Design of Orthogonal Uplink Pilot Sequences for TDD Massive MIMO under Pilot Contamination

Sajjad Ali, Zhe Chen, and Fuliang Yin
School of Information and Communication Engineering, Dalian University of Technology, Dalian 116023, China
Email: sajjad.ali@mail.dlut.edu.cn; {zhechen, flyin}@dlut.edu.cn

Abstract —Massive MIMO has been acknowledged as a promising technology to counter the demand for higher data capacity for wireless networks in 2020 and beyond. However, each Base Station (BS) requires good enough knowledge of Channel State Information (CSI) on both the uplink and the downlink as massive MIMO relies on spatial multiplexing. In Time Division Duplex (TDD) massive MIMO systems, this CSI is acquired using channel reciprocity. However, the use of non-orthogonal uplink pilot sequence due to limited coherence time leads to pilot contamination in TDD massive MIMO systems that results in inter-cell interference in the downlink data transmission. This paper proposes a design of orthogonal uplink pilot sequences for multi-cell TDD massive MIMO systems. We propose to use Zadoff-Chu Pilot Sequences (ZCPS) and eliminate pilot contamination during channel estimation process. In the proposed design, each BS is assigned with a specific orthogonal code and the set of ZCPS is multiplied element-wise at each BS with BS-specific orthogonal code to generate orthogonality among pilot sequences across neighboring cells. The proposed design eliminates pilot contamination during channel estimation process thus achieves significant sum-rate gains as verified by the simulation results.

Index Terms—Massive MIMO, channel state information, pilot contamination, Zadoff-Chu, orthogonal codes

I. Introduction

The massive MIMO is a promising technology for the Fifth Generation (5G) cellular networks due to its unprecedented high spectral efficiency [1]-[3]. However, the performance of massive MIMO systems depends critically on the precision of the Channel State Information (CSI), regardless whether the CSI is used for the uplink reception or for the downlink transmission [4], [5]. This CSI can be obtained either using Frequency-Division Duplex (FDD) or Time-Division Duplex (TDD). In FDD massive MIMO systems, the CSI is estimated by the Mobile Stations (MSs) and signaled back to the Base Station (BS). Thus the CSI signaling overhead scales linearly with the number of antennas deployed at the BS that shows the use of large antenna arrays impractical. However, in TDD massive MIMO systems, the CSI is obtained at the BS using the uplink pilot training that will be used for the downlink transmission, by exploiting reciprocity between the uplink and the downlink channels. Furthermore, in TDD massive MIMO systems, the signaling overhead imposed by the acquisition of the uplink CSI scales linearly with the number of MSs, which is typically much lower than the number of antennas deployed at the BS [6], [7]. However, the accuracy of the CSI obtained and thus the attainable system performance depends on having perfectly orthogonal uplink pilot sequences allocated to the different MSs across the network. Orthogonal uplink pilot sequences may only be assured for the MSs roaming within the same cell, but not for those scattered across the different cells, because of limited channel coherence time [8], [9]. The worst-case scenario is associated with the multi-cell systems relying on the pilot reuse factor of one when all cells use the same set of orthogonal pilot sequences at the same time. Then the pilot sequences for different MSs are correlated, the estimated CSI of an MS is contaminated by the CSI of neighboring MSs that results in inter-cell interference. This detrimental effect is known as pilot contamination that constitutes a much more severe impairment compared to the effect of additive white Gaussian noise (AWGN). Therefore, pilot contamination forms a performance bottleneck in massive MIMO communication systems [6], [7], [10].

A number of schemes have been proposed in the literature to eliminate the pilot contamination from the TDD massive MIMO systems [5], [6], [9], [11], [12]. Reference [5] and [11] proposed time-shifted pilots with a finite and an infinite number of BS antennas, respectively. However, time-shifted pilot scheme entails a central controller for managing the time-shifting of the pilot-intervals in all of the cells in order to protect their ‘orthogonality’ across different cells, which becomes a challenge for growing number of users and cells. Similarly, [6] consisted of an amalgam of downlink and uplink training phases, which are capable of eliminating pilot contamination at the cost of requiring a much longer training duration than the conventional simultaneous uplink training. More specifically, the scheme of [6] consists of $(L+3)$ training phases for an L-cell system. Therefore, it requires that the coherence interval of the channel is no less than $(L+3)\tau$, where τ is the length of the training sequences, which is assumed to be equal to

Manuscript received October 17, 2016; revised January 22, 2017.
This work was supported by National Natural Science Foundation of China (Nos. 61172107, 61172110), National High Technology Research and Development Program (863 Program) of China (No. 2015AA016306), Major Projects in Liaoning Province Science and Technology Innovation (No. 201302001), and Fundamental Research Funds for the Central Universities of China (No. DUT13LAB06).
Corresponding author email: flyin@ mail.dlut.edu.cn.
doi:10.12720/jcm.12.1.40-48

the number of users per cell. Whereas [9] and [12] proposed minimum mean-square error (MMSE) based precoding and scheduling methods, respectively. In [9], the precoding matrix at one BS is designed to minimize the sum of the squared error of its own users and interference to the users in all other cells. The distributed single-cell precoding method is shown to provide better performance than traditional single-cell zero-forcing precoding. However, this scheme [9] needs the knowledge of second-order statistics of all the uplink channels. Although, a BS estimates only its in-cell channels, not the interfering channels from the adjacent cells. Therefore, it is impractical to presume that the BS can obtain the second-order statistics of all the uplink channels without estimating them. Whereas, [12] presented the pilot scheduling under two MMSE criteria, and proposed a low complexity pilot scheduling algorithm motivated by the channel angle of arrival non-overlapping condition. However, this proposed scheduling algorithm entails the knowledge of angles of arrival (AOAs) of all the users, which is unrealistic to know in a practical environment.

Along with aforementioned schemes and extensive pilot contamination elimination literature has not focused on uplink pilot sequence design or selection features and such contributions are limited in the literature, e.g., [13] and [14], where uplink pilot sequences in neighboring cells are treated as noise. Anzhong *et al*. [13] proposed to phase shift a given Zadoff-Chu (ZC) sequence [15] among multiple cells and calculate the required phase shift to be used in each cell. Hien *et al*. [14] proposed a greedy sequence assignment algorithm, where the sequences are allowed to take random values and are not chosen from a predefined set of sequences. This makes a practical implementation rather challenging, as the complete sequences need to be transmitted from the BS to the respective users. Jae Won *et al*. [16] treat the inter-cell pilot interference problem with ZC sequences and find subsets of such sequences that minimize inter-cell pilot interference. Their approach treats sequences in neighboring cells as noise as well and assumes that pilots of each user occupy all available subcarriers, making the used framework not suitable for the TDD massive MIMO systems. Because in TDD massive MIMO system only a limited portion of the coherence interval can be used for the uplink training.

Different from [13], [14] and [16], [17] and [18] proposed a user capacity-achieving pilot sequence design together with power allocation for downlink transmission in a single- and multi-cell multiuser massive MIMO system, respectively. Both schemes [17] and [18], proposed to generate pilot sequences and the corresponding power allocation scheme to satisfy the signal-to-interference-plus-noise ratio (SINR) requirements of all the users in the system, not considering the rigorousness of pilot contamination. Furthermore, pilot sequence design of [17] and [18] is based on the rules of the Generalized Welch-Bound-Equality (GWBE) sequence design. Different from [17] and [18], in this work the proposed pilot sequence design uses ZC sequence and eliminates pilot contamination during channel estimation process.

Given the above background, this paper proposes an efficient and practical pilot contamination elimination scheme for multi-cell TDD massive MIMO systems. The proposed design of orthogonal uplink pilot sequences uses ZC pilot sequences (ZCPS) as uplink pilot sequences, which remain orthogonal within a cell due to their constant amplitude and zero autocorrelation (CAZAC) property (i.e., the correlation of a ZC sequence of any length with the circularly shifted version of itself is zero for non-zero shifts). The perfect circular autocorrelation property allows multiple orthogonal sequences to be generated from a ZC sequence. In fact, if the periodic autocorrelation of a ZC sequence provides a single peak at the zero lag, the periodic correlation of the same sequence against its cyclic shifted replica provides a peak at a lag L_{CS}, where L_{CS} is the number of samples of the cyclic shift. This creates a zero-correlation zone (ZCZ) between the two sequences [15]. To make pilot sequences orthogonal across the network, the orthogonal codes are used. Before random access, a set of ZCPS is multiplied element-wise with BS-specific orthogonal code row at each BS that will make ZCPS orthogonal across the network. The proposed scheme will eliminate pilot contamination during channel estimation process. Furthermore, the proposed scheme uses conventional simultaneous uplink pilot training and does not require any prior knowledge regarding either the MIMO channels or MS information.

The remaining sections of this paper are organized as follows. The multi-cell TDD massive MIMO system model, uplink training and pilot contamination problem are presented in Section II. Section III describes orthogonal codes, ZC sequences, proposed design of orthogonal ZCPS, eliminating pilot contamination during channel estimation process, downlink transmission, and achievable throughput rates. Section IV presents simulation and result discussions. Finally, some conclusions are given in Section V.

Notations: The notations used in this paper are as follows. The boldface variables denote the matrices or vectors. The transpose and the Hermitian transpose are denoted by $(.)^{\mathrm{T}}$ and $(.)^{\mathrm{H}}$, respectively. A $\mathrm{diag}\{\mathbf{d}\}$ symbolizes a diagonal matrix with diagonal entries equal to the components of vector $\mathbf{d}$ and '$*$' indicates element-wise multiplication. The trace and inverse operations are denoted by $\mathrm{tr}\{.\}$ and $(.)^{-1}$, respectively. The two-norm, expectation, and variance are symbolized as $\|\,.\,\|$, $\mathbb{E}[.]$ and $\mathrm{var}\{.\}$, respectively.

II. Multi-Cell TDD System Model

Consider a cellular network composed of L hexagonal cells, tagged by $l = 1,\ 2,\ ...,\ L$, where the BS of each cell contains an array of A antennas and serves U single-antenna MSs, where impliedly $A >> U$. All the BSs and MSs are synchronized and TDD operation is employed to estimate the CSI at the BS. Unity frequency reuse (UFR) is employed and ZCPS will be orthogonal within a cell due to their CAZAC property. The average powers during transmission at each BS and MS are ρ_{dl} and ρ_{up}, respectively. The propagation vector connecting the a-th BS antenna of the q-th cell and the u-th user of the r-th cell is $h_{rqua}\beta_{rqu}^{1/2}$, where h_{rqua} is a random variable with independent and identically distributed (i.i.d.) zero-mean, circularly symmetric complex Gaussian distribution $\mathbb{C}\mathcal{N}(\mathbf{0},\mathbf{1})$ and known to nobody, and β_{rqu} is a positive constant and supposed to be known to everybody. This multi-cell model is illustrated in Fig. 1, where the h_{rqua} variable model fast fading that presumed to be a constant for a duration of T symbols and can be defined as [4], [6]

$$h_{lqua} = \begin{bmatrix} e^{-j\psi_{lqua1}} e^{-j2\pi\frac{(l-1)\mathfrak{D}}{\lambda}\cos(\theta_{lqua1})}, \\ e^{-j\psi_{lqua2}} e^{-j2\pi\frac{(l-1)\mathfrak{D}}{\lambda}\cos(\theta_{lqua2})}, \\ \ldots,\ e^{-j\psi_{lquat}} e^{-j2\pi\frac{(l-1)\mathfrak{D}}{\lambda}\cos(\theta_{lquat})} \end{bmatrix} \quad (1)$$

where t is the number of i.i.d paths, ψ_{lquat} is the phase of the path and it is a random variable uniformly distributed in $[0, 2\pi)$, $\mathfrak{D}$ is the antenna spacing at the BS, λ is the wavelength of the carrier, and $\theta_{lquat} \in [0, \pi]$ is a random angle of arrival (AOA). Whereas β_{rqu} invariable model path-loss and shadow fading that change slowly over space [1], [2], [10].

A. Uplink Training

At the start of every coherence interval, all the MSs in the network synchronously transmit their uplink pilot sequences, which are the column vectors with τ length.

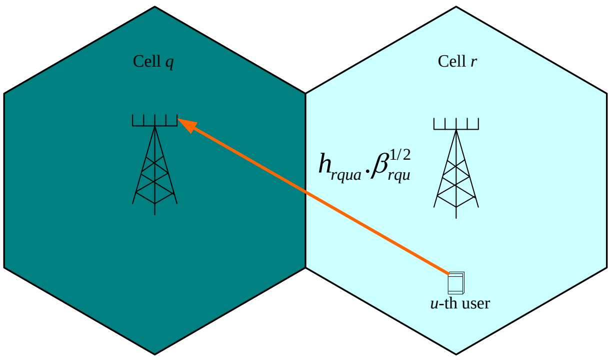

Fig. 1. The connectivity model between the a-th antenna of the q-th cell and u-th user of the r-th cell in a TDD M-MIMO System.

Define $\mathbf{z}_{ru}$ as the uplink pilot sequence transmitted by the u-th user in the r-th cell and it is denoted by $\mathbf{z}_{ru} = \left[z_{ru}^{[1]} \quad z_{ru}^{[2]} \quad \ldots \quad z_{ru}^{[\tau]}\right]^{\mathrm{T}}$, where $z_{ru}^{[b]}$ is the pilot sequence element, and without loss of generality, assume $\left|z_{ru}^{[b]}\right| = 1$, then $\mathbf{z}_{ru}^{\mathrm{H}}\mathbf{z}_{ru} = \tau$. In the BS of the q-th cell, the signal received during uplink training phase at the a-th antenna of the q-th BS is, [19], [20]

$$\mathbf{y}_{qa} = \sum_{l=1}^{L}\sum_{u=1}^{U}\sqrt{\rho_{up}\beta_{lqu}}\, h_{lqua}\mathbf{z}_{lu} + \mathbf{v}_{qa} \quad (2)$$

where $\mathbf{v}_{qa}$ is the i.i.d. AWGN with zero mean and unit variance.

Let $\mathbf{Y}_q = [\mathbf{y}_{q1} \quad \mathbf{y}_{q2} \quad \ldots \quad \mathbf{y}_{qA}]_{\tau\times A}$, $\mathbf{V}_q = [\mathbf{v}_{q1} \quad \mathbf{v}_{q2} \quad \ldots \quad \mathbf{v}_{qA}]_{\tau\times A}$. All L cells employ the same set of U uplink pilot sequences, represented by $\mathbf{Z} = [\mathbf{z}_1 \quad \mathbf{z}_2 \quad \ldots \quad \mathbf{z}_U]_{\tau\times U}$ satisfying constraint $\mathbf{Z}^{\mathrm{H}}\mathbf{Z} = \tau\mathbf{I}$, $\mathbf{D}_{lq} = \mathrm{diag}\{[\beta_{lq1} \quad \beta_{lq2} \quad \ldots \quad \beta_{lqU}]\}$, and $\mathbf{H}_{lq} = \begin{bmatrix} h_{lq11} & \cdots & h_{lq1A} \\ \vdots & \ddots & \vdots \\ h_{lqU1} & \cdots & h_{lqUA} \end{bmatrix}$.

Then, we have

$$\mathbf{Y}_q = \sqrt{\rho_{up}}\sum_{l=1}^{L}\sqrt{\mathbf{D}_{lq}}\mathbf{H}_{lq}\mathbf{Z} + \mathbf{V}_q \quad (3)$$

After the q-th BS receives the signal $\mathbf{Y}_q$, the channel $\mathbf{H}_{qq}$ is estimated with the MMSE estimator [9], [20]

$$\begin{aligned} \hat{\mathbf{H}}_{qq}^{\mathbf{MMSE}} &= \sqrt{\rho_{up}}\mathbf{D}_{qq}\left(\mathbf{I} + \rho_{up}\left(\sum_{l=1}^{L}\mathbf{Z}\mathbf{D}_{lq}\mathbf{Z}^{\mathrm{H}}\right)\right)^{-1}\mathbf{Y}_q\mathbf{Z}^{\mathrm{H}} \\ &= \sqrt{\rho_{up}}\mathbf{D}_{qq}\left(\mathbf{I} + \rho_{up}\tau\sum_{l=1}^{L}\mathbf{D}_{lq}\right)^{-1}\mathbf{Y}_q\mathbf{Z}^{\mathrm{H}} \end{aligned} \quad (4)$$

B. Pilot Contamination

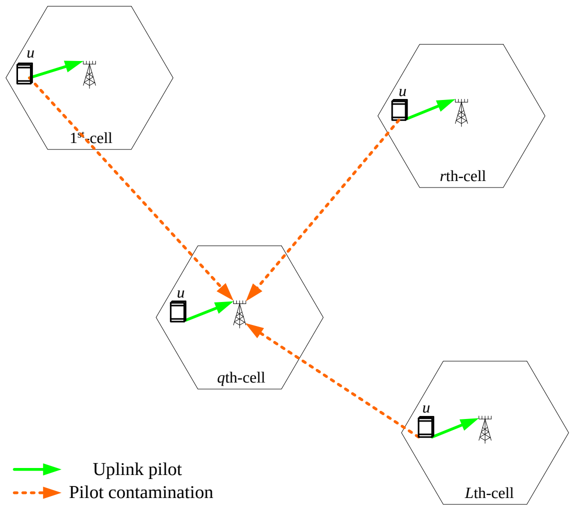

Fig. 2. The pilot contamination problem.

It is obvious from (4) that the q-th BS estimates the desired channel $\mathbf{H}_{qq}$ by correlating the received signal $\mathbf{Y}_q$ with the known pilot sequence. Since all L cells employ the same set of pilot sequences, which is the worst case scenario, therefore, this Channel Estimate (CE) is severely polluted by the MSs of adjacent cells, which are allocated same pilot sequences. This is so-called pilot contamination and is illustrated in Fig. 2. Thus, (4) of MMSE CE can be simplified as

$$\hat{\mathbf{H}}_{qq}^{\mathbf{MMSE}} = \left(\frac{\mathbf{I}}{\rho_{up}\tau}\mathbf{D}_{qq}^{-1} + \sum_{l=1}^{L}\mathbf{D}_{lq}\mathbf{D}_{qq}^{-1} \right)^{-1} \times \left(\mathbf{D}_{qq}^{1/2}\mathbf{H}_{qq} + \underbrace{\sum_{l\neq q, l=1}^{L}\mathbf{D}_{lq}\mathbf{H}_{lq}}_{\text{fourth term}} + \frac{1}{\sqrt{\rho_{up}}\tau}\mathbf{V}_q\mathbf{Z}^{\mathrm{H}} \right) \quad (5)$$

The fourth term of (5) shows the severity of the pilot contamination mutilation that result in a considerable estimation error.

III. Proposed Pilot Contamination Elimination Scheme

As shown in Section II-B, the CE relying on the uplink pilot sequences suffers from pilot contamination and the existing schemes [3], [5], [6], [9], [11] and [12] either require relatively large training duration or require prior knowledge regarding either the MIMO channels or MS information. Fortunately, the number of cells is limited compared to the number of MSs, therefore it is possible to allocate distinct orthogonal code rows to the BSs. These BS-specific orthogonal code rows can be exploited to eliminate pilot contamination during CE process. Before discussing the proposed scheme in detail, let us briefly explain the orthogonal codes and ZC sequences, respectively.

A. Orthogonal Codes

The orthogonal codes such as Orthogonal Variable Spreading Factor (OVSF) and Walsh-Hadamard codes can be used in the proposed design [21], [22]. The code rows of these two orthogonal codes are mutually orthogonal; hence element-wise multiplication of code rows with ZCPS will make ZCPS orthogonal across the network. Specifically, OVSF codes were first commenced for 3G systems to maintain the orthogonality among different uplink channels in a wireless communication system [21]. An 8-by-8 orthogonal matrix can be given as,

$$\mathbf{O}_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \quad (6)$$

where each row of the matrix can be represented as $\mathbf{O}_{8,w}$, where w indicates the row number.

B. Zadoff-Chu Sequences

The ZC sequences are a category of polyphase sequences defined as [15], [23]

$$\mathbf{z}_\mu = \left\{ z_\mu(n) = \exp(-\mathrm{j}\pi\mu\frac{n^2}{P}),\ n = 0,1,...,P-1 \right\} \quad (7)$$

where P (even) is the length of the sequence and $\mu \in \{1, 2, \cdots, P-1\}$ is the root index of the sequence.

ZCs of any length possess an ideal or "perfect" periodic autocorrelation property (i.e., the correlation with the circularly shifted version of itself is zero for any non-zero shifts)

$$\gamma_{\mu\mu}(\Delta\kappa) = \frac{1}{P}\sum_{n=0}^{P-1} z_\mu[n] \cdot z_\mu^{\mathrm{T}}[n+\Delta\kappa] = \delta[\Delta\kappa] \quad (8)$$

where $\Delta\kappa$ is the shift or correlation lag, and $-(P-1) \le \Delta\kappa \le P-1$.

C. Orthogonal ZCPS

The proposed pilot contamination elimination scheme multiplies ZCPS element-wise with the BS-specific orthogonal code rows. This element-wise multiplication will make ZCPS orthogonal across the network. The proposed scheme takes the length of ZCPS as the multiple of two i.e. $P = 2^m$, where $m \ge 2$. Furthermore, the rows of the orthogonal codes can be re-used in a similar pattern as that of the frequency reuse pattern in wireless communication systems.

A ZC sequence of length $P = 2^m$ with root index μ, can be given as

$$\mathbf{z}_\mu = \left\{ z_\mu(n) = \exp(-\mathrm{j}\pi\mu\frac{n^2}{2^m}),\ n = 0,1,...,2^m-1 \right\} \quad (9)$$

Then the set of cyclically shifted ZCPS of $\mathbf{z}_\mu$ can be represented as $\mathbf{z}_\mu(\Delta\kappa)$, where $-(2^m-1) \le \Delta\kappa \le 2^m-1$. Here, we consider a seven cell system; therefore, each BS of the seven-cell cluster can be assigned to a distinct orthogonal code row from the $\mathbf{O}_8$ matrix. Then, each of these assigned orthogonal code rows is multiplied element-wise with the set of cyclically-shifted ZCPS at each BS of the seven cell system as given below, refer to Fig. 3.

$$\begin{aligned} \mathbf{c}_{e,\Delta\kappa} &= \mathbf{z}_\mu(\Delta\kappa) * \mathbf{O}_{8,w} \\ &= \left\{ z_{e,\Delta\kappa}(n) = z_\mu(n+\Delta\kappa) * O_{8,w}(n+1),\ n = 0,1,...,2^m-1 \right\} \end{aligned} \quad (10)$$

where '*' denotes the element-wise multiplication, $e = 1, 2, ..., 7$ represents the number of the BS in the seven cell system. This element-wise multiplication will make $\mathbf{c}_{1,\Delta\kappa}, \mathbf{c}_{2,\Delta\kappa}, ..., \mathbf{c}_{7,\Delta\kappa}$ a set of sequences mutually orthogonal to each other. This is from the fact that $z \times z^{\mathrm{H}} = 1$ where z is an element of ZC sequence. This assertion is proved as follows.

$$\begin{aligned}\mathbf{X} &= \mathbf{c}_{1,\Delta k} \cdot \mathbf{c}_{2,\Delta k}^{\mathrm{H}} \\ &= \left[\mathbf{z}_{\mu}(\Delta\kappa) * \mathbf{O}_{8,0}\right] \cdot \left[\mathbf{z}_{\mu}(\Delta\kappa) * \mathbf{O}_{8,1}\right]^{\mathrm{H}} \\ &= \left[\mathbf{z}_{\mu}(\Delta\kappa) \cdot \mathbf{z}_{\mu}^{\mathrm{H}}(\Delta\kappa)\right] * \left[\mathbf{O}_{8,0} \cdot \mathbf{O}_{8,1}^{\mathrm{H}}\right]\end{aligned}$$

since

$$\mathbf{O}_{8,0} \cdot \mathbf{O}_{8,1}^{\mathrm{H}} = 0$$

we have

$$\mathbf{X} = 0 \tag{11}$$

Hence, proposed orthogonal ZCPS design can eliminate pilot contamination from TDD massive MIMO systems. Next section will show that the proposed orthogonal ZCPS design can eliminate pilot contamination during channel estimation process.

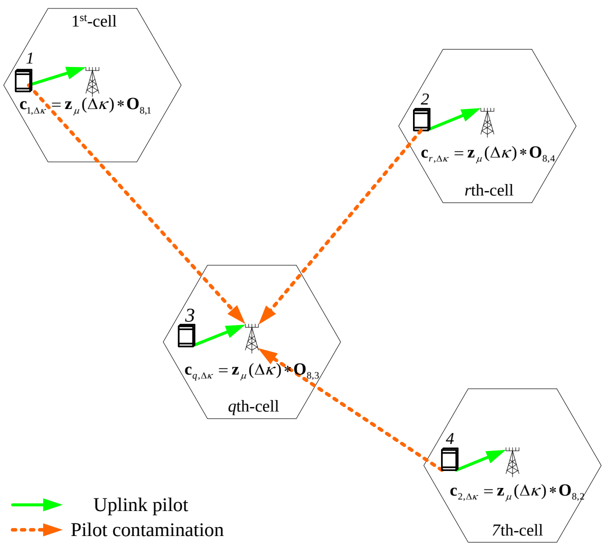

Fig. 3. Orthogonal ZCPS.

D. Eliminating Pilot Contamination During CE Process

Consider the scenario of the seven-cell cluster; refer to Fig. 3, where each cell has one user that transmits its uplink training sequence. Then, the uplink training signal received at the q-th BS can be given as

$$\mathbf{y}_{qa} = \sum_{l=1}^{L}\sum_{u=1}^{U}\sqrt{\rho_{up}\beta_{lqu}}\,h_{lqua}\mathbf{c}_{q} + \mathbf{v}_{qa} \tag{12}$$

After receiving the uplink training signal $\mathbf{y}_{qa}$, the q-th BS will estimate the channel h_{qq3a} with the MMSE estimator.

The MMSE estimate of the channel h_{qq3a} is

$$\hat{h}_{qq3a}^{\mathrm{MMSE}} = \sqrt{\rho_{up}}\,\beta_{qq3}\left(\mathbf{I} + \rho_{up}\left(\sum_{l=1}^{L}\sum_{u=1}^{U}\mathbf{c}_{q}\beta_{lqu}\mathbf{c}_{q}^{H}\right)\right)^{-1}\mathbf{y}_{q}\mathbf{c}_{q}^{\mathrm{H}} \tag{13}$$

$$\hat{h}_{qq3a}^{\mathrm{MMSE}} = \left[\begin{array}{l} 2^{m}\rho_{up}\beta_{qq3}\left(\mathbf{I} + 2^{m}\rho_{up}\left(\sum_{l=1}^{L}\sum_{u=1}^{U}\beta_{lqu}\right)\right)^{-1} \\ \times\left(\sqrt{\beta_{qq3}}\,h_{qq3a} + \dfrac{1}{2^{m}\sqrt{\rho_{up}}}\mathbf{v}_{qa}\mathbf{c}_{q}^{\mathrm{H}}\right)\end{array}\right] \tag{14}$$

Using matrix inversion lemma $(I+A)^{-1} = I - (I+A)^{-1}A$, (14) can be further simplified as

$$\hat{h}_{qq3l}^{\mathrm{MMSE}} = \left[\begin{array}{l} \left(\dfrac{2^{x}\rho_{up}\beta_{qq3}}{1 + 2^{x}\rho_{up}\sum_{l=1}^{L}\sum_{u=1}^{U}\beta_{lqu}}\right) \\ \times\left(\sqrt{\beta_{qq3}}\,h_{qq3l} + \dfrac{1}{2^{x}\sqrt{\rho_{up}}}\mathbf{v}_{ql}\mathbf{c}_{q}^{\mathrm{H}}\right)\end{array}\right] \tag{15}$$

Dropping user and antenna subscripts, then (15) can be rewritten as

$$\hat{h}_{qq}^{\mathrm{MMSE}} = \left(\frac{2^{m}\rho_{up}\beta_{qq}}{\eta_{q}}\right)\sqrt{\beta_{qq}}\,h_{qq} + \frac{1}{2^{x}\sqrt{\rho_{up}}}\mathbf{v}_{q}\mathbf{c}_{q}^{\mathrm{H}} \tag{16}$$

where $\eta_{q} = 1 + 2^{m}\rho_{up}\sum_{l=1}^{L}\beta_{lq}$.

From (16), it is obvious that the MMSE estimation of the channel h_{qq3a}, after implementing the proposed scheme, are clean from the pilot contamination.

E. Downlink Transmission

After estimating the uplink CE using the proposed scheme, the BSs can acquire the downlink CE by exploiting the channel reciprocity of the TDD protocol. Consider that the information symbols transmitted by the BS of the q-th cell to its users are $\mathbf{b}_{q} = [b_{q1}\ b_{q2}\ \ldots\ b_{qU}]^{\mathrm{T}}$ and the $A \times U$ linear precoding matrix is $\mathbf{E}_{q} = \Re(\hat{\mathbf{H}}_{qq})$, where denotes a particular linear precoding method performed at the BS and $\hat{\mathbf{H}}_{qq}$ are the MMSE CE. Then, $\mathbf{E}_{q}\mathbf{b}_{q}$ is the transmission precoding (TP) vector transmitted by the q-th BS. Furthermore, consider that the information symbols $\mathbf{b}_{q}$ and precoding method $\Re(.)$ satisfy $\mathbb{E}\left[\mathbf{b}_{q}\right] = 0$, $\mathbb{E}\left[\mathbf{b}_{q}\mathbf{b}_{q}^{\mathrm{H}}\right] = \mathbf{I}$ and $\mathrm{tr}\left\{\mathbf{E}_{q}^{\mathrm{H}}\mathbf{E}_{q}\right\} = 1$, which imply that the average power constraint at the BS is satisfied [2], [6].

The signal vector received by the users of the q-th cell is

$$\mathbf{F}_{q} = \sqrt{\rho_{dl}}\,\mathbf{D}_{qq}\mathbf{H}_{qq}\mathbf{E}_{q}\mathbf{b}_{q} + \mathbf{W}_{q},\ (U \times 1 \text{ vector}) \tag{17}$$

where $\mathbf{W}_{q}$ is the i.i.d. AWGN with zero mean and unit variance. Therefore, the signal received by the u-th user can be given as

$$F_{qu} = \sum_{k=1}^{U} \sqrt{\rho_{dl}\beta_{qqu}} \left[h_{qqu1} \quad h_{qqu2} \quad \ldots \quad h_{qquA} \right] \mathbf{e}_{qk} b_{qk} + w_{qk} \tag{18}$$

where $\mathbf{e}_{qk}$ is the k-th column of the precoding matrix $\mathbf{E}_q$ and w_{qk} is the k-th element of $\mathbf{W}_q$.

F. Achievable Throughput Rates

In order to show the effectiveness of the proposed scheme and the advantages of eradicating the pilot contamination, the lower bound of the achievable downlink throughput rate is derived using matched filter (MF) precoding given by [9]

$$\mathbf{E}_q = \frac{\hat{\mathbf{H}}_{qq}^{\mathrm{H}}}{\left\| \hat{\mathbf{H}}_{qq} \right\|} \tag{19}$$

Let $g_{qk} = \sqrt{\rho_{dl}\beta_{qqu}} \left[h_{qqu1} \quad h_{qqu2} \quad \ldots \quad h_{qquA} \right] \mathbf{e}_{qk}$, then (18) can be rewritten as

$$\begin{aligned} F_{qu} &= \sum_{k=1}^{U} g_{qk} b_{qk} + w_{qk} \\ &= \mathbb{E}\left[g_{qk} \right] b_{qk} + \left(g_{qk} - \mathbb{E}\left[g_{qk} \right] \right) b_{qk} + w_{qk} \end{aligned} \tag{20}$$

In (20), the effective noise is defined as

$$w'_{qk} = \left(g_{qk} - \mathbb{E}\left[g_{qk} \right] \right) b_{qk} + w_{qk} \tag{21}$$

Now, (20) can be written in the familiar form

$$F_{qu} = \mathbb{E}\left[g_{qk} \right] b_{qk} + w'_{qk} \tag{22}$$

where b_{qk} , F_{qs} , $\mathbb{E}\left[g_{qk} \right]$ and w'_{qk} are the input, output, known channel and additive noise, respectively. Now the achievable downlink throughput rate for (22) is [6], [9]

$$R_{qk} = \log_2 \left(1 + \frac{\left| \mathbb{E}\left[g_{qk} \right] \right|^2}{1 + \mathrm{var}\left\{ g_{qk} \right\}} \right) \tag{23}$$

IV. SIMULATION AND RESULT DISCUSSIONS

To verify the effectiveness of the proposed scheme, some simulation results are presented. These simulations are based on a multi-cell TDD M-MIMO system and simulation parameters are listed in Table I. The AOAs θ_{lquat} of all paths are the i.i.d Gaussian random variables with mean $\bar{\theta}_{AOA} = 90^{\circ}$ and standard deviation $\sigma_{AOA} = 90^{\circ}$. The achievable downlink throughput rate of the proposed scheme, given in (23), is evaluated by (16) of MMSE CE. Pilot-assisted CE represents the sophisticated combination of downlink training and scheduled uplink training to eradicate the pilot contamination proposed in [6]. The MMSE CE with aligned pilots, proposed in [11], represents the estimator that depends on all the MSs of all the cells simultaneously transmitting their uplink pilot sequences those occupy τ OFDM symbol lengths for its uplink training. The MMSE CE with the staggered pilot sequences represents the estimator, in which the MSs roaming in different cells transmit their uplink pilot sequences at non-overlapping instances proposed in [11]. The sum-rate performances of staggered pilots of [11] are evaluated using appendix given in [6]. The achievable downlink sum-rate performance versus different network parameters is evaluated for the proposed scheme and compared with those of [6] and [11].

TABLE I: VALUES OF PARAMETERS FOR SIMULATION

Parameter	Symbol	Value
Number of cells	L	7
Number of users per cell	U	4
Number of antennas at each BS	A	50
Average uplink transmit power	ρ_{up}	0dB
Average downlink transmit power	ρ_{dl}	10dB
Direct gain	β_{qqu}	1
Cross gain	β_{lqu} where $l \neq q$	0.3
Pilot length	$\tau = 2^m$	8
Frequency re-use factor	α	1
Mean of AOAs	$\bar{\theta}_{AOA}$	90°
Standard deviation of AOAs	σ_{AOA}	90°
Antenna spacing	$\mathfrak{D}$	$\frac{\lambda}{2}$
Number of paths	t	50

The achievable downlink sum-rate versus cross gain performance of different schemes is illustrated in Fig. 4. When the cross gain increases, the sum-rate of all the schemes decrease. We can observe from Fig. 4 that the sum-rate performances of the pilot assisted CE and MMSE with aligned pilots fall drastically from 23 bps/Hz to 2.8 bps/Hz and 18.1 bps/Hz to 0.6 bps/Hz, respectively, as the cross gain increases. Whereas the sum-rate of the proposed scheme with MMSE CE estimator drops from 31 bps/Hz to 22 bps/Hz, which is a drop of only 9 bps/Hz compared with the falls of 20.2 bps/Hz and 17.5 bps/Hz of pilot assisted CE and MMSE with aligned pilots, respectively. Therefore, the performance of the proposed estimators is far better than those of [6] and [11], which authenticates the effectiveness and superiority of the proposed MPSOACO scheme.

The achievable downlink sum-rate versus a number of cells performance of different schemes are illustrated in Fig. 5, where the values of the cross gain coefficients used between the q-th cell and its adjacent cells are

$\beta_{lqu} = 0.3$ and $\beta_{lqu} = 0.2$ for $L \leq 8$ and $L \geq 9$, respectively. As the number of cells increases, the sum-rate values of all the schemes decrease. It is observed from Fig. 5 that the sum-rate values of the pilot assisted CE and MMSE CE estimator with aligned pilots decrease drastically from 21 bps/Hz to 5 bps/Hz and 18.3 bps/Hz to 2.6 bps/Hz, respectively, as the number of cells increases. Whereas the sum-rate of the proposed scheme with MMSE CE drops from 32.5 bps/Hz to 22 bps/Hz, which is a drop of only 10.5 bps/Hz compared with the decreases in the sum-rate of 20.2 bps/Hz and 17.5 bps/Hz of pilot assisted CE and MMSE with aligned pilots, respectively. Therefore, the performance of the proposed estimators is far better than those of [6] and [11].

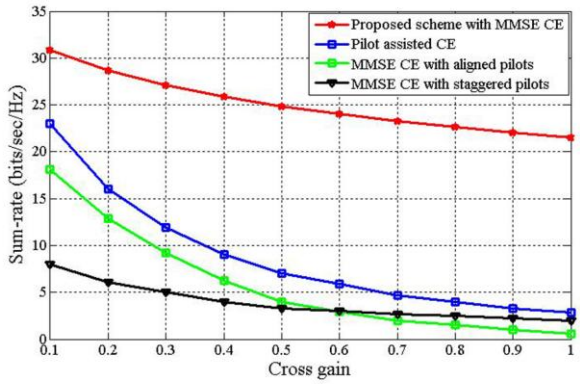

Fig. 4. Performance comparison of the proposed MPSOACO scheme with the schemes of [6] and [11] for the sum-rate versus the cross gain.

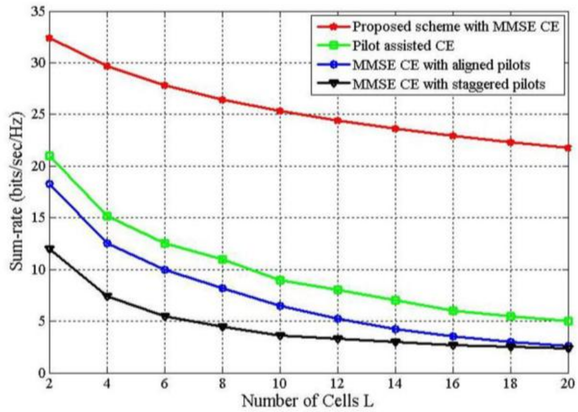

Fig. 5. Performance comparison of the proposed MPSOACO scheme with the schemes of [6] and [11] for the sum-rate versus the number of cells.

The achievable downlink sum-rate versus a number of antennas performances of different schemes are illustrated in Fig. 6. When the number of BS antennas increases, the sum-rate of all the schemes increases. It is observed from Fig. 6 that the sum-rate values of pilot assisted CE and MMSE CE with aligned pilots increase only by 7.7 bps/Hz and 6.5 bps/Hz, respectively when the number of antennas increases from 5 to 30. Whereas the sum-rate value of the proposed scheme with MMSE CE estimator increases by 10 bps/Hz, which is slightly higher than those of pilot assisted CE and MMSE CE with aligned pilots, respectively, as the number of antennas increases from 5 to 30. This shows that the performance of the proposed estimator is better than those of [6] and [11].

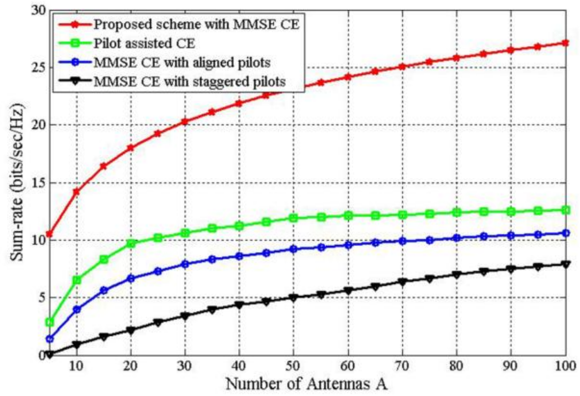

Fig. 6. Performance comparison of the proposed MPSOACO scheme with the schemes of [6] and [11] for the sum-rate versus the number of antennas.

The achievable downlink sum-rate versus standard deviation of AOAs performance of different schemes are illustrated in Fig. 7. We can see from Fig. 7 that the sum-rate performance of all the schemes is slightly changed by the standard deviation of AOAs while the sum-rate values of the proposed scheme are again far better than those of [6] and [11].

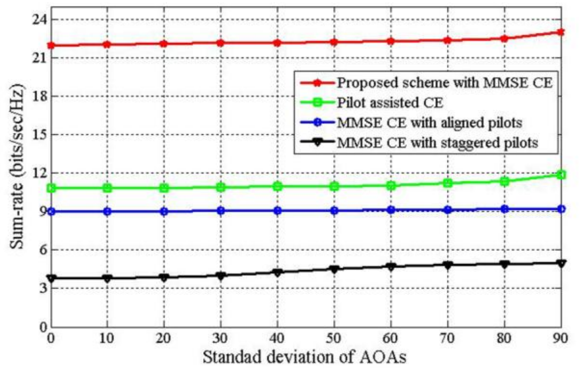

Fig. 7. Performance comparison of the proposed MPSOACO scheme with the schemes of [6] and [11] for the sum-rate versus standard deviation of AOAs.

V. CONCLUSIONS

A design of orthogonal uplink pilot sequences is proposed to eliminate pilot contamination from TDD massive MIMO systems. The proposed design uses Zadoff-Chu pilot sequences (ZCPS) and eliminates pilot contamination during channel estimation process. In the proposed design, each BS is assigned with a specific orthogonal code and a set of ZCPS is multiplied element-wise at each BS with BS-specific orthogonal code to generate orthogonality among pilot sequences across the neighboring cells. The proposed design uses conventional simultaneous uplink pilot training compared to the training overhead of $(L+3)\tau$ imposed by the pilot assisted scheme of [6]. Furthermore, the proposed design does not require any prior knowledge regarding either the MIMO channels or MS information. The MF precoding is employed for downlink transmission, which is a linear precoding and simple to implement compared to zero-forcing precoding. Simulation results show that the sum-rate performance of the proposed design significantly outperforms both the pilot assisted CE and MMSE CE with aligned and staggered pilots.

REFERENCES

[1] O. Elijah, C. Y. Leow, T. A. Rahman, S. Nunoo, and S. Z. Iliya, "A comprehensive survey of pilot contamination in massive MIMO—5G system," *IEEE Commun. Surveys & Tutorials*, vol. 18, no. 2, pp. 905-923, Second Quarter 2016.

[2] A. Khansefid and H. Minn, "Achievable downlink rates of MRC and ZF precoders in massive MIMO with uplink and downlink pilot contamination," *IEEE Trans. on Commun.*, vol. 63, no. 12, pp. 4849-4864, Dec. 2015.

[3] X. Zhu, Z. Wang, L. Dai, and C. Qian, "Smart pilot assignment for massive MIMO," *IEEE Commun. Letters,* vol. 19, pp. 1644-1647, 2015.

[4] L. Lu, *et al*., "An overview of massive MIMO: Benefits and challenges," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 742-758, Oct. 2014.

[5] S. Jin, X. Wang, Z. Li, K. K. Wong, Y. Huang, and X. Tang, "On massive MIMO zero-forcing transceiver using time-shifted pilots," *IEEE Trans. on Vehicular Technology*, vol. 65, pp. 59-74, 2016.

[6] J. Zhang, *et al*., "Pilot contamination elimination for large-scale multiple-antenna aided OFDM systems," *IEEE J. Sel. Topics on Signal Process.*, vol. 8, no. 5, pp. 759-772, Oct. 2014.

[7] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun*. *Mag*., vol. 52, no. 2, pp. 186-195, Feb. 2014.

[8] T. X. Vu, T. A. Vu, and T. Q. S. Quek, "Successive pilot contamination elimination in multiantenna multicell networks," *IEEE Wireless Commun. Letters*, vol. 3, no. 6, pp. 617-620, Dec. 2014.

[9] J. Jose, A. Ashikhmin, T. L. Marzetta, and S. Vishwanath, "pilot contamination and precoding in multi-cell TDD systems," *IEEE Trans*. *Wireless Commun*., vol. 10, no. 8, pp. 2640-2651, Aug. 2011.

[10] T. L. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," *IEEE Trans. Wireless Commun*., vol. 9 no. 11, pp. 3590-3600, Nov. 2010.

[11] F. Fernandes, A. Ashikhmin, and T. L. Marzetta, "Inter-cell interference in noncooperative TDD large scale antenna systems," *IEEE J. Sel. Areas Commun*., vol. 31, no. 2, pp. 192–201, Feb. 2013.

[12] L. You, X. Gao, X. G. Xia, N. Ma, and Y. Peng, "Pilot reuse for massive MIMO transmission over spatially correlated rayleigh fading channels," *IEEE Trans. on Wireless Commun.*, vol. 14, pp. 3352-3366, 2015.

[13] A. Hu, T. Lv, H. Gao, Y. Lu, and E. Liu, "Pilot design for large-scale multi-cell multiuser MIMO systems," in *Proc. IEEE International Conference on Communications*, Budapest, 2013, pp. 5381-5385.

[14] H. Q. Ngo, A. Ashikhmin, H. Yang, E. G. Larsson, and T. L. Marzetta, "Cell-free massive MIMO: Uniformly great service for everyone," in *Proc. 16th IEEE International Workshop on Signal Processing Advances in Wireless Communications*, Stockholm, 2015, pp. 201-205.

[15] M. Hua, M. Wang, K. W. Yang, and K. J. Zou, "Analysis of the frequency offset effect on Zadoff-Chu sequence timing performance," *IEEE Trans*. on *Commun*., vol. 62, no. 11, pp. 4024–4039, Nov. 2014.

[16] J. W. Kang, Y. Whang, H. Y. Lee, and K. S. Kim, "Optimal pilot sequence design for multi-cell MIMO-OFDM systems," *IEEE Trans. on Wireless Commun.*, vol. 10, no. 10, pp. 3354-3367, Oct. 2011.

[17] J. C. Shen, J. Zhang, and K. B. Letaief, "Downlink user capacity of massive MIMO under pilot contamination," *IEEE Trans. on Wireless Commun.*, vol. 14, no. 6, pp. 3183-3193, June 2015.

[18] N. Akbar, N. Yang, P. Sadeghi, and R. A. Kennedy, "Multi-cell multiuser massive MIMO networks: User capacity analysis and pilot design," *IEEE Trans. on Commun.*, vol. 64, no. 12, pp. 5064-5077, Dec. 2016.

[19] N. Shariati, E. Björnson, M. Bengtsson, and M. Debbah, "Low-complexity polynomial channel estimation in large-scale MIMO with arbitrary statistics," *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 815-830, Oct. 2014.

[20] H. Yin, D. Gesbert, M. Filippou, and Y. Liu, "A coordinated approach to channel estimation in large-scale multiple-antenna systems," *IEEE J. Sel. Areas Commun*., vol. 31, no. 2, pp. 264-273, Feb. 2013.

[21] Y. Tseng and C. Chao, "Code placement and replacement strategies for wideband CDMA OVSF code tree management," *IEEE Trans. on Mobile Comput*., vol. 1, no. 4, pp. 293-302, Oct.-Dec. 2002.

[22] A. Iossifides and S. Louvros, "A new aspect of Walsh-Hadamard coding over rayleigh fading channels," *IEEE Latin America Trans.*, vol. 10, no. 3, pp. 1680-1685, April 2012.

[23] B. M. Popovic, "Generalized chirp-like polyphase sequences with optimum correlation properties," *IEEE Trans. Inf. Theory*, vol. 38, no. 4, pp. 1406-1409, July 1992.

Sajjad Ali received his B.E. degree in Telecommunication Engineering and the M.E. degree in Communication Systems Networks from Mehran University of Engineering & Technology (MUET), Jamshoro, Pakistan, in 2007 and 2011, respectively. He joined Telenor Pakistan as O&M engineer in 2007. He then joined the Department of Telecommunication Engineering, MUET, as Lab-Lecturer in 2008, and became Assistant Professor in 2011. He is currently doing his Ph.D. in Dalian University of Technology (DUT), Dalian, China. His research interests are in the field of digital signal processing and broadband wireless communications.

Zhe Chen received his B.S. degree in Electronic Engineering, the M.S. degree in Signal and Information Processing, and the Ph.D. degree in Signal and Information Processing from Dalian University of Technology (DUT), Dalian, China, in 1996, 1999 and 2003, respectively. He joined the Department of Electronic Engineering, DUT, as a Lecture in 2002, and became an Associate Professor in 2006. His research interests include digital signal processing, speech processing, image processing, and broadband wireless communication.

Fuliang Yin received his B.S. degree in Electronic Engineering and the M.S. degree in Communications and Electronic Systems from Dalian University of Technology (DUT), Dalian, China, in 1984 and 1987, respectively. He joined the Department of Electronic Engineering, DUT, as a Lecturer in 1987 and became an Associate Professor in1991. He has been a Professor at DUT since 1994, and the Dean of the School of Electronic and Information Engineering of DUT from 2000 to 2009. His research interests include digital signal processing, speech processing, image processing, and broadband wireless communication.