An Autoregressive Model for Mobile-to-Mobile Rayleigh Fading Channel Simulation

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Abstract —A new autoregressive based simulation model for mobile-to-mobile (M2M) Rayleigh fading channels is proposed. The new model is capable of generating correlated flat-fading M2M envelope processes with the desired auto-correlation functions, cross-correlation functions, and probability density functions (pdfs). The utility of the method is demonstrated by examples involving the accurate synthesis of non-isotropic M2M fading channel models. Furthermore, it is shown that the proposed model yields the statistical properties of the reference model in only one simulation trial. Performance comparisons are then made with popular fading generation techniques to demonstrate the merits of the approach.

Index Terms—Mobile-to-Mobile (M2M), Autoregressive Rayleigh Model (AR), Spectrum Shape Filter (SSF)

I. INTRODUCTION

In wireless communication systems, the signals traverse the fading channels between the transmitter and the receiver. These fading channels degrade the performance of the wireless communications. Thus, in order to enable realistic performance assessments of these wireless channels, an accurate fading channel model plays a crucial role.

Mobile-to-mobile (M2M) communication systems, where both transmitter and receiver terminals are in motion, find applications in mobile and ad-hoc networks as well in intelligent transport systems [1]. Such M2M wireless systems differ from the conventional cellular fix-to-mobile (F2M) systems, where the base station is fixed in location and only the mobile user is moving. Thus, how mobility affects systems becomes an interesting subject for researchers to investigate.

Akki and Haber were the first to propose a reference model [2], [3] for M2M wireless channels. They showed that the received signal envelop is Rayleigh faded under non line-of-sight conditions, but the statistical properties differ from the F2M channels. A modified double-Ring model is used instead of the reference model to simulate M2M channels \cite{patel2005simulation}, assuming omni-directional antennas and isotropic scattering around the transmitter (T_x) and the receiver (R_x), which offers a faster convergence and lower variance at higher complexity.

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Recently, several methods have been proposed for the efficient generation of both single-input single-output (SISO) and multiple-input multiple-output (MIMO) M2M channels. Among these methods we find the well-known sum-of-sinusoids (SOS) fading approach [4]. Whereas, SOS methods are straightforward to implement, the statistical accuracy of this method has been found wanting [2], [3]. In the second major technique, called filter-based approach [5], [6], the fading variates are generated by passing uncorrelated samples of a zero-mean Gaussian process through a spectrum shaping filter (SSF) in a way that guarantees accuracy, but computational efficiency still depends from the number of taps of the filter.

Properties of Gaussian autoregressive (AR) models have been extensively investigated in [7], [8]. One of their useful properties is the matrix-valued Yule-Walker equation, which yields the parameters of the AR process for the given correlations. Because of these useful and well-formulated properties, the Gaussian vector AR process is selected in order to simulate M2M Rayleigh fading processes. To the best of our knowledge, no previous work has been conducted to apply such model to reproduce the behavior of an M2M Rayleigh channel.

In this paper, we extend the work in [7] to address the modeling of M2M fading channels where the transmitter and the receiver may experience changes in their velocities over time. We assume a stationary scattering environment and a narrow-band single-input single-output (SISO) M2M channel. Then through simulations results we approve that the proposed model is useful to describe the physical reality marked by rapid changes in realistic M2M communication scenarios. Compared to the SOS based model and the filter based model proposed in previous works [4], [6], AR model is more rapidly converging, has a lower variance of the auto-correlation functions and produces uncorrelated multiple faded envelopes.

The remainder of this paper is organized as follows. Section II presents the theory of SOS-based model for M2M channels, as well as the "double ring" concept. Section III describes the AR method of generating multiple M2M Rayleigh processes with specified second-order statistics. Our treatment briefly discusses the numerical problems that must be overcome for the accurate simulation of band-limited fading processes, which are of interest in wireless applications. Section IV provides some simulation results and compares the new model to previously proposed models. Finally, a conclusion and some perspectives are outlined in section V.

II. THE MATHEMATICAL REFERENCE MODEL

This section describes the SISO reference model for a narrow band M2M channel considered in this paper and reviews the first and second order statistical properties.

Assuming a single user SISO communication system with omnidirectional antenna element and a non-line-of-sight (NLOS) conditions, Akki and Haber's were the first to propose a mathematical reference model for mobile-to-mobile wireless channel [3]. Hence, the corresponding complex channel gain process can be expressed as in.

$$\begin{aligned} x(t) &= x_1(t) + jx_2(t) \\ &= \lim_{x\to\infty}\sum_{n=1}^{N}\sqrt{\frac{2}{N}}\exp[j2\pi(f_{T,\max}\cos(\alpha_n) \\ &\quad + f_{R,\max}\cos(\beta_n))t+\varphi_n] \end{aligned} \tag{1}$$

where N is the number of propagation paths, $f_{T,\max}$ and $f_{R,\max}$ are the maximum Doppler frequencies due to the motion of the transmitter (T_x) and the receiver (R_x), respectively α_n and β_n are the random angle of departure (AOD) and the angle of arrival (AOA) of the n^{th} path, respectively. Finally, ϕ_n is the random phase, which is uniformly distributed on $[-\pi,\pi[$.

For a sufficiently large value of N and by invoking the Central Limit Theorem we can confirm that the real and imaginary parts of $x(t)$ are zero mean Gaussian process [9]. Thus, the envelope $R(t)=|x(t)|$ is Rayleigh distributed and the phase $\Phi(t)$ is uniformly distributed. Thereafter, the statistical properties of the reference channel model can be summarized as in (2) and (3).

$$\begin{aligned} Rx_ix_i(\tau) &= E\{x_i(t)^* x_i(t+\tau)\} \qquad i=1,2 \\ &= \sigma^2 J_0(2\pi f_{T,\max}\tau)\cdot J_0(2\pi f_{R,\max}\tau). \end{aligned} \tag{2}$$

$$Rx_ix_j(\tau) = 0, \qquad i,j=1,2 \quad and \quad i\neq j \tag{3}$$

An important use of the autocorrelation function (ACF) resides in the fact that the power spectral density (PSD) function can be directly obtained from it. The general relation between the PSD S_{xx} of the process $x_i(t)$ and the ACF $\Gamma_{x_ix_i}$, $(i=1;2)$ is known as the Wiener-Khinchine relationship [10] Therefore, the M2M double-Doppler PSD of the process $x_i(t)$, $(i=1;2)$, is obtained as expressed in (4).

$$S_{x_ix_i} = \frac{\sigma^2}{\pi^2\sqrt{f_{T,\max}f_{R,\max}}}K\left[\left(\frac{(f_{T,\max}+f_{R,\max})^2-f^2}{4f_{T,\max}f_{R,\max}}\right)^{1/2}\right] \tag{4}$$

where K stands for the complete elliptic integral function of the first kind. Fig. 1 shows the Doppler PSD encountered in M2M fading channels with different transmit and receive Doppler frequencies $f_{T,\max}$ and $f_{R,\max}$.

Higher order statistics have been derived for the M2M Rayleigh fading model [2]. Namely, the expression of the level crossing rate (LCR) and average fading rate (AFD) of the M2M Rayleigh fading process $R(t)=\| x(t)\|$ are given by equations (5) and (6), respectively.

$$N_R(r) = \frac{\sqrt{2\pi}\left(f_{T,\max}^2+f_{R,\max}^2\right)^{1/2}}{\sigma} r\cdot exp\left[-\frac{r^2}{2\sigma^2}\right] \tag{5}$$

$$T_R(r) = \frac{\sigma}{\sqrt{2\pi}\left(f_{T,\max}^2+f_{R,\max}^2\right)^{1/2}}\frac{1}{r}exp\left[-r^2\right] \tag{6}$$

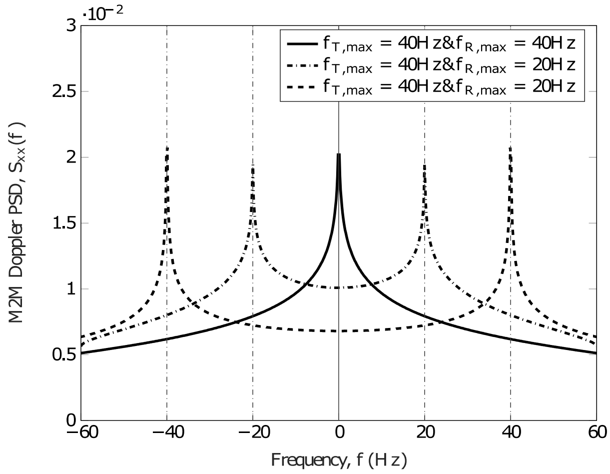

Fig. 1. The M2M double-Doppler PSD for various combinations of the maximum Doppler frequencies $f_{T,\max}$ and $f_{R,\max}$.

III. NEW AUTOREGRESSIVE SIMULATION MODEL

The auto-regression models are commonly used to estimate discrete-time random processes; this is due to their simplicity in which their parameters can be calculated as well as to their correlation matching property. Despite the fact that we aim the modeling of the propagation channels and not their parameter estimation, AR remains an effective method to do so. Thus, the basis of our approach utilizes the Gaussian vector autoregressive AR process as the driving process. Therefore, by adopting a vector AR model, the correlated complex Gaussian process $x[n]$ are assumed to evolve according to the p^{th} order auto-regression as shown in equation (7).

$$x[n] = -\sum_{k=1}^{p}a[k]x[n-k]+w[n] \tag{7}$$

where $a[k]$ is an $M\times 1$ vector containing the AR model coefficients. The driving noise $\omega[n]$ is a complex white Gaussian noise process with uncorrelated real and imaginary components. In M2M Rayleigh variate generation, $\omega[n]$ is temporally white zero-mean Gaussian vector process with a variance σ_p^2 The basic relationship between the desired model ACF $R_{xx}[k]$ and the AR $a[k]$ coefficient parameters, well known as the Yule-Walker equations , is given in [11] and expressed as in equation (8).

$$R_{xx}a = -\mathrm{v} \tag{8}$$

The matrices representing the correlation structure of the Gaussian vector AR are denoted by the $p \times p$ matrix in equation (9),

$$R_{xx} = \begin{bmatrix} R_{xx}[0] & R_{xx}[-1] & \cdots & R_{xx}[-p+1] \\ R_{xx}[1] & R_{xx}[0] & \cdots & R_{xx}[-p+2] \\ \vdots & \vdots & \ddots & \vdots \\ R_{xx}[p-1] & R_{xx}[p-2] & \cdots & R_{xx}[0] \end{bmatrix} \tag{9}$$

and the $p \times 1$ matrix in equation (10).

$$\mathrm{v} = -E\begin{bmatrix} x[n+1]x[n] \\ x[n+2]x[n] \\ \vdots \\ x[n+p]x[n] \end{bmatrix} = -\begin{bmatrix} R_{xx,1} \\ R_{xx,2} \\ \vdots \\ R_{xx,p} \end{bmatrix} \tag{10}$$

The coefficient matrix is denoted by the $p \times 1$ matrix

$$a = [a_1, a_2, \cdots, a_p] \tag{11}$$

Once $a[k]$ coefficients are computed, the variance of the white noise process can be calculated using [11] as expressed in equation (12).

$$\sigma_p^2 = R_{xx}[0] + \sum_{k=1}^{p} a[k]R_{xx}[-k] \tag{12}$$

In order to correspond to a wide sense stationary uncorrelated scattering (WSSUS) random process, the covariance matrix in equation (9) which is defined as $R_{xx} = E\left\{x[n]x^*[n]\right\}$ must be singular, a semi-definite block matrix and must have a block Toeplitz structure [11]. Subsequently, the Yule–Walker equations are guaranteed to have a unique solution. Thereafter, using the desired ACF sequence of the reference model expressed in (2), the AR coefficients filter can be determined by solving the set of p Yule-Walker equations $a = -R_{xx}^{-1}\mathrm{v}$. While $AR(1)$ and $AR(2)$ are adequate to model exponential correlations, higher order are required to model a band-limited process like M2M fading channels.

Therefore, due to the highly deterministic nature the M2M fading channels, the Yule-Walker equations in the band limited spectrum cases are plagued by a numerically ill-conditioned R_{xx} [7]. If the ill conditioning in R_{xx} is ignored, this will lead to a meaningless solution with an unstable infinite impulse (IIR) filter.

IV. Performance Evaluation

In order to validate the model presented in the previous section, complex Gaussian processes are generated and passed through the AR IIR filter. The results presented in Fig. 2, Fig. 3 and Fig. 4 show an excellent fitting of the analytical and simulation results. In Fig. 2, we measured the probability density function (pdf) for the amplitude of the generated samples. Note that the PDF accurately matches an ideal Rayleigh distribution.

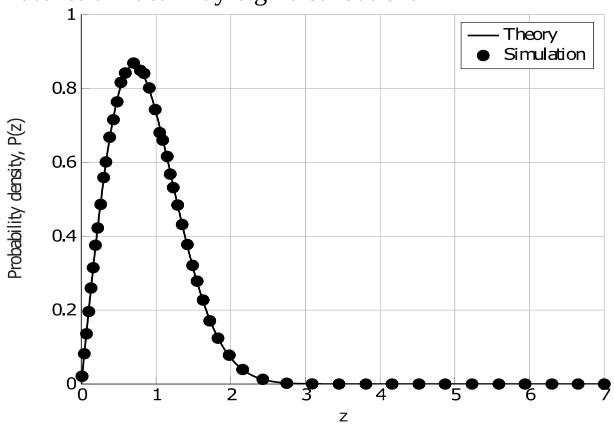

Fig. 1. The PDF $P_R(r)$ of the envelope of the double rayleigh process $\mu(t)$

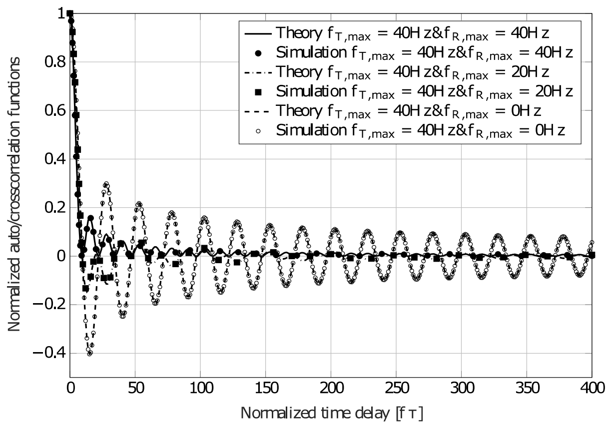

Fig. 2. Theoretical and simulated autocorrelation / cross-correlation functions of $2e^6$ generated Rayleigh fading samples

The first evaluation deals with the validation of the first statistical properties expressed by the ACF and CCF functions of the generated samples with the theoretical ones. Note that the auto-correlation function is derived in equation (2) while the theoretical cross-correlation function is always equal to zero. In Fig.3, we plot the real and imaginary parts of the auto-correlation of the simulated complex channel waveform along with different Doppler frequencies versus the theoretical auto-correlation and cross-correlation functions. A good agreement between the theoretical curve and simulated results proves the effectiveness of our model.

Another statistical characteristic often measured in Rayleigh fading waveforms is the LCR. This is defined in [12] as the expected rate at which the magnitude of the fading waveform crosses a specified signal level r in the positive direction.

In Fig. 4, we measured the Level crossing rate (LCR) for the amplitude of the generated samples. A good agreement between the theoretical curve expressed in equation (5) and simulated results proves the effectiveness of our model.

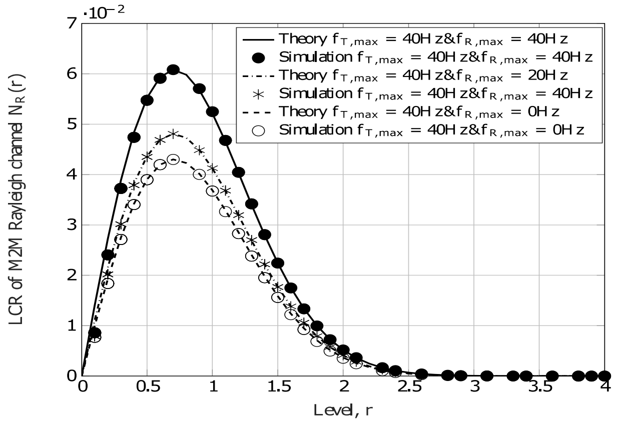

Fig. 3. The M2M double-Doppler PSD for various combinations of the maximum Doppler frequencies $f_{T,max}$ and $f_{R,max}$.

In order to demonstrate the effectiveness of our work, we compared simulation results of our autoregressive proposed model with a previously SOS and filter based models. The first model was proposed in [4]. It uses Random Walk Based distribution instead of uniformly distributed variables in order to generate the angle of departure (AOD) and angle of arrival (AOA), we refer to this model as Model I. The second model was also proposed in a previous work [6]. It applies the filter-based concept to simulate M2M double scattering channels; we refer to this model as Model II.

In the following simulations, we use a normalized Doppler frequency $f_\tau = 0.01$ at both sides T_x and R_x. In model I we used $N, M = 14$ sinusoids to obtain a complex envelope with uncorrelated quadrature components. In model II we model the magnitude response of the M2M channel with an infinite impulse response (IIR) filter of order $2K$ with. $K = 7$ Using these parameters, we have calculated the variance of the auto-correlation functions of the quadrature components for over 10^2 simulations, which provides a measure of the utility of the stochastic model. For Model I , Model II and the autoregressive model, the equivalent quantity can be defined as $\mathrm{Var}[R_{xx}(\tau)] = |\hat{R}((\cdot) - R(\cdot)|^2$ .The variance for the reference model is given in (13).

$$\mathrm{Var}[R_{xx}(\tau)] = [1 + J_0(2\omega_1\tau)J_0(2\omega_2\tau) - J_0^2(2\omega_1\tau)J_0^2(2\omega_2\tau)]/(2*N) \tag{13}$$

Fig. 5 shows that the three models do not perform as well as the reference model. Nevertheless, the AR statistical model converges faster than the other statistical models and has a lower variance of the auto-correlation functions ($\approx 10^{-6}$) than Models I and II .

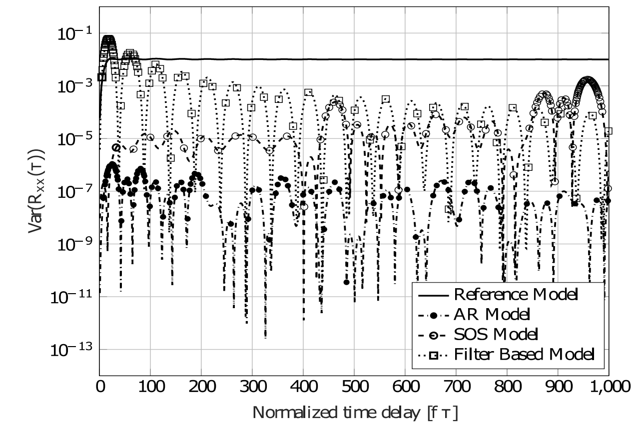

Fig 4. Variance of the auto-correlation function of the quadrature component

V. CONCLUSION

This paper proposes an autoregressive based model that enables accurate simulation of M2M Rayleigh fading channels. Performances of the proposed model are evaluated in terms of the auto-correlation and the cross-correlation of the channel waveform through theoretical and simulation results. The proposed model produces accurate statistical properties in each simulation run compared to the desired one. Furthermore, the variance of the MSE does not exceed 10^{-6} which confirms that the proposed model always overlies the statistical proprieties of the reference model. In addition, it is shown that the analytical results for the LCR compare very well with the statistical model.

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