Energy Efficiency Analysis of Multi-Antenna Heterogeneous Relay-Assisted Cellular Network via Stochastic Geometry

Yonghong Chen$^{1,2}$, Jie Yang$^{3}$, Xuehong Cao$^{1,3}$, and Rui Chen$^{3}$

$^1$Department of Communication and Information Engineering, Nanjing University of Posts and Telecommunications, Nanjing, China
$^2$Xinglin College, Nantong University, Nantong, China
$^3$Department of Communication Engineering, Nanjing Institute of Technology, Nanjing, China

Email: chenyh1107@ntu.edu.cn; yangjie@njit.edu.cn; caoxh@njupt.edu.cn

Abstract —Energy Efficiency (EE) of cellular networks has received considerable attention recently. However, EE of relay-assisted cellular networks has not been thoroughly addressed, especially that the MBSs adopt multi-antenna. In this paper, considering downlink transmission of multi-antenna relay-assisted cellular network where the MBSs and RSs are deployed in the Euclidean plane according to independent homogeneous Poisson point processes, meanwhile, an sleep mode strategy is used in MBS, a geometric model is built to derive the coverage probability for the MBS to user (UE), the MBS to Relay Station (RS), the RS to UE links and analyze the system EE. A metric defined as the ratio of average network throughput to average network power consumption is used to measure the average EE and its analytical expression is derived. Simulation results confirm the theoretical findings and demonstrate that the MBSs equipped with multi-antenna has better performance on energy efficiency and coverage probability of every link than that with single antenna.

Index Terms—Energy efficiency, multi-antenna, coverage probability, random sleep

1. INTRODUCTION

With the growth of mobile traffic, the energy consumption of mobile network becomes more and more serious. For the future mobile communication system, the research of network architecture and transmission mechanism to save energy consumption, and improve the energy efficiency, is a widespread concern of the academia and industry [1]-[2]. In order to study the energy efficiency of the whole cellular network, a cellular network deployment model is established. The traditional analysis is based on two-dimensional hexagonal network model network, increasing with the size and the random of the network, there is a more complicated interference correlation in wireless cellular networks, which makes it difficult to reach the requirements of the analysis model. In 2010, G. Andrews Jeffrey first modeled the distribution of the cellular network with the homogeneous Poisson point process in stochastic geometry [3]. Literature [4] first proposed using random geometric distribution of the cellular network with the homogeneous Poisson point process in stochastic geometry [3]. Literature [4] first proposed using random geometric theory to analyze the energy efficiency of single layer relay cellular network. In [5] different techniques based on stochastic geometry have led to results on the connectivity, the capacity and the outage probability. In [6]-[7], sufficient progress has been made in modeling single antenna HetNets.

In the cellular network, relay can be used to expand the coverage area, resist the fading and improve the spectrum efficiency and data rate [8]. With regard to the relay deployment scheme, the literature [9] pointed out that a minimum of two hop relay transmission scheme of the energy consumption in any asymmetric transmission rate requirements, the best position is in the middle position of the two relay. The effect of relay deployment on energy and cost in heterogeneous LTE-A networks is studied in [10]. Then, in [11], it is found that by introducing an appropriate number of relays in a cellular network, the energy efficiency can be improved without affecting the throughput of the system. Literature [12] studied the energy efficiency of single antenna relay-assisted cellular networks using tools from stochastic geometry and derived the coverage probability for the MBS to UE, the MBS to RS, and the RS to UE links. Literature [13] developed a general downlink model for multi-antenna heterogeneous cellular networks where base stations across tiers differ in terms of transmit power, SINR, deployment density, number of transmit antennas and the type of multi-antenna transmission, and compared three transmission mode, such as space division multiple access (SDMA), single user beamforming (SU-BF) and baseline single-input single-output (SISO) transmission. In [14], two sleep mode strategies were proposed, their effects on power consumption and energy efficiency were investigated.

But the analysis of multi-antenna relay-assisted cellular networks has not been reported. In this paper, we consider a multi-antenna relay-assisted cellular network,
Meanwhile, an sleep mode strategy is used in MBS. We analyze the energy efficiency of multi-antenna relay-assisted cellular networks by stochastic geometry and the expression of energy efficiency for the proposed algorithm is obtained.

The rest of this paper is organized as follows. Section II describes the system model. Section III presents the coverage probability of different links and the expression of system energy efficiency. Some simulation results are discussed in Section IV. Conclusions are stated in Section V.

II. SYSTEM MODEL

Consider a downlink of multi-antenna heterogeneous relay cellular network where the Macro base stations (MBSs) and relay stations (RSs) are deployed in the Euclidean plane according to independent homogeneous Poisson point processes (HPPPs) $\Phi_M$ and $\Phi_R$ with densities $\lambda_M$ and $\lambda_R$, respectively. In this network, the MBSs are equipped with multiple antennas, each RS connects to the closest MBS and has a circular coverage area with radius $R$. Users (UEs) are classified into two groups according to their locations: (i) Users that communicate directly to the MBS (M-UE), which connect with the closest MBS and are distributed according to an independent stationary point process. (ii) Users located in the nearest RSs' coverage area (R-UE) which communicate to the MBS with the help of RS are arranged to an independent HPPP in the circular area.

We suppose the distance between RS (or M-UE) and MBS is $r$, since the RS (or M-UE) connects to the nearest MBS, the interfering MBSs must be farther than $r$. So the probability density function (PDF) of $r$ is $f_r(r) = 2\pi \lambda_M r e^{-\lambda_M r^2}$. Since R-UE locates in a RS’s coverage, the distance $l$ between RS and R-UE follows the distribution $f_l(l) = 2l / R^2$. The transmit power from MBS to M-UE and RS is $P_{MU}$ and $P_{MR}$, respectively. The transmit power for all RSs is $P_{RU}$.

Adopting the signal transmission frame structure of literature [12], in each cell, MBS-UE link and MBS-RS-UE link work at non-overlapping spectrum with bandwidth of $\omega_M$ and $\omega_R$, respectively. For the MBS-RS-UE link, RS using half duplex communication mode, the transmission process is divided into two phases and the time of both phases is equal, in the first phase, MBS transmit information to the relay (RS) and in the second phase, the relay decodes the information and forwards them to the UE. We assume that each MBS serves only one RS and M-UE at any time, if there are multiple RSs or M-UEs in the cell, time division multiple access (TDMA) can be performed. Similarly, each RS serves only one R-UE at any time, if there are multiple R-UEs in the coverage area of the RS, TDMA can be used.

In this paper, we assume perfect CSI and the multi-antenna technique adopted by MBS is single user beamforming, under which for Rayleigh fading that the channel power distributions of both the direct and the interfering links follow the Gamma distribution [15]. We investigate a random sleeping policy which dynamically switching off MBS and model the sleeping strategy as a Bernoulli trial such that each MBS continues to operate with probability $q$ and switch off with probability $1-q$.

The power consumed by a switched off MBS is $P_{M_{sleep}}$. The random sleeping strategy is equivalent to modeling the active MBSs as a marked PPP with intensity of $q \lambda_M$, where the transmission power of the active MBSs is increased to $\beta P_{MU}$ or $\beta P_{MR}$, where $\beta$ represents the increase ratio of fixed power control.

III. ENERGY EFFICIENCY ANALYSIS

In this section, the definition of the coverage probability is given first, then the coverage probability of different links are derived. Finally, the power consumption model is given, and energy efficiency expression is derived.

A. Coverage Probability

Coverage probability is defined as the probability that the receive signal-to-interference-plus-noise ratio (SINR) is larger or equal to a certain threshold $T$, which can be written as $P_c = \Pr(SINR \geq T)$. The SINR is defined as the ratio between the received power from the tagged transmitter and the cumulative interference from all the other transmitters plus noise. The SINR can be expressed as

$$\text{SINR} = \frac{P h r^{-\alpha}}{I + \sigma^2}$$

where $P$ is transmit power, $h$ is the channel power gain, $r$ is the distance between the receiver and the tagged transmitter, $\alpha$ is the path loss exponent, $I$ is the cumulative interference from all the other transmitters, $\sigma^2$ is the variance of the additive white Gaussian background noise.

1) Coverage probability for the MBS-UE link

Suppose MBS contains $k$ antennas and serves one user at any time. The multi-antenna technique adopted by MBS is single user beamforming, under which for Rayleigh fading that the channel power distributions $h_i$ follow the Gamma distribution $\Gamma(k,1)$.

Coverage probability for the MBS-UE link is defined as (2), where $I_{MU} = \sum_{i=0}^{k-1} |P_{MU} h_i r_i^{-\alpha}$ is the aggregate interference from all the other transmitters.

To solve the $P[l_h \geq T_{MU} | I_{MU} + \sigma^2 r^2 P_{MU} | r]$, considering interference-limited HCNs, the background noise is ignored, $\sigma^2 \to 0$. 

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\[ P_{c,MU} = \Pr(\text{SINR}_{MU} \geq T_{MU}) = E_{\{r \in [0, \infty)\}} \left[ \int_{r_1}^{r} f_r(r') dr' \right] \]
\[ = \int_{r_1}^{\infty} P[\text{SINR}_{MU} \geq T_{MU} \mid r] f_r(r) dr = \int_{r_1}^{\infty} P[\frac{P_{MU} h_r r^{-\alpha}}{I_{MU} + \sigma^2} \geq T_{MU} \mid r] \exp(-\pi q_{\lambda_M} r^2) \cdot 2\pi q_{\lambda_M} r dr \]
\[ = \int_{r_1}^{\infty} P[h_0 \geq T_{MU} (I_{MU} + \sigma^2) r^{-\alpha} P_{MU} \mid r] \cdot 2\pi q_{\lambda_M} r dr \]
\[ P[\text{SINR}_{MU} \geq T_{MU}] = \Pr(h_0 \geq T_{MU} \mid r) = \Pr(h_0 \geq s I_{MU} \mid r, I_{MU}) \]

where \( s = T_{MU} r^{-\alpha} P_{MU}^{-1} \) and

\[ E_{\{r \in [0, \infty)\}} \left[ \sum_{i=0}^{\infty} \frac{1}{i!} \cdot \exp(-s) \cdot \left( s P_{MU} h_r r_i^{\alpha} \right)^{i} \right] \]
\[ L_{\text{MU}}(s) = E_{\{r \in [0, \infty)\}} \left[ \sum_{i=0}^{\infty} \frac{1}{i!} \cdot \exp(-s) \cdot \left( s P_{MU} h_r r_i^{\alpha} \right)^{i} \right] \]
\[ \lambda = \min(q_{\lambda_M}, \lambda_\rho) \]

Combined with the formula (5)-(7), we can get the expression of \( P_{c,MU} \).

**2) Coverage probability for the MBS-RS link**

The coverage probability of MBS-RS is defined as (6),

\[ P_{c,MRS} = \Pr(\text{SINR}_{MRS} \geq T_{MRS}) = \int_{r_1}^{\infty} P[\text{SINR}_{MRS} \geq T_{MRS} \mid r] f_r(r) dr = \int_{r_1}^{\infty} P[\frac{P_{MRS} g_r r^{-\alpha}}{I_{MRS} + \sigma^2} \geq T_{MRS} \mid r] \cdot \exp(-\pi q_{\lambda_M} r^2) \cdot 2\pi q_{\lambda_M} r dr \]
\[ = \int_{r_1}^{\infty} P[g_0 \leq T_{MRS} (I_{MRS} + \sigma^2) r^{-\alpha} P_{MRS} \mid r] \cdot \exp(-\pi q_{\lambda_M} r^2) \cdot 2\pi q_{\lambda_M} r dr \]

where

\[ I_{MRS} = \sum_{i=0}^{\infty} P_{MRS} g_i r_i^{\alpha} \]

\[ P_{c,RS} = \Pr(\frac{P_{RS} l_r r^{-\alpha}}{I_{RS} + \sigma^2} \geq T_{RS}) = \frac{1}{R^2} \int_{0}^{R} \exp(-\pi q_{\lambda_M} \cdot \varphi(T_{RS} \cdot \alpha)) \exp(-\frac{\lambda_{\varphi} u^{1/2} \sigma^3}{P_{RS}}) du \]

is the aggregate interference received at the RS.

Suppose MBS contains \( k \) antennas and servers one RS at any time. The channel power distributions \( g_0 \) follow the Gamma distribution \( \Gamma(k,1) \), evaluating at \( s = T_{MBS} r^{-\alpha} P_{MBS}^{-1} \), in the same way, we can easily obtain the power of MBS.

\[ \sum_{i=0}^{\infty} P_{MRS} g_i r_i^{\alpha} \]

Combined with the formula (6)-(8), we can get the expression of \( P_{c,MRS} \).

**3) Coverage probability for the RS-UE link**

The coverage probability of RS-UE is defined as

\[ P_{c,RS} = \Pr(\text{SINR}_{RS} \geq T_{RS}) \]

where \( \text{SINR}_{RS} = \frac{P_{RS} l_r r^{-\alpha}}{I_{RS} + \sigma^2} \),

\[ I_{RS} = \sum_{i=0}^{\infty} P_{RS} l_i r_i^{\alpha} \]

For the RS-UE link, the coverage probability is given by (9), where \( \varphi(T_{RS} \cdot \alpha) = \frac{T_{RS}}{1 + T_{RS}} \int_{0}^{\infty} \frac{1}{1 + u^{1/2}} du \)

\[ \lambda = \min(q_{\lambda_M}, \lambda_\rho) \]

**B. Power Consumption Model**

For the downlink, the power consumption at each MBS and RS can be given by \( P_{MBS} = \beta_{\Delta M} P_M + P_{MBS} \) and \( P_{RS} = \Delta_{\rho} P_R + P_{RS} \), where \( P_M \) and \( P_R \) are the transmit power of MBS and RS, \( 1/\beta_{\Delta M} \) and \( 1/\Delta_{\rho} \) denote the efficiency power amplifier of MBS and RS, \( P_{MBS} \) and \( P_{RS} \) are the static power consumption for MBS and RS, respectively.
After applying random sleeping at MBS, as area network at most only \( \lambda (\lambda = \min(q(\lambda_M, \lambda_R))) \) MBSs send information to the RSs at any time, so the density of \( P_{\text{MR}} \) is \( \lambda \).

The average power consumption of each MBS is given by

\[
P_{\text{Mtot}} = q[c_{\text{MR}} + \frac{1}{2} c_{\text{MU}} 1 + (1 - q) P_{\text{Mstep}}] 
\]

(10)

The average networks power consumption of the MBSs is

\[
P_{\text{Mtot}} = \lambda_M q c_{\text{MU}} P_{\text{MU}} + \frac{1}{2} \lambda_M c_{\text{MR}} P_{\text{MR}} + \lambda_M q c_{\text{MR}} + (1 - q) P_{\text{Mstep}} 
\]

(11)

At any time, the number of RS taking part in communication is at most equal to the number of MBSs and only the RS decode successfully and forward the information to R-UE at the second phrase, this number is \( \lambda P_{\text{cMR}} \), the other RS \( (\lambda - \lambda P_{\text{cMR}}) \) don’t send the information and the power consumption is static power consumption. So the average networks power consumption of the RSs is given by

\[
P_{\text{Rtot}} = \lambda P_{\text{cMR}} \frac{1}{2} \Delta_M P_{\text{RU}} + (\lambda - \lambda P_{\text{cMR}}) P_{\text{RU}} 
\]

(12)

C. Energy Efficiency

The energy efficiency (EE) of multi-antenna heterogeneous relay cellular network is defined as

\[ \eta_{EE} = \frac{\text{Average Network Throughput}}{\text{Average Network Power Consumption}} = \frac{\tau_M + \tau_R}{P_{\text{Mtot}} + P_{\text{Rtot}}} \text{(bps/Hz/W)} \]

(13)

where \( \tau_M \) and \( \tau_R \) are the average network throughput of MBS-UE links and MBS-RS-UE links. \( P_{\text{Mtot}} \) and \( P_{\text{Rtot}} \) are the average networks power consumption of the MBSs and RSs.

For the MBS-UE links, the average network throughput is defined as

\[ \tau_M = q c_{\text{MU}} P_{\text{MU}} \frac{\omega_M}{\omega_M + \omega_R} \log_2 (1 + T_{\text{MU}}) \]

(14)

For the MBS-RS-UE links, which divide into two phrases, the average network throughput is determined by the minimum throughput of the two phrases. Considering the coverage probability of each phrase, the average network throughput of MBS-RS-UE links is given by

\[ \tau_M = \frac{1}{2} \lambda P_{\text{cMR}} P_{\text{cMR}} \frac{\omega_R}{\omega_M + \omega_R} \times \min \{ \log_2 (1 + T_{\text{MR}}), \log_2 (1 + T_{\text{RU}}) \} \]

(15)

Combining the above equations, the expression of energy efficiency for the multi-antenna relay cellular network with random sleeping strategic is obtained.

IV. SIMULATION RESULTS

In this section, we provide some simulation results of coverage probability and energy efficiency of multi-antenna relay cellular network proposed in this paper, and compare them with the single antenna relay-assisted cellular network [12]. In order to better analyze the performance of the multi-antenna relay cellular network, MBSs with random sleeping strategy [16] is also simulated. In order to simplify the simulation, we suppose that each MBS equipped with two antennas, each RS and user has only one antenna. We use the default simulation parameters of the system model in Table I.

![Fig. 1. Coverage probability \( P_{\text{MU}} \) vs. \( \lambda_M \)](image)

Fig. 1 shows the relationship of the coverage probability \( P_{\text{MU}} \) and \( \lambda_M \) where non-random sleeping strategy and random sleeping strategy are employed in the MBSs. It is seen that the MBSs with multi-antenna offer an improvement over the MBSs with single antenna regardless of random sleeping or non-random sleeping.

![Fig. 2](image)

Fig. 2 shows the relationship of the coverage probability \( P_{\text{cMR}} \) and \( \lambda_M \) under the same condition of Fig. 1. The curve is divided into two parts according to the relationship between \( \lambda_M (q \lambda_M) \) and \( \lambda_R \), when \( \lambda_M (q \lambda_M) \) less than \( \lambda_R \), the \( P_{\text{cRU}} \) increase slowly with the...
MBSs adopted non-random sleeping, when MBSs adopt random sleeping strategy, $P_{c-RU}$ basically don’t change with the increase of the $\lambda_M$. When $\lambda_M$ ( $q\lambda_M$ ) larger than $\lambda_R$, $\lambda$ equal to $\lambda_R$, the curve approximate linear increase with the increase of the $\lambda_M$. We also see that in all cases, the MBSs with multi-antenna supply an improvement over the MBSs with single antenna.

Fig. 2 shows the relationship between coverage probability $P_{c-MR}$ and $\lambda_M$. According to the relationship between $q\lambda_M$ and $\lambda_R$ the curve is divided into two parts, when $q\lambda_M$ less than $\lambda_R$, $P_{c-RU}$ approximate linear decrease with the increase of the $\lambda_M$. When $q\lambda_M$ larger than $\lambda_R$, $\lambda$ equal to $\lambda_R$, then only intermediate variable $P_{c-MR}$ with $\lambda_M$ varies in the expression of $P_{c-RU}$, so the curve slow down with the increase of $\lambda_M$.

Fig. 4 shows the relationship between energy efficiency and awake fraction of MBS $q$ for different antenna number equipped by MBSs and various values of amplification factor $\beta$. We consider two cases, which MBSs equipped with single antenna or multi-antenna, from Fig. 5, we can derive that the energy efficiency of MBS equipped with Multi-antenna is better than that equipped with single antenna. It also can be seen that the energy efficiency increases with the density of MBSs and saturates when the density goes to infinity in both cases. The density of RSs increase, the energy efficiency drops, which means that the growth of average network throughput can’t make up for the growth of average network power consumption. It also can be seen that the energy efficiency of MBS equipped with Multi-antenna is better than that equipped with single antenna.

Fig. 5. Energy efficiency vs. awake fraction of MBSs $q$
Fig. 6 illustrates the relationship between energy efficiency and transmit power of MBS $P_{\text{MU}}$ for different values of RS densities. The figure reveals that while the density of RS increases, the energy efficiency drops, which means that the increase of average network throughput can’t offset the growth of average network power consumption. The MBSs would cause a lot of interference and can increase energy loss with the increase of MBS’s transmit power. The energy saved by random sleeping MBSs can’t compensate for the area network throughout decrement, so the MBSs which adopted random sleeping strategy decrease the energy efficiency of system. It is clearly that MBSs equipped with multi-antenna has better performance on energy efficiency than that with single antenna.

Fig. 6. Energy efficiency vs. transmit power of MBS $P_{\text{MU}}$.

V. CONCLUSIONS

In this paper, we investigate the energy efficiency of multi-antenna relay cellular network, which the MBSs adopting random sleeping strategy. We derive the coverage probability of MBS-UE links and MBS-RS-UE links. According to power consumption model and the average network throughput, the expression for energy efficiency is derived. Through the simulation we obtain energy saved by random sleeping MBSs can’t compensate for the area network throughout decrement, so the MBSs which adopted random sleeping strategy decrease the energy efficiency of system. The MBSs equipped with multi-antenna has better performance on energy efficiency and coverage probability of every link than that with single antenna.

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Yong Hong Chen was born in Jiangsu Province, China, in 1981. She received the B.S. degree in electronic information science and technology from Zhengzhou University of Light Industry, in 2004 and the M.S. degree in signal and information processing from Nanjing University of Posts and Telecommunications in 2007. She is currently a lecturer of Nantong University. Her research interests include communication signal processing and cooperative communications.

Jie Yang was born in Jiangsu Province, China, in 1979. She received the B.S. and M.S. degrees from Lanzhou University of Technology in 2000 and 2003, respectively. She received PhD degree from Nanjing University of Post and Telecommunications in 2015. Now she is the vice professor of Nanjing Institute of Technology. Her research interests include cooperative communications, relaying network, and resource allocation.

Xuehong Cao was born in Suzhou, China, in 1964. She received the B.S. and M.S. degrees from the Nanjing University of Posts and Telecommunications in 1985 and 1988, respectively, and the Ph.D. degree in electronic engineering from Shanghai Jiaotong University in 1999. From 2004 to 2005, she worked as a visiting professor at the Department of Electrical Engineering, Stanford University. Now she is a Professor and the vice president of Nanjing Institute of Technology. Her research interests include multicarrier modulation, cooperative communication system and information theory.