

# Low Rate QC LDPC Codes with Reconfigurable Structures for Space Information Networks

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**Abstract**—This paper presents the construction of the efficient low rate Quasi-Cyclic (QC) Low-Density Parity-Check (LDPC) codes and their reconfigurable structures for space information networks. The code is firstly produced by a seed Accumulate-Repeat-Accumulate (ARA) protograph and then extended to a QC generalized LDPC code, where a parity check node pair is replaced with a more powerful pre-coding Hamming node. The code matrix is optimized by both the Progressive Edge progressive (PEG) and the QC oriented modified approximate cycle extrinsic message degree (QC-MACE) algorithm. The QC form in the code saves much storage and reduces coding complexity greatly. The reconfigurable code structure is also given and it can be used in variable space channels. Simulation results show that the proposed code obtains 0.05 dB gain and has better error floor, when compared to the Turbo code in the Consultative Committee for Space Data Systems (CCSDS) with same parameters at a Bit-Error-Rate (BER) of  $10^{-5}$ . And the coding gain is also about 0.1 dB over that of the currently low rate and short QC LDPC codes at a BER of  $10^{-5}$ . Besides, it has good features of low decoding complexity and low latency and so on.

**Index Terms**—QC LDPC codes, protograph, PEG, QC-MACE

## I. INTRODUCTION

LDPC codes, originally proposed by Gallager in 1962 [1], were rediscovered by Mackay in 1995 [2]. They have received extensive attentions due to their ability to approach Shannon capacity. They usually have random check matrices, which lead to dense generator matrices of high encoding complexity in proportion to the square of code length. Some researches [3]-[5] have shown that the LDPC codes with random check matrices, by carefully chosen degree profiles, can achieve Shannon capacity closely, when decoded by the well-known Belief Propagation (BP) algorithm. However, they still adopt irregular and rather complex encoders, which hinder their implementation in practice. So it is necessary to construct

structured LDPC codes, *e.g.* Quasi-Cyclic (QC) LDPC codes [6], for efficient encoding and good error floor. Low rate LDPC codes can be applied in space information networks with high power efficiency. But it is still hard to construct short-length, low-rate and reconfigurable QC LDPC codes with both good thresholds and low floors [7]. The CCSDS committee has suggested a class of low rate (*e.g.* 1/6) Turbo codes with good performance at low Signal-to-Noise Ratio (SNR) [8]. But the Turbo codes have deficiencies of high decoding complexity and large decoding latency, which limit their implementations. So it is necessary to produce low rate QC LDPC codes with configurable structure for high power efficiency in the variable space channels.

LDPC codes can be represented by Tanner graphs for the ease of design and analysis. The variable and check nodes in a Tanner graph are related to the codewords and the parity-check equations, respectively [9]. Recently, the low rate structured LDPC codes, *i.e.* the Accumulate-Repeat-Accumulate (ARA) codes, are presented with good thresholds and low floors [10]. They were produced by a protograph based design methodology [11] for good performance. But they can't be easily adjusted to the reconfigurable structure to fit the variable space channels. G. Liva, *et al.* [7] suggested the low rate generalized LDPC codes with encodable QC structure [12] for good performance. But the codes can't adequately utilize QC encoding structure and they needed special encoding structure to solve the rank deficiency problem of the code matrix in the protograph based method, which increased the complexity of encoding. So we propose an efficient method of generating the codes based on the protograph doping scheme [7] by splitting the base template matrices by the Progressive Edge Growth (PEG) algorithm [13] and eliminating one minimum QC sub-matrix in the protograph code matrix to solve the rank deficiency problem without destroying the whole QC structure. And the PEG algorithm can also be used to avoid the short trapping sets, and thus improve the error floor in the code design [14]. By extending the approximate cycle extrinsic message degree (ACE) algorithm [15] for the QC LDPC codes to find the circulant offsets in the QC sub-matrices, the codes are designed to obtain perfect performance a little better than that of the Turbo code [8] with the same code rate. Besides, they obtain lower complexity and less latency in decoding. Due to QC structures, the codes also

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Manuscript received Aug. 31, 2015; revised March 7, 2016.

This work was supported by the Zhejiang Provincial Natural Science Foundation of China (No.LZ14F010003), the National Natural Science Foundation of China (No.61320106003, No.61471152), the China High-Tech 863 Plan under Grant No.2012AA01A506, the Open Research Fund of National Mobile Communications Research Laboratory, Southeast University (No.2014D02), the Zhejiang Provincial Science and Technology Plan Project (No.2015C31103) and the China Postdoctoral Science Foundation (No.2014M561555).

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doi:10.12720/jcm.11.3.255-262

have similar linear encoding complexity of the Turbo code with the same code length. In the design of the QC LDPC code, the technique of cyclic lifting of the code protograph can also be adopted to improve performance [16]. And it was implemented with proper permutation shifts to the circulant permutation sub-matrices in the parity check matrix after lifting by a greedy algorithm. In addition, the pre-lifted protograph method can overcome the restrictions of undesirable fixed upper limits on important code parameters, such as minimum distance and girth, to improve the code performance [17].

The paper is organized as follows. In Section II, the design of the reconfigurable low-rate QC LDPC codes is proposed. A typical code is generated with an extension of an ARA protograph by a Hamming node, which is further divided and optimized by the PEG algorithm. The QC circulant offsets are then searched by the QC-MACE algorithm. In Section III, the reconfigurable structures of the codes are given to be used in space information networks. The decoding complexity is also analyzed. In Section IV, the numerical simulations of the proposed codes and the contrast codes on an AWGN channel are presented to verify good performance of the code. Finally, conclusions are given in Section V.

## II. CONSTRUCTION OF THE QC LDPC CODES

### A. LDPC Codes and Their Tanner Graphs

An LDPC code can be represented by a  $m \times n$  low-density parity-check matrix  $\mathbf{H}$ , which corresponds to a code with rate  $(n-m)/n$ , where  $n$ ,  $k$  and  $m=n-k$  is the code length, number of the information bits and number of the parity check bits per codeword, respectively. So in the Tanner graph of the code, there are  $m$  check nodes, related to  $m$  '1' in each row of  $\mathbf{H}$ , and  $n$  variable nodes, related to  $n$  '1' in each column of  $\mathbf{H}$ . For the general case where  $\mathbf{H}$  has irregular row and column weight, the Tanner graph is characterized by the degree assignment sets  $\{d_v(i)\}_{i=1}^n$  and  $\{d_c(j)\}_{j=1}^m$ , where  $d_v(i)$  is the degree of the  $i$ -th variable node and  $d_c(j)$  is the degree of the  $j$ -th check node [4]. For an LDPC code, the variable and check degree profile, *i.e.*  $\lambda(x)$ ,  $\rho(x)$ , which influences the code performance, are depicted as

$$\lambda(x) = \sum_{i=2}^{d_v} \lambda_i x^{i-1} \quad (1)$$

$$\rho(x) = \sum_{i=2}^{d_c} \rho_i x^{i-1} \quad (2)$$

where  $d_v$  and  $d_c$  are the maximum variable and check node degree. The degree profile is assumed as the number of the edges connected to the variable or check nodes, respectively, where  $\lambda_i$  is the fraction of edges connected to variable nodes of degree  $i$  while  $\rho_i$  is related to check nodes of degree  $i$ . The degree profiles determine the theoretic decoding threshold of the codes, which can be optimized by the density evolution technique [3]. So the optimal Tanner graphs for the codes can be designed under the constraints of the degree profiles.

Since a bipartite graph gives a complete description of an LDPC code, for a QC LDPC code, the specifications of the component (linear) block are also required. Using the same terminologies in [7], we define  $V = \{V_j\}_{j=0}^{n-1}$  be the set of  $n$  variable nodes (VNs) and  $C = \{C_i\}_{i=0}^{m-1}$  be the set of  $m$  check nodes (CNs) in the bipartite graph of a G-LDPC code (Fig. 1). The connection between the CNs and VNs are summarized in a  $m \times n$  base matrix  $\mathbf{T}$ . Then,  $\mathbf{T}$  is extended and structured as an adjacency matrix  $\mathbf{\Gamma}$  with quasi-cyclic structure, where each "1" and "0" in  $\mathbf{T}$  being replaced with a cyclic sub-matrix  $\mathbf{\Pi}[e(i, j)]$  (short for  $\mathbf{\Pi}_{i,j}$ ) and a zero matrix of the same dimension. And the cyclic sub-matrix is usually derived from circulant shift of an identity matrix. So the relationship between the adjacency matrix  $\mathbf{\Gamma}$  and the parity check matrix  $\mathbf{H}$  for a QC LDPC code is determined, once each cyclic sub-matrix  $\mathbf{\Pi}_{i,j}$  is confirmed. And  $\mathbf{\Gamma}$  is an array of  $m \times n$  circulant permutation matrices in the form of

$$\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Pi}_{0,0} & \mathbf{\Pi}_{0,1} & \cdots & \mathbf{\Pi}_{0,n-1} \\ \mathbf{\Pi}_{1,0} & \mathbf{\Pi}_{1,1} & \cdots & \mathbf{\Pi}_{1,n-1} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{\Pi}_{m-1,0} & \mathbf{\Pi}_{m-1,1} & \cdots & \mathbf{\Pi}_{m-1,n-1} \end{bmatrix} \quad (3)$$

where  $\mathbf{\Pi}_{i,j}$  is either a  $q \times q$  circulant permutation matrix or a  $q \times q$  full zero matrix. So  $\mathbf{\Gamma}$  is just constructed by substituting circulant permutation matrices for the 1's in the base matrix. The 0's in  $\mathbf{\Gamma}$  are replaced by all-zero  $q \times q$  matrices. By this mean, short cycles can be avoided with circulant permutation expansion. So, the final check matrix  $\mathbf{H}$  is an optimized permutation case of the binary matrix  $\mathbf{\Gamma}$ , with size of  $mq \times nq$ , which is enlarged for  $q$  times of the base matrix in the dimension of both rows and cows. In summary, the check matrix  $\mathbf{H}$  of a QC-LDPC code can be obtained by the expansion of a base matrix  $\mathbf{T}$  in the form of the adjacent matrix  $\mathbf{\Gamma}$  with determined circulant permutation matrices, which are optimized for large cycles in their Tanner graphs.

### B. The Seed Protograph and Extended Hamming Node

To construction a low rate-1/6 QC LDPC code, the seed protograph of a rate-1/3 code with threshold -0.048 dB is adopted and shown in the top of Fig. 1. The protograph is then extended by a single bit check node replaced with a Hamming node as the generalization processing [7] shown in the bottom of Fig. 1. So the code rate of the extended protograph is drop to rate-1/6. Since there are much more coding constraints in the Hamming code than that of the original single parity-check (SPC) node pair shown in the top of Fig. 1, it brings much larger code distance and contribute a lot for the good coding performance due to the better pre-coding gain brought by the Hamming pre-coding. Also a higher code rate can be made by puncture the degree-1 node  $p_5$  or  $\{p_4, p_5\}$  for rate 1/5 and 1/4, respectively. The check matrix of the Hamming code uses the H(7,4,3) code and it is depicted in (4). Other low rate code can be also produced with the same 1/3 ARA protograph and other Hamming code

nodes, e.g. H(15,7,8) and so on, including their original or shorten form. Finally, the protograph of the code can be designed and shown in Fig. 2 with the rate-1/6 parity matrix listed in (5).

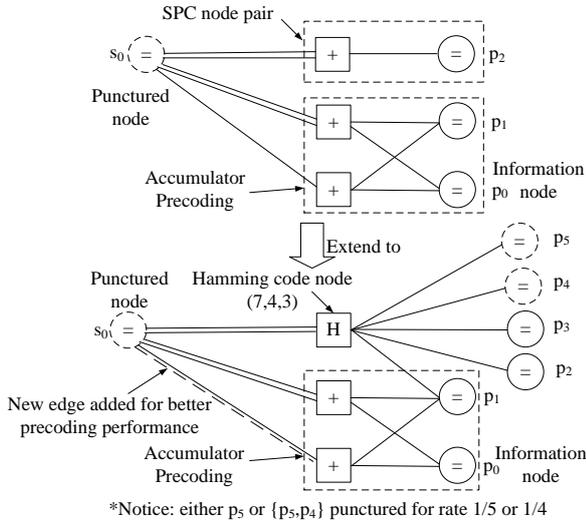


Fig. 1. A seed protograph extended by a hamming node

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

In Fig. 2, the variable nodes “s<sub>0</sub>” are information bit nodes to be punctured and other nodes are check bit nodes. In [7], the punctured node with the highest degree is assigned to the information bit node and the AR4A rate-1/3 code is selected for good performance. Similar to [7], we assign the same variable node “s<sub>0</sub>” of the highest degree, i.e. most reliable variable node, to the information bit node to guarantee the best performance. The variable node with highest degree obtains much more messages from the most linked check nodes, which leads to the high reliability of the node. The extended code matrix **T** is generated by (5) as (6), where all “1” and “2” are replaced with square circulant sub-matrix as {**I**<sub>1</sub>, **I**<sub>2</sub>}, shifted from first row of weight-1 and weight-2, respectively, and all “0” with square zero matrix.

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_2 & \mathbf{I}_1 & \mathbf{I}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I}_2 & \mathbf{I}_1 & \mathbf{I}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I}_1 & \mathbf{0} & \mathbf{I}_1 & \mathbf{I}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I}_2 & \mathbf{0} & \mathbf{I}_1 & \mathbf{0} & \mathbf{I}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{I}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_1 & \mathbf{0} \\ \mathbf{I}_1 & \mathbf{0} & \mathbf{I}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_1 \end{bmatrix} \quad (6)$$

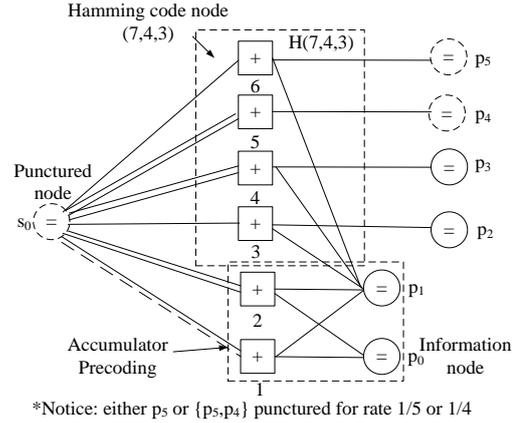


Fig. 2. Final base protograph of the proposed LDPC codes

In the design of the QC LDPC code, the extended code matrix **T** has to be further divided to improve the randomness of the matrix and thus enhance the code performance. In other words, the circular square sub-matrix **I**<sub>1</sub> and **I**<sub>2</sub> is needed to split and randomize the distribution of ‘1’ in the final check matrix. For instance, each square circulant matrix **I**<sub>1</sub> or **I**<sub>2</sub> in the base protograph is divided into a 4×4 square circulant matrix in (7) and (8), respectively, as long as it guarantees row and column weight in the original matrix **T**.

$$\mathbf{I}_1 = \begin{bmatrix} \mathbf{I}'_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}'_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}'_3 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}'_4 \end{bmatrix} \quad (7)$$

$$\mathbf{I}_2 = \begin{bmatrix} \mathbf{I}_{1,1} & \mathbf{0} & \mathbf{I}_{1,3} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{2,2} & \mathbf{0} & \mathbf{I}_{2,4} \\ \mathbf{0} & \mathbf{I}_{3,2} & \mathbf{I}_{3,3} & \mathbf{0} \\ \mathbf{I}_{4,1} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{4,4} \end{bmatrix} \quad (8)$$

In (7) and (8), {**I**<sub>i</sub>, **I**<sub>j</sub>} are the embedded sub-matrices after the split of {**I**<sub>1</sub>, **I**<sub>2</sub>}. The PEG algorithm [13] is then used to locate the optimal distribution of {**I**<sub>i</sub>, **I**<sub>j</sub>} in the sub-matrix {**I**<sub>1</sub>, **I**<sub>2</sub>} of **T**. And the detailed steps of the PEG algorithm are listed as follows.

The Tanner graph of the code can be reorganized as a tree structure, where the root node is chosen from an initial variable node. A typical tree with the root node, i.e. the variable node v<sub>j</sub>, is shown in Fig. 3.

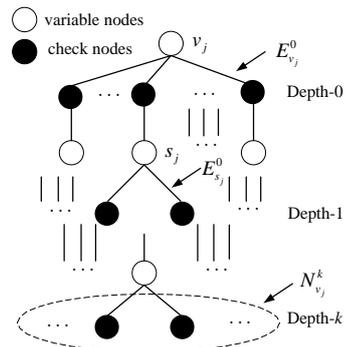


Fig. 3. A typical tree structure of a tanner graph expansion



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Initialization of all ACE parameters
Find its children set  $Ch(w_s)$ 
for every child  $\mu_i \in Ch(w_s)$ 
  begin
     $p_{temp} \leftarrow p(w_s) + ACE(\mu_i)$ 
    if  $p_{temp} + p(\mu_i) - ACE(v_i) - ACE(\mu_i) < \eta_{ACE}$ 
      exit with failure
    elseif  $p_{temp} \geq p(\mu_i)$ 
      Deactivate  $\mu_i$  in level- $k$  with current parent  $w_s$ 
    else
       $p(\mu_i) \leftarrow p_{temp}$ 
    end
  end
end
exit with success
if  $(ACE < \eta_{ACE}$  for a cycle of length  $2d_{ACE}$  or less)
  goto redo
end
end

```

Finally, we tentatively search possible combinations by the QC-MACE algorithm and the  $(d_{ACE}, \eta_{ACE})$  is selected as (6,3) (for short code length  $\leq 5000$ ), (8,4) (for short code length  $> 5000$ ) and so on by multiple tentative trials, which are optimized for the QC LDPC codes with the ACE criterion.

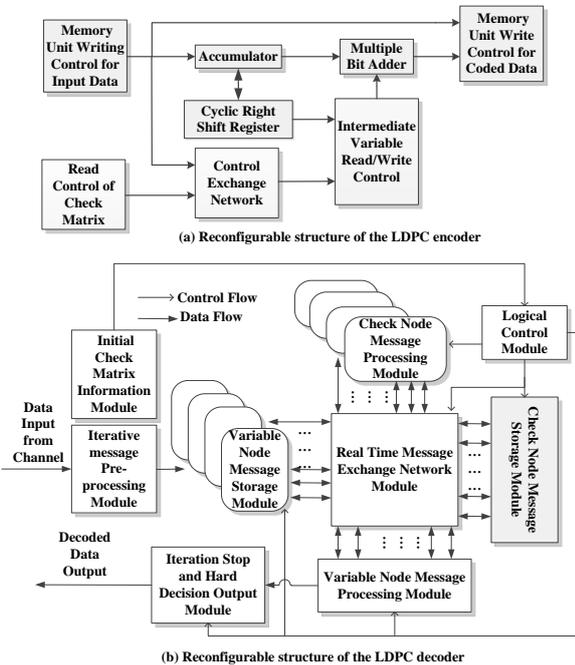


Fig. 4. Reconfigurable structure of LDPC encoder and decoder

### III. RECONFIGURABLE ENCODER & DECODER STRUCTURE

In space information networks, channel quality usually varies according to the orbit and relative speed of the aircrafts. Low rate LDPC codes are used on occasions of low Signal-to-Noise Ratio (SNR) for reliability, while a little high rate codes are suited for good high SNR channel conditions to achieve better spectrum efficiency. So the code rate need to be flexibly and adaptive to real variable space channel. Due to the QC structure, the LDPC encoder is realized with the module-2 addition between the input information bits and the data fetched from a generator matrix, easily obtained from the cyclic

shift register. The LDPC decoder also benefits from it, which is capable of easily addressing and accessing to the variable/check nodes involving in the iterative message propagations of the BP algorithm. Therefore, the general framework for the encoder and decoder is shown in Fig. 4.

From previous Fig. 2, the punctures of some check bit nodes are used to obtain higher code rate. The punctured nodes are actually related to some groups of the QC circulant sub-matrices. They refer to some columns of the matrix, which are related the columns in the generator matrix. So the variable rate LDPC encoder can be realized by puncturing several columns in the generator matrix. For the decoder, all the punctured nodes should be recovered in the Tanner graph for decoding and the missing messages due to the punctured encoding are set as 0. So the decoder is compatible for all low rate codes with puncture. Unfortunately, the generator and check matrix with different code rate usually haven't the same common data, because they need be optimized separately under different puncture situations. So the initial matrix data for different code rate should be stored in the memories and then be used according to the control instructions.

Generally, Turbo codes have high decoding complexity and decoding latency, since a more complex logarithm domain maximum posterior probability (LOG-MAP) decoding is more complex and needs much more memories to record the interleaving table [19]. Because they have two component convolutional decoders to finish the BCJR forward and backward decoding in turn with interleaving, which leads to more decoding delay. And the Turbo decoding is also very difficult to be parallel implemented [19]. LDPC codes suffer a lot from their high encoding complexity, which is in proportion to the square of the code length. But for the proposed code in QC form, it has just linear encoding complexity. In addition, it uses a linear feedback shift register to store the generated matrix, which is very proper to be implemented in a resource limited hardware platform. LDPC decoding can also be realized with the parallel BP algorithm for low complexity. The QC structure also makes the decoding more efficiently, since the addressing, *i.e.* finding the position of the variable and check nodes in the check matrix, is much easier and also saves many memories. And the decoding latency is also improved a lot due to the QC structure. From above analyses, given code length  $N$ , information bit length  $K$ , and parity check bit length  $M$  (*i.e.*  $N-K$ ), average row and column weight as  $w_r$  and  $w_c$ , iteration number  $L$ , the decoding complexity of the proposed code is listed in Table II.

TABLE II: THE DECODING COMPLEXITY OF THE LDPC CODE.

Additions	Module-2 operation (XOR)	LUTs
$8(2Lw_c - w_c + L)N + LM(w_r - 1)$	$LM(w_r - 1)$	$LMw_r$

### IV. NUMERICAL SIMULATION AND RESULT ANALYSES

To verify the effectiveness of the LDPC codes by the proposed method, the BER performance of the codes are

simulated and compared with the CCSDS standard Turbo code and the currently well designed low rate QC LDPC codes, respectively. The results are as follows.

A. Comparison of BER with the CCSDS Turbo Code

The BER simulation parameters are listed as follows. QC LDPC codes with moderate block length and low rate  $\{(10704,1/6), (8832,1/6), (6144,1/6), (5120,1/5) \text{ and } (5632,1/4)\}$  are designed with above proposed design method. In the contrast schemes, the CCSDS Turbo code  $(10704, 1/6)$  is adopted [8]. Simulation at each  $E_b/N_0$  runs until a specified number (*e.g.* 50) of error frames occur or a total of 0.8 million trials have been executed. The maximum iterations are 50 for each LDPC decoding. Finally, the BER for all codes are shown in Fig. 5.

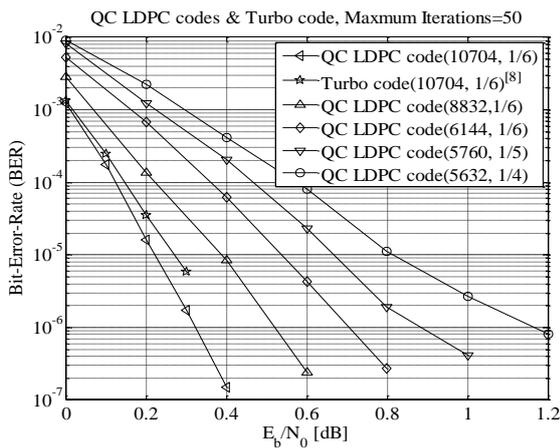


Fig. 5. BER of the proposed QC LDPC codes vs. a Turbo code.

From Fig. 5, it can be observed that the proposed QC LDPC code achieves rather good performance over 0.05dB of the contrast CCSDS Turbo code at BER of  $10^{-5}$  on an AWGN channel, where there is the error floor for the Turbo code. In addition, it shows good error floor up to BER of  $10^{-7}$  where there is obvious error floor for the Turbo code. Moreover, it has lower decoding complexity and less decoding latency. The shorter length code  $(8832,1/6)$  also achieve good performance within 0.1dB of the Turbo code. Other three codes with almost equal length and different rate also shown good error floor and have the gap of about 0.15 dB among rate 1/6, 1/5 and 1/4, respectively. And they all show good floor at BER of  $10^{-5}$ . From the above simulations, the proposed LDPC code obtains better performance than that of the Turbo code with the same simulation parameters. It mainly relies on the factor that the code structure can be much more flexible in the LDPC code design and the whole code matrix can be constructed globally. Therefore, both decoding threshold and error floor can be optimized for good coding performance. But in the Turbo code design, the code performance is mainly decided by two component convolutional codes and the interleaver, which are hard to be designed with larger code distance. So the Turbo codes are suffered from poor BER performance as well as high error floors. Since larger code length in an LDPC code can avoid small loops and

thus improve the girth properties. The long LDPC codes obtain better threshold and floor performance over that of the short codes, just as shown in Fig. 5.

The decoding complexity of the proposed code is much lower than that of the Turbo code. In practice, the LDPC decoding can even be implemented partial parallel given limited resources. In addition, the proposed methodology is flexible and easily adjusted to generate other low rate reconfigurable QC LDPC codes with efficient encoding and decoding. And it can also be accomplished by selecting a more proper seed protograph and a more powerful linear block or even non-linear convolutional code node for complex node extensions.

B. Comparison of BER with Low Rate QC LDPC Codes

The BER simulation parameters are listed as follows. QC LDPC codes with moderate and short code length and low rate  $\{(6144,1/6), (5120,1/5) \text{ and } (3000,1/5)\}$  are generated with the proposed algorithm. In the contrast schemes, the QC G-LDPC code  $(6144,1/6), (5120,1/5)$  in [7] and the rate compatible irregular LDPC code  $(3000,1/5)$  in [20] are adopted for comparison. Simulation termination condition is the same to that of part A in this section. The maximum iterations are up to 200 for each LDPC frame decoding. With all above parameters, the BER performance for all codes is shown in Fig. 6.

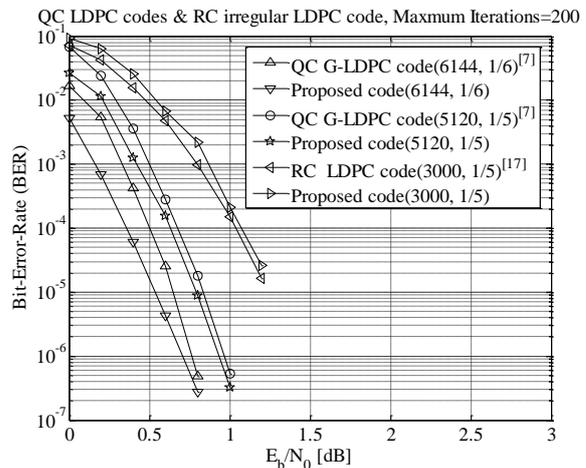


Fig. 6. BER of the proposed LDPC codes vs. current LDPC codes

From Fig. 6, the proposed QC LDPC codes also obtain rather good performance over 0.1dB of the contrast QC G-LDPC code at BER of  $10^{-5}$  on an AWGN channel. And the performance of the proposed code is closely near that of the RC irregular LDPC code [20] with low rate 1/5. But without the QC structure, the latter suffers a lot from the encoding complexity due to the random matrix. And the complexity is in proportion to the square of the code length, rather than the linear complexity caused by the QC structure. In addition, the codes also show good error floor. The possible reason for good decoding threshold and low floor is that the code is designed by a much powerful pre-coding Hamming node, as well as the matrix splitting for further code matrix randomness. The pre-coding enhances the accumulated code gain and the

increase of the code matrix randomness also promotes the code performance according to the Shannon's coding theory. In addition, the QC-MACE is also adopted to improve the loop relationship for better property of the stop set or the trapping set [19].

The complexity of the proposed code design method is moderate and easy to be applied in the practice. It mainly includes the procedures of the base template protograph optimization with good threshold property (or directly chosen from current literature), the Hamming node extension, the sub-matrix extension along with the position optimization by the PEG algorithm, and the search of the circulant offsets in all sub-matrices by the QC-MACE algorithm. The first two procedures are easy to be accomplished with negligible complexity. The complexity of the 3<sup>rd</sup> procedure is similar to the application of the PEG algorithm in the base matrix of equation (6). Since the scale of the matrix of equation (6) is much smaller than the extended final parity-check matrix, the complexity is also limited and in proportion to the length of the base matrix. In the last procedure, the QC-MACE algorithm is implemented in Table II, and it has complexity of approximate  $o(m \times n)$ , where  $m \times n$  is the size of the block element in the whole matrix shown in (9). So the complexity is also finite and acceptable. According to the above analyses of the proposed method, the calculation won't cost a lot of time and they are rather moderate when compared with the dimension of the check matrix of the final codeword. Then, the latency of the method is acceptable and can be used properly in the design of an efficient code in space information networks.

## V. CONCLUSIONS

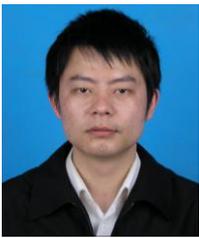
In this paper, we have presented the construction of the low-rate QC LDPC codes and their reconfigurable structures for space information networks. The low rate QC LDPC codes are produced by the extension of a good protograph with a Hamming node, followed by the two optimizations. A matrix splitting technique is used and the divided sub-matrices are optimized by a PEG algorithm to improve the randomness of the code matrix. And the QC-MACE algorithm is applied to improve the loop relationship. Simulation results show that the proposed rate-1/6 code obtain 0.05 dB performance gain and has better error floor, as well as lower decoding complexity and less latency, when compared to the CCSDS Turbo code at BER of  $10^{-5}$ . The new LDPC codes with short length also outperform the currently well designed QC LDPC codes for about 0.1 dB. The proposed codes are also reconfigurable and can be easily implemented in practice to fit the variable space channels. Therefore, the proposed code design method can be efficiently applied in the design of the QC LDPC codes for space information networks with good performance, low complexity and low latency.

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