Robust Power Allocation for OFDM-Based Cognitive Radio Networks under Signal-to-Interference-plus-Noise-Ratio Constraints

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Abstract—Traditional power allocation schemes in Orthogonal Frequency Division Multiplexing (OFDM) based Cognitive Radio Networks (CRNs) are achieved under perfect channel state information (i.e., exact parameter information). Due to channel estimation errors and feedback delays, however, channel uncertainties are inevitable in practical CRNs. In this paper, considering bounded channel uncertainties, a robust power allocation algorithm is proposed to minimize the total transmit power of Secondary Users (SUs) subject to the interference temperature constraint of primary user and the received Signal-to-Interference-plus-Noise Ratio (SINR) constraint of SU where the non-convex optimization problem is converted into a convex optimization problem that is solved by dual decomposition theory. Numerical results demonstrate the effectiveness of the proposed algorithm by comparing with the non-robust algorithm in the aspect of suppressing the effect of parameter uncertainties.

Index Terms—Cognitive radio, robust power allocation, parametric uncertainties, robustness

I. Introduction

With the rapid development of communication technique, more and more wireless communication devices need to access wireless transmission networks, which makes spectrum resource become more and more scarce. Cognitive Radio (CR) appeared in the end of last century is a good solution to deal with this problem by opportunistic spectrum access [1]-[4]. In CR Networks (CRNs), secondary users (also called cognitive users, unlicensed users) can use the licensed frequency bands owned by primary users (licensed users) as long as the interference power generated by Secondary Users (SUs) below the interference threshold which can well protect the Quality of Service (QoS) of Primary Users (PUs). Therefore, power allocation strategy of SUs becomes a key technology in CRNs for achieving spectrum resource sharing.

In recent years, Orthogonal Frequency Division Multiplexing (OFDM) has been considered as an effective transmission technique for CRNs due to its good spectrum efficiency, anti-multipath fading and flexibility in resource allocation (i.e., power allocation). In an OFDM-based CRN, the licensed frequency band is always divided into multiple orthogonal subcarriers which are used by both SUs and PUs. Based on different spectrum sharing frameworks (e.g., underlay, overlay) [5], SUs can optimize their power allocation strategies for achieving some objectives and simultaneously reducing the interference power to PUs. To strictly protect the performance of PUs, we consider the underlay spectrum sharing way in this paper.

Currently, power allocation problems for OFDM-based CRNs have been studied from different optimization objectives and mathematical tools. In [6], a subcarrier power allocation scheme via water-filling formula was proposed to maximize the sum capacity of SU subject to the constraints of total power, each subcarrier power and the aggregate interference power of PU. The authors address the single-user scenario. In [7], for an OFDM CRN with single-PU-multi-SU scenario, a hierarchic power allocation algorithm based on Stackelberg game theory was designed subject to the total power constraint of SU and the interference-to-signal ratio (ISR) constraint. In [8], the authors studied the subcarrier allocation and power allocation problem for an OFDM-based CRN where an opportunistic power allocation method was proposed to achieve the capacity maximization of SU by using Lagrange method. In addition, the outage probability of PU was analyzed to keep it within a tolerable range. In [9], the authors jointly considered the power and subcarrier optimization problem for a multiuser OFDM-based CRN. A dynamic resource allocation algorithm was proposed under the QoS constraint of PU and the proportional constraint of SU. All above aforementioned works consider power allocation problem under the assumption of perfect channel information. For example, SUs can exactly obtain the channel gains between SUs and PUs, channel gain among SUs and interference channel gains from PUs. However, in practical CRNs, these parameters can not be perfectly obtained due to the stochastic nature of wireless

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channel and delays. The effectiveness of the designed algorithms (e.g., [6]-[9]) may be degraded when the channel uncertainties and interference uncertainties are not taken into account.

To overcome the effect of parameter uncertainties and improve the robustness of algorithm, many scholars have focused on robust power allocation (i.e., resource allocation) problems for OFDM-based CRNs from the worst-case approach (i.e., bounded uncertainty) and stochastic optimization approach (i.e., probabilistic constraints or statistical model) [5]. In [10], considering an OFDM-based cellular CRN with uplink transmission scenario, the authors aimed to maximize the social utility of SUs while keeping the interference from SUs to each primary Base Station (BS) below a given threshold and meeting the QoS requirement of each SU-receiver (SU-Rx), and formulate the channel uncertainties as ellipsoid models. And the original NP-hard problem was converted into a geometric programming problem solved by using Lagrange dual decomposition in a distributed way. In [11], based on the concept of fractional programming, a robust power allocation algorithm with energy efficiency maximization (i.e., maximize throughput-to-power ratio) was proposed under the rate requirement constraint of SU and interference power constraint of PU where channel uncertainties among SUs are considered. Similarly, in [12], considering channel uncertainties between SUs and PUs, the authors also studied the energy efficiency maximization problem in multiuser OFDM-based CRNs. The probabilistic interference constraint was tackled by Bernstein approximation approach. In [13], for a downlink OFDM-based CRN with a secondary BS communicating with multiple cognitive mobile stations, a weighted sum rate maximization problem was studied where channel uncertainties from SUs to PUs are modeled by bounded uncertainty sets. To achieve distributed power allocation in OFDM-based CRNs, the authors in [14] studied the robust throughput maximization problem according to non-cooperative game theory where the ellipsoid approximation was used to formulate the imperfect channel state information (CSI). In [15], our previous work studied the robust rate maximization problem in OFDM-based CRNs suject to the probabilistic interference constraint and maximum transmit power constraint of SU. The problem was converted into a deterministic one by the assumption of exponential distribution of channel estimation errors. However, interference power uncertainty from PU transmitter to SU receiver and multiuser scenario are not considered.

Despite many literatures have done a lot of work about the robust power allocation problem for OFDM-based CRNs (e.g., capacity maximization/energy efficiency maximization), those studies do not consider the energy consumption problem and simultaneously ignore the channel uncertainty from PU transmitter to SU receiver. With the development of wireless communication, more and more devices will access into the communication system in a wireless way, therefore, green communication and energy consumption reduction become very important for prolonging the life of mobile terminal and reducing environmental pollution. Robust power allocation with the consideration of channel uncertainties from PUs to SUs also has been ignored. But the interference power from PU transmitters to SU receivers is affected by the channel uncertainties from PUs to SUs, which influences the received SINR at the SU receiver. For example, this uncertainty may cause the actual received SINR of SU below the target value.

In this paper, a robust power allocation algorithm for multiuser OFDM-based CRNs is designed to minimize the total transmit power of SUs under the transmit power limit of each SU, the SINR constraint of SU and interference power constraint of PU where all channel uncertainties are modeled by the bounded sets. According to the worst-case approach, the original infinite dimensional robust optimization problem is transformed into a deterministic and convex optimization problem which is solved by using Lagrange dual method. Several numerical results demonstrate the effectiveness of the proposed algorithm from the aspect of SU's SINR performance and the received interference power at the PU receiver. The contribution of the paper is summarized as:

- We consider multiuser resource allocation problem in OFDM-based CRNs subject to parametric uncertainties.
- Not only the robust interference constraint is considered in resource optimization problem, but also the robust SINR constraint is involved. Therefore, the QoS of all users (i.e., both of SUs and PUs) can be ensured under stochastic channel environment.
- The NP-hard problem is converted into a deterministic and solvable optimization problem solved by using Lagrange dual method. Simulation results show that the proposed algorithm can well protect the QoS of PU.

The rest of this paper is organized as follows. In Section II, traditional optimal power allocation problem in underlay OFDM-based CRNs is presented. In Section III, we introduce the bounded channel uncertainties into the original resource allocation problem, and give the transformation process and robust power allocation algorithm. The numerical results and performance analysis are given in Section IV. Finally, we conclude the paper in Section V.

II. Problem Formulation

Assume that there are K pairs of SUs and M pairs of PUs sharing with the spectrum resource by an underlay spectrum mode, as shown in Fig. 1. The licensed bands B are divided into N orthogonal sub-carriers. Both SUs and PUs are assumed to use the frequency band by exploiting OFDM framework. All the SUs can access and use the frequency band under the constraint that the total

interference power generated by SUs does not exceed the acceptable level [2], [3].

$$\sum_{k=1}^{K}\sum_{n=1}^{N} p_k^n g_{k,m}^n \le I_m, m=1,\cdots,M \tag{1}$$

where p_k^n denotes the transmit power of the kth SU-Tx over the subcarrier n. $g_{k,m}^n$ denotes the channel gain from the kth SU-Tx to the mth PU-Rx over the subcarrier n. I_m represents the maximum interference power that the mth PU-Rx can tolerant.

Consider the effect of PU's interference power and background noise, the SINR at the kth SU-Rx is given as

$$\gamma_k^n = \frac{p_k^n h_k^n}{\sum_{m=1}^{M} P_m^n G_{m,k}^n + n_k^n} \tag{2}$$

where h_k^n represents the direct channel gain of the kth SU over the subcarrier n. n_k^n is assumed to be the additive white Gaussian noise over subcarrier n. $G_{m,k}^n$ denotes the channel gain from the mth PU-Tx to the kth SU-Rx over the subcarrier n. In addition, P_m^n denotes the transmit power of the mth PU over the subcarrier n. Obviously, the first term of denominator in (2) denotes the sum interference power from PUs which can affect the value of actual SINR (i.e., γ_k^n).

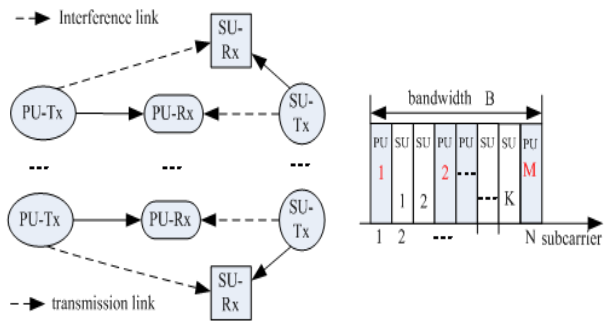


Fig. 1. A multiuser OFDM-based cognitive system. (PU-Tx: PU transmitter, PU-Rx: PU receiver, SU-Tx: SU transmitter, SU-Rx: SU receiver.)

In order to satisfy the SINR requirement of each SU, the actual received SINR of SU should be bigger than it's minimum QoS requirement. Therefore, we have the following constraint.

$$\gamma_k^n \ge D_k^n \tag{3}$$

where D_k^n denotes the minimum SINR requirement of SU k over subcarrier n. When the actual SINR γ_k^n is below the target value D_k^n, the normal communication of SU will be interrupted. As a result, this constraint is very important for resource allocation in CRNs.

Due to the limitation of battery capacity of mobile device, the transmit power of each SU should be lower than it's maximum transmit power. Thus, we have

$$\sum_{n=1}^{N} p_k^n \le p_k^{\max} \tag{4}$$

where $p_k^{\max}$ denotes the maximum transmit power for the kth SU-Tx.

In order to improve energy efficiency and prolong the operation time of mobile equipment, our optimization objective is to minimize the total transmit power of secondary network under the constraints on interference power of PU (1), SINR requirement of SU (3) and maximum transmit power of SU (4). Then the power allocation optimization problem with perfect CSI can be depicted as

$$\begin{aligned} &\min_{\forall p_k^n} \sum_{k=1}^{K}\sum_{n=1}^{N} p_k^n \\ &s.t.\ C1: \sum_{k=1}^{K}\sum_{n=1}^{N} p_k^n g_{k,m}^n \le I_m, m=1,\cdots,M \\ &\quad C2: \gamma_k^n \ge D_k^n, n=1,\cdots,N; k=1,\cdots,K \\ &\quad C3: \sum_{n=1}^{N} p_k^n \le p_k^{\max}, k=1,\cdots,K \end{aligned} \tag{5}$$

In the optimization problem (5), the constraint $C1$ denotes the interference temperature constraint for guaranteeing the normal communication of PU. The constraint $C2$ can keep the basic QoS of each SU. The constraint $C3$ denotes the maximum transmit power limitation due to the physical condition. Most of the existing works have concerned the throughput maximization problem under these three constraints under good knowledge of channel gains or interference power from PUs. Robust power allocation with total energy consumption minimization and interference uncertainty is never considered. For the non-robust optimization problem (i.e., nominal optimization problem (5)), obviously, it is a non-convex problem due to the constraint $C2$.

III. Robust Power Allocation Algorithm

To keep stable transmission performance in CRNs, the robustness of the power allocation algorithm should be taken into account ahead of time. In this section, we give the uncertainty formulation, robust power allocation problem and analytical solution of the proposed algorithm.

A. Uncertainty Formulation

With the consideration of estimation error, real channel gain is combined channel estimation value with the corresponding estimation error, i.e., $g_{k,m}^n = \hat{g}_{k,m}^n + \Delta g_{k,m}^n$. Since there is no cooperation between SUs and PUs, SUs is difficult to exactly obtain the channel gains from SU-Txs to PU-Rxs. As a result, we must consider this uncertainty in interference temperature constraint. To overcome the effect from all link uncertainties to the PU-Rx, channel uncertainty between SU-Tx and PU-Rx is formulated as the following ellipsoidal uncertainty set.

$$X=\left\{g_{k,m}^{n} \mid \hat{g}_{k,m}^{n}+\Delta g_{k,m}^{n}:\sum_{k=1}^{K}\left|\Delta g_{k,m}^{n}\right|^{2}\le(\delta_{m}^{n})^{2}\right\} \quad (6)$$

where X denotes the uncertainty set. $|\cdot|$ is the absolute value operation. δ_m^n is the upper deviation of uncertainty at the mth PU over subcarrier n. $\hat{g}_{k,m}^n$ and $\Delta g_{k,m}^n$ represents the channel estimation value and corresponding estimation error. In the nominal optimization problem, the estimated channel gain $\hat{g}_{k,m}^n$ is considered as the equivalent true physical channel $g_{k,m}^n$ which is impractical due to inevitable estimation errors $\Delta g_{k,m}^n$. Thus, the performance of non-robust algorithm is degraded in practice.

To guarantee the basic SINR requirement of each SU, the parameter uncertainties in SU's SINR constraint also need to be considered. Similarly, the uncertainty formulation of channel gain from PU-Tx to SU-Rx is given as

$$H=\left\{G_{m,k}^{n} \mid \hat{G}_{m,k}^{n}+\Delta G_{m,k}^{n}:\sum_{m=1}^{M}\left|\Delta G_{m,k}^{n}\right|^{2}\le(\omega_{k}^{n})^{2}\right\} \quad (7)$$

where ω_k^n denotes the maximum tolerant perturbation threshold which also determines the size of interference uncertainty from PUs. The perturbation channel gain (i.e., (7)) can determine the term of PU's interference, e.g., $\sum_{m=1}^{M} P_m^n G_{m,k}^n$. Therefore, we require to consider this interference uncertainty in SU's SINR.

Additionally, due to the effect of feedback delays and estimation errors, the direct channel gain of SU may be also uncertain. Thus, we have.

$$E=\left\{h_{k}^{n} \mid \hat{h}_{k}^{n}+\Delta h_{k}^{n}:\left|\Delta h_{k}^{n}\right|\le\varepsilon_{k}^{n}\right\} \quad (8)$$

where ε_k^n is the maximum upper boundary of channel uncertainty Δh_k^n.

B. Robust Power Allocation Problem

Combining with (6), (7) and (8), the transmission power optimization problem (5) (i.e., non-robust problem) can be formulated as the following robust optimization problem

$$\begin{aligned}
&\min_{\forall p_k^n}\sum_{k=1}^{K}\sum_{n=1}^{N}p_k^n\\
&s.t.\ C1:\sum_{k=1}^{K}\sum_{n=1}^{N}p_k^n g_{k,m}^n\le I_m, m=1,\cdots,M\\
&\quad C2:\gamma_k^n\ge D_k^n, n=1,\cdots,N; k=1,\cdots,K\\
&\quad C3:\sum_{n=1}^{N}p_k^n\le p_k^{\max}, k=1,\cdots,K\\
&\quad C4: g_{k,m}^n\in X, G_{m,k}^n\in H, h_k^n\in E
\end{aligned} \quad (9)$$

It is clear that problem (9) is a semi-infinite programming problem due to the infinite number of constraints (e.g., uncertain constraint). In order to resolve the problem (9), we need to transform it into a convex optimization problem with deterministic and finite number of constraints.

Robust optimization [16] is a well-known theory to deal with the robust constraints (i.e., the constraint with uncertainty). In other words, by using the worst case approach, the constraints can be guaranteed under the worst-case uncertainty (i.e., maximum harmful estimation error). According to the Cauchy-Schwarz inequality, we have

$$\begin{aligned}
\max_{g_{k,m}^n\in X}\sum_{k=1}^{K}(g_{k,m}^n-\hat{g}_{k,m}^n)p_k^n &= \max_{g_{k,m}^n\in X}\sum_{k=1}^{K}\Delta g_{k,m}^n p_k^n\\
&\le\sqrt{\sum_{k=1}^{K}(\Delta g_{k,m}^n)^2\sum_{k=1}^{K}(p_k^n)^2}\\
&\le\delta_m^n\sum_{k=1}^{K}p_k^n
\end{aligned} \quad (10)$$

Therefore, robust interference constraint $C1$ becomes

$$\sum_{k=1}^{K}\sum_{n=1}^{N}p_k^n(\hat{g}_{k,m^*}^n+\delta_{m^*}^n)\le I_{m^*} \quad (11)$$

where $I_{m^*}=\min_{\forall m} I_m$ denotes the minimum interference level of PUs. As a result, multiple PUs' interference constraint becomes a single PU constraint. From (11), when the perturbation term $\delta_{m^*}^n$ is zero, it denotes that the estimated channel gain $\hat{g}_{k,m^*}^n$ is equal to the true channel $g_{k,m}^n$. Under this condition, the optimal transmit power $p_k^{n,non}$ is bigger than that of our method $p_k^{n,rob}$, since the feasible region with non-robust scheme is more slacker than that of our robust scheme. When there is channel estimation error in the system, the term $\Delta g_{k,m}^n$ must exist. Therefore, the interference recieved by PU-Tx under non-robust scheme may exceed the interference threshold I_m^*, i.e., $I_{actual}=\sum_{k=1}^{K}\sum_{n=1}^{N}p_k^{n,nom}(\hat{g}_{k,m^*}^n+\Delta g_{k,m^*}^n)\ge\sum_{k=1}^{K}\sum_{n=1}^{N}p_k^{n,nom}(\hat{g}_{k,m^*}^n)=I_{m^*}$.

Combing with (7) and (8), the robust SINR constraint $C2$ becomes

$$\begin{aligned}
\frac{\sum_{m=1}^{M}P_m^n G_{m,k}^n+n_k^n}{p_k^n h_k^n} &= \frac{\sum_{m=1}^{M}P_m^n(\hat{G}_{m,k}^n+\Delta G_{m,k}^n)+n_k^n}{p_k^n(\hat{h}_k^n+\Delta h_k^n)}\\
&\le\max_{G_{m,k}^n\in H, h_k^n\in E}\frac{\sum_{m=1}^{M}P_m^n(\hat{G}_{m,k}^n+\Delta G_{m,k}^n)+n_k^n}{p_k^n(\hat{h}_k^n+\Delta h_k^n)}\\
&=\frac{\sum_{m=1}^{M}P_m^n\hat{G}_{m,k}^n+n_k^n+\max_{\Delta G_{m,k}n}\sum_{m=1}^{M}P_m^n\Delta G_{m,k}^n}{p_k^n(\hat{h}_k^n-\varepsilon_k^n)}\\
&\le\frac{\sum_{m=1}^{M}P_m^n\hat{G}_{m,k}^n+n_k^n+\omega_k^n\sqrt{\sum_{m=1}^{M}(P_m^n)^2}}{p_k^n(\hat{h}_k^n-\varepsilon_k^n)}
\end{aligned} \quad (12)$$

As a result, the upper bound of SINR constraint can satisfy the following constraint

$$\frac{z_k^n}{p_k^n(\hat{h}_k^n-\varepsilon_k^n)} \le \frac{1}{D_k^n} \tag{13}$$

where $z_k^n = \left(\sum_{m=1}^{M} P_m^n \hat{G}_{m,k}^n + n_k^n\right) + \omega_k^n \sqrt{\sum_{m=1}^{M}(P_m^n)^2}$ is the equivalent interference plus noise. Similarly, we can easily obtain the results how to produce impact on SINR of SU under channel estimaiton errors.

Combining (11) and (13), the deterministic convex optimization problem with finite constraints is

$$\begin{aligned}
&\min_{\forall p_k^n} \sum_{k=1}^{K}\sum_{n=1}^{N} p_k^n \\
&s.t.\ C1: \sum_{k=1}^{K}\sum_{n=1}^{N} p_k^n(\hat{g}_{k,m}^n + \delta_{m^*}^n) \le I_{m^*} \\
&\quad C2: \frac{z_k^n}{p_k^n(\hat{h}_k^n-\varepsilon_k^n)} \le \frac{1}{D_k^n}, n=1,\cdots,N; k=1,\cdots,K \\
&\quad C3: \sum_{n=1}^{N} p_k^n \le p_k^{\max}, k=1,\cdots,K
\end{aligned} \tag{14}$$

It is obvious that the problem (14) is a deterministic and convex optimization problem with linear constraints $C1$ and $C3$. In addition, the convexity of $C2$ can be proved by the second order partial derivative with the variable p_k^n.

C. Design of the Algorithm

The power allocation problem (14) is a convex optimization problem which can be efficiently solved by Lagrange dual method. We define the following Lagrange function as

$$\begin{aligned}
L\left(\{p_k^n\},\upsilon,\{\lambda_k^n\},\{\alpha_k\}\right) &= \sum_{k=1}^{K}\sum_{n=1}^{N} p_k^n \\
&+\upsilon\left\{\sum_{k=1}^{K}\sum_{n=1}^{N} p_k^n\left(\hat{g}_{k,m^*}^n+\delta_{m^*}^n\right)-I_{m^*}\right\} \\
&+\sum_{k=1}^{K}\sum_{n=1}^{N}\lambda_k^n\left(\frac{z_k^n}{p_k^n(\hat{h}_k^n-\varepsilon_k^n)}-\frac{1}{D_k^n}\right) \\
&+\sum_{k=1}^{K}\alpha_k\left(\sum_{n=1}^{N} p_k^n - p_k^{\max}\right)
\end{aligned} \tag{15}$$

where υ, λ_k^n and α_k are the corresponding Lagrange multipliers of constraints. Define

$$L_{\alpha_k} = \sum_{n=1}^{N} p_k^n - p_k^{\max} \tag{16}$$

$$L_{\lambda_k^n} = \frac{z_k^n}{p_k^n(\hat{h}_k^n-\varepsilon_k^n)} - \frac{1}{D_k^n} \tag{17}$$

$$L_{\upsilon} = \sum_{k=1}^{K}\sum_{n=1}^{N} p_k^n(\hat{g}_{k,m^*}^n+\delta_{m^*}^n) - I_{m^*} \tag{18}$$

The Lagrange multipliers can be calculated by subgradient updating method [17]. Thus, we have

$$\alpha_k(t+1) = \max\left(0, \alpha_k(t) + b_1 L_{\alpha_k}(t)\right) \tag{19}$$

$$\lambda_k^n(t+1) = \max\left(0, \lambda_k^n(t) + b_2 L_{\lambda_k^n}(t)\right) \tag{20}$$

$$\upsilon(t+1) = \max\left(0, \upsilon(t) + b_3 L_{\upsilon}(t)\right) \tag{21}$$

where t denotes the iteration number. And $b1$, b_2 and b_3 are the small positive step sizes. The optimal transmit power can be calculated by $\frac{\partial L\left(p_k^n,\upsilon,\lambda_k^n,\alpha_k\right)}{\partial p_k^n}=0$. Therefore, we have

$$p_k^n = \sqrt{\frac{\lambda_k^n z_k^n}{\left(1+\alpha_k+\upsilon\left(\hat{g}_{k,m^*}^n+\delta_{m^*}^n\right)\right)\left(\hat{h}_k^n-\varepsilon_k^n\right)}} \tag{22}$$

The outline of our proposed robust power allocation algorithm is given as follows.

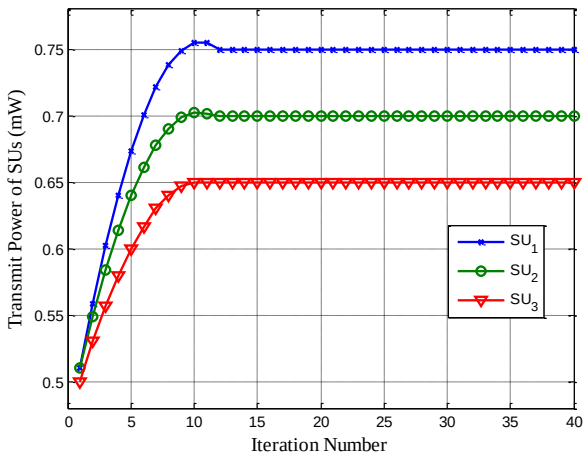
1) Initialization: Set $t=0$, $\lambda_k^n(0)>0$, $\alpha_k(0)>0$, $\upsilon(0)>0$, $m^*=\arg\min I_m$, $\delta_{m^*}^n>0$, $p_k^n(0)>0$.
2) Estimate direct channel gain h_k^n and measure actual received SINR at SU-Rx. Calculate Lagrange multiplier by (20) and feedback available $\hat{h}_k^n$ and the multiplier λ_k^n to the SU-Tx.
3) Receive channel gain $\hat{h}_k^n$ and Lagrange multiplier λ_k^n by control channel. Update Lagrange multipliers α_k and υ by (19) and (21). Update transmit power $p_k^n(t+1)=\min\{p_{new}(t+1), p_k^{\max}\}$, where p_{new} is calculated by (22).
4) Go to step 2).

IV. Numerical Results

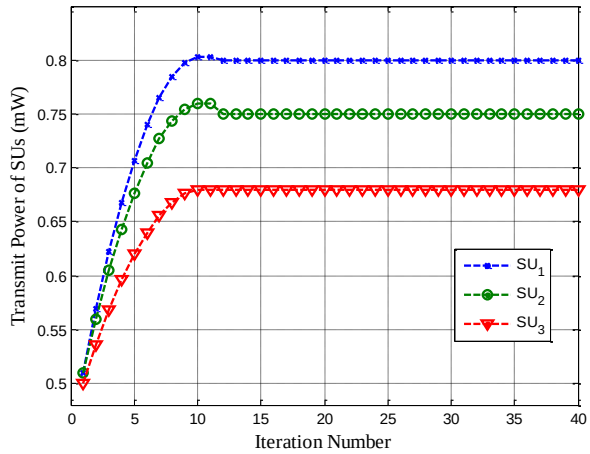
In this section, several computer experiments are given to support the theoretical results of previous sections. Assume that there are three active SUs, one PU and each user occupies one subcarrier for data transmission. The target SINR is defined as $D^{\min}=[2;2.5;2.5]$ dB. The maximum transmit power of each SU transmitter is $p_k^{\max}=1$ mW. The uncertain parameters are $\omega_k^n=5\%$ and $\varepsilon_k^n=10\%$ for $\forall m,k,n$. The interference threshold at PU receiver is 0.065 mW. The background noise n_k^n and interference gains $g_{k,m}^n$ are randomly chosen from the intervals (0,0.1/(N-1)) and (0,1/(N-1)) respectively [18].

Fig. 2 gives the convergence of the transmit power of SUs under the non-robust power allocation algorithm (i.e., $\Delta G_{m,k}^n=0, \Delta h_k^n=0, \Delta g_{k,m}^n=0$ for $\forall m,k,n$) and our proposed robust power allocation algorithm. Obviously, each user can obtain a good convergence performance

and quickly arrive at an equilibrium point. In addition, the transmit power of SUs under the proposed algorithm has a little higher than that of the non-robust scheme. This reason is that SUs under uncertain environment try to improve their transmit power to overcome the effect of uncertain parameters (i.e., $\Delta h_k^n \neq 0$) which will be shown in the following simulation results.

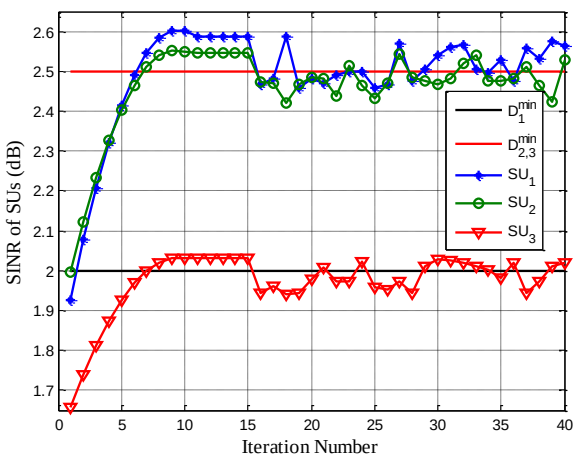


(a)

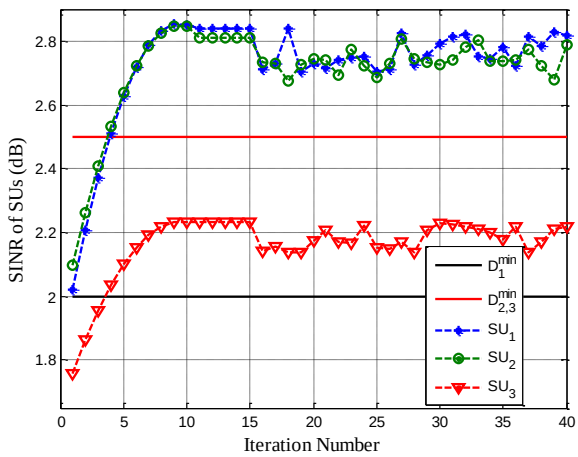


(b)

Fig. 2. Transmit power of SUs under different schemes. (a) Non-robust scheme. (b) Proposed robust scheme.



(a)



(b)

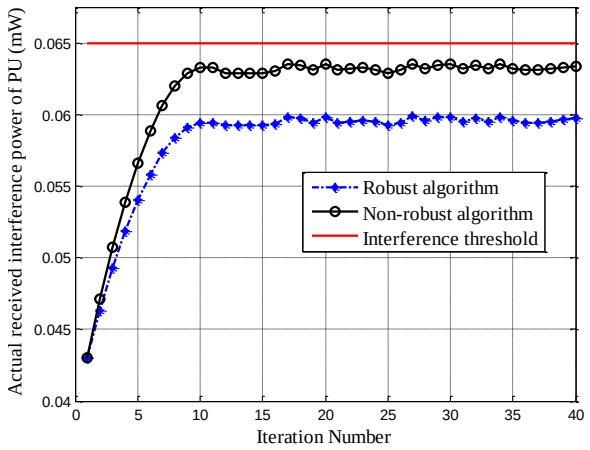
Fig. 3. Received SINRs of SUs under different schemes. (a) Non-robust scheme. (b) Proposed robust scheme.

Fig. 3 presents the performance of SINRs of SUs under different algorithms while channel uncertainties happen at the 15th iterations. Before the 15th iterations, the received SINRs of SUs under the non-robust algorithm (i.e., Fig. 3(a)) are above their threshold values (i.e., $D_k^{\min}$ for $\forall k$), since there is no parameter uncertainty in the CR systems. Namely, all SUs can perfectly estimate channel gains and interference power from PUs. Assuming that the cognitive systems have some stochastic channel uncertainties, however, after the 15th iterations, the SINRs of SUs under the non-robust algorithm can not keep the QoS requirement for each user while the outage events of SUs will happen. And the performance loss depends on the size of uncertain boundary (i.e., ε). Obviously, the effect of uncertainty of SU1 is weaker than that of SU2 and SU3. Despite this, the proposed robust power allocation algorithm can well protect the QoS of SUs under parameter perturbation due to the consideration of uncertainties ahead of time.

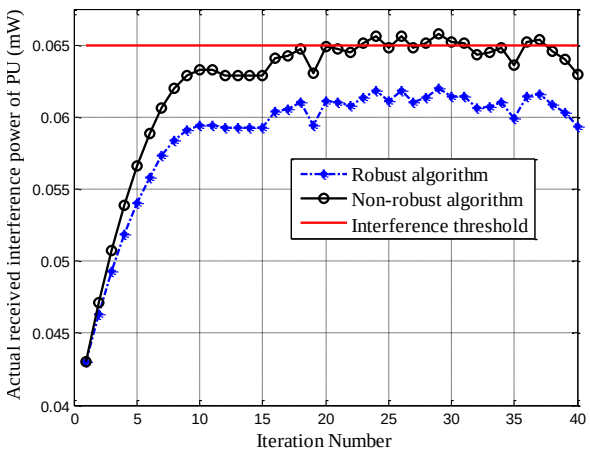
In order to reflect the impact of uncertain channel gains on the interference power of PUs, Fig. 4 shows the received interference power at the PU-receiver under some uncertainty (i.e., $\Delta g_{k,m}^n \neq 0$). For the first case, if channel uncertainty is very small, namely, the estimated channel gain ($\hat{g}_{k,m}^n$) is greatly close to the actual channel gain $g_{k,m}^n$.

From Fig. 4(a), it is clear known that both of the non-robust algorithm and the proposed algorithm do not interfere the normal communication of PU where the interference power from SUs is kept under the interference threshold (i.e., interference temperature level). Whereas, from Fig. 4(b), we know that the received interference power at the PU-receiver under the non-robust algorithm becomes above the allowable interference power level under the strong perturbation (i.e., big channel uncertainty). As a result, the QoS of PU will be degraded, which is not allowed for the licensed networks (i.e., primary user network). And the outage

probability of PU can be easily calculated by $\Pr_{out}^{PU} = \max\left(0, \left(\sum_{k=1}^{K}\sum_{n=1}^{N} \tilde{p}_k^n g_{k,m}^n - I_m\right) / I_m\right)$ which is omitted in here, where $\tilde{p}_k^n$ denotes the optimal transmit power of the non-robust resource allocation problem.



(a)



(b)

Fig. 4. Actual received interference power of PU. (a) Weak perturbation ($\delta_m^n = 1\%$). (b) Strong perturbation ($\delta_m^n = 5\%$).

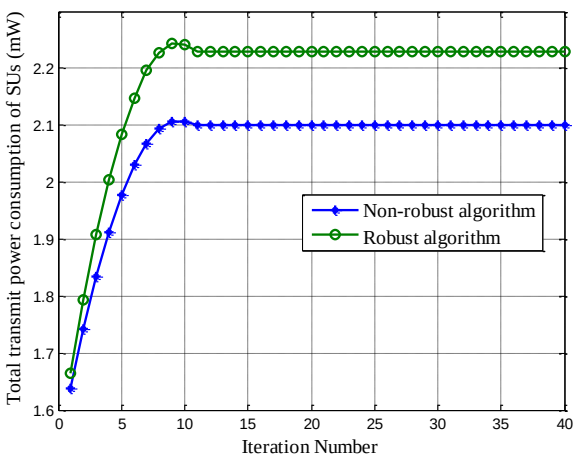


Fig. 5. Energy consumption of SUs under different algorithms.

Fig. 5 depicts the total energy consumption of SUs under the two methods. From the figure, we clearly know that total power consumption of SUs under the proposed robust power allocation algorithm is bigger than that of the non-robust power allocation algorithm since the proposed algorithm can well protect the QoS of each SU at the cost of more energy consumption.

In addition, to verify the performance of our algorithm under the changing minimum SINR, we give the simulation result in Fig. 6. We define that the uncertainties of different links are assumed to be same, e.g., $\varepsilon = \varepsilon_k^n, \forall k, n$ and $\omega = \omega_k^n, \forall k, n$. The minimum SINR of SU is defined as $D^{\min} = D_k^n, \forall k, n$. From Fig. 6, it is clear to know that the total power consumption of secondary network increases with the bigger target SINR of each SU, such as $D^{\min}$, since the bigger target SINR requirement means the user equipment (UE) is far away from the base station (BS) so that the increasing transmit power can guarantee the basic communication quality. Additionally, the transmit power under non-robust algorithm is the same under different degree of uncertainty, because the optimal power is not a function with the variables ε_k^n and ω_k^n. The bigger ε and ω cause more power consumption, which means the increasing transmit power is used to overcome the effect of parametric uncertainties. As a result, it aims to reduce the outage probability of user under parameter perturbation.

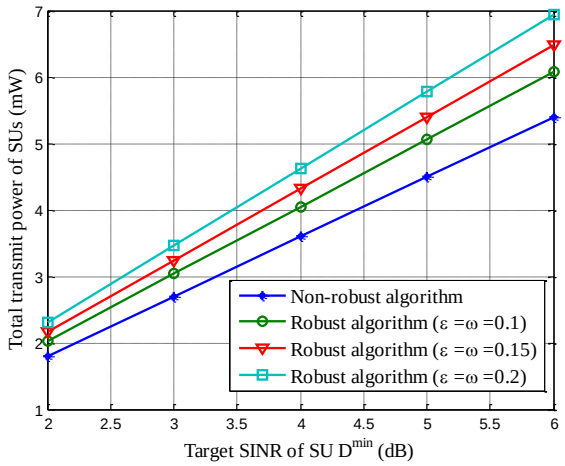


Fig. 6 Total transmit power versus target SINR under different uncertainties.

V. Conclusions

In this paper, for a multiuser OFDM-based CRN, a robust power allocation algorithm is designed to achieve the total transmit power minimization of SUs while the effect of uncertainties are suppressed effectively. Both the robust SINR constraint and robust interference temperature constraint are converted into the convex constraints by a relaxing method (i.e., consider the worst-case uncertainty). The original NP-hard robust power allocation problem is transformed into a convex one efficiently solved by using convex optimization techniques. Simulation results verify the proposed robust power allocation algorithm can fast reach the equilibrium

point under the consideration of channel uncertainties and well protect the QoS of SUs and PUs simultaneously.

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REFERENCES

[1] H. Liu, S. Hua, X. J. Zhuo, "Cooperative spectrum sharing of multiple primary users and multiple secondary users," *Digital Commun. and Networks*, vol. 2, no. 4, pp. 191-195, 2016.

[2] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun*., vol. 23, no. 2, pp. 201-220, Feb. 2005.

[3] F. Gao, R. Zhang, and Y. C. Liang, "Design of learning-based MIMO cognitive radio systems," *IEEE Trans. Veh. Technol*., vol. 59, no. 4, pp. 1707-1720, Jan. 2010.

[4] L. Zhang, C. H. Bai, F. Zhuo, H. T. Xu, and W. Huang, "Dynamic rate allocation for multipath routing under path stability and prioritized traffic session constraints for cognitive radio ad hoc networks with selfish secondary users," *Journal of Commun*., vol. 11, no. 5, pp. 462-470, May 2016.

[5] Y. J. Xu, X. H. Zhao, and Y. C. Liang, "Robust power control and beamforming cognitive radio networks: A survey," *IEEE Commun. Surv. & Tutor*., vol. 77, no. 4, 1834-1857, April 2015.

[6] D. Bepari and D. Mitra, "Improved power loading scheme for orthogonal frequency division multiplexing based cognitive radio," *IET Commun*., vol. 9, no. 16, pp. 2033-2040, Nov. 2015.

[7] T. Zhang, W. Chen, and Z. Han, "Hierarchic power allocation for spectrum sharing in OFDM-based cognitive radio networks," *IEEE Trans. Veh. Technol*., vol. 63, no. 8, pp. 4077-4091, Feb. 2014.

[8] L. Tang, Q. B. Chen, and G. Y. Wang, "Opportunistic power allocation strategies and fair subcarrier allocation in OFDM-based cognitive radio networks," *Telecommun. Syst.*, vol. 52, no. 4, pp. 2071-2082, May 2013.

[9] K. V. Krishnan, R. M. Sajith, and S. Khara, "Dynamic resource allocation in OFDM based cognitive radio system considering primary user QoS and secondary user proportional constraints," *Journal of Commun. Technol. Electr.*, vol. 60, no. 11, pp. 1269-1275, Sept. 2015.

[10] S. Parsaeefard and A. R. Sharafat, "Robust distributed power control in cognitive radio networks," *IEEE Trans. Mobile Comput*., vol. 12, no. 4, pp. 609-620, Jan. 2013.

[11] E. Bedeer, O. Amin, O. A. Dobre, M. H. Ahmed, and K. E. Baddour, "Energy-efficient power loading for OFDM-based cognitive radio systems with channel uncertainties," *IEEE Trans. Veh. Technol*., vol. 64, no. 6, pp. 2672-2677, July 2015.

[12] S. W. Wang, W. Shi, and C. Wang, "Energy-efficient resource management in OFDM-based cognitive radio networks under channel uncertainty," *IEEE Trans. Commun*., vol. 63, no. 9, pp. 3092-3102, July 2015.

[13] S. J. Kim, N. Y. Soltani, and G. Giannakis, "Resource allocation for OFDMA cognitive radios under channel uncertainty," *IEEE Trans. Wireless Commun*., vol. 12, no. 7, pp. 3578-3587, July 2013.

[14] P. Jun, H. Zhen, Z. Zheng, Y. W. Tao, and W. R. Li, "A worst-case robust distributed power allocation in OFDM-based cognitive radio networks," in *Proc. IEEE Chinese on Control Conference*, Xi'an, 2013, pp. 6422-6427.

[15] Y. J. Xu and X. H. Zhao, "Robust rate maximization for OFDM-based cognitive radio networks," in *Proc. IEEE Global Conference on Signal and Information Processing*, Atlanta, 2014, pp. 1195-1198.

[16] B. T. Aharon, E. G. Laurent, and N. Arkadi, *Robust Optimization*, Princeton University Press, 2009, ch. 2.

[17] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004, ch. 5, pp. 215-289.

[18] P. Setoodeh and S. Haykin, "Robust transmit power control for cognitive radio," *Proc. of the IEEE*, vol. 97, no. 5, pp. 915-939, May 2009.

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