Optimal Power Allocation Based on Transmission Completion Time Minimization for Energy Harvesting Relay Networks

Zhenwei Xie[1] and Qi Zhu[2]
[1] The Key Wireless Laboratory of Jiangsu Province, School of Telecommunication and Information Engineering, Nanjing University of Posts and Telecommunications, Nanjing 210003, China
[2] Key Laboratory on Wideband Wireless Communications and Sensor Network Technology of Ministry of Education, Jiangsu Nanjing 210003, China
Email: 1015010333@njupt.edu.cn; zhuqi@njupt.edu.cn

Abstract —In this study, an algorithm to optimize the power allocation by minimizing the transmission completion time in energy harvesting wireless relay networks is proposed. The algorithm considers the energy harvesting relay which uses Decode-and-Forward (DF) mode and assumes that the data packets sizes and the harvested energy amounts have been got before the data transmission begins. According to the queue lengths of data and harvested energy, the algorithm optimizes the power policy of the source and relay by minimizing the time that all data is transmitted limited by the causality constraints of data and harvested energy. Finally, the power output capability of each node is considered. Simulation results verify that the proposed algorithm can minimize the transmission completion time of the data transmission.

Index Terms—Energy harvesting; power allocation; decode-and-forward relay

I. INTRODUCTION

In wireless communication systems, energy harvesting technology can continuously harvest energy from the surrounding environment, which greatly extends the life cycle of equipment [1]-[3] and improves the performance of wireless networks. In addition, relay technology extends the coverage of wireless networks and ensures the communication quality of cell-edge users. Therefore, the study on the resource allocation problem in energy harvesting wireless relay networks is very significant.

A lot of research efforts have been done on the power allocation problem for energy harvesting wireless networks. In [4], the authors assumed that the source can utilize conventional energy and harvested energy, and harvested energy spends less than conventional energy. The problem of the total energy cost minimization was solved by the theory of linear programming limited by a minimum capacity constraint and the causality constraints of harvested energy. In [5], the problem of throughput maximization was studied by the theory of dynamic programming in a two-node energy harvesting wireless network. In [6], the authors assumed that both the source and destination are energy harvesting nodes. The system goes into sleep mode to harvest and store energy for some time, and goes into active mode to flow energy and communicate between the source and destination for the other time. The optimal proportion factor and power allocation are obtained by maximizing the throughput. But in [4]-[6], there was always enough data to send, without considering the case that data packets gradually arrive in the process of the data transmission. In [7], the cases that all data is in the queue before the data transmission and data packets gradually arrive in the process of the data transmission were considered. The optimal power allocation algorithm was presented and proved by minimizing the time that all data is transmitted limited by the causality constraints of data and harvested energy. However, the above papers don't consider relay technology. In [8]-[10], the problems about the full-duplex and half-duplex relay networks were studied. But energy harvesting technology isn't considered. In [11], the authors assumed that the source and relay are both energy harvesting nodes, and obtained the joint power allocation of the two nodes over time by maximizing the throughput, without considering the case that data packets gradually arrive in the process of the data transmission.

In this paper, a power allocation algorithm with considering the case that data packets gradually arrive in the process of the data transmission is proposed. In the network, the source and relay are both energy harvesting nodes, and the data packets sizes and the harvested energy amounts of each timeslot have been got before the transmission starts. The source equips with an infinite capacity data buffer and battery. The DF relay just equips with a battery that has infinite capacity. Meanwhile, the relay can't store data, i.e., it needs to send the received data within the next timeslot. The proposed algorithm jointly optimizes the power allocation of the two nodes, and then considers the power output capability of each node. First, the transmission completion time lower bound with unlimited available energy in the relay and that with unlimited available energy in the source are

Manuscript received September 25, 2016; revised February 23, 2017.
This work is supported by National Natural Science Foundation of China (NO. 61571234, 61631020.)
Corresponding author email: zhuqi@njupt.edu.cn.
doi:10.12720/jcm.12.2.81-90

obtained. Then, the power allocation policy of the two nodes within the transmission completion time lower bound is obtained. Finally, the power allocation policy in the last timeslot of the data transmission is respectively discussed from the source harvesting energy constraint and the relay harvesting energy constraint.

This paper has the following structure. In Section II, we present the system model, and formulate the problem. In Section III, we put forward the power allocation algorithm to solve related problem. In Section IV, the power output capability of each node is considered. Section V gives simulation results. Finally, Section VI gives the related conclusions.

II. System Model and Problem Formulation

The classic three-node wireless network where the source and relay are both energy harvesting nodes is considered as shown in Fig. 1. Assuming that the direct link is in a deep fading state, and the relay is necessary to complete the communication. The timeslot is the minimum transmission unit and the corresponding transmission duration is denoted as τ . Assuming that both data packets and harvested energy arrive before the data transmission of each timeslot starts with known amounts. Both links are in two orthogonal frequency bands, and the same bandwidth is denoted as W . We denote the data packets sizes and harvested energy amounts of the source at the i-th timeslot as $B_S(i)$ and $E_S(i)$, and the harvested energy amounts of the relay at the i-th timeslot as $E_R(i)$. Assuming that the Gaussian channel fading coefficients of the two links are h_{sr} and h_{rd} , and the noise power spectral density at the source and relay is N_0 . Moreover, the smaller value between two numbers x and y is $\min(x,y)$ and the larger value is $\max(x,y)$; $floor(x)$ denotes round down to the nearest integer.

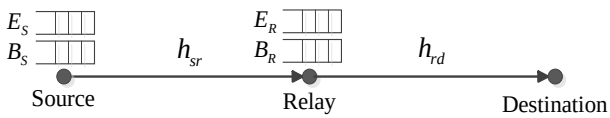

Fig. 1. A three-node wireless communication system model.

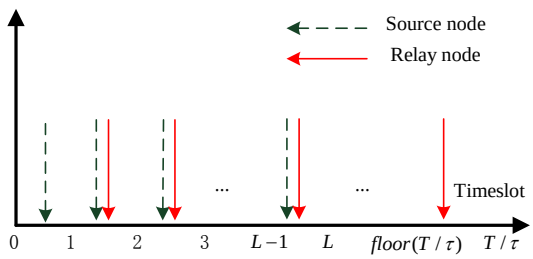

Fig. 2. Schematic diagram of system communication process.

The network works in a full-duplex mode, and the data transmission timeslots of the two nodes are shown in Fig. 2, where we assume that the minimum transmission completion time T corresponds to the (T/τ)-th timeslot and L denotes the timeslot that the last data packet arrives at the source. Then, in the 1st timeslot, data is sent from the source to the relay; in the 2nd timeslot, the relay sends the data received in the 1st timeslot, meanwhile, data is also sent from the source to the relay; until the last timeslot of the data transmission, the relay sends the data received in the last timeslot to the destination, i.e., except the first and last timeslots of the data transmission, the source and relay work simultaneously. Moreover, the relay can't store data, i.e., the relay needs to send the received data within the next timeslot.

First, in the i-th timeslot, assuming that the source works with power $P_S(i)$ and rate $R_S(i)$, and then cumulative energy consumption is denoted as:

$$E_S^c(i) = \sum_{k=1}^{i} P_S(k)\tau \tag{1}$$

The amounts of the data sent before the i-th timeslot are

$$B_S^c(i) = \sum_{k=1}^{i} R_S(k)\tau \tag{2}$$

Then, in the $(i+1)$-th timeslot, the relay works with power $P_R(i+1)$ and rate $R_R(i+1)$ based on the "random binning" technique [12], and cumulative energy consumption is denoted as:

$$E_R^c(i+1) = \sum_{k=1}^{i} P_R(k+1)\tau \tag{3}$$

The amounts of the data sent before the $(i+1)$-th timeslot are

$$B_R^c(i+1) = \sum_{k=1}^{i} R_R(k+1)\tau \tag{4}$$

Hence, under the criterion of transmission completion time minimization, we can formulate the problem as:

$$\begin{aligned}
\min_{\{P_S(i)\},\{P_R(i+1)\}} \quad & T = M\tau \\
s.t. \quad & E_S^c(i) \le \sum_{k=1}^{i} E_S(k) \\
& E_R^c(i+1) \le \sum_{k=1}^{i} E_R(k+1) \\
& B_S^c(i) \le \sum_{k=1}^{i} B_S(k) \\
& B_R^c(M) = \sum_{k=1}^{L} B_S(k) \\
& B_R^c(i+1) \le B_S^c(i) \\
& 0 \le P_S(i) \\
& 0 \le P_R(i+1)
\end{aligned} \tag{5}$$

where $M = T/\tau$ represents the timeslot corresponding to time T , and M may be non-integer value; the first three constraints represent the causality constraints of data and

harvested energy; the 4-th constraint represents the data forwarded by the relay in the next timeslot cannot beyond the data sent by the source; the 5-th constraint represents in the M-th timeslot all data must be transmitted to the destination; the last two constraints denote that power must be non-negative. Note that the data rate of the source cannot exceed the capacity of the source, i.e., $R_S(i) \le W\log_2\left(1+\frac{P_S(i)h_{sr}}{N_0W}\right)$. Similarly, we can obtain $R_R(i+1) \le W\log_2\left(1+\frac{P_R(i+1)h_{rd}}{N_0W}\right)$ for the relay.

III. Proposed Power Allocation Algorithm

Since the relay needs to send the received data within the next timeslot, system is reliable only if

$$R_S(i) \le \min\{C(P_S(i)), C(P_R(i+1))\} \tag{6}$$

where

$$C(P_S(i)) = W\log_2\left(1+\frac{P_S(i)h_{sr}}{N_0W}\right),$$

$$C(P_R(i+1)) = W\log_2\left(1+\frac{P_R(i+1)h_{rd}}{N_0W}\right).$$

In the process of the optimal power allocation, equation (6) must take the equal sign, i.e., the source sends data at the maximum transmission rate can be sent, otherwise T is not the minimum transmission completion time. If $P_S(i) \ge P_R(i+1)(h_{rd}/h_{sr})$, i.e., $C(P_S(i)) \ge C(P_R(i+1))$, the relay can't send the received data within the next timeslot, and we can reduce $P_S(i)$ until $P_S(i) = P_R(i+1)(h_{rd}/h_{sr})$ holds, without reducing the rate in the i-th timeslot of the source. Then, if $P_S(i) < P_R(i+1)(h_{rd}/h_{sr})$, i.e., $C(P_S(i)) < C(P_R(i+1))$, the data sent by the source in one timeslot can't reach the capacity of the relay in the next timeslot, so some energy is wasted at the relay, and we can reduce $P_R(i+1)$ until $P_R(i+1) = P_S(i)(h_{sr}/h_{rd})$ holds.

Therefore, in the optimal allocation strategy, $R_S(i) = C(P_S(i)) = C(P_R(i+1))$ and $P_S(i)h_{sr} = P_R(i+1)h_{rd}$ are always valid except the last timeslot of the data transmission. The last timeslot will be discussed later. Then, the constraints of $R_S(i)$ and $P_R(i+1)$ can be transformed into the constraints of $P_S(i)$. Hence, we firstly discuss the case without considering the causality constraints of the data of the source, and the causality constraints of the relay harvesting energy.

From [7], in order to get the optimal power allocation within M timeslots of the source, the problem, in which just the causality constraints of the source harvesting energy is considered, can be transformed into the minimum slope problem within M timeslots of the cumulative distribution function curve of harvested energy, as shown in Fig. 3.

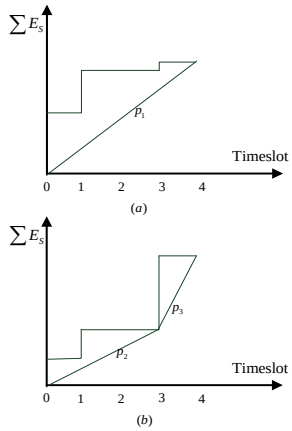

Fig. 3. The cumulative distribution function curves of energy harvesting and energy consumption.

Assuming that $M = 4$, the step-shaped curve represents the cumulative distribution function curve of harvested energy, and the other curves represent the cumulative distribution function curve of energy consumption. In Fig. 3 (a), we search from i_0=1, the slope of the minimum slope curve that connects the coordinate origin with the cumulative distribution function curve of harvested energy is p_1, so the optimal power within the four timeslots is p_1. In Fig. 3 (b), we search from i_0=1, the slope of the minimum slope curve is p_2, so the optimal power within the first three timeslots is p_2. Then, we search from $i_1 = 4$, the slope of the minimum slope curve is p_3, so the optimal power within the last timeslot is p_3.

Similarly, if the three causality constraints are considered simultaneously, we can transform the optimal power allocation problem within M timeslots into the minimum slope problem of three cumulative distribution function curves, two of which are obtained by substituting $P_S(i) = P_R(i+1)(h_{rd}/h_{sr})$ and $P_S(i) = C^{-1}(R_S(i))$ into the second and third causality constraints of (5). Then the optimal allocation strategy must satisfy (7) and (8), and the strategy that satisfies (7) and (8) must also the optimal allocation strategy, the corresponding proof is given in Appendix.

$$i_n = \arg\min_{i_{n-1}<i\le(\frac{T-\varepsilon_t}{\tau})} \left\{ \begin{array}{l} \dfrac{\sum_{k=1}^{i} E_S(k) - E_S^c(i_{n-1})}{(i-i_{n-1})\tau}, \\ \dfrac{\sum_{k=1}^{i} E_R(k+1) - E_R^c(i_{n-1}+1)}{(i-i_{n-1})\tau}\dfrac{h_{rd}}{h_{sr}}, \\ \left(2^{\frac{\sum_{k=1}^{i} B_S(k) - B_S^c(i_{n-1})}{(i-i_{n-1})\tau W}} - 1\right)\dfrac{N_0W}{h_{sr}} \end{array} \right\} \tag{7}$$

where $i_n (n = 0,1,...)$ is defined as the n-th allocated power change point of the source and relay, meanwhile, it is the n-th change point of the minimum slope in Fig. 3, and $i_0 = 0$; T - ε_t is the time to send all data from the source to the relay, and its value will be discussed later.

Therefore, the optimal policy in $i_{n-1} < i \le i_n$ is as follows:

$$P_S(i) = p_n = \min\left\{ \frac{\sum_{k=1}^{i_n} E_S(k) - E_S^c(i_{n-1})}{(i_n - i_{n-1})\tau}, \frac{\sum_{k=1}^{i_n} E_R(k+1) - E_R^c(i_{n-1}+1)}{(i_n - i_{n-1})\tau} \frac{h_{rd}}{h_{sr}}, \left(2^{\frac{\sum_{k=1}^{i_n} B_S(k) - B_S^c(i_{n-1})}{(i_n - i_{n-1})\tau W}} - 1 \right) \frac{N_0 W}{h_{sr}} \right\} \tag{8}$$

$$R_S(i) = C(P_S(i)) = C(P_R(i+1)) \tag{9}$$

$$P_R(i+1) = P_S(i) h_{sr} / h_{rd} \tag{10}$$

We assume that m_S denotes the timeslot that the last harvested energy arrives at the source and m_R denotes that timeslot at the relay, and let $u = \max(L, \max(m_S, m_R))$. Equation (7) can't be used if the exact T value isn't known, so, the transmission completion time lower bound T_{S_n} to transmit all data for the source-relay link with unlimited available energy in the relay, and the transmission completion time lower bound T_{R_n} to transmit all data for the relay-destination link with unlimited available energy in the source are obtained, separately.

First, assuming that the relay has unlimited available energy at the beginning of the data transmission, and the optimal power of the source has been allocated to the $i_{n-1} (n = 1, 2,...)$-th timeslot. So, the relay can send the received data within the next timeslot, i.e., the relay harvesting energy is no limit to the optimal power allocation of the source. If $L < m_S$, the data not all can be transmitted to the relay before the L-th timeslot. We search for the minimum energy A_i required to transfer all the remaining data to the relay from the $i (i = L)$-th $(L \le i \le m_S)$ timeslot, and A_i can be obtained by

$$A_i = \frac{(2^{\frac{\sum_{k=1}^{L} B_S(k) - B_S^c(i_{n-1})}{(i - i_{n-1})\tau W}} - 1) N_0 W}{h_{sr}} (i - i_{n-1})\tau \tag{11}$$

Then, if $A_i \le \sum_{k=1}^{i} E_S(k) - E_S^c(i_{n-1})$, we set $\tilde{i}_n = i$; if it is not satisfied, we set $i = i + 1$, and continue the forward search; if it is not satisfied until the m_S-th timeslot, we set $\tilde{i}_n = m_S$, which indicates the data not all can be sent to the relay before the $(\tilde{i}_n - 1)$-th timeslot , just by the harvested energy before the $(\tilde{i}_n - 1)$-th timeslot. If $m_S \le L$, the source harvesting energy must be all used up so that T_{S_n} is minimized, and we set $\tilde{i}_n = m_S$. Then T_{S_n} can be obtained by

$$W \log_2 \left(1 + \frac{\frac{\sum_{k=1}^{\tilde{i}_n} E_S(k) - E_S^c(i_{n-1})}{T_{S_n} - i_{n-1}\tau} h_{sr}}{N_0 W}\right)(T_{S_n} - i_{n-1}\tau) = \sum_{k=1}^{L} B_S(k) - B_S^c(i_{n-1}) \tag{12}$$

Similarly, assuming that the source has unlimited available energy at the beginning of the data transmission, so no matter how much data arrives at the source in one timeslot, it can be sent to the relay immediately in the next timeslot, i.e., the source harvesting energy is no limit to the power allocation of the relay. According to the above discussion, it is easy to obtain j_n and T_{R_n} from the relay to the destination. Then T_{R_n} is the solution of

$$W \log_2 \left(1 + \frac{\sum_{k=1}^{j_n} E_R(k+1) - E_R^c(i_{n-1}+1)}{T_{R_n} - (i_{n-1}+1)\tau} \frac{h_{rd}}{N_0 W}\right) \times (T_{R_n} - (i_{n-1}+1)\tau) = \sum_{k=1}^{L} B_S(k) - B_S^c(i_{n-1}) \tag{13}$$

Afterwards, we equally allocate the harvested energy $\sum_{k=1}^{\tilde{i}_n} E_S(k) - E_S^c(i_{n-1})$ over the time $i_{n-1}\tau \to T_{S_n}$, and obtain $\bar{p}_{S_n}$. Hence we have

$$\bar{p}_{S_n} = \frac{\sum_{k=1}^{\tilde{i}_n} E_S(k) - E_S^c(i_{n-1})}{T_{S_n} - i_{n-1}\tau} \tag{14}$$

Similarly, we equally allocate the harvested energy $\sum_{k=1}^{j_n} E_R(k+1) - E_R^c(i_{n-1}+1)$ over the time $(i_{n-1}+1)\tau \to T_{R_n}$, and obtain

$$\bar{p}_{R_n} = \frac{\sum_{k=1}^{j_n} E_R(k+1) - E_R^c(i_{n-1}+1)}{T_{R_n} - (i_{n-1}+1)\tau} \tag{15}$$

From the discussion of the following (6), if $\overline{p}_{S_n} > \overline{p}_{R_n}\frac{h_{rd}}{h_{sr}}$, we set $\overline{p}_{S_n} = \overline{p}_{R_n}\frac{h_{rd}}{h_{sr}}$; if not, we remain $\overline{p}_{S_n}$ unchanged.

Finally, we compare $\overline{p}_{S_n}$ with p_n obtained by (7) and (8) in $i_{n-1} < i \le k_n$, where $k_n = \max(\tilde{i}_n\text{-}1, j_n\text{-}1)$, and k_n indicates the data not all can be sent to the destination before the k_n-th timeslot, just by the harvested energy before the k_n-th timeslot. If $\overline{p}_{S_n} \le p_n$, then, maintaining $\overline{p}_{S_n}$ is feasible, and the optimal power allocation of the source is $\overline{p}_{S_n}$, meanwhile, the power allocation process is completed. Otherwise, maintaining $\overline{p}_{S_n}$ is not feasible because the causality constraints of data or harvested energy are violated. Such the optimal power allocation of the source is p_n. This procedure will be repeated until all data are delivered.

We give an illustration of the algorithm for the source as shown in Fig. 4. Assuming that the process of power allocation has reached the $i_{n-1}(n=1,2,...)$-th timeslot, and we don't consider the causality constraints of the data of the source, and the causality constraints of the relay harvesting energy, because the constraints of $R_S(i)$ and $P_R(i+1)$ can be transformed into the constraints of $P_S(i)$ through (9) and (10). T_n and T_n' are two cases of T_{S_n} obtained by (12), and $\overline{p}_{S_n}$ is corresponding to $\overline{p}_n$ and $\overline{p}_n'$. If $\overline{p}_{S_n} > p_n$, i.e., $\overline{p}_n > p_n$, it is against the causality constraints of harvested energy, and we set $P_S(i) = p_n$, $i_{n-1} < i \le i_n$. If $\overline{p}_{S_n} \le p_n$, i.e., $\overline{p}_n' \le p_n$, it is the minimum slope curve in the $(i_{n-1} < i \le T_n'/\tau)$-th timeslot, and we set $P_S(i) = \overline{p}_{S_n}$, $i_{n-1} < i \le T_n'/\tau$, and all the data has been sent to the relay.

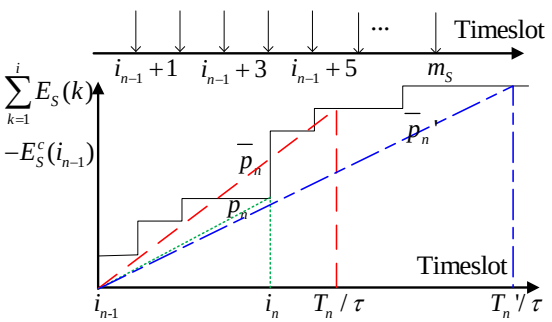

Fig. 4. An illustration of the algorithm.

In the optimal power allocation strategy, $P_S(i)h_{sr} = P_R(i+1)h_{rd}$ are always valid except the last timeslot of the data transmission. In the last timeslot, if $T_{S_n} + \tau > T_{R_n}$, it indicates that the source harvesting energy limits the optimal power allocation, and the relay harvesting energy should be all used to send the received data in the last timeslot, so that the transmission completion time can be minimized.

The data amounts of the relay in the last timeslot is

$$B_l = W\log_2\left[1+\frac{\overline{p}_{S_n}h_{sr}}{N_0W}\right]\left[T_{S_n} - floor\left(\frac{T_{S_n}}{\tau}\right)\tau\right] \quad (16)$$

In the last timeslot, the amounts of the remaining energy at the relay is

$$E_l = \sum_{k=1}^{floor(\frac{T_{S_n}}{\tau})} E_R(k+1) - E_S^c(floor(\frac{T_{S_n}}{\tau})+1) \quad (17)$$

The time δ_l that B_l is sent to the destination by E_l is the solution of

$$W\log_2\left(1+\frac{E_l}{\delta_l}\frac{h_{rd}}{N_0W}\right)\delta_l = B_l \quad (18)$$

Then, $T = floor(T_{S_n}/\tau)+1+\delta_l$, and the optimal power allocation is

$$\begin{aligned} &P_S(i) = \overline{p}_{S_n}, i_{n-1} < i \le \frac{T_{S_n}}{\tau} \\ &P_R(i+1) = \begin{cases} P_S(i)\frac{h_{sr}}{h_{rd}}, i_{n-1} < i \le floor\left(\frac{T_{S_n}}{\tau}\right) \\ \frac{E_l}{\delta_l}, floor\left(\frac{T_{S_n}}{\tau}\right) < i \le floor\left(\frac{T_{S_n}}{\tau}\right)+\delta_l \end{cases} \end{aligned} \quad (19)$$

Similarly, if $T_{S_n} + \tau \le T_{R_n}$, it indicates that the relay harvesting energy limits the optimal power allocation. But even the source harvesting energy is all used to send data in the last timeslot, the transmission completion time doesn't change. So we set $P_S(i) = P_R(i+1)(h_{rd}/h_{sr})$.

Then, $T = T_{R_n}$, and the optimal policy is

$$\begin{aligned} &P_S(i) = \overline{p}_{S_n}, i_{n-1} < i \le (\frac{T_{R_n}}{\tau}-1) \\ &P_R(i+1) = P_S(i)\frac{h_{sr}}{h_{rd}}, i_{n-1} < i \le (\frac{T_{R_n}}{\tau}-1) \end{aligned} \quad (20)$$

Hence, ε_t can be given of (7) as:

$$\varepsilon_t = \begin{cases} \tau & T_{S_n} + \tau \le T_{R_n} \\ \delta_l & T_{S_n} + \tau > T_{R_n} \end{cases} \quad (21)$$

The optimal power allocation algorithm is denoted as Algorithm I in Table I.

In the 3rd line of Algorithm I, $\tilde{i}_n$ and j_n are calculated according to Algorithm II in Table II.

TABLE I: THE OPTIMAL POWER ALLOCATION ALGORITHM

Algorithm I Optimal Power Allocation Algorithm

Initialize $i_0 = 0, B = \sum_{k=1}^{L} B_S(k), n = 0$;
while $B > 0$ **do**
  Compute $\tilde{i}_n$ and j_n according to Algorithm II;
  Compute $T_{S_n}, \overline{p}_{S_n}, T_{R_n}$, and $\overline{p}_{R_n}$ by (12), (13), (14) and (15);
  if $\overline{p}_{S_n} > \overline{p}_{R_n} \frac{h_{rd}}{h_{sr}}$
    Let $\overline{p}_{S_n} = \overline{p}_{R_n} \frac{h_{rd}}{h_{sr}}$;
  end if
  Update i_n and p_n by (7) and (8), where $(T - \varepsilon_t)/\tau$ is substituted by $k_n = \max(\tilde{i}_n - 1, j_n - 1)$;
  if $\overline{p}_{S_n} \le p_n$
    if $T_{S_n} + \tau > T_{R_n}$
      Compute δ_l by (16), (17) and (18), and compute P_S and P_R by (19), $T = floor(T_{S_n}/\tau) + 1 + \delta_l$;
      break;
    else
      Compute P_S and P_R by (20), $T = T_{R_n}$;
      break;
    end if
  else
    $P_S(i) = p_n, P_R(i+1) = P_S(i)\frac{h_{sr}}{h_{rd}}, i_{n-1} < i \le i_n$;
  end if
  $B = B - \sum_{i_{n-1}+1}^{i_n} W \log_2(1 + \frac{P_S(i)h_{sr}}{N_0 W})\tau$;
end while

TABLE II: COMPUTE THE $\tilde{i}_n$ AND j_n IN TABLE I

Algorithm II Compute $\tilde{i}_n$ And j_n

for $i = L:u$
  $A_i = \frac{(2^{\frac{B}{(i-i_{n-1})\tau W}} - 1)N_0 W}{h_{sr}}(i - i_{n-1})\tau$;
  if $A_i \le \sum_{k=1}^{i} E_S(k) - E_S^c(i_{n-1})$
    $\tilde{i}_n = i$;
    break;
  else
    $\tilde{i}_n = i$;
  end if
end for
for $i = L:u$
  $B_i = \frac{(2^{\frac{B}{(i-i_{n-1})\tau W}} - 1)N_0 W}{h_{rd}}(i - i_{n-1})\tau$;
  if $B_i \le \sum_{k=1}^{i} E_R(k+1) - E_R^c(i_{n-1}+1)$
    $j_n = i$;
    break;
  else
    $j_n = i$;
  end if
end for

IV. CONSIDERING THE PEAK POWER LIMIT

In this section, we consider the power output capability of the source and relay. Assuming that the peak power limits of the source and relay are $P_S^{\max}$ and $P_R^{\max}$, and then the allocated power should satisfy:

$$P_S(i) \le P_S^{\max} \tag{22}$$

$$P_R(i+1) \le P_R^{\max} \tag{23}$$

In Appendix, we prove that in the optimal allocation strategy, the allocated power must monotonically increase. After considering the peak power limit, we can still prove this conclusion in a similar manner. Meanwhile, $R_S(i) = C(P_S(i)) = C(P_R(i+1))$ and $P_S(i)h_{sr} = P_R(i+1)h_{rd}$ are always valid except the last timeslot since the relay can't store data. Then, $P_S(i)$ should satisfy:

$$P_S(i) \le \min(P_S^{\max}, P_R^{\max}(h_{rd}/h_{sr})) \tag{24}$$

When the peak power limit is not reached, we can solve the problem (5) with the peak power limit according to Algorithm 1. When the peak power limit is reached, we can obtain $P_S(i) = \min(P_S^{\max}, P_R^{\max}(h_{rd}/h_{sr}))$ by (24) and $P_R(i+1) = P_S(i)(h_{sr}/h_{rd})$. Meanwhile, the allocated power of the two nodes after this timeslot must not be less than $\min(P_S^{\max}, P_R^{\max}(h_{rd}/h_{sr}))$ and $\min(P_S^{\max}(h_{sr}/h_{rd}), P_R^{\max})$ according to the allocated power monotonically increases. Therefore, before the last timeslot, we have:

$$P_S(i) = \min(P_S^{\max}, P_R^{\max}(h_{rd}/h_{sr})) \tag{25}$$

$$P_R(i+1) = P_S(i)(h_{sr}/h_{rd}) \tag{26}$$

In the last timeslot, the source can be allocated power according to the above method. But the relay must use all remaining energy to forward the received data so that the transmission completion time can be minimized. Then, we can let:

$$P_R(i+1) = \min(P_R^{\max}, E_l/\delta_l) \tag{27}$$

Up to now, the optimal power allocation is completed.

V. SIMULATION AND ANALYSIS

In this section, the numerical results of the proposed algorithm are presented, and the proposed algorithm is compared with the maximum capacity transmission algorithm to explain that optimization power allocation is necessary. The simulation parameters mainly refer to [7]. Assuming that both links are in two orthogonal frequency bands, with the same bandwidth $W = 1\text{MHz}$, and the channel fading coefficients are $h_{sr} = 110\text{dB}$ and $h_{\text{rd}} = 100\text{dB}$. Moreover, we assume that the transmission duration of each timeslot is τ=1ms, and the noise power

spectral density is $N_0 = 10^{-19}$ W/Hz. The data packets sizes and the harvested energy amounts of the two nodes in each timeslot are given in Table III. Meanwhile, we temporarily do not consider the power output capability of each node so that the power change points can be easily seen.

TABLE III: SIMULATION PARAMETERS

Timeslot	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
$E_S / 10^{-5} J$	3	0	1	0	1	0	0	0	0	0	1	0.8	0	2	0	10	0	0	0
$B_S / Kbits$	1	0	0	0	1	0	0	0	6	0	0	8	0	0	0	0	0	0	0
$E_R / 10^{-6} J$	0	0.2	1	0	1	0	0	4	4	0	0	0	0	0	0	0	0	5	3

Fig. 5 shows the curves of the source harvesting energy and consumed energy among different algorithms, and the slope of the consumed energy curve denotes the allocated power of the source. From the figure we can see that the proposed algorithm has two power change points because of the causality constraints of the source harvesting energy, which are in the 13-th and 15-th timeslots. The time to transmit all data to the relay is 18.8ms. The source harvesting energy isn't all used up, because the relay harvesting energy limits the optimal power allocation in the last timeslot. It's clear from the figure that the consumed energy amounts of the proposed algorithm always are less than the maximum capacity transmission algorithm. Meanwhile, the maximum capacity transmission algorithm is finished earlier than the proposed algorithm, which does not mean that the time that the maximum capacity transmission algorithm transmits all data to the destination is shorter than the proposed algorithm. On the contrary, the data not all can be sent to the destination because the relay harvesting energy has been all used up, and the system doesn't have the ability to continue data transmission.

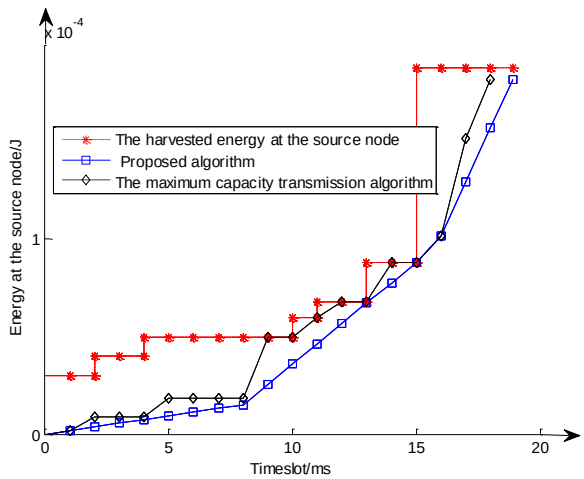

Fig. 5. The curves of the source harvesting energy and consumed energy among different algorithms.

Fig. 7 shows the curves of the relay harvesting energy and consumed energy among different algorithms. From the figure we can see that the proposed algorithm has two power change points due to the causality constraints of the relay harvesting energy, which are the 2nd and 17-th timeslots. The time to transmit all data to the destination is $T = 19.8$ms. Similarly, the consumed energy amounts of the maximum capacity transmission algorithm always are more than the proposed algorithm.

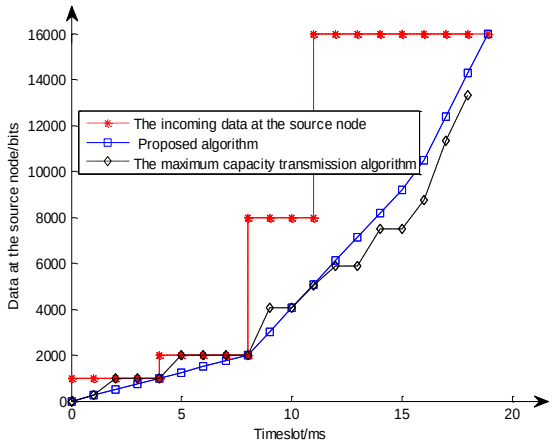

Fig. 6. The curves of the incoming data and the transmitted data at the source among different algorithms.

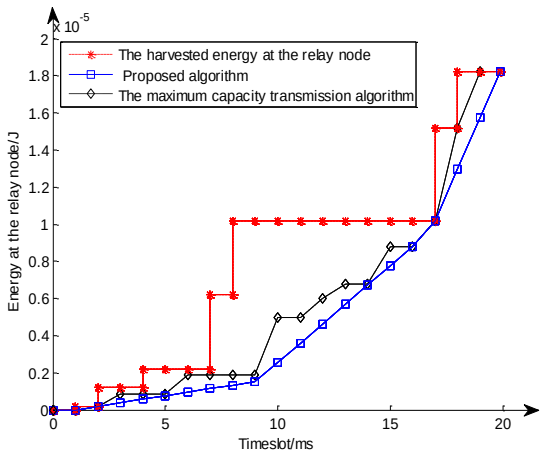

Fig. 7. The curves of the relay harvesting energy and consumed energy among different algorithms.

Fig. 6 shows the curves of the incoming data and transmitted data among different algorithms. From the figure we can see that the proposed algorithm has two power change points due to the causality constraints of the incoming data at the source, which are the 4-th and 8-th timeslots. Before the 10-th timeslot, the transmitted data amounts of the maximum capacity transmission algorithm are more than the proposed algorithm. After the 11-th timeslot, the transmitted data amounts of the maximum capacity transmission algorithm are less than the proposed algorithm. Especially in the 18-th timeslot,

i.e., in the 19-th timeslot of the relay, the data not all can be sent to the destination because the relay harvesting energy has been all used up, from which we can know the necessity to optimize power allocation.

In the above we give only one implementation of the simulation to verify the feasibility of the algorithm. Now, we give the simulation results that are averaged over randomizations of the simulation parameters. Assuming that the harvested energy of the two nodes in each timeslot is uniformly distributed between $[0, E_S^{\max}]$ and $[0, E_R^{\max}]$, where $E_S^{\max}=10^{-4}$ J, $E_R^{\max}=10^{-5}$ J. The data packets sizes are uniformly distributed between $[0, B_S^{\max}]$. Meanwhile, there are 20 timeslots data to be sent, and the peak power limits of the source and relay are $P_S^{\max}=0.5\times10^{-4}$ J, $P_R^{\max}=0.5\times10^{-5}$ J.

Fig. 8 shows the minimum transmission completion time comparison among different algorithms. When $B_S^{\max}$ is relatively small, two algorithms have similar performance. With the increase of $B_S^{\max}$, the system performance of the proposed algorithm is better.

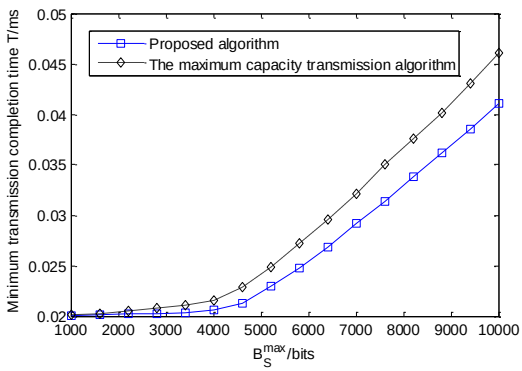

Fig. 8. The minimum transmission completion time comparison among different algorithms.

VI. Conclusions

This paper studies a power allocation problem by minimizing the transmission completion time for an energy harvesting relay wireless network. According to the queue lengths of data and harvested energy, the algorithm jointly optimizes the power allocation of the two nodes limited by the causality constraints of data and harvested energy. The algorithm considers the energy harvesting relay. Moreover, the power allocation in the last timeslot is respectively discussed from the source harvesting energy constraint and the relay harvesting energy constraint. Simulation results verify that the power optimization sometimes is necessary in energy harvesting relay wireless networks, and the proposed algorithm can minimize the transmission completion time.

Appendix

We can transform the problem (5) into (28) by substituting $P_S(i)=P_R(i+1)(h_{rd}/h_{sr})$ and $P_S(i)=C^{-1}(R_S(i))$ into the second and third causality constraints of (5), where if $M=10.5$, then $P_S(i)>0$ in the first half of the 11-th timeslot and $P_S(i)=0$ in the latter half of the 11-th timeslot. Then, we will prove the necessity and sufficiency of (7) and (8) by reductio ad absurdum.

$$\begin{aligned}
&\min_{\{P_S(i)\}} \quad T=M\tau \\
s.t.\ &\sum_{k=1}^{i} P_S(k)\tau \le \sum_{k=1}^{i} E_S(k) \\
&\sum_{k=1}^{i} P_S(k)\frac{h_{rd}}{h_{sr}}\tau \le \sum_{k=1}^{i} E_R(k+1) \\
&\sum_{k=1}^{i} W\log_2(1+\frac{P_S(k)h_{sr}}{N_0W})\tau \le \sum_{k=1}^{i} B_S(k) \\
&\sum_{k=1}^{M} W\log_2(1+\frac{P_S(k)h_{sr}}{N_0W})\tau = \sum_{k=1}^{L} B_S(k) \\
&0\le P_S(i)
\end{aligned} \tag{28}$$

where $B_R^c(i+1)\le B_S^c(i)$ can be omitted due to $P_S(i)=P_R(i+1)(h_{rd}/h_{sr})$.

First, we will prove the necessity that the optimal allocation strategy must satisfy (7) and (8). Assuming that the optimal allocation strategy satisfies (7) and (8) in the $(0, i_{n-1}]$-th timeslots, and it doesn't satisfy (7) and (8) after the i_{n-1}-th timeslot, i.e., there exists the i_n'-th timeslot after the i_{n-1}-th timeslot, such that:

$$p_n > \min\left\{\frac{\sum_{k=1}^{i_n'} E_S(k)-E_S^c(i_{n-1})}{(i_n'-i_{n-1})\tau},\ \frac{\sum_{k=1}^{i_n'} E_R(k+1)-E_R^c(i_{n-1}+1)}{(i_n'-i_{n-1})\tau}\frac{h_{rd}}{h_{sr}},\ \frac{(2^{\frac{\sum_{k=1}^{i_n'} B_S(k)-B_S^c(i_{n-1})}{(i_n'-i_{n-1})\tau W}}-1)N_0W}{h_{sr}}\right\}=p' \tag{29}$$

When $p'=\frac{(2^{\frac{\sum_{k=1}^{i_n'} B_S(k)-B_S^c(i_{n-1})}{(i_n'-i_{n-1})\tau W}}-1)N_0W}{h_{sr}}$, then if $i_n'<i_n$, the total amount of the data sent by the source in the $(i_{n-1}, i_n']$-th timeslots satisfies:

$$\begin{aligned}
&\sum_{k=i_{n-1}+1}^{i_n'} W\log_2(1+\frac{P_S(k)h_{sr}}{N_0W})\tau = \\
&\sum_{k=i_{n-1}+1}^{i_n'} W\log_2(1+\frac{p_nh_{sr}}{N_0W})\tau > \\
&\sum_{k=i_{n-1}+1}^{i_n'} W\log_2(1+\frac{p_n'h_{sr}}{N_0W})\tau = \sum_{k=1}^{i_n'} B_S(k)-B_S^c(i_{n-1})
\end{aligned} \tag{30}$$

It is more than the total amount of the data in the source before the i_n'-th timeslot, and this strategy is infeasible.

If $i_n' > i_n$, there must exist $q(q \geq 1)$ timeslots, and the power of the source over these timeslots is less than p_n. Then we will prove it by reductio ad absurdum: assuming that the power of the source in all timeslots is greater or equal to p_n, and then the total amount of the data sent by the source in the $(i_{n-1}, i_n']$-th timeslots must greater than $\sum_{k=1}^{i_n'} B_S(k) - B_S^c(i_{n-1})$ by (30), which is obviously infeasible. As shown in Fig. 9, the case is simply illustrated. We set $R_n = W\log_2(1+\frac{p_n h_{sr}}{N_0 W})$, $R' = W\log_2(1+\frac{p' h_{sr}}{N_0 W})$, so there must exist some timeslots which satisfy $R'' < R_n$.

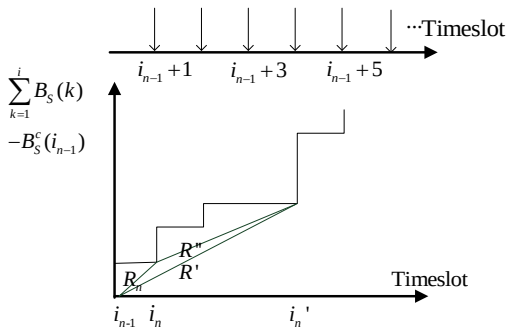

Fig. 9. An illustration of the algorithm about the data of the source.

In the optimal allocation strategy, the allocated power must monotonically increase. Similarly, we will prove it by reductio ad absurdum: assuming that there are two powers p_n and p_{n+1}, which satisfy $p_{n+1} < p_n$, and the corresponding durations are $l_n = (i_n - i_{n-1})\tau$ and $l_{n+1} = (i_{n+1} - i_n)\tau$. Let $p_n' = p_{n+1}' = \frac{p_n l_n + p_{n+1} l_{n+1}}{l_n + l_{n+1}}$, and we can obtain that $(l_n + l_{n+1})W\log_2(1+\frac{p_n' h_{sr}}{N_0 W}) > l_n W\log_2(1+\frac{p_n h_{sr}}{N_0 W}) + l_{n+1} W \times \log_2(1+\frac{p_{n+1} h_{sr}}{N_0 W})$, because the log is a concave function. Meanwhile, it is easy to verify that the causality constraints of data and harvested energy are still satisfied in this case. So in the optimal allocation strategy, the allocated power monotonically increases.

In the $(i_n, i_n']$-th timeslots, there must exist $q(q \geq 1)$ timeslots where the power of the source over these timeslots are less than p_n, and it contradicts with the allocated power monotonically increases in the optimal allocation strategy. So the hypothesis is not true when $p' = \frac{(2^{\frac{\sum_{k=1}^{i_n'} B_S(k) - B_S^c(i_{n-1})}{(i_n' - i_{n-1})\tau W}} - 1)N_0 W}{h_{sr}}$. Similarly, we can prove the hypothesis is not true when $p' = \frac{\sum_{k=1}^{i_n'} E_S(k) - E_S^c(i_{n-1})}{(i_n' - i_{n-1})\tau}$ or $p' = \frac{\sum_{k=1}^{i_n'} E_R(k+1) - E_R^c(i_{n-1}+1)}{(i_n' - i_{n-1})\tau}\frac{h_{rd}}{h_{sr}}$. In summary, the strategy that satisfies (29) is definitely not the best strategy.

In short, the optimal allocation strategy must satisfy (7) and (8).

Then, we will prove that the strategy that satisfies (7) and (8) must also the optimal allocation strategy. Assuming that the strategy that satisfies (7) and (8) is denoted as power vector $\mathbf{p}$ and duration vector $\mathbf{l}$. We can find another allocation strategy with $\mathbf{p}', \mathbf{l}'$, which can achieve a smaller transmission completion time. Both two strategies have the same power and duration in the $(0, i_{n-1}]$-th timeslots, but they are different after the i_{n-1}-th timeslot, which are denoted as p_n, l_n and p_n', l_n', respectively.

We can know $p_n < p_n'$ based on (7) and (8). First, the case of $p_n = \frac{(2^{\frac{\sum_{k=1}^{i_n} B_S(k) - B_S^c(i_{n-1})}{(i_n - i_{n-1})\tau W}} - 1)N_0 W}{h_{sr}}$ is discussed.

If $l_n < l_n'$, we can get $l_n p_n' > l_n p_n$, i.e., $l_n W\log_2(1+\frac{p_n' h_{sr}}{N_0 W}) > l_n W\log_2(1+\frac{p_n h_{sr}}{N_0 W}) = \sum_{k=1}^{i_n} B_S(k) - B_S^c(i_{n-1})$. The strategy $\mathbf{p}', \mathbf{l}'$ is infeasible because it is more than the total amount of the data in the source.

If $l_n > l_n'$, since in the optimal allocation strategy, the allocated power monotonically increases, then we can get $p_{n+1}' \geq p_n'$ and $p_n' l_n' + p_{n+1}'(l_n - l_n') > p_n l_n$, so the strategy $\mathbf{p}', \mathbf{l}'$ is also infeasible. Similarly, we can prove the hypothesis is not true when $p' = \frac{\sum_{k=1}^{i_n} E_S(k) - E_S^c(i_{n-1})}{(i_n - i_{n-1})\tau}$ or $p' = \frac{\sum_{k=1}^{i_n} E_R(k+1) - E_R^c(i_{n-1}+1)}{(i_n - i_{n-1})\tau}\frac{h_{rd}}{h_{sr}}$.

In short, the strategy that satisfies (7) and (8) must also be the optimal allocation strategy.

ACKNOWLEDGMENT

This work is supported by National Natural Science Foundation of China (NO. 61571234, 61631020.), and National Basic Research Program of China (NO. 2013CB329005.)

REFERENCES

[1] S. Ulukus, A. Yener, E. Erkip, O. Simeone, M. Zorzi, and K. Huang, "Energy harvesting wireless communications: A

review of recent advances," *IEEE Journal on Selected Areas in Communications*, vol. 33, no. 3, pp. 360-381, March 2015.

[2] X. Lu, P. Wang, D. Niyato, D. I. Kim, and Z. Han, "Wireless networks with RF energy harvesting: A contemporary survey," *IEEE Communications Surveys & Tutorials*, vol. 17, no. 2, pp. 757–789, 2015.

[3] M. L. Ku, W. Li, Y. Chen, and K. J. R. Liu, "Advances in energy harvesting communications: Past, present, and future challenges," *IEEE Communications Surveys & Tutorials*, vol. 18, no. 2, pp. 1384–1412, 2016.

[4] X. Kang, Y. Chia, C. K. Ho, and S. Sun, "Cost minimization for fading channels with energy harvesting and conventional energy," *IEEE Transactions on Wireless Communications*, vol. 13, no. 8, pp. 4586-4598, August 2014.

[5] C. Ho and R. Zhang, "Optimal energy allocation for wireless communications powered by energy harvesters," in *Proc. IEEE International Symposium on Information Theory*, June 2010, pp. 2368-2372.

[6] W. Ni and X. Dong, "Energy harvesting wireless communications with energy cooperation between transmitter and receiver," *IEEE Transactions on Wireless Communications*, vol. 63, no. 4, pp. 1457–1469, April 2015.

[7] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system," *IEEE Transactions on Wireless Communications*, vol. 60, no. 1, pp. 220-230, January 2012.

[8] Z. Zhang, Z. Ma, Z. Ding, M. Xiao, and G. K. Karagiannidis, "Full-Duplex two-way and one-way relaying: Average rate, outage probability, and tradeoffs," *IEEE Transactions on Wireless Communications*, vol. 15, no. 6, pp. 3920-3933, June 2016.

[9] K. Vardhe and D. Reynolds, "The performance of multistage cooperation in relay networks," *Journal of Communications and Networks*, vol. 17, no. 5, pp. 499–505, October 2015.

[10] B. Yu, L. Yang, X. Cheng, and R. Cao, "Power and location optimization for full-duplex decode-and-forward relaying," *IEEE Transactions on Wireless Communications*, vol. 63, no. 12, pp. 4743–4753, December 2015.

[11] C. Huang, R. Zhang, and S. G. Cui, "Throughput maximization for the gaussian relay channel with energy harvesting constraints," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 8, pp. 1469-1479, August 2013.

[12] Y. B. Liang and V. V. Veeravalli, "Gaussian orthogonal relay channels: Optimal resource allocation and capacity," *IEEE Transactions on Information Theory*, vol. 51, no. 9, pp. 3284-3289, September 2005.

Zhen-Wei Xie was born in Henan Province, China, in 1991. He received the B.E. degree in Communication Engineering from Jiangnan University, Wuxi, in 2015. He is now pursuing master's degree in the department of Communication and Information Engineering. He researches on the area of resource management, relay selection and subcarrier paring in cooperative networks.

Zhu Qi (corresponding author**)** was born in Suzhou, Jiangsu, China, in 1965. She received the M.S. degree in radio engineering from Nanjing University of Posts and Telecommunications in 1989. Now she is a professor in the Department of Telecommunication and Information Engineering, Nanjing University of Posts and Telecommunications, Jiangsu, China. Her research interests focus on technology of next generation communication, broadband wireless access, OFDM, channel and source coding, dynamic allocation of radio resources.