Study of Constellation Design of Pseudolites Based on Improved Adaptive Genetic Algorithm

Jiancai Song1,2, Chunping Hou3, Guixiang Xue1,3, and Maode Ma4
1Hebei University of Technology, Tianjin, 300401, China
2Tianjin University, Tianjin, 300072, China
3Hebei Key Laboratory of Dig Data Calculation, Tianjin, 300401, China
4Nanyang Technological University, Singapore

Abstract—Global Navigation Satellite System (GNSS) is vulnerable to interferences and has other shortcomings such as unreliable signals in locations that are indoors, in urban canyons, and deep mines. Therefore, the pseudolite (pseudo-satellite) positioning technology, which has gained wide attention in recent years, is used to complement and enhance GNSS. The constellation layout of pseudolites creates geometrical benchmarks in spatial positioning, which in turn affects the receiver positioning accuracy by impacting the Dilution of Precision (DOP) value of each location point within the service area. The constellation layout design of pseudolites poses a combinatorial optimization problem with multiple constraints, given the service area and station layout area, such as the distance between the pseudolite and the receiver, making the indicators of spatial configuration and redundancy maintenance and the availability within the service area to achieve best. An improved Adaptive Genetic Algorithm (AGA) is presented based on the mathematical modeling of pseudolite positioning and constraints. The simulation results show that the novel algorithm can effectively solve the optimization problem associated with the constellation layout of pseudolites under a variety of constraints.

Index Terms—Pseudolite configuration, adaptive genetic algorithm, radio navigation, DOP

I. INTRODUCTION

GPS satellite signals cover a wide range, have high precision in position and velocity measurement, have all-weather work capabilities, and other advantages [1]; however, the signals are weak in urban canyons, deep mining areas, and other indoor locations [2], [3]. Therefore, the pseudolite positioning technology is used to complement Global Navigation Satellite System (GNSS) and enhance the performance; this technology gained much attention among research institutions and scholars in recent years. Borio proposed a general pulsing signal scheme that could be used to reduce the near-far effect based on the principle of Time Division Multiple Access (TDMA) [4]. Several literatures focus on using pseudolites in indoor locations [5], [6], and much research has been carried out to improve the performance of GPS or other GNSS systems [7], [8].

With a principle similar to GPS, the time and spatial reference of pseudolite positioning system is the most important factor in determining the positioning accuracy. Time reference indicates the time taken to measure the synchronization accuracy between pseudolites that determine the receiver’s pseudorange measurement accuracy; spatial reference indicates the geometric distribution relationship of pseudolites and calibration accuracy, which determines the geometric dilution of precision (GDOP) of the entire system services area. However, GDOP is the magnification factor between the measurements error and positioning error. Spatial reference means that the pseudolite constellation includes the number and relative position of pseudolites. In order to make the pseudolite system work independently in the service area, you will require at least four pseudolites. If you want your receiver to support autonomous integrity monitor testing, then you need at least five pseudolites; if you include fail detection, then six satellites are considered as the minimum configuration. Under normal circumstances, for pseudolites deployed in Near-Earth space, the height difference between pseudolites is relatively small, and so the vertical receiver positioning accuracy will degrade due to poor vertical DOP. To improve the vertical receiver positioning accuracy, simply increase the number of pseudolites and improve the optimization constellation layout design.

The constellation layout of pseudolites is a belt-constrained combinatorial optimization problem, which is mainly focused on basic simulation. Chang Hao researched the configuration of indoor positioning system using pseudolites, which was mainly about determining the horizontal dilution of precision [9]. Duan Wenwei proposed a niche Genetic Algorithm (GA) for the distribution of pseudolites [10]. Crawford studied the optimal geometric deployment of a ground-based pseudolite navigation system to track a landing aircraft [11]. An optimum distribution of the near-space pseudolite regional independent network system was found by analyzing the method of calculating the DOP.
and improving the empirical positioning algorithm of regional satellite optimum distribution [12]. Tiwary proposed a novel method to improve the GDOP in pseudolite-only navigation system using GA [13]. However, the main drawback in all approaches is that they are dependent on the empirical layout configuration such as the layout of a star, the letter Y, or a honeycomb, and the Optimization method does not consider various practical engineering constraints, such as elevation angle and pseudo random code isolation resulted into near-far interference. In contrast with the traditional method, this paper proposes an improved adaptive genetic algorithm (AGA) to solve the multiple constraints associated with the constellation layout of pseudolites; the GA is based on the rule of “survival of the fittest” and parallel mechanisms. Using the global optimization characteristics for constellation layout optimization of pseudolites, the coding format, fitness function, selection, crossover, and mutation operator were designed based on the principle of spatial configuration and usability evaluation. The simulation experiments show that the novel algorithm can effectively solve this problem under multiple constraints that have much better feasibility.

This paper is organized as follows. In Section II, the DOP and some evaluation standards are introduced. The proposed algorithm, i.e., the constellation layout scheme of pseudolites based on AGA, is elaborated in Section III; its simulation experiments and the performance evaluation are shown in Section IV. Finally, we conclude this brief in Section V.

II. SYSTEM BACKGROUND

The GPS localization algorithm is usually based on the mathematics of geometric principle whereas the pseudolite positioning system uses pseudorange differential positioning in order to reduce the timing precision of the pseudolite base station that influences the receiver positioning accuracy.

\[
\sqrt{(X_k - x)^2 + (Y_k - y)^2 + Z_k - z^2} = t_k - T_{0u} \tag{1}
\]

where, \(X_k, Y_k, Z_k\) denote the coordinates of pseudolite transmitter antenna, \(x, y, z\) represent the unknown coordinates of the receiver, \(t_k\) is the transmission time, in general, \(T_{0u}\) is actually a time-varying parameter that includes a receiver’s clock error and receiver time delay. In equation (1), pseudolite base station 1 was assumed as the reference station and the difference in the distance between the reference station and other base stations can be obtained as follows:

\[
\frac{\sqrt{(X_k - x)^2 + (Y_k - y)^2 + Z_k - z^2}}{c} - \frac{\sqrt{(X_1 - x)^2 + (Y_1 - y)^2 + Z_1 - z^2}}{c} = t_k - T_{0u} \tag{1}
\]

Obviously, \(T_{0u}\) was automatically eliminated in equation (2); in this way, the uncertainty \(T_{0u}\) of the receiver has no influence on the position precision. So,

\[
\Delta R_k(x, y, z) = \sqrt{(X_k - x)^2 + (Y_k - y)^2 + Z_k - z^2} - \sqrt{(X_1 - x)^2 + (Y_1 - y)^2 + Z_1 - z^2} \tag{2}
\]

Assuming that the target position is \(x_0, y_0, z_0\), we can get,

\[
R_{0k} = \sqrt{(X_k - x_0)^2 + (Y_k - y_0)^2 + (Z_k - z_0)^2} \tag{3}
\]

and the direction cosine between the pseudolite and the receiver is depicted below:

\[
\begin{align*}
\alpha_k &= \frac{X_k - x_0}{R_{0k}} \\
\beta_k &= \frac{Y_k - y_0}{R_{0k}} \\
\gamma_k &= \frac{Z_k - z_0}{R_{0k}}
\end{align*} \tag{4}
\]

Assuming that

\[
G = \begin{bmatrix}
\alpha_1 & \alpha_2 & \beta_1 & \beta_2 & \gamma_1 & \gamma_2 \\
\alpha_1 & \alpha_3 & \beta_1 & \beta_3 & \gamma_1 & \gamma_3 \\
\alpha_1 & \alpha_4 & \beta_1 & \beta_4 & \gamma_1 & \gamma_4 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\alpha_1 & \alpha_N & \beta_1 & \beta_N & \gamma_1 & \gamma_N
\end{bmatrix}
\]

\[
\Delta X = \begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix}, \quad \Delta R = \begin{bmatrix}
\Delta R_2 \\
\Delta R_4 \\
\vdots \\
\Delta R_N
\end{bmatrix}
\]

the least squares solution can be obtained as:

\[
\Delta X = (G^T G)^{-1} G^T \Delta R \tag{6}
\]

The dimension of symmetric matrix \(H = G^T G^{-1}\) is \(3 \times 3\). The positioning error is proportional to the distance error. Difference in position DOP (PDOP) used in traditional satellite navigation such as GPS or Glonass is that relative PDOP (RPDOP) was used to represent the magnification level measurement error in pseudorange difference positioning algorithm. Therefore,

\[
\begin{bmatrix}
RPDOP \\
RHDOOP \\
RVDOOP
\end{bmatrix} = \begin{bmatrix}
\text{tr}(G^T G) \\
\text{tr}(H) \\
R_{11} + R_{22} + R_{33}
\end{bmatrix}
\]

\[
\begin{bmatrix}
RPDOP \\
RHDOOP \\
RVDOOP
\end{bmatrix} = \begin{bmatrix}
R_{11} + R_{22} + R_{33}
\end{bmatrix}
\]

\[
\begin{bmatrix}
RPDOP \\
RHDOOP \\
RVDOOP
\end{bmatrix} = \begin{bmatrix}
R_{11} + R_{22} + R_{33}
\end{bmatrix}
\]
Spatial configuration and availability are the key performance indicators of the navigation constellation. The availability is defined as the percentage of coverage for RPDOP:

\[ A_R = \frac{N^n_R}{N^R} \times 100\% \]  \hspace{1cm} (8)

where
- \( A_R \) Denotes the availability of the constellation;
- \( N^R \) Specifies the number of sampling;
- \( N^n_R \) Refers to sampling calculation statistics;
- \( n \) Represents the threshold of RPDOP that meets the requirement.

III. PROPOSED ALGORITHM

Holland from the University of Michigan proposed the GA [14], which uses selection, crossover, and mutation operators to involve the evolutionary process of organisms [15-17]. With powerful parallel processing and global optimization ability, this study presented an improved AGA to realize the optimization of the constellation layout design of pseudolites under multiple constraints.

A. Coding Format

This study uses the floating-point encoding method as it is relatively simple and can maintain the diversity of the population. The space of service area allows the pseudolite to be arranged as shown in Fig. 1. Using the grid split method, the service area was divided into small boxes each of size \(50 \times 50 \times 50 \text{ m}^3\); the center point of the box was used to calculate the RPDOP value. Similarly, assuming that the pseudolite is lying on the ground within a small square area \(50 \times 50 \text{ m}^2\), each grid can be represented by \(\text{Num}_{L_i}, \text{Num}_{B_i}, \text{Num}_{H_i} \) in three-dimensional form. Therefore, a pseudolite constellation layout can be expressed as:

\[ x_i = \left\{ \text{Num}_{L_1}, \text{Num}_{B_1}, \text{Num}_{H_1}, \text{Num}_{L_2}, \text{Num}_{B_2}, \text{Num}_{H_2}, \ldots, \text{Num}_{L_N}, \text{Num}_{B_N}, \text{Num}_{H_N} \right\} \]  \hspace{1cm} (9)

B. Fitness

The fitness function represents the individual’s character with a monotonous and non-negative property. Usually, the fitness function is better if it is bigger in value.

The design goal of the pseudolite constellation layout is the minimal average RPDOP value of each location within the service area. According to the requirements of positioning accuracy and the accuracy of the pseudorange, the DOP can be a reasonable range as estimated below:

\[\text{RPDOP} < 8\]  \hspace{1cm} (10)

Equation (11) shows that for the positioning system, the smaller the RPDOP value is, the better it is; however, the fitness function is monotonically increasing, and so we can derive the corresponding fitness function as follows:

\[ \text{Fit}_i = \begin{cases} 2 - \frac{\text{RPDOP}_i^2}{8} & \text{RPDOP}_i < 8 \\ 0 & \text{RPDOP}_i \geq 8 \end{cases} \]  \hspace{1cm} (11)

The principle of pseudolite positioning system is based on CDMA, which like GPS has a large pseudolite signal power strength that may cause near-far interference. It is reasonable to assume that the signal power difference among all receiver signals is less than 10 dB and this will meet the following distance constraint:

\[ g_x = \sqrt{\sum_{k=1}^{N} \left( (X_k - x)^2 + (Y_k - y)^2 + (Z_k - z)^2 \right)} \]  \hspace{1cm} (12)

The area constraints associated with the constellation layout of pseudolites mainly refer to the geological terrain or traffic that is needed in order to ensure the safety of the equipment and personnel. Given one or more small regions, the total regional area can be expressed as:

\[ D = \bigcup_{i=1}^{m} D_i \]

Combinatorial optimization problems with inequality constraints of objective function can be shown mathematically as:

\[ \text{Maximize } \sum_{i \in D} \text{Fit}_i \]  \hspace{1cm} (13)

C. Selection Operator

Selection operator produces the offspring with the probability \( P_s \), the higher of the fitness, the bigger of the selection probabilities.
\[
P_{T_s} X = x_i = \frac{f_{x_i}}{\sum_{k=1}^{N} f_{x_k}}
\]  

(14)

\(f_{x_i}\) denotes the fitness function value of individual \(x_i\) in population \(X\), which satisfies the following constraints:

\[
\begin{aligned}
P x_i & \geq 0 \\
\sum_{i=1}^{N} P x_i &= 1
\end{aligned}
\]  

(15)

The chromosomes are selected randomly into the next generation with a probability proportional to its fitness, which may damage or lose the best individual in the current population. The elite reservation strategy was introduced to improve the efficiency and the convergence speed of the algorithm.

D. Adaptive Crossover Operator

Crossover is a critical operation to create a new population by generating offspring and inheriting the merits from the “parents.” Single-point crossover operator \(T_c\) is widely used with probability \(P_c\).

\[
P_{T_c} x_1, x_2 = Y = \begin{cases} 
\frac{k_p}{l} Y = x_1 \\
1 - p_c + \frac{k_p}{l} Y = x_1 
\end{cases}
\]  

(16)

where \(l\) is the length of chromosome.

The first step in the evolution of AGA is that if the best individual is more suitable than other optimal chromosomes that have a higher probability of being selected, then the GA can converge to the local minimum. In order to maintain the diversity of population, an adaptive crossover operator is proposed.

\[
P_c = \begin{cases} 
P_{C1} - \frac{P_{C1} - P_{C2}}{f_{max} - f_{avg}} f' \geq f_{avg} \\
P_{C1} & f' < f_{avg}
\end{cases}
\]  

(17)

where

- \(f_{max}\) Represents the maximum fitness value;
- \(f_{avg}\) Represents the average fitness value;
- \(f'\) is the largest value between the pair of parents;
- \(P_{C1}\) Denotes the maximum probability of crossover;
- \(P_{C2}\) Denotes the minimum probability of crossover.

E. Adaptive Mutation Operator

The mutation operator is introduced to change the gene with a given probability that can maintain the diversity of population, improve the global search ability, and prevent the premature convergence of the GA.

The mutation rate was used to control the individual gene reproduction probability, which should maintain a reasonable value range of \(0.001 \sim 0.1\). A bigger value will lead to the degeneration of random search, whereas a small value will not generate good genes. In order to avoid premature convergence to local minima and improve the ability of global optimization, the mutation probability \(P_m\) is introduced for dynamic adjustment according to the fitness function value of the population; this will ensure the diversity of population.

\[
P_m = \begin{cases} 
P_{m1} - \frac{P_{m1} - P_{m2}}{f'_{max} - f_{avg}} f' \geq f_{avg} \\
P_{m1} & f' < f_{avg}
\end{cases}
\]  

(18)

where,

- \(f_{max}\) Represents the maximum fitness value;
- \(f_{avg}\) Represents the average fitness value;
- \(f'\) is the largest value between the pair of parents.
- \(P_{m1}\) Represents the maximum mute probability;
- \(P_{m2}\) Represents the minimum mute probability.

F. Improved Adaptive Genetic Algorithm

AGA usually retain the best individual where the algorithm is very likely to fall into local optimum in the search space. This is because the cross rate between individuals is very small, which results in slow evolution, and so, the whole algorithm appears as a “premature” phenomenon. To have a population with the largest fitness value of individuals, the crossover and mutation rate were increased, which also improves the performance of the individual crossover and mutation rate.

\(f_{max}\) and \(f_{min}\) ratios reflect the degree of approximation of the whole population. The closer the GA is, the more likely it is to fall into local optima. \(f_{avg}\) and \(f_{max}\) ratios reflect the internal distribution of population fitness.

The \(p_c\) and \(p_m\) make adaptive changes according to the degree of concentration of the population, the population keep the original value unchanged in this generation when the conditions are not met.

IV. SIMULATION EXPERIMENTS

In our experiments, we test the performance of the proposed algorithm of the constellation layout of pseudolites based on the improved AGA. The result was compared to the GA. The key performance of the AGA depends on the parameters selected. Without loss of generality, the pseudolite can be placed on Earth’s ground, and this is called as “ground pseudolite.” It is assumed that the time synchronization between each pseudolite has been completed.

A. Main Parameters

The convergence speed and accuracy of GA were affected by evolutionary parameters such as the
population size, crossover, and mutation rate. All parameters are shown in Table I.

<table>
<thead>
<tr>
<th>Symbolic</th>
<th>Physical meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>Population size</td>
<td>500</td>
</tr>
<tr>
<td>$P_s$</td>
<td>Selection probability</td>
<td>[0,1] random</td>
</tr>
<tr>
<td>$T$</td>
<td>Maximum iteration</td>
<td>20</td>
</tr>
<tr>
<td>$x_i$</td>
<td>Chromosome layout solution</td>
<td></td>
</tr>
<tr>
<td>$l$</td>
<td>Chromosome length</td>
<td>gene number</td>
</tr>
<tr>
<td>$f(x_i)$</td>
<td>Fitness of individual $x_i$</td>
<td>fitness value</td>
</tr>
<tr>
<td>$P_{c1}$</td>
<td>Maximum cross probability</td>
<td>0.9</td>
</tr>
<tr>
<td>$P_{c2}$</td>
<td>Minimum cross probability</td>
<td>0.6</td>
</tr>
<tr>
<td>$P_{m1}$</td>
<td>Maximum mute probability</td>
<td>0.01</td>
</tr>
<tr>
<td>$P_{m2}$</td>
<td>Minimum mute probability</td>
<td>0.001</td>
</tr>
</tbody>
</table>

**B. Simulation Analysis**

In order to simplify the analysis without loss of generality, assume that the service area is a rectangular area and the coordinates of pseudolite are represented as:

$$D_{Service} = \begin{cases} 
  x : 5km \sim 40km \\
  y : -12km \sim 12km \\
  z : 1km \sim 20km
\end{cases}$$

(19)

Pseudolites were allowed to be arranged in the rear surface area, which had the launch coordinate system; the ranges can be expressed as:

$$D_{Layout} = \begin{cases} 
  x : -10km \sim 15km \\
  y : -10km \sim 20km \\
  z : 0m \sim 50m
\end{cases}$$

(20)

Using the improved AGA, after many iterations, for the global optimization of the constellation layout, the optimal three-dimensional space coordinates of pseudolite constellation can be obtained for this scenario.

Using the gradual convergence of AGA, the average fitness of population geometric growth vs. iteration number is depicted in Fig. 2. The excellent genes become the main part of the new population, and gradually tend to the optimal solution. In general, the AGA outperforms GA in the aspect of convergence speed.

![Fig. 2. The curve of fitness value vs. iteration](image)

**Fig. 3. Distribution map of RPDOP in service area**

![Fig. 4. Positioning error on a single static point (Unit:m)](image)

**Fig. 5. Scatter of dynamic positioning error: east error vs. north error (Unit:m)**

The cube of RPDOP distribution of the service area is shown in Fig. 3. Different colors represent different values of the DOP in the figure; if the DOP value is small, then it falls in the blue spectrum, and if the DOP value is large, then it is shown in the red spectrum. We have carried out the slice analysis of the RPDOP cube in different dimensions, and can see from Fig. 3 that the...
DOP value of the vast majority of points is small, which can satisfy the high precision positioning requirements of the user receiver. However, the DOP value is larger in the bottom and posterior service areas where the positioning accuracy can get degraded. Assuming that the receiver is located in the geometric center of the service area, in the static positioning mode, the three-dimensional positioning accuracy of the receiver is shown in Fig. 4, the horizontal positioning error scattered point view is shown in Fig. 5. In dynamic positioning mode, we randomly select a continuous trajectory of service area, for example, the three-dimensional positioning error of the receiver is shown in Fig. 6. Each GDOP, HDOP, and VDOP value corresponding to the point of the curve is shown in Fig. 7. In addition to the service area in the last end and low-end, the DOP values of each point can basically meet the requirement of receiver positioning accuracy, and therefore, increasing the number of pseudolites can significantly improve the entire service area of DOP values, i.e., one pseudolite deployed at the back end of the service area can dramatically improve the positioning precision of the receiver.

Fig. 6. Dynamic positioning error (Unit:m)

Fig. 7. GDOP, HDOP, and VDOP curves

V. CONCLUSION

The constellation layout of pseudolite is a constrained nonlinear combinatorial optimization problem. To solve this problem, in this study, a novel AGA that makes the full use of the inner parallel mechanism and has fast convergence was proposed. Comprehensive simulation and analysis show that the algorithm is effective and can quickly search for the optimal layout scheme, which improves the positioning precision and the system availability.

ACKNOWLEDGMENT

The authors would like to thank the Associate Editor and anonymous reviewers for their valuable comments, which improved the presentation of this paper.

REFERENCES

Wright-Patterson AFB oh School of Engineering and Management, 2006.


JianCai Song received the B.S. degree in computer science and technology from HeBei University of Technology, TianJin, China, in 2003. He is currently pursuing the Ph.D. degree, with a major in communication engineering at TianJin University. His research interest include artificial intelligence, evolutionary computation, and their applications in design and optimization on the integrity navigation application of multi-constellation GNSS with INS.

ChunPing Hou received the M.Eng. and Ph.D.degrees, both in electronic engineering, from Tianjin University, Tianjin, China, in 1986 and 1998, respectively. She was a Post-Doctoral Researcher with the Beijing University of Posts and Telecommunications, Beijing, China, from 1999 to 2001. Since 1986, she has been with the faculty of the School of Electronic and Information Engineering, Tianjin University, where she is currently a Full Professor and the Director of the Broadband Wireless Communications and 3-D Imaging Institute. Her current research interests include wireless communication, 3-D image processing, and the design and applications of communication systems.

Gui Xiang Xue received the M.Eng. and Ph.D.degrees, both in electronic engineering, from Tianjin University, Tianjin, China, in 2006 and 2008, respectively. She has been with the faculty of the School of Computer Science and Technology, HeBei University of technology, where she is currently a teacher. Her current research interests include Artificial Intelligence, cloud computing, and the design and applications of communication systems.

Dr. Maode Ma received his Ph.D. degree in Department of Computer Science from Hong Kong University of Science and Technology in 1999. Now, Dr. Ma is an Associate Professor in the School of Electrical and Electronic Engineering at Nanyang Technological University in Singapore. He has extensive research interests including network security and wireless networking. Dr. Ma has over 300 international academic publications including more than 140 journal papers and over 170 conference papers. He currently serves as the Editor-in-Chief of International Journal of Computer and Communication Engineering and International Journal of Electronic Transport. He also serves as a Senior Editor for IEEE Communications Surveys and Tutorials, and an Associate Editor for International Journal of Security and Communication Networks, International Journal of Wireless Communications and Mobile Computing and International Journal of Communication Systems. Dr. Ma is the Fellow of IET and a senior member of IEEE Communication Society and IEEE Education Society. He is the Chair of the IEEE Education Society, Singapore Chapter. He is serving as an IEEE Communication Society Distinguished Lecturer from 2013 to 2016.