

Improvement of Compressive Sampling and Matching Pursuit Algorithm Based on Double Estimation

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Abstract—Compression sampling matching pursuit algorithm (CoSaMP) is widely applied to image reconstruction owing to its high precision of reconstruction, robustness, and simple operation. In this paper, we propose a new method, the CoSaMP based on double estimation, to properly overcome the shortcomings in CoSaMP for choosing too much optional atoms and imprecise choice. The concept of the maximum estimation, which is called *maxValue*, is proposed as a key point. The *maxValue* is calculated from the largest relevant $2k$ atoms of selected atoms in each iteration using least square solution method. At the next step, the *maxValue* is regarded as a selection condition for the optional atoms to decrease the number of candidate atoms and increase its accuracy. The simulation results show that this algorithm can precisely reconstruct the original signal. Under the same sampling rate, compared to the CoSaMP, the proposed method can greatly improve the PSNR and reconstruction performance.

Index Terms—Compressed sensing, double estimation, sparse reconstruction, atomic precision filter

I. INTRODUCTION

The traditional sampling system based on Shannon theorem wastes a lot of sampling data when compressing data. Compressive Sensing (CS) [1]-[3] is a new sampling theory, which is a new method of signal acquisition and reconstruction based on the signal sparse representation, the non-correlation of the measurement matrix, and approximation theory. It was put forward by Donoho, Tao Zhexuan and E. Candes in 2006. The core idea of CS is to make compression and sampling run at the same time, which collects non-adaptive linear projection of the signal (measured) firstly, and then reconstructs the original signal from the measured value according to the corresponding reconstruction algorithm.

CS was proposed, immediately caused extensive concern of academia and industry. In information theory, image processing, earth science, optical/microwave imaging, pattern recognition, wireless communications, atmosphere, geology and other fields has been highlighted [4]-[7].

CS can obtain the signal of discrete samples under the condition of far less than the Nyquist sampling rate and ensure the reconstruction signal without distortion. CS theory mainly includes three aspects: 1) the sparse representation of signals, 2) the design of the measurement matrix, and 3) signal reconstruction algorithm. One of the most important aspects is the design of the reconstruction algorithm, namely how to fast and efficiently recover the high dimension original signal from the observation of the low dimensional data. At present, the studies of domestic and foreign scholars have proposed many reconstruction algorithms, such as Matching Pursuit (MP) algorithm, Orthogonal Matching Pursuit algorithm (OMP) [8] and Regularized Orthogonal Matching Pursuit algorithm (ROMP) [9], etc.

MP algorithm has low convergence speed, long reconstruction time, and large computational complexity. OMP algorithm convergence speed is faster than MP algorithm, but it just adds one atom to signal support set in each iteration. As a result, the atom selection efficiency is low. BP [10] algorithm measures value of the minimum required, but its computation complexity is high and need to know the signal sparsity in advance. ROMP algorithm is based on the idea of regularization, the efficiency atom picking is higher with prior information of the signal sparse degree. However, it is hard to know in practice.

In this paper, we propose a new method which is called compression sampling matching pursuit algorithm based on double estimation. This method can overcome the shortcomings in CoSaMP for choosing too many optional atoms and imprecise choice. The new method offers a comparably theoretical guarantee as a better optimization-based approach. Its numerical results are even more attractive as it outperforms the CoSaMP algorithms in extensive simulations.

II. COMPRESSION PERCEPTION MODEL

Whether the signal is sparse or similar to sparse is the premise and key of using compressed sensing theory. In real life there are many signals can be regarded as approximate sparse, such as digital image signal, video signal, and so on, which creates good conditions for the application of the compression perception.

Mathematically speaking, a vector $\mathbf{f} \in \mathbf{R}_n$ which we expand in an orthonormal basis: the original signal \mathbf{f} can

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be represented under an orthonormal basis (such as a wavelet basis) $\Psi = \{\psi_1, \psi_2, \psi_3, \dots, \psi_N\}$ as follows:

$$\mathbf{f} = \sum_{i=1}^N \psi_i \alpha_i \quad (1)$$

where α_i is the coefficient sequence. It will be convenient to express \mathbf{f} as $\Psi\alpha$ (where Ψ is the $N \times N$ matrix with $\psi_1, \psi_2, \psi_3, \dots, \psi_N$ as columns). The final obtained formula is as follows:

$$\mathbf{y} = \Phi\mathbf{f} = \Phi\Psi\alpha \quad (2)$$

where \mathbf{y} is the observation vector of length M , Φ is the observation matrix, is an $M \times N$ random matrix, \mathbf{f} is a K -sparse signal of length N , K is the number of nonzero elements in \mathbf{f} , with $K < M < N$.

This vector is sparse in a strict sense since all but a few of its entries are zeros; we will call S -sparse if this signal has at most S nonzero entries.

Measurement matrix falls broadly into three categories: random measurement matrix, partial random measurement matrix, and deterministic measurement. They all satisfy the restricted isometric principle (RIP) [11], [12]. The commonly used measurement matrixes include the Gaussian random matrix, Bernoulli random matrix, Fourier random matrix, Belize random matrix, etc. By using the measurement matrix the original signal can be reduced to a lower dimension. In order to accurately reconstruct the \mathbf{f} , generally, $M \ll N$, by solving the minimum l_0 norm for problems to get exact solution of \mathbf{f} :

$$\min \|\alpha\|_0 \quad s.t. \quad \mathbf{y} = \Phi\mathbf{f} = \Phi\Psi\alpha \quad (3)$$

Equation (3) leads to solving the l_0 -minimization problem, which unfortunately is NP-hard in general. Subsequently, Chen, Donoho puts forward that when the measurement matrix Φ meets the null space property (NSP) and the *restricted isometry property*, l_1 -minimization will perform an exact reconstruction.

$$\min \|\alpha\|_1 \quad s.t. \quad \mathbf{y} = \Phi\mathbf{f} = \Phi\Psi\alpha \quad (4)$$

III. DOUBLE ESTIMATION RECONSTRUCTION ALGORITHM

This section presents a summary of CoSaMP and the improved algorithm. As shown in Fig. 1 (a)-(b). Here, in the k -th iteration, \mathbf{r}_k and \mathbf{F}_k represent the residue and the estimated signal's support (called *finalist*), respectively. The CoSaMP uses two different tests in each iteration. But, the size of their finalist is fixed. However, as shown in Fig. 1 (b), the size of the improved algorithm's finalist keeps changing.

In this paper, since the choice of original algorithm atoms are not precise enough, we propose a double estimation based compression sampling matching pursuit algorithm.

As shown in Fig. 1 (a), the CoSaMP [13] adopts the maximal correlation test, and just $2k$ (k -the sparsity of

input signal) candidates are added in each iteration. As a result, the number of \mathbf{F}_k is fixed between $2k$ and $3k$, and the number of incorrect atoms is far greater than the right one's. In fact, the number accurate atoms is no more than k in optional atom set. Because the atoms which are participated in the least squares in each iteration are not precise enough, the accuracy of the recovery signal is reduced. In this situation, this paper proposes a double estimation in atom selection. In Fig. 1 (b), we use the largest $2k$ relevant atoms selected in each iteration and the \mathbf{r}_k to solve least square solution to get the maximum estimation which is called *maxValue*. The concept of the *maxValue*, is proposed as a key point. The *maxValue* is saved as a second choice of a reference value. Then use least square solution to calculate the \mathbf{y} (noisy sample vector) and the candidate set which is composed of $2k$ selected atoms and backtracking atoms, and finally get current operation value. If the current operation value is no less than $u * maxValue$ (u is the threshold value between 0 and 1), the corresponding position of the atoms will be added to *finalist*. At last, the \mathbf{y} and the *finalist* use least square to get approximation of the target signal and \mathbf{r}_k . In this paper, we regard the reconstruction of peak signal-to-noise ratio (PSNR) and probability of exact recovery signal as the judgment basis for the accuracy of signal reconstruction.

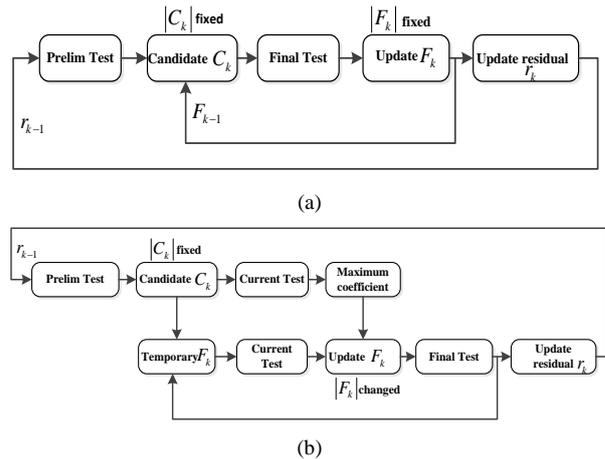


Fig. 1. A conceptual diagram of (a) The CoSaMP; (b) The proposed CoSaMP

According to the relevant literature showing that when $\Phi\Psi$ satisfies the *restricted isometry property*, CoSaMP algorithm can accurately reconstruct the original signal. This paper, based on the experimental conclusion proposed by Donoho $M \approx 4k$ [14] of image dilution degrees and image observation, proves the effectiveness of the proposed method through improving the algorithm simulation.

A. CoSaMP Image Reconstruction Algorithm

Currently, there are a lot of compressed sensing image reconstruction algorithms, such as the minimization of

norm l_1 algorithm, matching pursuit algorithm, iterative threshold algorithm, gradient algorithm and so on. CoSaMP algorithm is one of matching pursuit algorithms used for compressed sensing image reconstruction. The following are concrete steps of the CoSaMP algorithm.

TABLE I: CoSaMP ALGORITHM

Input: Sampling matrix Φ , noisy sample vector \mathbf{u} , sparsity level s	
Output: An s -sparse approximation \mathbf{a} of the target signal	
1) Initialization:	
$\mathbf{a}^0 \leftarrow 0$	{ Trivial initial approximation }
$\mathbf{v} \leftarrow \mathbf{u}$	{ Current samples = input samples }
$k \leftarrow 0$	
2) repeat	
(1) $k \leftarrow k + 1$	
(2) $\mathbf{y} \leftarrow \Phi^T \mathbf{v}$	{ Form signal proxy }
(3) $\Omega \leftarrow \text{Supp}(\mathbf{y}_{2s})$	{ Identify large components }
(4) $T \leftarrow \Omega \cup \text{Supp}(\mathbf{a}^{k-1})$	{ Merge supports }
(5) $\mathbf{b} _T \leftarrow \Phi^+ T$	{ Signal estimation by least-squares }
(6) $\mathbf{b} _{T^c} \leftarrow 0$	
(7) $\mathbf{a}^k \leftarrow \mathbf{b}_s$	{ Prune to obtain next approximation }
(8) $\mathbf{v} \leftarrow \mathbf{u} - \Phi \mathbf{a}^k$	{ Update current samples }
until halting criterion true	

The improved CoSaMP algorithm based on double estimation is summarized in Table II.

TABLE II: IMPROVED CoSaMP ALGORITHM

Input: Sampling matrix Φ , noisy sample vector \mathbf{u} , $s = (1/4) * \text{length}(\mathbf{u})$, Threshold selecting μ	
Output: An s -sparse approximation \mathbf{a} of the target signal	
1) Initialization:	
(1) $\mathbf{a}^0 \leftarrow 0$	{ Trivial initial approximation }
(2) $\mathbf{V} \leftarrow \mathbf{u}$	{ Current samples = input samples }
(3) $k \leftarrow 0$	
2) repeat	
(4) $k \leftarrow k + 1$	
(5) $\mathbf{y} \leftarrow \Phi^T \mathbf{V}$	{ Form signal proxy }
(6) $\Omega \leftarrow \text{Supp}(\mathbf{y}_{2s})$	{ Identify large components }
(7) $\mathbf{b} _\Omega \leftarrow \Phi_\Omega^T \mathbf{V}$	{ Signal estimation by least-squares }
(8) $F \leftarrow \Omega \cup \text{Supp}(\mathbf{a}^{k-1})$	{ Merge supports }
(9) $\mathbf{b} _F \leftarrow \Phi_F^T \mathbf{u}$	{ Signal estimation by least-squares }
(10) $R \leftarrow \text{Supp}(\mathbf{b} _F > \mu * \mathbf{b} _\Omega)$	{ merge final supports }
(11) $\mathbf{b} _R \leftarrow \Phi_R^T \mathbf{u}$	{ signal estimation by least-squares }
(12) $\mathbf{b} _{R^c} \leftarrow 0$	
(13) $\mathbf{a}^k \leftarrow \mathbf{b}_s$	{ Prune to obtain next approximation }
(14) $\mathbf{V} \leftarrow \mathbf{u} - \Phi \mathbf{a}^k$	{ Update current samples }
until halting criterion true	

From the steps of CoSaMP algorithm, the running efficiency of the entire CoSaMP algorithm is determined by (3) and (7). In (3), it identifies $2k$ positions of the maximum absolute value of the elements of the vector which is formed by the inner product of each column of Φ and \mathbf{v} in order to add it to signal support set in the next step; In fact, in the $2k$ selected atoms, at most k

atoms are optimal, and most of the rest of the atoms are false. These false atoms can not only increase operation error, but also will reduce the recovery precision of the signal. Aiming at this situation, the method of the double estimation of atoms proposed by this paper, can effectively reduce the dimension of the matrix and improve the signal sparse approximation accuracy.

From the steps of the improved CoSaMP algorithm, the size of the improved algorithm's finalist keeps being changed by step (10). In (10), it can make the finalist more accurate by using the threshold μ .

IV. SIMULATION AND ANALYSIS

Experimental platform was MATLAB R2013b. We selected the international standard test image lena.bmp (256*256*8bit) as the testing image. The wavelet transform base [15]-[16] is used as sparse basis, and Gaussian distribution matrix is used as measurement matrix. At the same time, PSNR is used to evaluate the quality of the reconstructed image.

Fig. 2 shows that the proposed algorithm in the signal recovery PSNR is significantly better than the OMP, CoSaMP algorithm, etc. The image reconstruction effect is more obvious. Below are reconstructed images under different sampling rate and the original image.

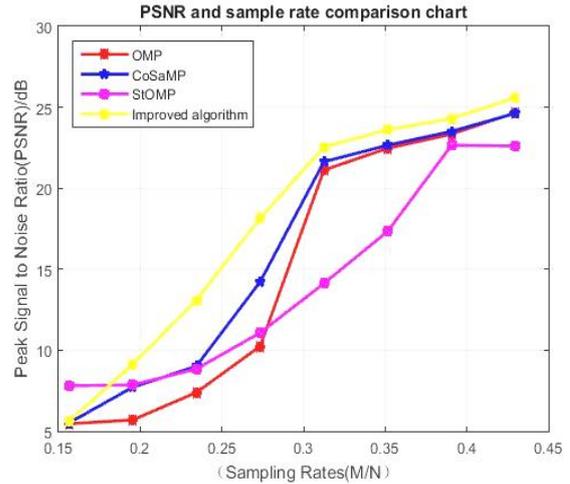


Fig. 2. Different algorithms SNR and sample rate comparison chart

From Fig. 2, it is obvious that the PSNR of the improved CoSaMP reconstructed image is always greater than standard CoSaMP's at different sampling rates, that is to say, the quality of the reconstructed image of the improved CoSaMP is better than standard CoSaMP. As shown in Fig. 3 and Fig. 4, the reconstructed image becomes increasingly clearer with the increase of the sampling rate. The reconstructed image of the improved CoSaMP is clearer than the standard CoSaMP when they are at the same sampling rate. The improved algorithm has higher accuracy in image sparse approximation. PSNR is improved by 1~1.5dB. Compared with the original algorithm, our algorithm can accurately reconstruct sparse signal, and can greatly improve calculation accuracy.

According to the analysis of the Fig. 2, when the sampling rate is less than 0.4, the PSNR of the proposed algorithm is more superior than the original algorithm. Under the low sampling rate, signal reconstruction effect is more accurate than the standard CoSaMP.

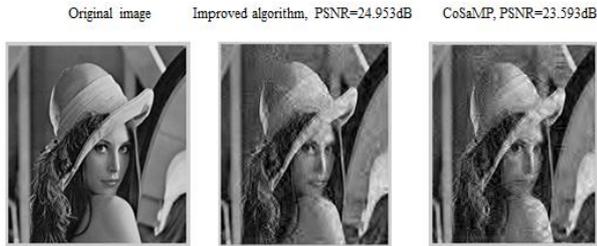


Fig. 3. Reconstruction comparison chart (M/N=0.4)



Fig. 4. Reconstruction comparison chart (M/N=0.6)

Fig. 5 shows the curves of the probability of exact recovery signal with the proposed algorithm. 1000 simulations were conducted for each pair of sparsity k (i.e., 4,12,20,28,36 non-zero entries.) and the number of measurements M . Fig. 5 shows the proposed algorithm has higher probability of exact reconstruction.

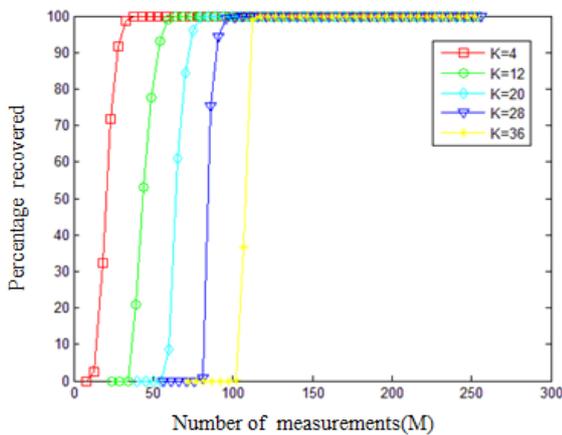


Fig. 5. Probability of exact recovery signal with proposed algorithm. the original signal has length of $N = 256$ with $k = 4, 12, 20, 28, 36$ non-zero entries.

This experiment investigates the probability of exact recovery signal. For each value of the number of measurements (M), we generate a signal \mathbf{x} of sparsity k and its measurement $\mathbf{y} = \Phi \mathbf{x}$. Then we use the proposed algorithm and StOMP algorithm to recover \mathbf{x} . This procedure is repeated 1000 times for each value of M . We then calculate the probabilities of exact reconstruction. Fig. 6 depicts these probability curves of Gaussian and sparse signals, respectively. One can see clearly that the

performance of the proposed algorithm rises quickly under the same sparsity and the number of measurements.

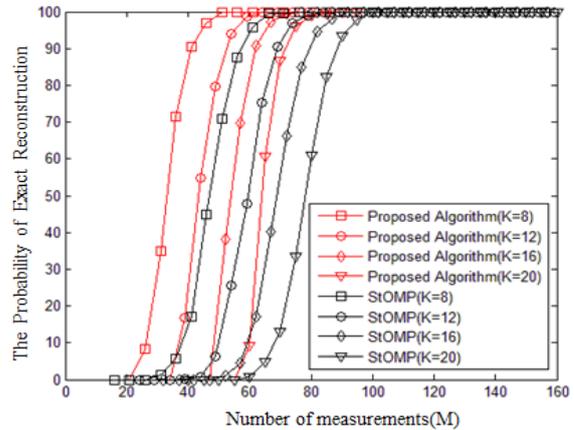


Fig. 6. Probability of exact recovery signal with StOMP and proposed algorithm. the original signal has length of $N = 256$ with $k = 8, 12, 16, 20$ non-zero entries.

The measurement matrix Φ is Gaussian distribution matrix. 1000 simulations were conducted for each pair of sparsity k and the number of measurements M . Fig. 5 shows the curves of the probability of exact recovery signal with proposed algorithm under different k . Fig. 7 shows the probability of exact reconstruction with CoSaMP and the proposed algorithm, the original signal has length of $N = 256$ with $k = 8, 12, 16, 20$ (i.e., 8,12,16,20 non-zero entries.). One can see clearly that the performance of the proposed algorithm rises quickly under the same sparsity and the number of measurements.

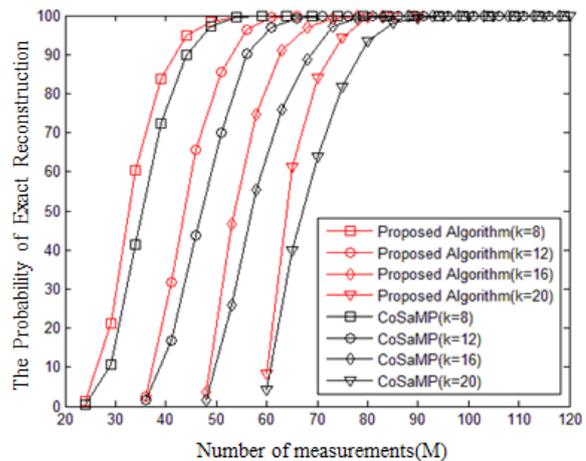


Fig. 7. Probability of exact recovery signal with CoSaMP and proposed algorithm. the original signal has length of $N = 256$ with $k = 8, 12, 16, 20$ non-zero entries.

Fig. 7 also shows the proposed algorithm has higher probability of exact reconstruction. Therefore, through double estimation, it reduces operation dimensions of the matrix and improves reconstruction accuracy.

V. CONCLUSIONS

Based on the CoSaMP in each iterative process, the number of redundant atoms being selected is too much

and the selection accuracy of matching atoms is not high. Thus leads to worse reconstruction effect of image. Meanwhile, it also increases the complexity of calculation. Aiming at those problems mentioned above, based on the atoms selection of the original algorithm, the double estimation method is proposed in this paper to carry out the relatively accurate secondary scanning to atoms. Thereby, it reduces operation dimensions of the matrix and improves reconstruction accuracy. In terms of the method proposed by this paper, the precision of recovery is greatly improved, and the image restoration effect is obviously enhanced. Therefore, through double estimation, it can significantly reduce the redundancy of selected atoms and improve the number of accurate matching atoms to achieve a better reconstruction effect. In addition, this method has great practical value to those systems which have a higher requirement on accuracy.

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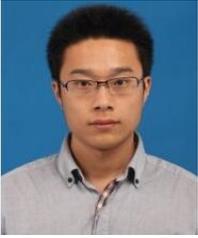
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