Performance Analysis of Noise Signal Reduction Using Novel MUSIC Method of Adaptive Arrays

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Abstract —This paper presents a performance analysis of a direction-of-arrival (DOA) estimation algorithm that is based on a modified MUSIC algorithm. We present the description, comparison performance and high resolution analyses of this algorithm. The signal estimator is based on a linear algebraic relation between the standard subspace model of the array correlation matrix and a special signal interference model. This algorithm is not a subspace weighted MUSIC algorithm, because the scaling depends on the eigen-structure of the estimated signal subspace. The proposed modified MUSIC algorithm has the advantage of simultaneously estimating the DOA and the power of each source. Estimates of the sampled channel impulse response are derived using of the channel symbols. The channel response samples are separately processed to recover the DOA of the relative paths. Through simulations, we compare the DOA estimator using the modified MUSIC algorithm, based on these representations. Numerical results both demonstrate the superior performance of the modified MUSIC algorithm relative to the traditional MUSIC algorithm and confirm the validity of the results.

Index Terms—DoA, MUSIC algorithm, eigen value, array antenna, subspace, estimation

I. Introduction

In recent year, adaptive array antennas have become an important component in various wireless applications, such as radar, sonar, and mobile communication. These antennas lead to an increase both in the detection range of radar and sonar systems, and in the capacity of mobile communication systems. Array antennas are used as spatial filters for receiving desired signals coming from specific directions while minimizing the reception of unwanted signals emanating from other directions [1]. One of the most popular algorithms for performing Direction-of-Arrival (DOA) estimation is the MUSIC algorithm : its attractiveness is due to the fact that it provides good resolution while limiting the search for incoming signals to a single dimension. Well-known subspace based methods that dependent on the decomposition of the observation space into a signal subspace and a noise subspace, can provide high resolution DOA estimations with high estimation accuracy. However, traditional subspace based methods such as MUSIC-type [2] methods, involve the estimation of the covariance matrix and its eigen decomposition. As a result, traditional subspace-based methods are computationally intensive, especially for the case in which the model orders in these matrices are large. In MUSIC vector are the mathematical description of the interaction between the direct and the indirect signals as seen by the array antenna [3]. The presence of height ambiguities leads us to consider the simultaneous use of many frequencies in the bandwidth as in the exit algorithm. We derivation will closely follow the approach of Bienvenu [4], who has derived a MUSIC estimator for multipath frequencies.

We make the standard assumptions underlying the MUSIC algorithm, i.e., we have stationary processes, we have known noise covariance matrices, the number of sources, is less than the number of sensors, and the number of snapshots is greater than the number of sensors. In this study, we consider the case of snap shots or data vectors taken from an element array. The case in which some or all of the frequencies are the same is included as a special case of the model. The techniques presented are applicable in situations involving multiple antennas and, unlike traditional methods, are asymptotically optimal at high SNR even when multiple overlapping copies of the signal are received. This observation motivates the development of subspace-based techniques similar to those in [5], which provide closed-form solutions for the linear parameters.

In the past, when these models were used, research focused on only the single signal path case. Other recently proposed techniques for the case of a single signal arrival include the wideband ambiguity function method and the structured covariance estimator [6]. A recent paper presented a de-convolution approach for resolving multiple delayed and Doppler shifted paths but only over a quantized parameter.

The key features of the methods proposed below are that they provide continuous-valued estimates of the time delays and Doppler shifts for multiple signal arrivals, and that they are parametric estimators with asymptotic accuracy that is equivalent to that of the maximum likelihood approach. The outline of the remainder of the paper is as follows. In the next section, we present time and frequency domain versions of the data model used in this work. By interchanging the roles of the samples in

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space and time, we show that the time delay and Doppler estimation problem can be formulated in the well-studied framework of DOA estimation.

Specifically, we draw parallels between the array manifold in space that arises in DOA estimation and the signal manifold in time that we employ in this work.

II. Beamformain Signal Model

A representative member of the eigenvector methods is the MUSIC algorithm. However the presence of a highly correlated source, namely the target image, renders the conventional MUSIC in effective for low angle tracking over a smooth sea and air.

We propose to extend MUSIC by replacing the direction of arrival search vector with the refined propagation model vector, here, the vector represents the wave front shape as sampled by the array geometry when specular multipath is present. In the conventional MUSIC, the DOA vector contains the classical exponentials representing the delay between the signal received by one sensor compared to a reference.

We define a single observation vector, which is the concatenation of data snap shots observed at the array output at the intervals. Corresponding to this observation vector x_i, we defined a noise vector n_i, a signal vector s_i for the ith source, i=1 to P sources, and transfer matrix G_i from the signal source to the array outputs. The observation vector is then written as follows[7,8]

$$\mathrm{X} = \sum_{i=1}^{P}(s_i G_i + n_i) \tag{1}$$

where s , G , and n are single , amplitude, and noise, respectively. The deterministic matrix is the transfer matrix between the ith source and the signal component of the array output vector. The matrix contains the directional information on the source positions, the phase delay at the lth snapshot between the source and the array output signals. The covariance matrix is defined as follow

$$\mathrm{R} = \mathrm{E}[\{n_i n_i^H\} + \sum_{i=1}^{P} E\{G_i s_i s_i^H G_i^H\}] \tag{2}$$

$$= R_n + \sum_{i=1}^{P} G_i R_i G_i^H \tag{3}$$

The correlation matrix R_i is Toeplitz matrix since it is a stationary discrete time stochastic process. The matrix R_i may be diagonal by the singular value decomposition where the matrix Q_i that is used to diagonal R_i has as its columns an orthonormal set of eigenvectors for R_i. The resultant diagonal matrix ∇_i, has as its diagonal elements, the eigenvalues of R_i. We can be written

$$R_i = Q_i \, \nabla_i \, Q_i^H \tag{4}$$

It is quite difficult to fine analytically the eigenvalues and eigenvectors of a Toeplitz matrix. To circumvent that difficulty we use the fundamental theorem of [9] on the asymptotic behavior of the eigenvalue distribution of Toeplitz matrices. This theorem relates the properties of Toeplitz matrices to those of circular matrices.

Circular matrices are an especially tractable class of matrices since the eigenvalues of such matrices can easily be found exactly as the discrete fourier transform of their first row and all circular matrices have the same set of eigenvectors.

The eigenvalues of R_i are derived by construction an asymptotically equivalent circular matrix C_i as described in [10]. This is done using two criteria the strong norm and the weak known.

A. Sysytem Second order Signal Model

If we consider source to be the source of interest, then the signal noise ratio model may be written as the following signal interference noise model as follow [11]

$$\mathrm{y} = \mathrm{A}(\theta_i)\alpha_i + B(\theta_i)\beta_i + n \tag{5}$$

where Both $A(\theta)_i$ and $\mathrm{B}(\theta)_i$ are the steering vector to source ith at angle θ_i, Both β_i and α_i are the complex amplitude of source and interference , respectively, and n is complex white noise of covariance.

The steering matrix is$\mathrm{A} = [A(\theta)_1, \cdots, A(\theta)_P]$. In this model, the steering matrix $B(\theta)_i = [B(\theta)_1, \cdots, B(\theta)_P]$ contains the $(\mathrm{P}-1)$ interfering sources. The second order model from equation (4) can be written

$$\mathrm{R} = \mathrm{E}[yy^H] \tag{6}$$

$$= \sum_{i=1}^{P} s_i A(\theta_i)\, A^*(\theta_i) + \sigma^2 I \tag{7}$$

$$= \mathrm{AS}A^* + \sigma^2 I \tag{8}$$

where $\mathrm{S} = \mathrm{diag}[s_1, \cdots, s_P]$ is the diagonal matrix of powers for the uncorrelated source. Each term $s_i a(\theta)_i a^*(\theta)_i$ is a rank-1 covariance matrix for a radiating source. The beamforming problem can be formulated as follow

$$\min_{W} \nabla(W) = E[(d(t) - y(t))(\mathrm{d(t)} - \mathrm{y(t)})^H] \tag{9}$$
$$\text{subject to } \bar{a}^H \, \mathrm{W}=1$$

where $\mathrm{d(t)}$ is desired signal, $\mathrm{y(t)}$ is receive signal on array antenna Where λ is a Largrange multiplier. $\mathrm{r(t)} = \mathrm{E}[\mathrm{d(t)} - \mathrm{y(t)}]$,$\mathrm{R}_{rr} = E[r(t)r(t)^H]$. We can obtain the optimal weight vector as follow:

$$\mathrm{W}_p = E[X(t)r(t)^H] + R\, a(\theta) \frac{1 - \{\mathrm{p}(\theta|\mathrm{X})\, \mathrm{a}(\theta) \mathrm{R}\, E[X(t) r^H(t)]\}}{\mathrm{p}(\theta|\mathrm{X})\, \mathrm{a}(\theta)^H\, \mathrm{p}(\theta|\mathrm{X})^H\, \mathrm{R}\, \mathrm{a}(\theta)} \tag{10}$$

The second order model can also be written to be from equation (4)

$$\mathrm{R} = s_i\, A(\theta_i)\, A^*(\theta_i) + H_i B(\theta_i)\, B^*(\theta_i) + \sigma^2 I \tag{11}$$

where $H_i = \mathrm{E}[\beta\, \beta^*]$ is diagonal matrix of the interfering sources` powers. Equation(5) and (8) are model based representations for the measurement covariance matrix. Correlation matrix singular value decomposition can be written

$$\mathrm{R} = [U_1 U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \sigma^2 I \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}^H \tag{12}$$

$$= U_1\Sigma_1^2U_1^* + \sigma^2 I \quad (13)$$

where U_1 is the signal subspace, U_2 is the noise subspace. We denote the orthogonal projection matrices into the signal and noise subspaces by P_1 and P_2. P_1 denote the orthogonal projection matrix with signal range space. P_2 denote the orthogonal projection matrix with noise range space.

Let us consider the least squares source separation of the component $\mathrm{A}(\theta_i)\alpha_i$ from the measurement y

$$\mathrm{A}(\theta_i)\ \hat{\alpha}_i = E[\mathrm{A}(\theta_i)\ B(\theta_i)\ \mathrm{y}] \quad (14)$$

$$= \mathrm{A}(\theta_i)\ \left(A^*(\theta_i)\ P_2(\theta_i)\ \mathrm{A}(\theta_i)\right)^{-1} A^*(\theta_i)\ P_2(\theta_i)\ \mathrm{y} \quad (15)$$

The mean of $\mathrm{A}(\theta_i)\ \hat{\alpha}_i$ is $\mathrm{A}(\theta_i)\alpha_i$ and the second moment can be written

$$\mathrm{E}\left[\mathrm{A}(\theta_i)\hat{\beta}_i\hat{\beta}_i^*\ A^*(\theta_i)\right] = E[A(\theta_i)B(\theta_i)](R_A + \sigma^2 I)E[A(\theta_i)B(\theta_i)]^* \quad (16)$$

When the angle between the subspace $\mathrm{A}(\theta_i)$ and $\mathrm{B}(\theta_i)$ is small, then the noise gain Trace $E[A(\theta_i)B(\theta_i)]$ $E[A(\theta_i)B(\theta_i)]^*$ can be large. Equation(12) can be written again

$$E[A(\theta_i)B(\theta_i)(R_A - \sigma^2 I)E[A(\theta_i)B(\theta_i)]^* = A(\theta_i)s_i a^*(\theta_i) \quad (17)$$

The covariance for the interfering sources may be extracted as follow:

$$E[A(\theta_i)B(\theta_i)(R_A - \sigma^2 I)E[A(\theta_i)B(\theta_i)]^* = B(\theta_i)H_i B^*(\theta_i) \quad (18)$$

Equation(17) and (18) can be combined to write

$$R_A = A(\theta_i)s_i a^*(\theta_i) + B(\theta_i)H_i B^*(\theta_i) \quad (19)$$

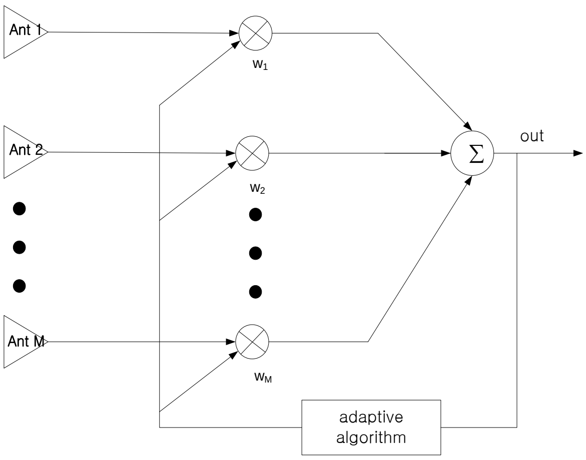

Fig. 1. Direction of arrival estimation with MUSIC algorithm.

III. Estimation of Subspace

A. Subspace Time Varing Channels

In this section, a second order analysis is carried out. Finite sample effects and calibration errors are not considered. Thus, only the effects of angular spreading are studied. The perturbation of the covariance matrix caused by the angular spreading is first related to the perturbation of the estimated signal and noise subspaces. These results are then used to find the perturbation of the estimated DOAs. For the case where the local scattering cause no angular spreading, but only variations of the received signal powers, $\nabla_i = 0$. The nominal covariance matrix of the observation, R is can be written

$$\mathrm{R} = A(\theta_i)s_i a^*(\theta_i) + B(\theta_i)H_i B^*(\theta_i) + \sigma^2 I \quad (20)$$

A basis for the nominal signal subspace may be defined from the eigenvalue decomposition of R

$$\mathrm{R} = E_s\Lambda_s E_s^* + E_I\Lambda_I E_I^* + E_n E_n^* \quad (21)$$

where E_s, E_I, and E_n are signal, interference and noise subspace, respectively. The estimates calculated with this covariance matrix will coincide with the nominal DOAs. With angular spread, the sample covariance matrix can be written

$$\bar{R} = [A + \bar{A}\,]\Lambda\, S\, \Lambda^*[A + \bar{A}\,]^* + [B + \bar{B}\,]\Lambda\, S\, \Lambda^*[B + \bar{B}\,]^* + \sigma^2 I \quad (22)$$

The estimated basis for the nose subspace is defined from the eigenvalue decomposition ($\bar{R}$). The de-correlation between long observation period to use the following approximation

$$B(\theta_i)\ B^*(\theta_i) \cong 0 \quad (23)$$

Note that the assumption of sequence de-correlation with co channel interference is basic for the derivation of the proposed method and that in practice this assumption is all the more valid if the training duration is long. Then, Equation(17) can be rewritten follow

$$\mathrm{R} = A(\theta_i)s_i a^*(\theta_i) + \sigma^2 I \quad (24)$$

B. DoA Estimation

The DOA are obtained as peaks in the following spectrum

$$P_i(\theta) = \frac{A(\theta)A(\theta)^H}{A(\theta)\ (I - V_iV_i^H)\ A(\theta)^H} \quad (25)$$

where $(I - V_iV_i^H)$ denotes the orthogonal projector on the noise subspace relative to the i$^{\text{th}}$ sample of the channel response. In the case of uniform linear array, this search can be avoided by polynomial rooting. Here $V_iV_i^H$ is the orthogonal projector on the source subspace and V_i is obtained as the dominant eigenvector of R.

IV. Computer Simulation

We employ N = 6 uniform linear sensor array antennas with equal power signals arriving in the half wavelength space. Output form these beams are based on

50 snapshots. We assume both that there are uncorrelated signal sources arriving from all direction and that there are equal power interferers in all directions.

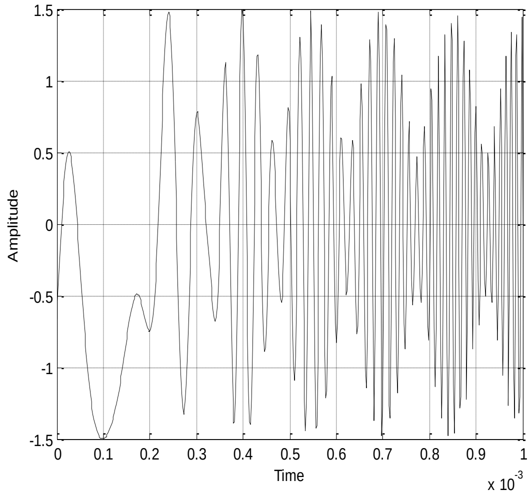

Fig. 2. Received channel signal.

In Fig. 2, we see the response of the source, interference, and noise signal. In Fig. 3, we see the filtered signal at the linear array antennas. In Fig. 4, the graph shows the general MUSIC algorithm used to estimate the desired signal. Fig. 4 shows the estimation of three signals, and which does not correctly estimate the desired. Fig. 5 shows the application of the modified MUSIC algorithm proposed in this paper. Fig. 5 correctly estimates the desired signal, and both interference and noise signal.

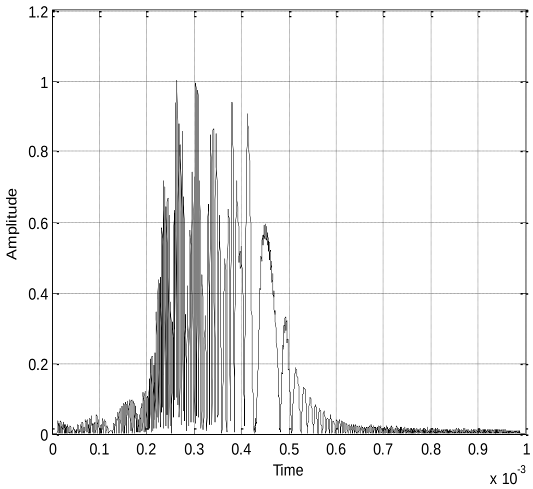

Fig. 3. Sampling signal.

The proposed modified MUSIC algorithm shows superior estimation of the desired signal relative to the general MUSIC algorithm.

We increased the number of sources to five; these sources are located at -35^o, -25^o, 0^o, 5^o, and 15^o. Fig. 6 shows the five directions of arrival, i.e., -35^o, -25^o, 0^o, 5^o, and 15^o, for the method proposed in this paper. This method correctly estimates the five DOAs.

Fig. 7 shows a graph of DOA estimation as the MUSIC: it shows errors of roughly 2^o. The proposed method can estimate the target direction based on reference signal and a phase shift in the case of adding more array antenna element than the signal sample number. This method is able to find the DOA in a wireless channel. Fig. 7 shows the estimated DOA for the conventional MUSIC algorithm at -5^o, 0^o, and 5^o.

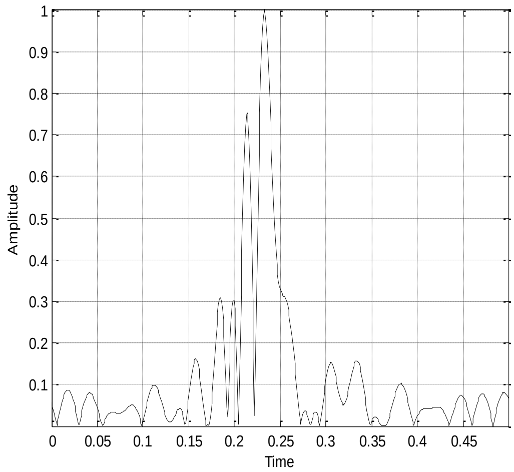

Fig. 4. General MUSIC algorithm.

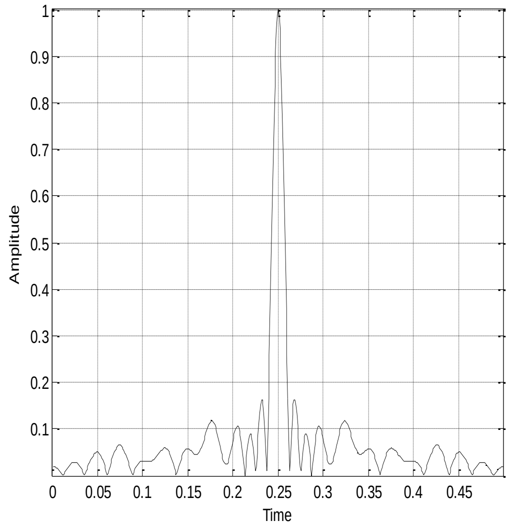

Fig. 5. Modified MUSIC signal

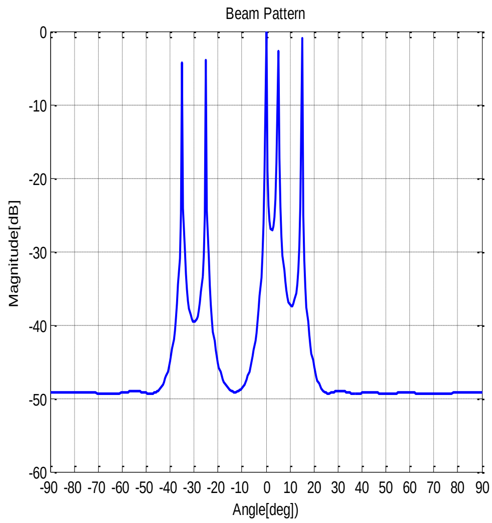

Fig. 6. DoA estimation of general MUSIC algorithm.

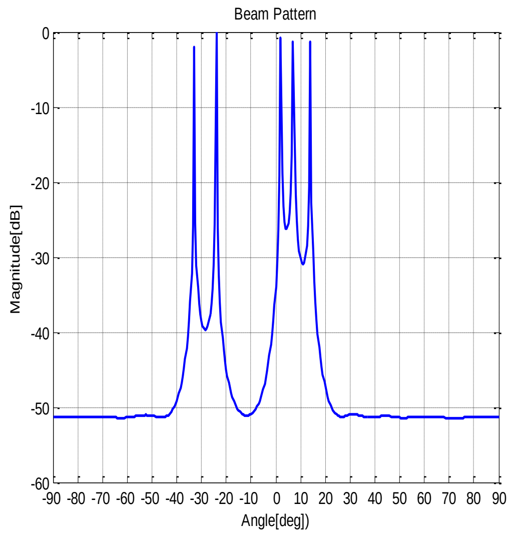

Fig. 7. DoA estimation of proposed MUSIC alogorithm.

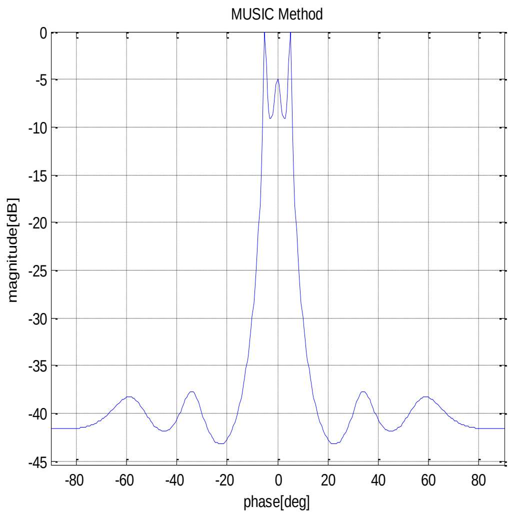

Fig. 8. DoA estimation of general MUSIC algorithm

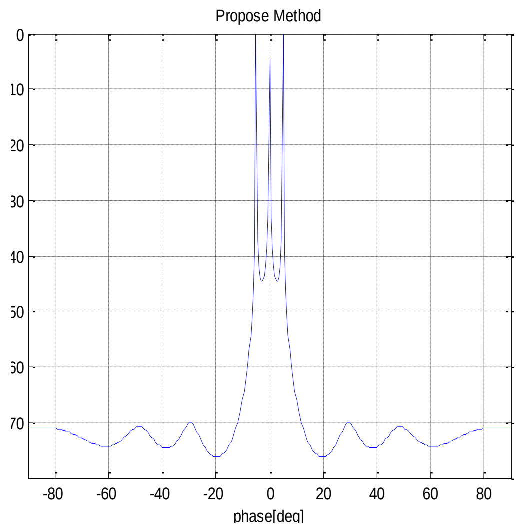

Fig. 9. DoA estimation of proposed MUSIC algorithm.

In Fig. 8, we saw that for [0^o], the signal amplitude peak decreased. Fig. 8 shows that the conventional method estimates two number signals, because it cannot estimate a 0^o signal when the threshold amplitude is below -5dB.

Fig. 9 shows the estimated direction of arrival with the algorithm proposed in this paper. Fig. 9 shows correct estimation DOA for three signals at -5^o, 0^o, and 5^o.

V. Helpful Hints

In this paper, we proposed a modified MUSIC algorithm to estimate the desired signal in a wireless channel. The proposed algorithm removes interference and noise signals in the time impulse channel. The proposed algorithm uses an acquisition covariance matrix, before removing noise and interference signals, to find the desired signal. We must find the covariance matrix to divide the subspace so as to determine the desired signal. The subspace is divided in to a signal subspace and a noise subspace. It is necessary to detect the number of sources before estimating the DOA. The results of computer simulation showed that our modified MUSIC algorithm has better performance than the general MUSIC method when finding desired signals and removing interference and noise signal in wireless communication

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