End-To-End Network Throughput Optimization Via Physical-Layer Network Coding

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Abstract—Physical-Layer Network Coding (PNC) is a promising technique that has great potentials for improving the achievable data rates of end-to-end flows through higher packet transmission rates, thereby increasing the overall network throughput. The number of IoT devices and its applications are dramatically increased therefore the necessity of higher throughput is essential. In this paper, we study the performance of the PNC transmission techniques for unidirectional end-to-end flows in multi-hop wireless networks and compare it with that of the traditional transmission techniques. We first derive the bit-error rate (BER) that the PNC transmission technique achieves, and then using that, we evaluate the network throughput measured as the aggregate throughput of multiple unidirectional end-to-end flows. Using simulations, we show that PNC increases the overall achievable network throughput, especially under medium to high signal-to-noise ratios.

Index Terms—IOT, Network coding, Physical-Layer Network Coding (PNC), Throughput, Bit-Error Rate (BER)

I. INTRODUCTION

The need for higher data rates and faster connection speeds of exchanging information in wireless networks have prompted researchers to think of new, efficient techniques that do so by making efficient use of the available wireless resources. Physical-Layer Network Coding (PNC) is one technique that has great potentials for improving the aggregate throughput of end-to-end flows through effective use and exploitation of wireless resources [1]. The idea of network coding was first introduced in 2000 by Ahlsweda [2], and then used in many other works (e.g., [3]–[6]) and showed great promises for throughput improvements over traditional transmission techniques. Later, PNC, emerged also as a promising technique, is shown to improve the performance of three-node bidirectional networks [1].

At the physical layer, data is transmitted through electromagnetic (EM) waves, and PNC takes advantage of the additive nature of simultaneous arrivals of multiple EM waves to reduce the number of packet transmissions, thus improving the overall network throughput. By using a proper modulation, the addition of EM signals can be mapped to $GF(2^n)$ additions of digital bit streams [2], [5]. Symbol-level and carrier-phase synchronization and the use of power control are then assumed in order to be able to receive the two signals with the same phase and amplitude.

For the sake of illustration, we explain the general idea of PNC using an unidirectional single-flow network. For simplicity, we assume a fixed distance between any two neighbor nodes, and consider an unidirectional five-node flow, where every node is equipped with an omni-directional antenna. The wireless channel is assumed to be half duplex, meaning that the transmission and reception must occur in different time slots. Furthermore, we consider the Decode-and-Forward relaying approach [7] in this work.

Fig. 1 illustrates the traditional transmission technique in a single unidirectional flow network. Here, node 1 and node 4 can both transmit their signals at the same time without interfering with one another, but node 1 and node 3 cannot transmit simultaneously, due to interference. Fig. 2 illustrates the unidirectional PNC transmission technique. Unlike the case of the traditional transmission technique, node 1 and node 3 here can transmit concurrently (i.e., node 1 sends $X_1$ while node 3 is sending $X_3$), and provided that node 2 has already received $X_3$, it can then perform PNC to recover the intended signal/packet coming node 1, even in the presence of the signal coming from node 3. In this case, the performance gain of the PNC technique over that of the traditional technique lies in the fact that the number of transmissions to deliver a packet successfully is expected to be lesser under the PNC technique than under the traditional one. However, due to interference, the Bit-Error Rate (BER) under the PNC technique is, on the other hand, expected to be worse than that under the traditional one. The objective of this paper is then to investigate whether the degraded BER due to interference pays off by reducing number of needed transmissions, thereby leading to an increased overall end-to-end network throughput.

The rest of this paper is organized as follows. Section II describes the network model. Section III derives the BER performance under the PNC technique and uses...
The multi-hop wireless network is modeled as a random graph \( G = (N, H, F) \), where \( N \) is the set of all nodes in the network. Each node is equipped with an omni-directional antenna and an infinite-capacity buffer. Each node is also characterized by a transmission range defined as the furthest distance that the node’s transmitted signal can reach. Nodes are generated and placed randomly in an area \( A \). \( H \) is the set of all pairs \((u,v)\) (hops) of distinct nodes in \( N \) such that \( u \) and \( v \) are within each other’s transmission range. That is, for any pair \((u,v)\) \( \in N_2 \), \((u,v) \in H \) (i.e., nodes are neighbors) if \( d_{uv} < d_m \), where \( d_{uv} \) is the distance between nodes \( u \) and \( v \), and \( d_m \) is node \( u \)’s maximum transmission range. We refer to node \( u \) as the transmitter and node \( v \) as the receiver. The hop is said to be active if \( u \) is currently transmitting to \( v \); otherwise, the hop is said to be inactive (idle). \( F \) is the set of all unidirectional end-to-end (multi-hop) flows in the network.

Each end-to-end flow consists of multiple hops connecting the source/sender node and destination/receiver node. We assume that the source node has an infinite number of packets that needs to be sent to the destination node. Furthermore, we assume that each packet has to be resent repeatedly until it is delivered successfully. Any node not belonging to one of these flows is considered to be idle. A destination or intermediate node belonging to a flow will be able to receive a packet correctly only if no other nodes located within the node’s transmission range are transmitting concurrently with the node’s reception.

In this section, we derive the bit-error rate (BER) for unidirectional end-to-end flows using the physical-layer network coding (PNC) transmission technique, and compare it with that of the traditional transmission technique. We assume an additive white Gaussian noise with power density \( N_0/2 \), and assume that the received signal energy of one bit (\( E_b \)) is unity. We also assume perfect carrier-phase synchronization, and consider the QPSK modulation technique. For the traditional transmission technique, the BER is the standard \( Q(2/N_0) \) [8], where \( Q(.) \) is the complementary cumulative distribution function of the zero-mean, unit-variance Gaussian random variable. Let us refer to the example of Fig. 2 again to illustrate the derivation of the BER of the PNC transmission technique. Using the PNC technique, both nodes 1 and 3 are allowed to transmit concurrently; i.e., at a given time slot, node 2 receives two signals at the same time: \( X_1(t) \) coming from node 1 and \( X_3(t) \) coming from node 3, although intended for node 4. As a result, the combined bandpass signal \( r_2(t) \) received by node 2 during one symbol period is

\[
r_2(t) = X_1(t) + X_3(t)
\]

which can also be expressed as

\[
r_2(t) = [i_1 \cos(wt) + q_1 \sin(wt)] + [i_3 \cos(wt) + q_3 \sin(wt)]
\]

where \( i_j \) and \( q_j \) are the QPSK modulated information bits of node \( j \) for \( j = 1,3 \), and \( w \) is the carrier frequency. Thus, node 2 receives two baseband signals, in-phase (I) and quadrature-phase (Q):

\[
I = i_1 + i_3 \quad \text{and} \quad Q = q_1 + q_3
\]

Here, node 2 encodes the combined bit, \((X_1+X_3)\), with the already received (stored) bit, \( X_3 \), to recover the intended bit, \( X_1 \); i.e., \((X_1@X_3)@X_3 = X_1\). Note that \( X_3 \) was already received by node 2 at an earlier transmission time, i.e., when \( X_3 \) was transmitted from node 1 to node 2.

The QPSK data stream can basically be considered as two BPSK data streams: an in-phase stream and a quadrature-phase stream. In Table I, we illustrate the PNC mapping, where \( X_j \in \{0,1\} \) and \( i_j \in \{-1,1\} \) for \( j = 1,3 \) represents the in-phase data bit.

As shown in Table I, there are three possibilities of the in-phase space, \(-2,0,2\), with corresponding probabilities of 0.25, 0.5, 0.25, respectively. Applying the maximum posterior probability criterion [8] and using Table I, \( i_2 = -1 \) for \( i_1 + i_3 = 2 \). Since the error occurs when this criterion is not met, the average probability of error is

\[
\frac{1}{3} \left( \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \right) = \frac{1}{8}
\]
calculated for all possible cases, and the BER can be written as follows

\[ BER_{PNC} = \frac{1}{4} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(r+2)^2}{N_0}\right) dr + \frac{1}{2} \int_{-\infty}^{0} \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{r^2}{N_0}\right) dr + \frac{1}{2} \int_{0}^{+\infty} \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{r^2}{N_0}\right) dr + \frac{1}{4} \int_{-\infty}^{0} \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(r-2)^2}{N_0}\right) dr \]  

(1)

When the received signal is less than \( \alpha_1 \), \( i_1 + i_3 \) is declared to be -2, and when it is greater than \( \alpha_2 \), \( i_1 + i_3 \) is declared to be 2. Otherwise, it is assumed to be 0. After some algebraic manipulations, the optimal values of \( \alpha_1 \) and \( \alpha_2 \) are derived respectively as

\[
\alpha_1 = -1 - N_0 \left(1 + \sqrt{1 - \exp\left(-\frac{8}{N_0}\right)}\right)
\]

\[
\alpha_2 = 1 + N_0 \left(1 + \sqrt{1 - \exp\left(-\frac{8}{N_0}\right)}\right)
\]

TABLE I: PNC MAPPING ILLUSTRATION

<table>
<thead>
<tr>
<th>Mapping at N3 and N1</th>
<th>Demodulation at N2</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(i)1</td>
<td>X(i)3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
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<td>0</td>
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In Fig. 4, we show the BER of both the PNC and traditional transmission schemes under various values of the signal-to-noise ratio (SNR). The figure shows that the BER of PNC scheme is slightly worse than that of the traditional transmission scheme. However, even though the BER gets worse under PNC, as will be shown and illustrated in the following section, the PNC technique is expected to improve the performance of the system in terms of the overall end-to-end flow throughput by reducing the number of transmissions needed to successfully send packets along the end-to-end flow.

IV. SINGLE-FLOW THROUGHPUT ANALYSIS

In this section, we evaluate the throughput of both PNC and traditional techniques for an unidirectional single-flow wireless network with \( n \) nodes. Nodes are labeled as node 1, node 2, ..., node \( n \), where node 1 and node \( n \) are the source node and the destination node, respectively. We assume that the source node has an infinite number of packets that needs to send to the destination node.

We also assume that a packet is received successfully by the destination node when all the bits are each received correctly, any erroneous packet is to be retransmitted again and again until it is correctly received. This is done on a per-link basis.

A. Traditional Transmission Technique

The flow of packets in the traditional transmission technique when \( n = 5 \) nodes is illustrated in Fig. 5.

Assuming that the packet success probability over a link is \( p_c \) and that a packet is to be resent repeatedly until it is delivered successfully, the average number of needed transmissions until a packet is successfully delivered is \( 1/p_c \). The average transmission time over a link is then \( L/(p_c \times C) \), where \( C \) is the capacity of the wireless link and \( L \) is the length of the packet.

Throughout this work, we assume that each packet transmission occurs in one time slot, and hence the length of a time slot is \( L/(p_c \times C) \).

Now in order to avoid interference, under the traditional transmission technique (as shown in Fig. 5), node 1 cannot transmit concurrently with node 3. But when node 4 starts forwarding packet i, node 1 can then transmit packet \( i + 1 \) concurrently with node 4’s transmission. This leads to a packet reception rate at the destination node of one packet every three time slots, resulting in a long-term average achievable end-to-end flow throughput of

\[ Th = \frac{1}{3} p_c C \]
where $p^L_e$ is the packet success rate over a link when the traditional technique is used. For a packet of length $L$ bits, the packet-success rate $p^L_e$ is $\left(1 - p^L_e\right)^L$, where $p^L_e$ is the BER under the traditional technique.

B. PNC Transmission Technique

The unidirectional PNC transmission technique is illustrated in Fig. 6. In this case, node 1 and node 3 can send concurrently, and, as explained in previous sections, node 2 will perform PNC to recover the intended packet coming from node 1, even in the presence of the signal/interference coming from node 3. Also, even though the BER experienced under the PNC technique degrades due to the concurrent transmissions (as shown in Fig. 3), the performance gain of the PNC transmission technique over that of the traditional technique comes from the fact that it requires fewer number of transmissions than what the traditional technique does to deliver a packet successfully.

As shown in Fig. 6, the concurrent transmissions lead to a packet reception rate at the destination of one packet every two time slots, resulting in a long-term average throughput of

$$T_h^{PNC} = \frac{1}{2}p_e^{PNC}C$$

where $p_e^{PNC}$ is the packet success rate over a link when the PNC technique is used. For a packet of length $L$ bits, the packet-success rate $p_e^{PNC}$ is $\left(1 - p_e^{PNC}\right)^L$, where $p_e^{PNC} = BER_{PNC}$ (BERPNC is given in Eq. (1)) is the BER under the PNC technique.

In Fig. 7, we show the normalized (w.r.t. the capacity of the link) average throughput of the traditional and PNC transmission techniques under various values of the SNR. The throughput basically depends on the packet success rate, which in turn depends on the bit-error rate. Observe that under low SNR values, the throughput obtained under the traditional transmission technique is slightly higher than that obtainable under the PNC technique. But under medium to high values of SNR, the PNC throughput is significantly greater than the traditional one.

V. ANALYSIS MULTI-FLOW THROUGHPUT

We have previously studied the performance of the PNC technique in an unidirectional single flow wireless network context. Now, we study and investigate the performances of the technique when considering multiple flows in multi-hop wireless networks. We already discussed the network model in Section II. In this section, we specifically evaluate and compare the aggregate throughput obtained using the PNC technique with that obtained using the traditional one in multi-hop wireless networks. We study the impact of various network parameters, such as the SNR, the contention window size (Cw), and the number of end-to-end flows (F), on the achievable performances.

A. Simulation Setting and Method

We use MATLAB to simulate and evaluate both techniques: PNC and traditional. In order to do that, we use and implement a mechanism similar to IEEE 802.11 CSMA/CA DCF MAC [9] for controlling access to the wireless medium. The readers are referred to [9] for more details on how CSMA/CA protocol works. In this paper, we use the same MAC terminologies (like Contention Window) that IEEE 802.11 protocol uses. In our experiments, the average number of transmissions 1/pc over a wireless link/hop depends on the BER value. For simplicity, we assume that the BER does not change with respect to the distance between the sender and the receiver.

The various network parameters used in the simulation are summarized in Table II.
We randomly generate 50 nodes in an area of 150 x 150 m² with a maximum transmission range of 40 m, and show in Fig 8 three end-to-end multi-hop flows (yellow, red, and green) each with four hops for the sake of illustration. The aggregate throughput that these three flows can achieve under both the traditional and the PNC techniques as provided in the next section.

B. Simulation Results

During our simulations, we fix the number of nodes (N), the area (A), and the maximum transmission range (dm), and measure the aggregate throughput observed over the entire duration of the simulation.

We evaluate and compare the PNC technique with the traditional one by studying 1) the impact of signal to noise ratio, 2) the impact of contention window size, and 3) the impact of the number of flows (F) on the performances of both techniques.

1) Impact of signal to noise ratio

First, we evaluate performance for various values of SNR. We set other parameters, such as F to 3 flows and Cw to 3. Then, we calculate the aggregated throughput at each SNR value.

Fig. 9 plots the aggregate throughput as a function of SNR. We observe that under low SNR values, the aggregate throughput obtained under the traditional transmission is slightly higher than that obtained under the PNC technique. But under medium to high values of SNR, the aggregate throughput of the PNC technique is significantly greater than that of the traditional one.

2) Impact of contention window size

We now seek to understand how the performance of the PNC technique behaves under different contention window sizes. In this case, we fix F to 3 flows and SNR to 12 dB and evaluate and measure the aggregate throughput of the both techniques for various sizes of contention window.

Fig. 10 shows the achievable throughput of both techniques under different contention window sizes. We see that the PNC technique consistently yields better throughput than the traditional one, and this is regardless of the contention window size. Furthermore, the aggregate throughput reaches its maximum when the window size is about 8, and then starts decreasing as we keep increasing the contention window size. This is because as the contention window size increases, the chances of nodes being idle (no node gains access to the medium) increases as well, resulting in waiting the medium by not using it which affects then the overall achievable throughput.

3) Impact of the number of flows

We have previously studied the impact of both SNR and Cw on the performances. Now, we are interested in studying the impact of the number of flows (i.e., the network load) on the overall achievable throughput performance. In this simulation, we fix Cw at 3 and SNR at 12 dB, and vary the number of flows from 3 to 8.

Fig. 11 depicts the aggregate throughput as function of the number of flows. We observe that the number of flow has no significant impact on the overall achievable network throughput. This is because as we increase the number of flows, the number of generated packets that need to be sent also augments on one hand, but this, on
the other hand, also results in more collision due to higher interference levels.

Fig. 11. Impact of number of flows on throughput (N=150, A=200×200m², dm=40 m, Cw=3, and SNR=12 dB)

VI. RELATED WORK

Network coding (NC) first introduced in [2] is now well-recognized for its great network throughput potentials [10]. As a result, many practical NC-based techniques have been developed to improve network throughput performance [11]–[13]. Random network coding (RNC) [13] is one effective technique that received a considerable attention due to its practical simplicity [10]. Briefly, RNC consists of having intermediate nodes (i) wait until receiving multiple packets, say n packets, (ii) construct one or more linear combinations (coded copies) of these n packets with coefficients to be chosen randomly from a large finite field, and (iii) send these linear combinations in lieu of individual packets. Upon receiving n linear combinations, a receiver recovers the n original packets by solving a set of linear equations. RNC has several attractive features: (i) eliminates the need for traditional single-path routing methods; nodes may continue constructing coded packets and sending them to random neighbors independently of their destinations, (ii) solves the out-of-order packet delivery problem, and (iii) balances traffic loads across the network.

Research efforts on NC was mainly focused at first on the theoretic aspects [14]–[17]. In [14], Yeung shows how NC can outperform routing in a simple network topology, known as Butterfly network. Li et al. [15] establishes analytic results for linear NC techniques, and constructs algorithms for optimal linear network codes. In [16], the authors present a number of examples that illustrate the insufficiency of linear NC and reveal the inherent difficulties of multi-session NC. Security and error detection are important subjects that attracted significant attention.

In [17], Zhang propose network error correction codes, which extend classical error correction coding in the time domain to new classes of codes in the space domain. In [18], Cai et al. present linear secure network coding. In this work, they explore the fundamental limit for confidential communication in networks in the presence of malicious eavesdroppers.

More recently, researchers have focused on the practical aspects of NC [19]–[22]. In [19], Fragouli connects the NC theory with its practical application and provides a number of examples on how NC can be practically used. In [20], Dimakis et al. propose a new application scenario in which the network coding is beneficial. In [21], the authors study the ability to apply NC on the most popular adapted transport layer protocol, transmission control protocol (TCP). Furthermore, the authors present the feasibility of applying the NC in the Internet without any changes in TCP. In [22], Baocun et al. bring the theoretical benefits of NC to practical systems. For example, peer-to-peer network application may be considered to be the most promising scenario for network coding.

Along the same line, Nazer et al. [22] propose to use network coding at physical layer (i.e., physical-layer network coding (PNC)), where interference from different signals can be treated and taken advantage of as a network code. The idea of PNC is first described in [1] and applied on a bidirectional three-node wireless linear network. A detailed capacity analysis in [24] proves that PNC improves the throughput of wireless networks compared with conventional relaying techniques (CNC). In addition, the paper reveals that the PNC technique can achieve the minimum delay and can provide confidentiality to the signals sent on the physical layer. In order to avoid the phase synchronization issue, Katti et al. [?] present the concept of Analog network coding (ANC) which basically depends on Amplify-and-Forward relay node.

PNC can use QPSK to increase performance. In [25], Lu et al. investigate symbol error rate (SER) for BPSK and QPSK, but the approaches can be generalized to another constellation technique as well. The closed-form SER results are derived over AWGN channels. In [26], Sorensen et al. propose using FSK modulation instead of BPSK to avoid phase tracking. The result shows that BFSK in De-Noise and Forward (DNF) yields a lower performance compared to BPSK in DNF, thus requiring a higher SNR before communication becomes even possible.

PNC technique can further improve the throughput of a wireless network but not before addressing some challenges. The key challenge in applying PNC to practical scenarios is the phase-level synchronization. Some recent papers, however, show that PNC can be practically feasible by for example relying on beamforming [27] to solve the phase synchronization issue. In [28], a round-trip carrier synchronization technique is implemented on a prototype for acoustic distributed beamforming.

Although PNC technique is more suited, by nature, for multicast communications, it can also be used for unicast
communications [29] and is shown to achieve performance gains as well. In [30], Katabi et al. propose a technique that is capable of dealing with multiple pairs of colliding packets in IEEE 802.11 WLANs. Here, the receiver can decode two consecutive signals from two colliding packets and successfully receive both packets. PNC technique can take advantage of the ZigZag decoding in 802.11 WLAN [31] to reduce transmissions.

In this work, we first derive the BER performances of PNC with QPSK modulation for a single flow wireless network, and then apply PNC technique on multiple concurrent flows in multi-hop wireless networks. By assuming phase-level synchronization, symbol level and power control, our results show that the PNC technique achieves higher overall network throughput when compared with the traditional

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Sofean A. Maeouf received the M.S. degree from the Oregon State University (OSU) in 2012 and the B.S. degree from the University of Tripoli (Al Fateh University) in 2008. His research interests include: Network coding, Wireless communication, femto-Macro cell and IOT. He is currently working with LTE/VOLTE team at Qualcomm Inc.