. The simulation employed WiMAX 802.16e standard and a fixed clock frequency of 500 MHz. The throughput ranged from 372 Mbps for rate 1/2 codes to 915 Mbps for rate 5/6 codes. An increasing of 1.4–2.2 times was obtained compared to the standard LDPC codes.

Index Terms—5G channel coding, WiMAX LDPC code, code flexibility, throughput, latency, computational and implementation complexity, BER performance.

I. Introduction

The recent interest in channel coding of 5G communication systems has directed to the Low-Density Parity-Check (LDPC) codes [1]. LDPC codes were invented by Gallager in 1960 and rediscovered by MacKay and Neal in the late of 1990s [2], [3]. LDPC codes are linear block codes and defined by a binary matrix called parity-check matrix (H-matrix). They are called low-density parity check because the density of their non-zero elements (ones) in H-matrix is very low. Because of high performance and low complexity, LDPC codes have been selected to provide channel coding for the next generations of communication systems [4]. The WiMAX 802.16e standard that was developed by IEEE uses LDPC codes as a channel coding scheme to meet the requirements of mobile communication systems, where this standard is the next step toward very high throughput wireless systems [5]. Recently, a lot of research has confirmed that LDPC codes achieve decoding with better bit error rate (BER) performance and lower computational complexity than turbo codes that are used in 3G and 4G systems; however, other features such as decoder flexibility and interconnection complexity still need to be improved. LDPC codes have become a key component of many communication systems such as WiFi (IEEE802.11n), WiMAX (IEEE802.16e), DVB-S2, CCSDS, and ITU G.hn [6]. The LDPC codes used in WiMAX standards are designed under hardware constraints; however, providing LDPC codes that have different lengths and rates imposes significant challenges on the LDPC decoder realization [5]. To overcome the problems of implementation and to meet the channel code requirements of 5G, several parameters such as throughput, flexibility, and complexity need to be improved. In this paper, two architectures for encoder and decoder are proposed based on WiMAX 802.16e standard to provide codes with high flexibility, improved throughput, and low complexity. The paper is organized as follows; Section II: introduces standard LDPC code. Section III: shows an overview on turbo code. The model proposed is depicted in section IV. Section V shows the valuation of the proposed code, while the simulation results with their discussions are detailed in Section VI. Finally, Section VII documents some conclusions based on the results of this work.

II. LDPC Codes

LDPC codes are fully characterized by a binary matrix called parity-check matrix (H-matrix), and also can be represented by bipartite graphs called Tanner graph in which non-zero elements (ones) of each row and column in H-matrix are represented by two sets of nodes; check node (CN) and variable node (VN) respectively. Variable and check nodes are connected by edge connections where the number of edges in the graph is equal to the number of ones in H-matrix. Fig. 1 shows an example of Tanner graph for H matrix given below:

$$H = \begin{bmatrix} 1&1&0&0&0&1&0&1 \\ 0&1&1&0&1&0&1&0 \\ 0&1&1&1&0&0&1&1 \\ 1&0&0&1&1&1&0&0 \end{bmatrix} \quad (1)$$

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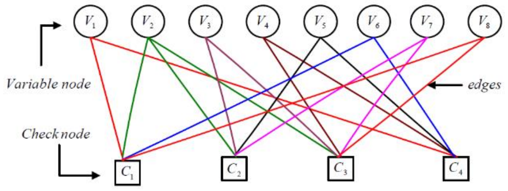

Fig. 1. Tanner graph representations of LDPC code

LDPC code, x must satisfy the equation below

$$Hx^T = 0\ (mod\ 2), \forall\ x\ \in c \quad (2)$$

An LDPC code of N length and K information is always represented by a parity-check matrix of size (M x N) and code rat of K/N, where M is the number of parity bits equivalent to M=N-K. The node degree is the number of edges on each node in the H-matrix. In addition, it represents the number of connections between VN and CN and defines two types of LDPC codes. If the weight distribution of column weights (d_v) and weight distribution of rows (d_c) are constant, the code is regular LDPC code. otherwise the code is irregular LDPC code. Each LDPC code that is used in WiMAX standards is defined by a sparse H-matrix which is, in turn, expanded from base matrix (H_b-matrix) of size m_b x n_b, where m_b=M/m_f and n_b=N/n_b with expansion factor Z_f. H-matrices of WiMAX standards can be found using the following equation

$$\mathrm{H} = \begin{bmatrix} P_{1,1} & P_{1,2} & \dots & P_{1,nb} \\ P_{2,1} & P_{2,2} & \dots & P_{2,nb} \\ \dots & \dots & \dots & \dots \\ P_{mb,1} & P_{mb,2} & \dots & P_{mb,nb} \end{bmatrix} \quad (3)$$

where $P_{i,j}$ is circularly right-shifted identity matrix (permutation matrix) of size Z_f x Z_f. For WiMAX 802.16e, the expansion factor Z_f varies from 24 to 96 with a granularity of 4 leading to support 19 different code sizes. The two parameters n_b and m_b take constant and maximum values of 24 and 12 respectively. Since permutation matrix tends to be rather large, the H-matrix can be written in a smaller form. This can be accomplished by labeling the circular identity matrix by the number of shift the matrix contained. Currently, there are six different codes with four different code rates that are used in WiMAX 802.16e standard based on six classes of base matrix (1/2, 2/3(A, B), 3/4(A, B), 5/6) [7, 8]. For an expansion factor of 96 and a code rate of 1/2, the code length is 2304 and the number of parity bits is 576. For the same expansion factor and code rate of 5/6, the code length is 2304 while the number of parity bits is 384. The following matrices are the H-matrices corresponding to 1/2 and 5/6 code rates.

Rate 1/2:

$$\begin{vmatrix}
-1 & 94 & 73 & -1 & -1 & -1 & -1 & -1 & 55 & 83 & -1 & -1 & 7 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & 27 & -1 & -1 & -1 & 22 & 79 & 9 & -1 & -1 & -1 & 12 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & 24 & 22 & 81 & -1 & 33 & -1 & -1 & -1 & 0 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
61 & -1 & 47 & -1 & -1 & -1 & -1 & -1 & 65 & 25 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & 39 & -1 & -1 & -1 & 84 & -1 & -1 & 41 & 72 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & 46 & 40 & -1 & 82 & -1 & -1 & -1 & 79 & 0 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & -1 \\
-1 & -1 & 95 & 53 & -1 & -1 & -1 & -1 & -1 & 14 & 18 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 \\
-1 & 11 & 73 & -1 & -1 & -1 & 2 & -1 & -1 & 47 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & -1 \\
12 & -1 & -1 & -1 & 83 & 24 & -1 & 43 & -1 & -1 & -1 & 51 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 \\
-1 & -1 & -1 & -1 & -1 & 94 & -1 & 59 & -1 & -1 & 70 & 72 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & -1 \\
-1 & -1 & 7 & 65 & -1 & -1 & -1 & -1 & 39 & 49 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 \\
43 & -1 & -1 & -1 & -1 & 66 & -1 & 41 & -1 & -1 & -1 & 26 & 7 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 0
\end{vmatrix}$$

Rate 5/6 code:

$$\begin{vmatrix}
1 & 25 & 55 & -1 & 47 & 4 & -1 & 91 & 84 & 8 & 86 & 52 & 82 & 33 & 5 & 0 & 36 & 20 & 4 & 77 & 80 & 0 & -1 & -1 \\
-1 & 6 & -1 & 36 & 40 & 47 & 12 & 79 & 47 & -1 & 41 & 21 & 12 & 71 & 14 & 72 & 0 & 44 & 49 & 0 & 0 & 0 & 0 & -1 \\
51 & 81 & 83 & 4 & 67 & -1 & 21 & -1 & 31 & 24 & 91 & 61 & 81 & 9 & 86 & 78 & 60 & 88 & 67 & 15 & -1 & -1 & 0 & 0 \\
50 & -1 & 50 & 15 & -1 & 36 & 13 & 10 & 11 & 20 & 53 & 90 & 29 & 92 & 57 & 30 & 84 & 92 & 11 & 66 & 80 & -1 & -1 & 0
\end{vmatrix}$$

All six matrices can be used when the expansion factor, Z_f is less than the maximum expansion (Z_0=96). In this case, each matrix can be expanded according to the following equations [9]

$$P_{f,i,j} = P_{i,j} \qquad \text{if } P_{i,j} \leq 0 \quad (4)$$

$$P_{f,i,j} = P_{i,j} * Z_f / Z_0 \qquad \text{if } P_{i,j} \geq 0 \quad (5)$$

The number of edges is also equal to the number of ones in the binary H-matrix. The degree distribution of the variable nodes and check nodes can expressed by $f[d_v^{max}, \dots, 3, 2]$ and $g[d_c^{max}, d_c^{max} - 1]$ respectively [5]. The six H-matrix classes of WiMAX 802.16e with its their degree distributions for VNs and CNs are summarized in Table I.

TABLE I: DEGREE DISTRIBUTIONS OF THE WIMAX 802.16E LDPC CODE

Code	f	g
$1/2$	$[2, 3, 6] = \{11/24, 1/3, 5/24\}$	$[6, 7] = \{2/3, 1/3\}$
$2/3$ A	$[2, 3, 6] = \{7/24, 1/2, 5/24\}$	$[10] = \{1\}$
$2/3$ B	$[2, 3, 4] = \{7/24, 1/24, 2/3\}$	$[10, 11] = \{7/8, 1/8\}$
$3/4$ A	$[2, 3, 4] = \{5/24, 1/24, 3/4\}$	$[14, 15] = \{5/6, 1/6\}$
$3/4$ B	$[2, 3, 6] = \{5/24, 1/2, 7/24\}$	$[14, 15] = \{1/3, 2/3\}$
$5/6$	$[2, 3, 4] = \{3/24, 5/12, 11/24\}$	$[20] = \{1\}$

The two functions, f and g also give the fraction of VNs and CNs with a certain degree. At the decoder, many decoding algorithms are known to recover information [12]: 1) Sum-Product algorithm (SPA), 2) Min-Sum algorithm, 3) Min-Max algorithm, 4) Message-Passing algorithm and 5) Bit-Flipping Decoding algorithm. The SPA is based on belief propagation and provides a better performance compared to the other algorithms. It has a scheduling of two phases, therefor all VNs ae updated in phase 1 and all CNs are updated in phase 2. In each phase, individual nodes are parallelized, so that they are independently processed. Each VN of degree d_v determines an update of message m by the following relation, where each message update is a Log-Likelihood Ratio (LLR).

$$\lambda_k = \lambda_{ch} + \sum_{l=0}^{d_v - 1} \lambda_l - \lambda_k^{old} \quad (6)$$

where λ_{ch} is LLR of a corresponding channel that is represented in the variable node and λ_l corresponds to LLRs of the incident edges. In this paper, all simulation results are obtained using the Sum-Product algorithm.

III. AN OVERVIEW ON TURBO CODE

Turbo codes are the best available error correction technique in the last few years. They have been standardized in 3G and 4G standards because they can achieve a performance near Shannon limit [10], [11]. Turbo code is a combination of two codes that work together. It is formed from a parallel concatenation of two convolutional encoders separated by an interleaver. These encoders are recursive and systematic in practice [12]. Turbo codes achieve better performance compared to convolutional codes when the length of the interleaver is very large. Therefore, for improved code performance, a large block size random interleaver is required. However, the delay and the computational complexity of the encoder and decoder are increased when length of the code is increased. At a short block length code, the BER performance of turbo code is worse than that of a convolutional code while the computational complexity is similar. For many applications such as mobile systems, complexity and delay are two important issues to consider when choosing code length. Many of interleaver designs are suggested to improve performance of short length codes [13]. Turbo code is the only channel coding that is used to process data in Long Term Evolution (LTE) and LTE-Advanced standards. These standards provide a mobile access technology for the so-called 4 and 4.5 mobile communication systems, and are investigated to increase capacity and improve performance of these systems [14]. LTE is designed to be scalable, (i.e., it can be updated without disrupting current services) [15]. In LTE, different coding and modulation schemes are used to improve decoding throughput and achieve high data rates. The modulation schemes that are used in LTE standards are QPSK, 16QAM, and 64QAM depending on channel quality. The LTE turbo code consists of three streams. The first stream represents systematic bits, while the second and third streams refer to the two parity bit streams. Each convolutional code contains a tail of 4 bits as trellis termination. For input data bits of size K, the output of each convolutional encoder is a stream of length K+4, and the coding rate of the LTE code slightly less than 1/3. LTE code uses188 values for K. The smallest and largest value of K are 40 and 6144 respectively [14]. For the smallest input block size (K=40), the corresponding code length is 152 bits. At the receiver, all operations of the encoder are inverted. Two convolutional decoders and two interleavers in a feedback loop are performed to decode information in the receiver. The same trellis structure and interleaver performed in the encoder are also used in the decoder but with an iterative operation, in which, BER performance and complexity of the decoder directly relate to the number of iterations.

IV. The Proposed Model

The WiMAX 802.16e LDPC decoder combines all the 19 block code lengths and the 4 code rates in one area chip. The most challenge is how to enhance the decoder flexibility, in other words, how to provide decoding with a wide variety of block lengths and coding rates. The most flexible code provides decoding with low complexity and high throughput, and so meets the requirements of 5G. The WiMAX 802.16e standard provides 76 sets of LDPC codes coming from using 4 H-matrices of different rates and 19 sets of block lengths. Although WiMAX code is more flexible than WiFi code, which supports only 12 combinations [16], the number of its combinations is lower than that required to achieve the throughput specified in 5G systems. The aim of this work is providing highly flexible LDPC code can support any block length and any coding rate by demonstrating an efficient decoder architecture performed by one class of H-matrix. Using one H-matrix instead of using the four will also minimize hardware implementation of the decoder and reduce the computational complexity of the decoding. The encoder and decoder of the proposed code can be detailed as follows:

A. Encoder Structure

The WiMAX LDPC codes are systematic codes [17], that is, the information is a part of the code. At the encoder, the generator matrix G is derived from the parity-check matrix as follows:

Systematic H-matrix can be written as

$$\mathrm{H} = \left[\, \mathrm{P}_{(\mathrm{N-K,K})} \middle| \, \mathrm{I}_{(\mathrm{N-K,N-K})} \right] \tag{7}$$

where P is a matrix includes parity-check equations, and I is an identity matrix. The corresponding generator matrix is

$$\mathrm{G} = [\, \mathrm{I_K} \mid \mathrm{P^T_{K,N-K}}] \tag{8}$$

$$\mathrm{HG^T} = 0 \tag{9}$$

Each generated code, x must satisfy

$$\mathrm{xH} = 0 \tag{10}$$

If s represents a message vector, then the code, x can also be formed using the following equation:

$$\mathrm{x = G^T s} \tag{11}$$

The code rate of the LDPC code is calculated by

$$\mathrm{R = K/N}, \text{ where } \mathrm{K = N - M} \tag{12}$$

The systematic LDPC code comprises two parts: information bits of length K, and parity bits of length M. So, the length of this code is

$$\mathrm{C = [K + M] = [K + N - K]} \tag{13}$$

Consider $K = K_a + K_z$ where K_a is the actual information and K_z is a sequence of well-known bits such as a set of zeros. Consequently, Eq. (13) can be rewritten as

$$\mathrm{C = [Ka + Kz + N - Ka - Kz]} \tag{14}$$

Since K_z is a known-value parameter for the receiver, it can be discarded from the code and the resulting code size can be written as

$$\mathrm{C = [Ka + N - Ka - Kz]} \tag{15}$$

The first term represents data bits while the other terms correspond to parity bits. The size of the code is exactly equal to

$$C = [N - Ka] \quad (16)$$

The code rate can be determined using Eq. (14) as

$$Rc = Ka / (Ka + N - Ka - Kz) = Ka / (N - Kz) \quad (17)$$

It is clear that all the code parameters, including block size, coding rate, and BER performance as well as computational operations are affected by Kz parameter, which ranges from 0 to K. When Kz increases, many code properties such as BER performance and computational complexity significantly improve while the code rate gradually decreases. For instance, if Kz = 0, then K = Ka and Rc = R. Therefore, no reduction will be in code rate, and the generated code is similar to the WiMAX LDPC code. When R = 1/2 and Ka = Kz, K = R*N=N/2 and Rc = Ka / (4Ka – Ka) = 1/3. In this case, the transmitted information and code size also changed to K-Kz and N-Kz respectively. Therefore, there are K of codes can be constructed through changing Kz between 0 and K. This means that the generated code supports a large number of length and rate combinations, and also has a flexible structure therefore it can be called Fully Flexible LDPC code (FF-LDPC code). Fig. 2 shows the main components of FF-LDPC encoder. In general, it comprises WiMAX LDPC encoder, size controller, and a bit selector of size (N – Kz).

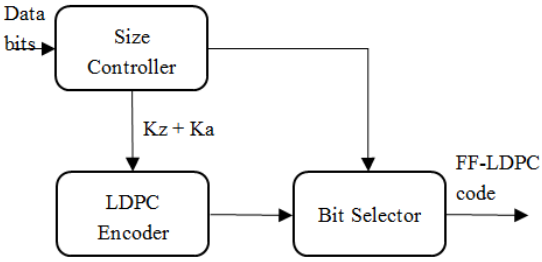

Fig. 2. Typical structure for the FF-LDPC encoder

The LDPC encoder is constructed by using only a single class of H-matrix instead of using the four classes. This approach will lead to minimized implementation area and reduced computational complexity in both the encoder and decoder. The size controller allows selecting the appropriate size for Kz that is enough to achieve the required performance according to the channel conditions, while the bit selector is employed to exclude Kz from the transmitted code. Therefore, code performance is directly related to Kz size. When Kz increases, BER performance is improved but the coding rate is decreased. The reduction in the coding rate, Cr can be calculated by

$$Cr = \frac{Rc}{R} = \frac{N*Ka}{(N-Kz)*K} = \frac{Ka}{(1-Kz/N)} \quad (18)$$

Cr represents the reduction in the rate of the FF-LDPC code relative to the WiMAX LDPC encoder used in the system.

B. Decoder Structure

At the receiver, the proposed codes can be decoded using a decoder with a simple structure. The typical FF-LDPC decoder mainly consists of three components: WiMAX LDPC decoder, size controller, and a bit selector. This decoder can easily be implemented using the structure shown in Fig. 3. Again, WiMAX decoder architecture is constructed using a single class of parity-check matrix. This matrix is the same as the matrix used in the encoder. The value of Kz can easily be determined by the size controller using the following relation.

$$Kz = N - C \quad (19)$$

where C is the size of received code. By adding Kz to the received code, the WiMAX LDPC code reconstructed again. The decoded stream comprises the actual information Ka and the un-requested bits Kz that is finally excluded by the bit selector. This means the required number of computational operations in the decoding will be limited to processing N-Kz of the VNs instead of processing all the VNs.

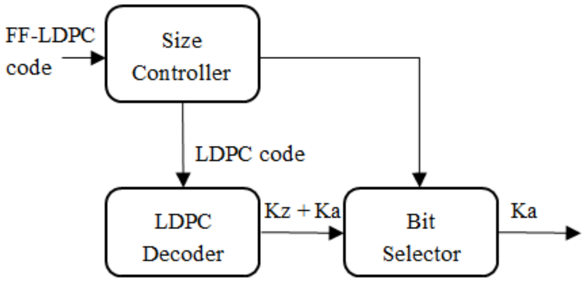

Fig. 3. Typical structure for the FF-LDPC decoder

In the WiMAX 802.16e code, the expansion factor, Z changes from 24 to 96 by an increment of 4, achieving 19 block lengths, ranging from N = 576 to N = 2304. However, by FF-LDPC code, the coding rate changes according to the value of Kz that can range from 0 to K. Theoretically, the number of rates and lengths that can be achieved by FF-LDPC encoder is calculated by

$$Nc = 19 * K \quad (20)$$

Nc also represents the number of coding rate and block length combinations that FF-LDPC code can provide. Therefore, the maximum number of combinations obtained using a structure of single parity-check matrix of R = 1/2 and Z = 96 (N = 2304 and K = 1152) is 19 * 1152 = 21,888, which is much larger than 76 combinations that achieved by WiMAX LDPC code. As mentioned previously, large values of Kz result in a significant reduction in the coding rate, so this reduction imposes a trade-off between the coding rate and the number of combinations. Therefore, Kz needs to be defined in a specific range, in which the resulting coding rate is useful and feasible. Using Eq. (17), Kz can be found as

$$Kz = N - Ka/Rc \quad (21)$$

By using $\mathrm{Ka} = \mathrm{K} - \mathrm{Kz}$,

$$\mathrm{Kz} = \mathrm{N} - (\mathrm{K} - \mathrm{Kz})/\mathrm{Rc}$$
$$= \mathrm{N} - \frac{\mathrm{K} - \mathrm{Kz}}{\mathrm{Rc}}$$
$$= \frac{\mathrm{N}*\mathrm{Rc} - \mathrm{K}}{\mathrm{Rc}+1} \quad (22)$$

By substituting $\mathrm{K} = \mathrm{N} * \mathrm{R}$, Kz can be written as

$$\mathrm{Kz} = \mathrm{N}\,\frac{\mathrm{R} - \mathrm{Rc}}{1-\mathrm{Rc}} \quad (23)$$

if an H-matrix of rate 5/6 and expansion factor of 96 (R=5/6, N=2304) is used for achieving coding rates ranging from 5/6 to 1/2, the maximum value of Kz is

$$2304 * (1/2 - 5/6)/((1 - 1/2)) = 1536.$$

Therefore, Kz can be changed from 0 to 1536. Now, if the matrix is fully expanded, the total number of combinations provided by FF-LDPC code is Nc = 19 * 1536 = 29,184.

V. Code Evaluation

Currently, LDPC codes are among the best error-correction codes that are utilized to meet the requirements of 5G communication systems [18]. These requirements were set to address the so - called 1000x challenge [19]. However, these requirements impose greater challenges upon the selected code employed in the system. Therefore, several key parameters, such as throughput, latency, error correction capability, flexibility, and implementation complexity should be investigated in code evaluation [6, 20]. Many researchers have proven that LDPC codes have throughput, delay, and computational complexity fulfill the 5G requirements [21]. However, some other challenges, include flexibility and interconnect complexity can be addressed by the proposed code in this work.

A. Throughput and Latency

The throughput or capacity is the number of bits that can be processed by the decoder, while latency is the time required for decoding a complete code, which can be roughly determined by dividing the code length by the associated throughput. Thus, the high throughput and low latency are two significant constraints are needed to meet the 5G requirements. Two different types of decoder architectures are used to decode the standard LDPC code. The first architecture is called two-phase decoder [22], and the second architecture is called layered decoder [23]. Both decoders have a partly parallel structure; however, the layered decoding achieves advantages in terms of implementation complexity and latency. The layered decoding is represented by two code classes in the WiMAX 802.16e standards, the R =1/2 and R =2/3B code. The decoding throughput of two-phase and layered architectures is calculated using the following two equations respectively.

$$T_{2ph} = N * R * f/\left(\left(5 + d_c^{max} + \frac{Edges}{Z}\right) * It\right) \quad (24)$$

$$T_{lay} = N * R * f/\left(5 + d_c^{max} + \frac{Edges}{Z} * It\right) \quad (25)$$

where f is a clock frequency and It is the maximum number of iterations (number of exchanging between VNs and CNs required in the decoding). Equation (24) and (25) calculate decoding throughput for standard LDPC code based on standardized WiMAX parameters. To calculate throughput of FF-LDPC code using H-matrix of size (Mf, Nf), several parameters need to be modified as follows

1- Replacing N with $C = Nf - K$.

2- Replacing R with $Rc = Ka/(Nf - Kz)$.

3- Subtracting edges corresponding to Kz from the original WiMAX edges, so that Ez = Edges - (kz/24) *(Edges/Z), where Edges/Z represents the number of non-zero elements.

Thus, for each type of decoding, the throughput of FF-LDPC decoder can be calculated by rewriting Eqs. (24) and (25) as follows

$$T_{2ph} = (Nf - Kz) * Rc * f/\left(\left(5 + d_c^{max} + \frac{Edges}{Z}\left(1 - \frac{Kz}{24*Z}\right)\right) * It\right) \quad (26)$$

$$T_{lay} = (Nf - Kz) * Rc * f/\left(5 + d_c^{max} + \frac{Edges}{Z}\left(1 - \frac{Kz}{24*Z}\right) * It\right) \quad (27)$$

As pointed out, FF-LDPC code can achieve any code rate by using one class of H-matrix. For the simulation, consider an FF-LDPC code constructed based on H-matrix of rate 5/6 and expansion factor of 96. Table 2 summarizes the throughput and latency of each rate specified in the WiMAX 802.16e standard compared to the LDPC code. The throughput of proposed code is calculated by Eq. (26) and Eq. (27), and the number of Kz that achieves the desired Rc is calculated using Eq. (23). H-matrix that is used in FF-LDPC code has a constant expansion factor of 96, so the code size is 2304, and the number of edges is 7680. The code size ranges according to the required code rate, while clock frequency, f is set to 500 MHz. In addition to the improvement obtained in throughput and latency, the FF-LDPC decoder achieves better performance than WiMAX LDPC decoder at a low number of iterations, resulting in a reduction of the number of logical operations required in the decoder. Results in Table II represent mathematical calculations. However, throughput can further be increased by 1) Increase of code length by expanding the decoder structure. 2) Increase of clock frequency. 3) Reduce the number of iterations required in decoding.

TABLE II: THROUGHPUT, AND LATENCY OF THE FF-LDPC AND WIMAX802.16E LDPC DECODERS

Common Parameters			LDPC Code					FF-LDPC Code		
Code Rate	Lay.	Code Size	Iter.	Exp.	Edges	Throughput Mbps	Latency µs	Kz	Throughput Mbps	Latency µs
1/2	**Yes**	**768**	**15**	**32**	**2432**	**166.67**	**13.82**	**1536**	**372**	**2.06**
2/3A	No	1152	15	48	3840	270	4.27	1152	591	1.95
2/3B	**Yes**	**1152**	**10**	**48**	**3888**	**465**	**2.478**	**1152**	591	1.95
3/4A	No	1536	10	64	5440	548.5	2.8	768	735	2.09
3/4B	No	1536	10	64	5632	533.3	2.88	768	735	2.09
5/6	No	2304	10	96	7680	915	2.52	0	915	2.52

B. *Flexibility*

The code flexibility is the key to meet the 5G requirements. The best error-correction codes are the codes that facilitate high decoding throughput at a lower implementation complexity. A code is flexible if it supports a wide range of block lengths and coding rates. Although WiMAX LDPC decoder is able to achieve the required throughput, it may be considered poor flexible since it supports only 76 combinations of block lengths and coding rates. The most important feature of FF-LDPC code, that its coding rate is not related to the expansion factor. The coding rate and block size of FF-LDPC code are only related to the size of Kz used in the encoding and decoding. Kz can change from 0 to K providing combinations of 19K when all the 19 sizes are used. However, to achieve coding rates from 1/2 to 5/6, the maximum number of combinations is 19 * 1536 = 29,184, which is much larger than 76 and 400 that are achieved by the WiMAX 802.16e LDPC and turbo decoders respectively [21].

C. *Interconnect Complexity*

Lower interconnection complexity is also one of the 5G requirements. It represents the implementation complexity of interconnection networks that depends on the decoding flexibility and the number of block lengths and code rates that can be supported by the decoder. In general, the interconnection network complexity of standard LDPC decoder can be determined by K*mean[d_v]/R, where mean[d_v] is the mean value of the variable node distributions [21]. In the case of WiMAX LDPC decoder, the decoder includes all the four code rate classes, each class contributes a certain amount of this kind of complexity. For instance, 1/2 code rate has an interconnection complexity of 7.3K since the mean value of VNs distribution is about 3.67. While in the case of FF-LDPC code, the interconnection complexity will be reduced to 3.6K if using 5/6 H-matrix, where the mean value of distributions is 3.

D. *BER Performance*

The 5G system aims to reduce the error to less than 1 bit for each 100,000 bits. BER performance of LDPC codes depends on the structure of parity-check matrix, type of decoding algorithm, number of iterations, and SNR of a received code. Kz bits used in the decoding increase SNR of the code, which in turn improves the BER performance and reduces the number of iterations required in the decoding. This advantage leads to reducing the computational complexity and also increasing decoding throughput. Results showed that FF-LDPC codes have good BER performance and also outperform LDPC codes when they simulated under same conditions.

E. *Implementation Complexity*

The implementation complexity of a decoder specifies hardware requirements and energy consumption. The hardware efficiency of a decoder is quantified by the ratio of throughput to its implementation area, and measured by bps per mm^2. Recently, a lot of research has been carried out to increase decoding throughput relative to the area of a decoder [24]-[26]. In that research, the authors have proposed different structures for the decoder based on improved decoding algorithms, multi-size cyclic-shifters structure, or using non-refresh eDRAM to achieve higher throughput as well as reducing the area of decoding. In the case of the FF-LDPC code, the implementation area of its decoder is a quarter of the area of the WiMAX LDPC decoder. Thus the hardware efficiency of FF-LDPC decoder is about four times the hardware efficiency of the WiMAX decoder. In other words, the FF-LDPC decoder can achieve four times that achieved by the standard LDPC decoder.

F. *Computational Complexity*

The encoding and decoding complexities of WiMAX LDPC codes directly relate to the code length and code rate. The encoding complexity of WiMAX codes can effectively be avoided by the use of a dual-diagonal matrix structure [3]. In general, the computational complexity is specified by maximum, minimum and addition operations (MMA) that are required in the encoder and decoder [27]. In a LDPC decoder, the computation complexity is given by $N * (6 * mean[dv] - 9)/(R + 6)$ MMA operations per iteration [21]. However, in the case of FF-LDPC code, the MMA is equivalent to $(Nf - Kz) * (6 * mean[dv] -$

$9)/(R+6)$. Table III shows the number of MMA per iteration for standard LDPC and FF-LDPC decoders when the code sizes are identical ($N = Nf - Kz$) and the proposed code is constructed by using the parity-check matrix of rate 5/6 and expansion factor of 96.

TABLE III. COMPARISON OF FF-LDPC CODE WITH LDPC CODE IN TERMS OF COMPUTATIONAL COMPLEXITY

Common Parameters		**LDPC Decoder**		**FF-LDPC Decoder**	
Code Rate	Code Size	Mean[dv]	MMA	Mean[dv]	MMA
1/2	768	3.667	1536	3	1011
2/3A	1152	3.667	2264	3	1517
2/3B	1152	3	1555	3	1517
3/4A	1536	3	2048	3	2023
3/4B	1536	3.667	2958	3	2023
5/6	2304	3	3035	3	3035

Results in Table III denote that the number of MMA per iteration required in the FF-LDPC decoder is less than that required in the LDPC decoder when they process codes of the same sizes.

VI. SIMULATIONS AND RESULTS

The BER performance of proposed code was evaluated and compared with that of standard WiMAX LDPC and turbo codes. All simulation results were obtained by using the standardized encoding method, Sum-Product decoding algorithm, AWGN channel, and QPSK modulation scheme. Since the proposed code is designed to meet the requirements of 5G systems, it is necessary to compare it to that of turbo codes that are used in the LTE and Advanced-LTE systems. Fig. 4 shows the BER performance of FF-LDPC code over different code lengths and rates based on 1/2 code rate H-matrix and 96 expansion factor ($Nf = 2304$). As shown in the figure, the performance is improved as the coding rate decreases. The Kz can take any value between 0 and K, and thus a large number of codes can be achieved. Each code corresponds to a specific coding rate and certain block length. It also can be seen that the bit error rate that is targeted by the 5G systems (one bit per 10^5 bits or more from received bits) was achieved at low values of SNR, meaning that the proposed decoder provides an improvement in power consumption Fig. 5 shows the improvement in the performance compared to standard WiMAX LDPC codes when the coding rate set at 1/3.

Two improved LDPC codes, based on an improved parity-check matrix of a size (16 x 24) derived from the standard WiMAX 1/2 code rate, were used to show this improvement. The value of Kz corresponds to the coding rate of 1/3 is 576. As shown in Fig. 5, the proposed code achieves better BER performance compared to the two LDPC codes although one of them is longer than the FF-LDPC code. This improvement confirms that the proposed code can be utilized to provide high-performance, low complexity error-correction codes over a wide range of block lengths and coding rates. For verifying whether the proposed code is capable of meeting the requirements of 5G systems, it is necessary to evaluate and compare its performance to that of the turbo codes. Fig. 6 shows the performance of FF-LDPC code compared to turbo codes when their rates are 1/3 and lengths are equal or very close.

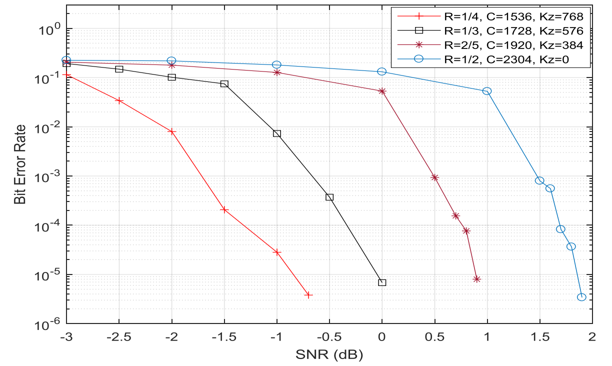

Fig. 4. Performance of FF-LDPC code for different coding rates using H-matrix of size (1152 x 2304)

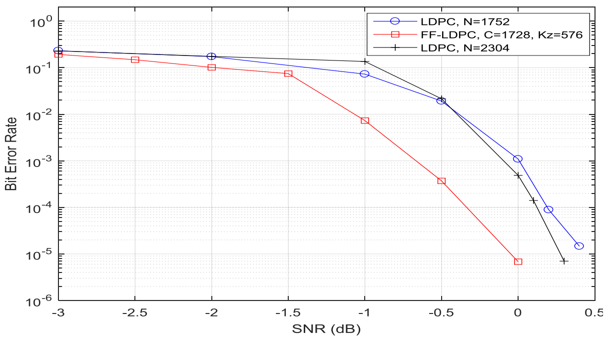

Fig. 5. Performance of FF-LDPC and LDPC codes when R = Rc =1/3

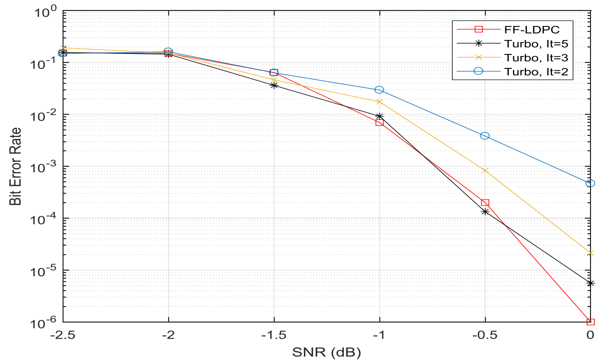

Fig. 6. Performance of FF-LDPC and turbo codes with different number of iterations

The performance and computational complexity of turbo decoders are increased directly with increased iteration number, while the results show that the proposed code outperforms turbo codes even when the number of iterations is increased.

For designing a fully flexible code that is capable of providing any coding rate defined in WiMAX 802.16e standard, the H-matrix of rate 5/6 and expansion factor of Z=96 was used in encoder and decoder construction.

Figs. 7 and 8 show the performance of LDPC and FF-LDPC codes for each coding rate specified in the WiMAX 802.16e standard. As shown in the figures, FF-LDPC codes perform similarly to that of LDPC codes. Table I showed the throughput and latency for all codes depicted in the figures. In addition to throughput and latency, this design also provides significant improvements in terms of computational and implementation complexities as presented in previous sections.

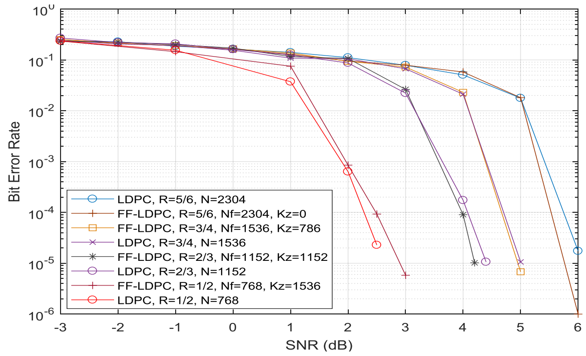

Fig. 7. Performance of FF-LDPC and LDPC codes (5/6, 3/4A, 2/3A, 1/2)

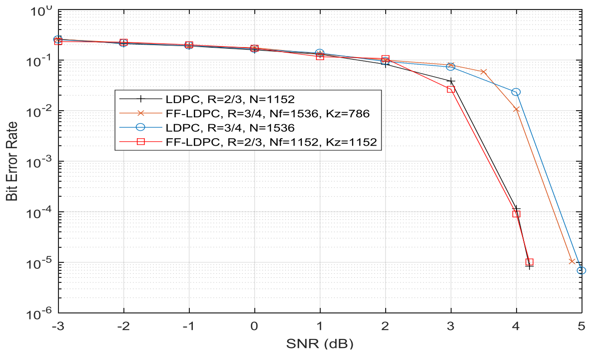

Fig. 8. Performance of FF-LDPC and LDPC codes (3/4B, 2/3B)

VII. Conclusion

Due to their high decoding throughput and low complexity, LDPC codes have been selected to provide channel coding in 5G communication systems. However, some work related to its ability in providing a wide range of block lengths and coding rates is still required to meet all requirements of 5G systems. Although LDPC codes achieve most of these requirements, including high throughput, low latency, and reduced computational complexity, their decoder flexibility and its implementation complexity still need to be improved. In this paper, a very high flexible error-correction code is designed and constructed in order to support a wide range of code lengths and code rates. The proposed code can be constructed and implemented based on any type or standard of LDPC codes including IEEE 802,16e, IEEE 802.11n and DVB-S2 systems. Since the proposed code can provide a large number of coding rates and block lengths, it is considered a multi-rate and variable-length code. The typical system introduced was constructed based on WiMAX 802.16e standard and then evaluated and compared to that of LDPC codes. The number of coding rates that can be achieved by the proposed code is about 19K which is much larger than 76 that are provided by WiMAX 802.16e standard. This feature allows the proposed code to achieve throughput and latency targeted by 5G systems. The most important features of this code in addition to its high flexibility are higher throughput, lower latency, reduced computational complexity, good hardware efficiency, and low interconnection complexity. Also, it can support different coding scheduling methods, namely two-phase and layered decoding methods. The interconnection complexity of its encoder and decoder summarized by a structure of single H-matrix instead of four H-matrix used in the standard WiMAX encoders and decoders. Furthermore, the simulation results showed that the code outperforms turbo codes in the BER even when the number of iterations used in the turbo decoder is high. At R=1/2, the code increases throughput by 2.23 and reduces latency by 6.7, while at R=3/4 compared to the LDPC code of a rate 3/4A, the throughput is increased by 1.34 and the latency is decreased by 1.34. In addition, the number of logical operations required to process a code has a rate of 1/2 is 1011 which is less than1536 that is required by LDPC decoder.

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