Community Detection in Weighted Networks via Recursive Edge-Filtration

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Abstract —In this paper, we present a Weighted Filtration Coefficient (WFC) ψ and a corresponding filtration method to detect the communities in weighted networks. In our method, a weighted network can be divided into groups by recursive filtration operations, and the dividing results are evaluated by the ψ. We prove that optimization on local ψ enables us to obtain maximal global weighted modularity Q^w, which corresponds to the correct communities. For a weighted network with m edges and c communities, the weighted communities can be detected in time $O((c+1)m)$, which is in linear scale time with the number of edges. Furthermore, the local weighted communities can be detected in an increasing order according to the edge weights between them. This division can reveal different levels of close connections between the nodes.

Index Terms—Weighted networks, weighted filtration coefficient, communities

I. Introduction

As an efficient tool, complex networks have already been used in many large complex systems such as the Web [1], [2], social networks [3], citation networks [4], and so on. It has been found that the complex networks have an important property, the community structure [5], which can efficiently describe the relationships between nodes and groups and give researchers a deep insight into the structure of the networks. Recently, community detection methods have attracted great interests.

In unweighted networks, the widely accepted definition of communities is groups of nodes with denser connections within them and sparse connections between them [5]. The communities can reflect perfectly the close connections between functional units, and a large number of algorithms have been proposed to detect the communities recently. These methods can be classified into several categories, such as divisive algorithms [6], optimization methods [7], dynamic algorithms [8], and so on.

Most of these methods have been developed to detect the separated communities in which each node belonging to one community. However, in real networks, nodes are often shared between communities, which lead to overlapping communities [9]. In the last two years, overlapping-community detection has become quite popular [10], [11]. Zhou *et al.* [10] presented an ant colony based overlapping community detection algorithm, which mainly includes ants' location initialization and movement. Zhan *et al.* [11] developed an effective encoding scheme for overlapping communities, and introduced two measures for the informativeness of nodes.

Besides overlapping characteristic, the community structure of real world networks often exhibits multiple scales [12], [13]. In this case a network can be divided at different hierarchical levels that constitute the different topological description. Chen *et al.* [12] proposed a novel hierarchical structure to dig finer information by partitioning the members into several levels, according to the coefficients that they belonged to. Lin *et al.* [13] presented a novel integer programming approach that was designed to detect hierarchical community structures in social networks.

However, for real networks, apart from the interactions between nodes described by edges, it is essential to realize that the interactions are intrinsically weighted, which is not considered in many recent studies. For instance, in social networks the social ties between individuals may be strong or weak [14], and in the WWW the data traffic between websites may be heavy or light [1]. Furthermore, the weights have great impact on the properties or functions of these networks in many situations, e.g., motif statistics [15] and synchronization dynamics [16], [17] and so on. It can be seen that the edge weights, which can describe the strength of the interactions between nodes, are important to the functions of weighted networks. Newman defined weighted communities as groups of nodes where the weights on the edges are relatively larger than the external weights between them, and proposed a weighted modularity denoted Q^w [18]. Recently, several algorithms have been proposed based on optimizing Q^w [18], such as the WGN algorithm [18], the WEO algorithm [19], [20], the random walk-based method [21], etc. The WGN algorithm is generalized from its unweighted version GN algorithm [5] by calculating the weighted edge betweenness [18], and the WEO algorithm is generalized from its unweighted version EO algorithm [19] by using

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Q^w [18]. For WGN and WEO, only one edge is deleted in each step to split a network, and the modularity function Q^w [18] must be calculated after each division, which leads to high computational cost. Random walk-based method is based on a transition probability matrix, which is obtained by calculating the conductance between any two nodes. When the method is applied to dense weighted networks, the problem of high cost inevitably occurs [21], [22].

In this paper, we present a method that can divide a weighted network into groups through deleting many edges in each filtration operation. In the filtration process, we present a weighted filtration coefficient (WFC) ψ to evaluate the divided results. We proved that recursive optimization on local ψ allow us to obtain maximal global modularity Q^w, which corresponds to correct community structure. The method can divide the network in parallel and the computation complexity of the method is $O((c+1)m)$ for a network with m edges and c communities. Furthermore, local weighted communities can be detected in an increasing order according to the edge weights on the external links, which reveals different levels of close connections between nodes and enables us to obtain more information about network structure.

II. The Method

By using a simple mapping from weighted networks to unweighted multigraphs, Newman proposed the weighted modularity function as [18]

$$Q^w = \frac{1}{2T}\sum_{ij}\left[w_{ij} - \frac{T_i T_j}{2T}\right]\delta(c_i, c_j) \tag{1}$$

where w_{ij} denotes the edge weights between the two nodes i and j. $T_i = \sum_j w_{ij}$ denotes the weight of node i. $T = \sum_{i,j}^{n} w_{i,j}/2$ is the total edge weights in the networks. $\delta(c_i, c_j) = 1$ if c_i and c_j are identical to each other, and $\delta(c_i, c_j) = 0$ otherwise. c_i represents the label of the community that node i belongs to.

The modularity measures the edge weights within communities minus the expected edge weights [18]. We can obtain the communities by maximizing Q^w, which means the distribution of the edge weights between communities is similar to that of the expected edge weights. Therefore, a weighted network may be divided into several sub-networks if the edges with weights obeying the expected distribution are deleted.

According to (1), for a weighted network with m edges, the expected weights on the edges between nodes i and j are distributed by the function [18]

$$P_{ij}^w = \frac{T_i T_j}{2T} \tag{2}$$

Equation (2) indicates that the larger the P_{ij}^w is, the higher the expected weights on the edge between nodes i and j. Inspired by this idea, we can construct a corresponding random-weighted network model described by the adjacent matrix $\mathbf{B}^w$ as follows:

(I) Construct the matrix $\mathbf{P}^w$ with the elements $P_{ij}^w = T_i T_j / 2T$ and $P_{ij}^w = 0 (i = j)$.

(II) Record k_i nodes pairs (i, j) according to the value of P_{ij}^w in decreasing order, and let the elements B_{ij}^w be w_{ij}, others are zero.

(III) Let $B_{ij}^w = B_{ji}^w = 0$ if $B_{ij}^w \neq B_{ji}^w$, others are unchanged.

The adjacent matrix $\mathbf{B}^w$ representing the random-weighted network model should be symmetrical, i.e. $B_{ij}^w = B_{ji}^w = w_{ij}$ if there is one edge connecting the nodes i with j, and $B_{ij}^w = B_{ji}^w = 0$ otherwise. Here, step (III) makes $\mathbf{B}^w$ symmetrical and allows it to represent a network.

The weighted adjacent matrix $\mathbf{W}$ with the elements w_{ij} represents the weighted network. According to the filtration operation described by (3), some of the weighted edges between communities can be deleted. We use a weighted adjacent matrix $\mathbf{C}^w$ with elements C_{ij}^w to represent the obtained networks.

$$C_{ij}^w = w_{ij} - B_{ij}^w \tag{3}$$

In (3), if $w_{ij} - B_{ij}^w \le 0$, $C_{ij}^w = 0$.

After the filtration operation, two problems should be considered: one is how to evaluate the local communities and the other is how to ensure the network is divided.

A. Evaluate the Local Communities

If a network with n nodes and m edges is filtrated by (3) and divided into c sub-networks, we use $n_k (k = 1, \ldots, c)$ to represent the number of nodes in the kth sub-network. If we define nw_i as the sub-network that node i belongs to, then $\delta(nw_i, nw_j) = 1$ if nodes i and j belong to the same sub-network and 0 otherwise. We propose a weighted filtration coefficient (WFC) ψ to evaluate the dividing results.

$$\psi = \frac{\frac{1}{2}\sum_{k=1}^{c}\sum_{i,j}^{n_k} C_{ij}^w \delta(nw_i, nw_j)}{\frac{1}{2}\sum_{i,j}^{n} w_{ij}} \tag{4}$$

We can see that the denominator is the total of the edge weights in the original weighted network, and the numerator is the total of the edge weights in the sub-networks after division. w_{ij} is the edge weight between nodes i and j in the original weighted network, and the total of the edge weights in the original network can be given by

$$\begin{aligned}\frac{1}{2}\sum_{i,j}^{n} w_{ij} &= \frac{1}{2}\sum_{k=1}^{c}\sum_{i,j}^{n_k} w_{ij}\delta(nw_i, nw_j) + \\ &\quad \frac{1}{2}\sum_{i,j}^{n} B_{ij}^{w}[1-\delta(nw_i, nw_j)] \\ &= \frac{1}{2}\sum_{k=1}^{c}\sum_{i,j}^{n_k}(C_{ij}^{w}+B_{ij}^{w})\delta(nw_i, nw_j) + \\ &\quad \frac{1}{2}\sum_{i,j}^{n} B_{ij}^{w}[1-\delta(nw_i, nw_j)] \\ &= \frac{1}{2}\sum_{k=1}^{c}\sum_{i,j}^{n_k} C_{ij}^{w}\delta(nw_i, nw_j) + \frac{1}{2}\sum_{i,j}^{n} B_{ij}^{w}\end{aligned} \tag{5}$$

We use ψ_0 to denote the WFC of the original network, and after combining it with (4), (5) can be written as

$$\frac{1}{2}\sum_{i,j}^{n} w_{ij} = \frac{1}{2}\psi_0\sum_{i,j}^{n} w_{ij} + \frac{1}{2}\sum_{i,j}^{n} B_{ij}^{w} \tag{6}$$

Combined with (6), the statement of (1) implies

$$\begin{aligned} Q^{w} &\sim \sum_{k=1}^{c}\sum_{i,j}^{n_k}(w_{ij}-B_{ij}^{w})\delta(nw_i, nw_j) \\ &\sim \sum_{k=1}^{c}\sum_{i,j}^{n_k} C_{ij}^{w}\delta(nw_i, nw_j) \sim \psi_0\sum_{i,j}^{n} w_{ij}\end{aligned} \tag{7}$$

If the lth ($l \in [1, c]$) sub-network with n_l nodes is filtrated by (3) and is divided into d sub-networks again, we can calculate the WFC of the lth sub-network, denoted by $\psi_{0,l}$. The label on the right corner of $\psi_{0,l}$ represents the index of which network is divided and its division level. For instance, $\psi_{0,m,n}$ denotes the WFC of the nth sub-network, which is divided from the mth sub-network. Then the weighted modularity $Q^{w'}$ can be written as

$$\begin{aligned} Q^{w'} &\sim \sum_{k=1}^{l-1}\sum_{i,j}^{n_k}(w_{ij}-B_{ij}^{w}) + \sum_{k=1}^{d}\sum_{i,j}^{n_k}(w_{ij}-B_{ij}^{w}) + \\ &\quad \sum_{k=l+1}^{c}\sum_{i,j}^{n_k}(w_{ij}-B_{ij}^{w})]\delta(nw_i, nw_j) \\ &\sim \psi_0\sum_{i,j}^{n-n_l} w_{ij} + \psi_{0,l}\sum_{i,j}^{n_l} w_{ij}\end{aligned} \tag{8}$$

where n_l is number of nodes in the lth sub-network, and according to (7) and (8), we obtain

$$\begin{aligned} \Delta Q^{w} &= (Q^{w'} - Q^{w}) \\ &\sim [(\psi_0\sum_{i,j}^{n-n'} w_{ij} + \psi_{0,l}\sum_{i,j}^{n'} w_{ij} - \psi_0\sum_{i,j}^{n} w_{ij}) \\ &\sim (\psi_{0,l} - \psi_0)\sum_{i,j}^{n'} w_{ij}\end{aligned} \tag{9}$$

We can see from (9) that the weighted modularity increases when $\psi_{0,l} > \psi_0$ and decreases otherwise, which indicates that optimization on the local WFC is equivalent to obtaining the maximal modularity.

B. Keep Dividing the Network

For the case that the weighted network isn't divided by the filtration operation described by (3), we propose a disturbance to deal with it. By using a simple mapping from weighted networks to unweighted multigraphs, we construct the Laplace matrix ($\mathbf{L}$ matrix) of a weighted network [18], and a $n \times p$ matrix $\mathbf{U}$ is composed by the eigenvectors corresponding to $\mathbf{L}$ matrix's eigenvalues $\lambda_k (k = 1, \ldots, p)$ which are close to 0. For node vector $\mathbf{r}_i (i = 1, \ldots, n)$ of dimension p, the jth component of $\mathbf{r}_i$ is U_{ij}. The Euclidean distance between two connected nodes i and j is $d_H(i, j) = |\mathbf{r}_i - \mathbf{r}_j|$, and can be calculated with the eigenvectors corresponding to the main eigenvalues. The two connected nodes are considered in different communities if they have the largest Euclidean distance [23]. The disturbance is defined as the deletion of the edge between the two connected nodes with the largest Euclidean distance.

C. The Method for Detecting Weighted Community Structure

We present our method as follows:

(I) Construct the random-weighted network model of the network.

(II) Apply the filtration operation described by (3) to the network. If the network is divided go to step (III). Otherwise, apply a disturbance to the network before filtration and go to step (I).

(III) Calculate the WFC of the network. If the values of WFC stop increasing, the network is considered as a local community. Otherwise, each sub-networks divided from the network is considered as a new network and go to step (I).

We can see that our method works in a recursive way until all local maximal WFCs are found.

III. The Computation Complexity

For a weighted network with n nodes and m edges, it takes $O(m)$ to construct its random-weighted network model. If the weighted network has c communities, it only needs to be filtrated at most $c+1$ times. Calculating the main eigenvectors is the most communicational cost step in our method. Since the matrix $\mathbf{L}$ is usually a sparse matrix and not all eigenvalues are required, we can use the variants of Lanczos method [24]. We primarily concentrate on the leading eigenvector which can be obtained in time $O(m/(\lambda_3 - \lambda_2))$ [24], where λ_2 and λ_3 are the second and third main eigenvalues respectively. The number of disturbance is decided by the network topology and is much less than m. Thus, for a weighted network with c communities and m edges, the total computational cost of our method is $O((c+1)m)$, which is in linear scale time with the number of edges.

IV. Experiments

In this section, we first apply our method to a simple weighted network to show its performance. Then we use special designed computer-generated weighted networks with a pre-determined community structure to investigate the accuracy of our method. Finally, we apply our method to several real-world weighted networks.

For illustrative purposes, we first consider a simple weighted network with three communities. The sizes of edges indicate their weights on the edges. Fig. 1 shows the results of our method. The process of detecting the weighted community structure can be described by a dendrogram as shown in Fig. 1 (b).

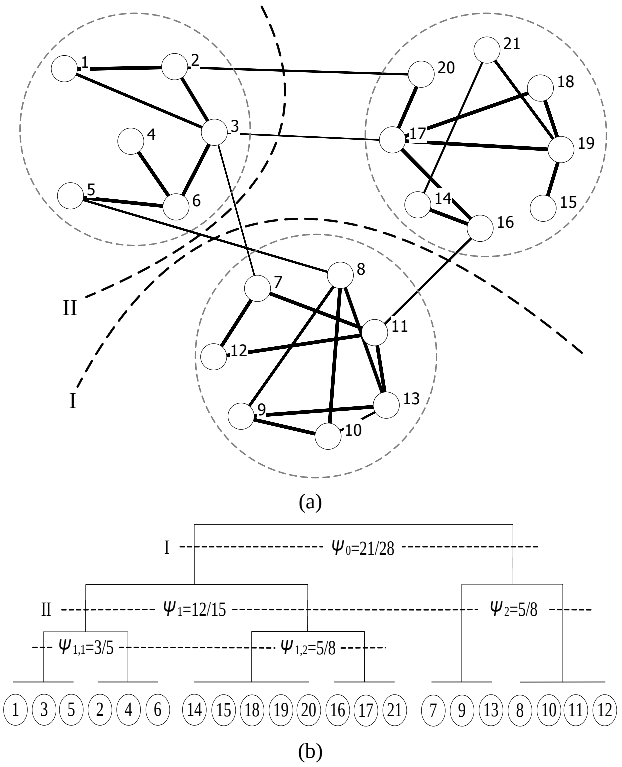


Fig. 1. The division of the simple weighted network: (a) The simple weighted network. (b) The detection process described by a dendrogram.

From Fig. 1 we can see that the network is divided in parallel with three steps indicated by three dotted lines (I, II, III). Nodes 7 to 12 are considered as a local community because $\psi_2 < \psi_0$. Another local maximal WFC is ψ_1, so nodes 1 to 6 and nodes 14 to 21 are considered as local communities. Furthermore, it is worth noting that the external links with smaller weights are deleted preferentially because they have higher probability P_{ij}^{w} . We can see that local communities are detected in an increasing order according to the edge weights between them.

A. The Analysis of Accuracy

We use a set of realistic benchmark networks called LFR benchmark networks [25] to test our method. The LFR networks have heterogeneous degrees distributions and community sizes, and allow communities to overlap, which are in accordance with the features of real networks, and have been widely used to test the community detection methods in recent years [26], [27]. The adjustable topology parameters for the LFR benchmark networks are network size N , average degrees $\langle k \rangle$, maximum degrees $k_{\max}$, minimum and maximum community size $c_{\min}$ and $c_{\max}$, the degree distributions γ, the community size distributions β, and the mixing parameter μ. In this case, we set the mixing parameter μ to be 0.5. In this case, it is difficult to correctly find communities if we ignore the edge weights. Without changing the network topology, we randomly assign integer weights in the range from 1 to 10 to the edges between groups and a weight $w \geq 1$ to the edges within groups. We increase w from a starting value of 1 to investigate the accuracy of our method. We use the NMI (Normalized Mutual Information) [28] to measure the accuracy of our method.

The NMI measures the difference between true partition A and found partition B , which is written as [28]:

$$NMI(A,B) = \frac{2\sum_{i=1}^{C_A}\sum_{j=1}^{C_B} N_{ij}\log\left(\frac{N_{i.}N_{j.}}{N_i N_j}\right)}{\sum_{i=1}^{C_A} N_{i.}\log\left(\frac{N_{i.}}{N}\right) + \sum_{j=1}^{C_B} N_{.j}\log\left(\frac{N_{.j}}{N}\right)} \quad (8)$$

where $\mathbf{N}$ is a matrix with N_{ij} being the number of nodes from real group i that were in found group j . C_A and C_B are the numbers of real communities and detected communities respectively, $N_{i.}$ and $N_{.j}$ are the sums over row i and column j of matrix $\mathbf{N}$ respectively. The range of NMI is between 0 and 1. Large value of NMI corresponds to accurate community division.

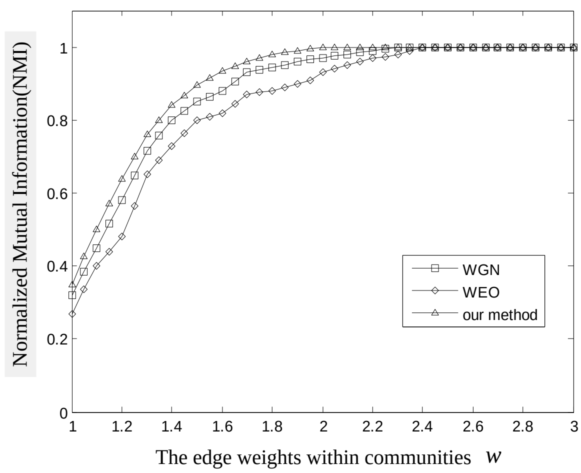
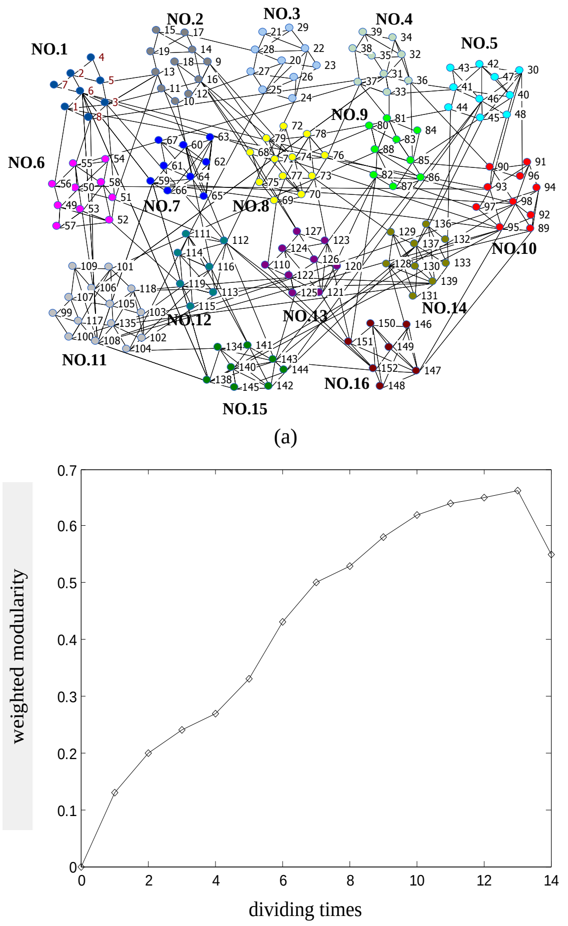


Fig. 2. Results on the LFR networks, as a function of the edge weights within communities w.

We show the results of our method on the LFR networks in Fig. 2. The WGN algorithm and WEO algorithm are also investigated for making comparisons. Each data point is obtained by averaging over 400 networks. From Fig. 2 we can see that when $w < 2$, a tiny increment of the within-community weight leads to large improvements in performances for all the methods. When $w > 2$, all methods perform very well and they can identify more than 90% of all nodes correctly. Our method performs better than the WGN and WEO. The reason is that our method is a heuristic search method, which allows the weighted modularity to be optimized in a recursive way.



(a)

(b)

Fig. 3. The results of our method applied to the scientific collaboration network. (a)The collaboration network and the detected communities (best viewed in color). (b)The variety of the weighted modularity obtained by our method as a function of the dividing times.

B. A Collaboration Network to Show the Implemention Process of Our Method

As a real example, we apply our method to a collaboration network of teachers in a university [20]. The network has 152 nodes and each node represents a research member. If two research members coauthored articles during the year 1992, the two corresponding nodes are connected. The network edges are weighted in accordance with the number of articles they coauthored .Results of our algorithm are plotted in Fig. 3.

The network has been divided for 14 times, and the number of disturbance is 5. The maximal Q^w is 0.662, achieved by finding 16 communities which corresponds to 16 research groups. We also apply the WGN and WEO method to the network and obtain similar results. In the network, if the researchers in a group collaborate seldom with the members in other groups, the weights on the corresponding edges are small and will be deleted preferentially, so the group will be detected at first. Whereas, when the researchers in a group collaborate frequently with the members in other groups, the weights on the corresponding edges are large, and the group will be detected later. The local communities are detected in the order NO.3→NO.2→NO.4→NO.1→NO.5→NO.16 →NO.6→NO.15→NO.7→NO.11→NO.13→NO.13→ NO.7→NO.12→NO.14→NO.9→NO.10. This is in accord with the weights of external edges to them in increasing order. It well reflects different levels of close connections between nodes and cannot be achieved by the existing weighted methods.

C. Other Real Applications

To further test our method, we also apply it to other real examples including the Zachary karate network [29], the American college football network [30], and an electronic circuits network [31]. The Zachary network has 78 links representing friendships among 34 members. The network has been widely used to test community detection methods, and is usually divided into two groups with 16 nodes and 18 nodes. The American college football network represents the schedule of games between American college football teams in a single season, which has 115 teams and 616 competitions. The network does not have apparent center node and is usually divided into 8-12 team communities. The circuit's network has 512 nodes and 819 edges in which node s are electronic components and edges are wires. Table I shows the results for the WGN method, the WEO method, and our method. As Table I shows, the weighted modularity obtained by our method is higher than that obtained by the WGN and WEO methods, which indicates that our method performs best.

TABLE I: THE RESULT FOR THE REAL NETWORKS. THE NUMBER OF COMMUNITIES C_n AND THE GLOBAL MODULARITY Q ARE PRESENTED.

Name	WGN: O_n , Q	WEO: O_n , Q	Our: O_n , Q
Karate	2, (0.31)	3, (0.34)	3, (0.37)
Football	8,(0.48)	10, (0.51)	11,(0.53)
E. Circuit	13,(0.475)	14, (0.49)	14, (0.51)

V. CONCLUSIONS

The communities in weighted networks can be used to reveal close connections between the nodes in different groups, and have attracted considerable interests within complex networks. However, most existing detection methods are divisive and agglomerative methods, which delete only one edge each time to split the network or agglomerating only one node each time until no individual node remains. In this paper, we propose a method to split the networks in parallel by deleting many edges in each filtration operation, and we also propose a Weighted-Recursive Filtration Coefficient (WFC) ψ instead of the traditional Q^w to quantify the splitting results. We proved that recursive optimizing of the local ψ is equivalent to acquiring the maximal global Q^w corresponding to correct community structure. For a network with m edges and c communities, our method can detect the community structure in time $O((c+1)m)$, which is in linear scale time with the number of edges. This is lower than many other existing methods. We also analyze the accuracy of our method and compare it with other existing methods. The results show that our method performs better than the WGN and WEO methods. In the end, we apply our method to four real weighted networks, and the results show that our method can correctly detect the weighted communities. Furthermore, a collaboration network is used to show in detail the implementation process of our method. The results show that our method can detect local communities according to the edge weights between them in an increasing order, which reveals different levels of close connections between the nodes.

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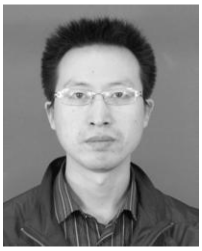

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