A Fast Adaptive Control Algorithm for Slotted ALOHA

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Abstract — The control algorithm which is in order to achieve the aim of keeping throughput stability is needed in the Slotted ALOHA(S-ALOHA) protocol. The core of the control algorithm is to derive the accurate number of nodes. The classic control algorithm suffers performance loss when the system nodes change. In this paper, we addressed a Fast Adaptive (FA) algorithm to promote the performance of ALOHA. The channels are grouped into two states, idle channel and non-idle channel, by the mode of status run. When 7 continuous idle channel slots are detected, the number of nodes becomes 1/2 the number of estimated nodes at fiducially interval 0.95 and updates the probability of transmitting data on twice the pace. Similarly, channel states are also divided into collision and non-collision. If 7 continuous collision slots are detected, the estimated number of nodes will be doubled and the probability of transmitting data will be updated as 1/2 times. In other cases, the p-persistent control algorithm (pPCA) or Pseudo-Bayesian Control Algorithm (PBCA) are adopted. Simulation results show that the performances of fast adaptive p-persistent control algorithm (FA-pPCA) and fast adaptive pseudo-Bayesian control algorithm (FA-PBCA) outperform those of pseudo-Bayesian control algorithm and p-persistent control algorithm.

Index Terms—Run-length of channel state, slotted ALOHA, fast adaptive, pseudo-Bayesian control algorithm, p-persistent control algorithm

I. INTRODUCTION

Because of its simplicity, S-ALOHA and its improved protocol [1] have been widely applied to environments with long delays [2]-[4] such as satellite communication, GSM digital cellular network and tag identification [5], cognitive radio network [6]-[8], vehicle network [9] and underwater sensor network. However, S-ALOHA in its essence is unstable. For the purpose of solving this instability problem, literature [10] proposed that different retransmission probabilities \( P \) may be utilized to acquire stable system under different overall input loads. In literature [11]-[14], Sarker investigated how the constraint on the number of re-transmissions which influence the system stability under the environments where the system. Performance is analyzed according to continuous states of system and cooperation among nodes in literature [15]-[18]. Game theory is also introduced into channel competition so as to study the utilization rate and throughput of the channel of slotted ALOHA system. Both Pseudo-Bayesian control algorithm proposed by Rivest [19] and \( p \) persistent control algorithm [20] presented by Ivanovich realize the goal of adjusting stability of system through the way of accurately estimating actual communication nodes and modulating data transmission probability of each node. In order to track the nodes of system, Wu address the Fast Adaptive S-ALOHA scheme (FASA) using access history which converge the estimate of the number active devices fast to the true values and can gain the stable throughput [21].

When the number of nodes drastic changes, a large number of idle or collision time slots would appear which causes a quite low channel contention. Research shows that system throughput would be higher than 60% of the maximum theoretical throughput if the ratio of the estimated number of nodes to the actual number of the nodes is between 0.5 and 2. According to this conclusion, the overall throughput of the system can be improved through using some sort of control algorithm to maintain the ratio within [0.5,2] and further applying some precision adjustment algorithms. In literature [22], based on the universal S-ALOHA system and the theory of detecting fixed sizes \( c \) of idle slots to exponentially adjust backoff time which is proposed and the performance of the algorithm are simulated as \( c = 4.8 \). Aiming at the problem of \( c \) values, the study in literature [23] show that the system can acquire optimal performance as \( c = 7 \).

But, literature [22], [23] can not proof that why the system can gain optimal performance as \( c = 7 \). This paper introduces the channel state-run, utilizes dichotomy model to obtain the difference equation of run length \( k \) and estimates the occurrence probability of channel state. Then, a fast adaptive algorithm is proposed based on the channel state-run information, which is combined with Pseudo-Bayesian control algorithm and p-persistent control algorithm to form new algorithms of FA-PBCA and FA-pPCA.

II. P-PERSISTENT CONTROL ALGORITHM AND PSEUDO-BAYESIAN CONTROL ALGORITHM

System with limited network nodes uses S-ALOHA to share wireless channel. The time axis is divided into time slot with the same size, whose length is just the time to send one data frame and each node can only be allowed to send data at the beginning of time slot. When two or more nodes send data simultaneously, there will be collision, and the collided data packet will be retransmitted at the subsequent time slot. Therefore, the information channel includes three kinds of state at any time slot: idle, successful and collision, expressed in...
\[0,1,\infty]\] separately. The fundamental of slotted Aloha is shown in Fig. 1.

![Fig. 1. The system principle diagram of slotted Aloha](image)

Assuming the actual number of node is \(N\) at one time slot \(t_0\), but the estimated number of node is \(M\). Each node transmits data at the probability of \(P = 1/M\) so as to gain the maximum throughput. So, the probabilities of being idle, successful transmission and collision are:

\[
\begin{align*}
P_{idle} &= \left(\frac{N}{0}\right)(1 - P_0)^N = \left(1 - \frac{1}{M}\right)^N \approx e^{-N/M} = e^{-\beta} \\
P_{succ} &= \left(\frac{N}{1}\right)P_0(1 - P_0)^{N-1} = N\frac{1}{M}\left(1 - \frac{1}{M}\right)^{N-1} \approx \beta e^{-\beta} \\
P_{coll} &= 1 - P_{idle} - P_{succ} = 1 - (1 + \beta)e^{-\beta}
\end{align*}
\]

where \(\beta = N/M\) is the ratio of actual number of node and estimated number of node. When \(\beta = 0.5\), \(P_{idle} \approx 0.61, P_{coll} \approx 0.09\). And when \(\beta = 2\), \(P_{idle} \approx 0.14, P_{coll} \approx 0.60\). The smaller \(\beta\) is, the larger the channel idle possibility \(P_{idle}\) will be and the collision possibility \(P_{coll}\) will be smaller; conversely, \(P_{idle}\) should be small and \(P_{coll}\) should be larger as \(\beta\) increase.

Take the natural logarithm of \(P_{idle}\) equation and we will get the actual number of node \(N\),

\[
\ln P_{idle} = -\frac{N}{M} \Rightarrow N = -M \ln P_{idle}
\]

where \(\ln x\) is the natural logarithm of \(x\). According to (2), the actual number of nodes \(N\) can be gained by counting up the probability of idle time slot in the channel during a certain time \((RW)\), which can be defined as the Renew Window of this period) and combining the assumed number of node \(M\). In the next \(RW\), each node can transmit data by adopting new probability \(p = 1/N\). Formula (2) can be further simplified and the sending probability \(P'\) of each node in the next \(RW\) can be gained,

\[
P' = P_0/\ln P_{idle} = P_0/(1/P_{idle})
\]

According to formula (3), pPCA Algorithm Control process can be designed as follows:

(i) At a starting time \(t_0\), the value of initialized number of node is assumed as \(n_0\). \(p = 1/n_0\) is calculated and the value of \(RW\) shall be set. A variable \(S\) shall be defined to count up the number of time slot, which shall be initialized as 0;

(ii) \(S\) shall be judged whether it equals to \(RW\); if so, \(S = 0\) and it shall go to (iv);

(iii) \(S\) adds 1 to produce \((S - 1)\) the even-distributed random number \(x\). \(x \leq p\) shall be judged; if so, data shall be sent to go to (ii);

(iv) The number of \(RW\) free time slot shall be counted up to calculate \(P_{idle}\). If \(P_{idle} = 0\) or \(P_{idle} = 1\), \(p\) remains unchanged; otherwise, \(p = -p/\ln P_{idle}\) and it shall go to (iii).

During the algorithm design, \(RW\) is a very important parameter. If its setting value is too small, the probable error will increase and the regulation precision will decrease; if \(RW\) is too large, the reregulation process will be delayed.

In slotted ALOHA system, when Pseudo-Bayesian Control Algorithm (PBCA) is used, the sending probability shall be adjusted on the base of current state of time slot. The procedure of algorithm implementation is shown as follows:

(i) At time slot \(v\), the number of system node is assumed as \(N_v\), each of which shall be sending data and grouping at the probability of \(q_v(N_v) = \min[1/N_v]\);

(ii) The number of node \(N_{v+1}\) for sending data at the next time slot shall be estimated by the formula (4):

\[
N_{v+1} = \begin{cases} 
\max(\lambda, N_v + \lambda - 1), & \text{idle or success} \\
N_v + \lambda + (e - 2)^{-1}, & \text{collision}
\end{cases}
\]

where \(\lambda\) is the mean arrival rate of new packet in a time slot.

(iii) Nodes send data packet with probability \(1/N_{v+1}\) in slot time \(v+1\).

III. THE CHANNEL STATE RUN OF SLOTTED ALOHA

**Definition 1:** In the system of slotted ALOHA with \(N\) nodes, the length of a successive status in the channel is considered to be the channel status run and the channel status switch with successive length of \(x\) is called channel state run of \(x\).

Dichotomies Model is used to divide the channel state into idle (or collision) state \(E\) and non-idle (non-collision) state \(\bar{E}\). The incidence probability of \(E\) is set as \(p\), \(t\) denotes time slot number and \(s(t)\) shows the channel state run from the start of time slot \(t\) in the \(RW\). Supposing that each time slot channel state is independent, \(s(t)\) is one-dimensional Markov chain. According to the definition of run, run is set as \(k \geq 1\). However, state 0 is increased in order to demonstrate the transfer process of channel state. The run state transition probability of the channel state in the slotted ALOHA system is shown in Fig. 2.
In this Markov chain, the only non null one-step transition probabilities are:
\[
P_{i_0} = 1 - p, \quad i = 0, 1, \ldots, RW
\]
\[
P_{i+1} = p, \quad i = 0, 1, \ldots, RW - 1
\]
(5)

A. The Difference Equation of Channel State Run

To study the run distribution of channel state at different nodes and transfer probabilities in slotted ALOHA system, the number of sampling elements is \(H(H \geq 2)\) and each sample value takes random experiment \(X\) of uniform distribution \((P = 1/H)\) as reference model in literature \([24, 25]\) to analyze the run distribution of a certain state \(E\) (supposing \(X = 1\)). In the independent experiments of \(n\) times, the total sample number is \(H^n\) and the sample number of event \(E\) with run \(R\) can be described by the arrangements in Fig. 3 and Fig. 4, among which “□” stands for the value of active elements (excluding event \(E\)), and “○” stands for the sequence of run \(R\), “" stands for the whole element.

Taking \(\xi = n - k\), \(G(\xi)\) shows all the samples with run \(k\) in the whole sample space. From the Fig. 3, it can be seen that \(G(\xi)\) has the value of:

(i)when \(k = n\), \(G(0) = 1\);

(ii)when \(k = n - 1\), there is only one active element, whose value doesn’t include the event \(E\), so there are two possible positions for the sequence with run of \(k\), and \(G(1) = 2(H - 1)\);

(iii)when \(k = n - 2\), \(G(2) = 3H^2 - 4H + 1\);

(iv)when \(k < n - 2\), the run sample number \(G(\xi)\) can be decomposed by the model in Fig. 4, from which it can be seen that \(G(\xi + 2)\) can be divided into two \(G(\xi + 1)\) that minus two free elements (the value can be any of symbols) and one \(G(\xi)\) arrange. That is
\[
G(\xi + 2) - 2HG(\xi + 1) + H^2G(\xi) = 0
\]
(6)

Work out the difference equation of formula (6) and take in the initial value of \(G(1), G(2)\), thus acquiring the following one.

\[
G(\xi) = \begin{cases} 
1 & \xi = 0 \\
\frac{H^2 - 1}{H^2} \times \left(\frac{H - 1}{H^2}\right)^{k-\xi} & 1 \leq \xi < n 
\end{cases}
\]

(7)

\(k\) is used to replace variable \(\xi\) and \(RL(k)\) is used to express the possible occurrence number of run \(k\) in \(n\)-times experiment, then
\[
RL(k) = \frac{1}{(1 - P)(2 + (1 - P)(n - k - 1))} P^k, \quad 1 \leq k < n
\]

(8)

In \(n\)-times independent experiment, \(H\) kinds of value can be evaluated and the sum of sample is \(z(n) = H^n = p^n\), then the number of run with the length of \(k\) in each sample is:
\[
g(k) = \frac{p^n}{(1 - P)(2P + (1 - P)(n - k - 1))} P^k, \quad 1 \leq k < n
\]

(10)

In addition, if \(\xi\) in formula (6) is replaced by \(n - k\) and both ends are divided by \(H^n\), the difference equation of the run number \(g(k)\) with the sample length of \(k\) (\(g(k)\) is called the frequency of run with the length of \(k\)).
\[
g(k) - 2pg(k + 1) + p^2g(k + 2) = 0
\]

(11)

Work out the formula (11) and put in the initial state, then (10) can be gained.

In the difference equation about the state run by combination method, the number of each event is integer and appears in uniformity; that is the event element is valued as \(H \geq 2\) and the corresponding event probability \(p = 1/H \leq 0.5\) . But in the slotted ALOHA system, although the channel state has three states of idle, successful transmission and collision, the distribution is not equal probable.

Dichotomy model can be used to divide the probability space into two parts \([0, p]\) and \([p, 1]\) . In MATLAB, Monte Carlo can be used to simulate this random experiment and then the frequency \(g'(k)\) with run \(k\) in each sample can be counted up to compare with the conclusion from formula (11). The difference of the two is defined as \(\delta(k)\). That is:
The parameter setting of simulated situation is: sample size \( n=100 \), the sample number (the repeated times in simulation) \( m=100000 \). The related result is shown in Table I.

The numerical result of Table I shows that when the sample size is large, the simulated result is quite close to the theoretical result of formula (10) and with the increase of run \( k \), the difference \( \delta(k) \) between the two becomes smaller and smaller until 0, which shows that \( g(k) \) and \( g'(k) \) meet the same difference equation (11).

It can be concluded that if a certain event has the probability of \( q \), the event probability of mutually exclusive events \( E \) is \( 1-q \). In the \( n \)-times independent experiment, each sample has the number of run \( k \) to meet difference equation.

\[
g(k) - 2q g(k+1) + q^2 g(k+2) = 0
\]

### Table I. The Comparison of Run Statistic and Theoretical Value Generated by Dichotomy

<table>
<thead>
<tr>
<th>( k )</th>
<th>( g(k) )</th>
<th>( g'(k) )</th>
<th>( \delta(k) )</th>
<th>( g(k) )</th>
<th>( g'(k) )</th>
<th>( \delta(k) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14.8330</td>
<td>14.8260</td>
<td>0.0070</td>
<td>6.5878</td>
<td>6.5940</td>
<td>0.0062</td>
</tr>
<tr>
<td>2</td>
<td>4.4087</td>
<td>4.4037</td>
<td>0.0050</td>
<td>4.5640</td>
<td>4.5717</td>
<td>0.0077</td>
</tr>
<tr>
<td>3</td>
<td>1.3067</td>
<td>1.3079</td>
<td>0.0012</td>
<td>3.1724</td>
<td>3.1693</td>
<td>0.0031</td>
</tr>
<tr>
<td>4</td>
<td>0.3918</td>
<td>0.3884</td>
<td>0.0034</td>
<td>2.1951</td>
<td>2.1969</td>
<td>0.0018</td>
</tr>
<tr>
<td>5</td>
<td>0.1152</td>
<td>0.1153</td>
<td>0.0002</td>
<td>1.5219</td>
<td>1.5227</td>
<td>0.0008</td>
</tr>
<tr>
<td>6</td>
<td>0.0345</td>
<td>0.0342</td>
<td>0.0002</td>
<td>1.0577</td>
<td>1.0553</td>
<td>0.0024</td>
</tr>
<tr>
<td>7</td>
<td>0.0100</td>
<td>0.0102</td>
<td>0.0002</td>
<td>0.7329</td>
<td>0.7313</td>
<td>0.0016</td>
</tr>
<tr>
<td>8</td>
<td>0.0029</td>
<td>0.0030</td>
<td>0.0001</td>
<td>0.5064</td>
<td>0.5067</td>
<td>0.0003</td>
</tr>
<tr>
<td>9</td>
<td>0.0010</td>
<td>0.0009</td>
<td>0.0001</td>
<td>0.3509</td>
<td>0.3511</td>
<td>0.0002</td>
</tr>
<tr>
<td>10</td>
<td>0.0002</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.2444</td>
<td>0.2432</td>
<td>0.0012</td>
</tr>
<tr>
<td>11</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0000</td>
<td>0.1670</td>
<td>0.1685</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

**B. Probability Distribution of Channel State Run**

In wireless network system which utilizes S-ALOHA protocol to share channels, each of which has three states: idle, successful transmission and collision. Dichotomy principle can be used to divide the slotted ALOHA channel state into idle state \( E \) and non-idle state \( \overline{E} \), successful state \( E \) and non-functional status \( \overline{E} \), or collision state \( E \) and non-collision state \( \overline{E} \). In the later description, it is called channel state \( E \) and mutual exclusion \( \overline{E} \). Taking the channel state of each slot time in RW as a sample, the run of a certain state means the appearance of \( E \) in the sample.

**Definition 2:** The ratio of run (with the length of \( k \)) expectation times in state \( E \) of each sample and the frequency of total expectation times in this state is considered to be the probability density function \( f(k) \) in a certain state with the length of \( k \). That is:

\[
f(k) = g(k) / \sum_{x=1}^{k} g(x)
\]

While \( S = \sum_{i=1}^{k} i \cdot (p' + (1 - p')(n - i) - 1) \cdot p' \)

\[
= p' + 2(1-p')(n-1) \sum_{i=1}^{k} i \cdot p' \sum_{i=1}^{k} (n-i-1) \cdot p' \\
= p'[1+(1-p)(n-1)]
\]

Put it into formula (14) and

\[
f(k) = \begin{cases} 
\frac{p^{k-1}}{1+(1-p)(n-1)}, & k = n \\
\frac{p^{k-1}}{1+(1-p)(n-1)[2+(1-p)(n-k-1)]}, & 1 \leq k < n
\end{cases}
\]

When \( n \to \infty \), and \( k \ll n \).

**C. Recurrence Period of Channel State k Run**

\( g(k) \) describes the average occurrence number of the run with the length of \( k \) in the channel state samples with the length of \( n \) after \( n \) times data transmission. Formula (11) shows that the bigger \( n \) is, the larger \( g(k) \) will be.

**Definition 3:** when the channel state probability is \( p \), if the independent transmission of \( T(p,k) \) slot time is carried out, there will be at least once channel state run
with the length no less than \( k \), which is known that \( T(p,k) \) is the renewed period with the run no less than \( k \) in this channel status.

In the sample space of channel state formed by \( n \) times transmitting procedure, the number of run with the length no less than \( k \) included in each sample is:

\[
Q(k) = g(n) + \sum_{i=k}^{n-1} g(i) = p^k + (1-p)[2 + (1-p)(n-k-1)]p^k = [1 + (1-p)(n-k)]p^k
\]

(20)

According to the definition of \( T(p,k) \), the value refers to the sample length \( n \) divided by average time of channel state run with the length no less than \( k \) in each sample. Then

\[
T(p,k) = \frac{n}{Q(k)} = \frac{n}{[1 + (1-p)(n-k)]p^k}
\]

(21)

When the sample length \( n \) equals to the renewed period \( T(p,k) \) in channel state run, the times of channel state run with the length no less than \( k \) in the samples \( Q(k) = 1 \). That is \([1 + (1-p)(n-k)]p^k = 1\), and then:

\[
T(p,k) = k + \frac{p^{k-1} - 1}{1-p}
\]

(22)

IV. FAST ADAPTIVE CONTROL ALGORITHM

The slotted ALOHA system of \( p \)-persistent Control Algorithm is adopted and through the transmission of \( RW \) slot time, the idle probability of \( RW \) has been counted up. Then, each node in the system shall be calculated by formula (3) in the next \( RW \) to achieve the probability \( P^r = P_0 / \ln(1/P_{\text{Adv}}) \) and to send data. It can be seen from the formula (3) that:

(i) When \( P_{\text{Adv}} \rightarrow 1, \ln(1/P_{\text{Adv}}) \rightarrow 0 \), \( P^r \) increase and tends to 1; but when \( P_{\text{Adv}} = 1 \), formula (3) has no meaning. Actually, when \( P_{\text{Adv}} = 1 \), the whole \( RW \) has idle channel and the nodes have no data to transmit. From formula (1), it is obvious that \( N \) is small while \( M \) is large. That is, the system nodes change from a large value into small one at a certain time.

(ii) When \( P_{\text{Adv}} \rightarrow 0 \), \( \ln(1/P_{\text{Adv}}) \rightarrow \infty \), \( P^r \) decreases and tends to 0; however, when \( P_{\text{Adv}} = 0 \), formula (3) has no meaning as well. But when \( P_{\text{Adv}} = 0 \), each slot time is sending data and it is known from formula (1) that \( N \) is very large while \( M \) is small. That is, the system nodes change from a large value into small one at a certain time.

A. Theorem of Fast Adaptive Algorithm

Theorem 1: In the slotted ALOHA system, channel has been divided into idle state \( E \) (with the probability of \( p \)) and non-idle state \( \bar{E} \). After \( n (n > 7) \) times slot-time transmission, if a run sequence with length of 7 at channel idle state is detected, \( p \geq \exp(-0.5) \approx 0.61 \) at one-sided confidence interval of 0.95 and the actual number of system node is less than 1/2 of estimated number of node.

Demonstration: Variable \( R \) represents run of channel idle state of \( E \). \( P[R < k] \geq 1-\alpha \) is \( 1-\alpha \) confidence interval of \( R \) and there is correspondingly \( P[R \geq k] \leq \alpha \).

And

\[
P[R < k] = 1 - P[R \geq k] = 1 - F(k)
\]

(23)

From formula (17), it is known that if \( P[R < k] \geq 1-\alpha \), \( F(k) \) should meet the demands that:

\[
F(k) = P[R \geq k] \leq 1 - 0.95 = 0.05
\]

(24)

When \( n \) is relatively large, put \( p = \exp(-0.5) \approx 0.61 \) into formula (19), and the in equation of formula (23) has been changed into:

\[
F(k) = p^{k-1} \leq 0.05
\]

(25)

From formula (19), \( F(k) \) is the monotone increasing function of \( p \in [0,1] \). And formula (25) shows that when \( p < \exp(-0.5) \), the probability of detecting channel idle state \( E \) with the length \( k \geq 7 \) is \( F(k) < 0.05 \). The smaller \( p \) is, the smaller \( F(k) \) will be. Conversely, when \( p \geq \exp(-0.5) \), the probability of detecting channel idle state \( E \) with the length \( k \geq 7 \) is \( F(k) \geq 0.05 \). The larger \( p \) is, the larger \( F(k) \) will be.

Therefore, at the one-side confidence interval of 0.95, when the run of \( k = 7 \) is detected, the event probability of \( E \) is \( p \geq \exp(-0.5) \approx 0.61 \). From formula (1), when \( p = P_{\text{Adv}} = \exp(-N/M) \geq \exp(-0.5) \),

\[
N \leq 0.5 \Rightarrow N \leq M / 2
\]

(26)

Theorem 1 has been proved.

In the \( p \)-persistent Control Algorithm of slotted ALOHA, the rational choice of \( RW \) is crucial. From Theorem 1, when channel idle state run with the length of \( k = 7 \) is detected, the number of system node will be less than 1/2 of the actual number of node. And then the index will be adjusted. From formula (9), the larger the slot time number \( n \) used for transmitting procedure is, the larger the number of idle state run with the length of \( k \) will be. On the other hand, when \( n \) is too small, even the probability of channel idle state is large \( p > 0.61 \), the idle state run with the length of 7 cannot be obtained, so the adjustment cannot be performed in a quick way.

The formula (22) shows that \( k \) determines the situation. And the larger the event probability of channel idle state \( E \) is, the run recurrence interval \( T(k) \) with the length of
$k$ will be smaller; when $p$ is certain, the smaller $k$ is, and the smaller $T(k)$ will be. MATLAB computing equipment can be used to obtain the relationship between $T(k)$ and event probability $p$ of channel idle state $E$, shown in Fig. 5, from which it can be seen that when $p = 0.7$, a run sequence with the length no less than 7 every 40 times experiment, $T(0.7, 7) \approx 40$.

A run sequence with the length no less than 6 every 30 times experiment, $T(0.7, 6) \approx 30$. But when $p = 0.61$, $T(0.61, 7) \approx 80$, $T(0.61, 6) \approx 50$. The bigger the event probability of channel idle state is, the smaller the run renewed period with the length of $k$ will be. When the event probability of channel idle state is $p = e^{-n/M} \approx 0.7$, from formula (1), $N/M \approx 0.37$, the estimated number of node will be three times larger that the actual number of node. In the application FA algorithm system, a run with the length no less than 7 must appear more than once in a $RW$. But when the estimated nodes are three times more than actual ones, there can only be one fast adjustment. Therefore, more $RW$ values shall be realized to the greatest extent.

![Fig. 5. The relation schema $T(k)$ vs. probability $p$](image)

$T(0.7, 6) < RW \leq T(0.7, 7) \Rightarrow 30 < RW \leq 40$ \hspace{1cm} (27)

In pPCA, the $RW$ value shall be set as 32.

**B. Design of Fast Adaptive Algorithm**

The former theorem demonstrates that if seven idle states are continuously detected in $RW$ (the run with the length of 7 in channel idle state). At one-sided confidence interval of 0.95, channel idle probability $P_{idle} \geq 0.61$ and the actual number of system node shall be less than 1/2 of the estimated node in the current window. The estimated number of node $M$ is renewed to be 1/2 of the initial one (that is $M = M/2$), and meanwhile, the node adopts new probability $P = 1/M$ to send data. In a similar way, the channel state is divided into collision state and non-collision state, seen in formula (1). When the actual number of system node $N$ is twice larger than the estimated number of node, the channel collision probability $P_{coll} \approx 0.6$. Besides, the larger $\beta = N/M$ is, the larger $P_{coll}$ will be. When the collision state run with the length of 7 is detected, the actual number of node $N$ is regarded to be twice larger than the estimated number of node $M$ at confidence interval of 0.95. Control algorithm can adjust the sending probability of node to be $1/2$ of the original.

The index regulating process based on the detected channel state run is known as adaptive control algorithm (FA: Fast Adaptive). The executing process of fast adaptive p-persistent control executing process algorithm (FA-pPCA) combined with p-persistent control is shown as follows:

(i) Before a starting time $t_0$, the number of node in stable system is $n_0$, and each node is sending data at the probability of $p = 1/n_0$; at first, initialization of variable shall be carried out, including the $RW$ value, simulation slotted time $t$; the actual number of node after $t_0$ is $n$; the estimated number of node in $RW$ is $M$ and the variables in idle and collision runs are $S_{n_0}, S_n$;

(ii) Judge $mod(t, RW) = 0$? If so, go to (v);

(iii) Slot time counter adds 1($t = t + 1$). Count up the current transmission node of slot time $S_t$ and determine the channel state $S_t$;

(iv) Fast algorithm process;

(v) Count up the number of idle slotted time in $RW$ and calculate $P_{idle}$ and the node sending probability $p = -p / \ln P_{idle}$ in the next $RW$, then turn to (iii).

In the same way, the implementation step of fast self-adaption Pseudo-Bayesian control algorithm (FA-PBCA) is as follows:

(i) In slot time $v$, it is supposed that the node number in the system is $N_v$, and each node is sending data group at the probability of $q_v(N_v) = \min \{1, 1/N_v\}$;

(ii) The treatment of fast adaptive algorithms: if the idle state of run with 7 is detected, $N_{v+1} = N_v / 2$; if the collision state of run with 7 is detected, $N_{v+1} = 2N_v$ and jump into (iv);

(iii) The normal treatment of Pseudo-Bayesian control algorithm; the node number in the next slot time, which will send data, shall be estimated by the following formula:

$$N_{v+1} = \max(\lambda N_v + \lambda - 1, \begin{cases} \text{idle or success} & \lambda N_v + \lambda + (e-2)^{-1}c, \\ \text{collision} & \end{cases})$$ \hspace{1cm} (28)

where $\lambda$ is the arrival rate of new packet.

(iv) Each node of next slot time shall send request to group at the probability of $1/N_{v+1}$.

**C. Simulation and Verification of Algorithm**

The throughput of system is the key index to evaluate network performance. In the MAC agreement based on competition, the high throughput also means the low delay. MATLAB tools can be used to emulate FA-pPCA in the aspect of throughput capacity during stability adjustment and the adjust time. The environment setting of simulation is: at the starting moment $t_0 = 0$, the node
number changes from \( m \) to \( n \), with the simulation time of 100 slot time.

From the charts, it can be seen that, when \( m \) sharply changes into small node number (2, 5, 10). From slot time of \( t_0 \), the stability adjustment process and the throughput capacity of FA-pPCA and pPCA are shown in Fig. 6 (c), (d). The simulation result shows that FA-pPCA algorithm can rapidly adjust the node number \( m \) into the scope of actual node number \( n \) [0.5, 1]. During this process, the throughput capacity of FA-pPCA is larger than that of pPCA. Because of the effect of \( RW \), when the node number is very large, the channel collision in pPCA control system will be intensified and the throughput of system will be lowered. Generally, the stable maximum throughput of approximation theory will be gained after four adjustment windows (about 128 slot times). But FA-pPCA can restrain the estimated node number into the actual number of node \( n \) [0.5, 1] after several fast adjustments. And then one \( RW \) adjustment can basically make the maximum throughput close to theoretical value. It should be noticed that the specific value \( \beta = m/n \) of estimated node number \( m \) and the actual node number \( n \) shall be moderated. During a certain slot time period, the throughput capacity of FA-pPCA shall be slightly lower than that of pPCA. It is because that after the fast adjustment, the slot time counter of FA-pPCA leave over \( 7k \) (k is the times of fast adjustment) slot times. So, the adjustment of pPCA is also delayed by \( 7k \) slot times. When \( \beta \) has a probable value, idle probability statistics is relatively accurate and the stable maximum throughput of system can mostly reach the theoretical value.

Fig. 7 is the throughput performance comparison chart of FA-PBCCA and PBCA algorithms. Where, Fig. 7 (a), (b) show the average throughput of each time during the regulating process on the states that the initial node number of system changes from the maximum (e.g. \( m=50,100 \)) to the minimum(eg,2,5,10). The simulation results in Fig. 7 (a) and (b) show that the larger the initial node number is, the smaller the node number will be, the smaller the throughput of PBCCA will be and the longer the time to reach the stabilization of maximum throughput will take. But FA-PBCCA can quickly adjust the throughput of system into stable maximum throughput.

Fig. 7 (c) and (d) are the regulation performance comparison charts of FA-PBCCA and PBCCA algorithms when the initial node number changes from the small value (e.g. \( m=5,10 \)) into large one (eg, 20, 50, 100). From the simulation result, it can be seen that FA-PBCCA can obviously enhance the regulation performance of the system.

From the comparison of Fig. 7, the accommodation time of node number changing from large value to small
one is longer than that of the opposite process. To measure the system throughput performance of FA-PBCA, PBCA, FA-pPCA and pPCA at different node changing states, simulation is carried out at MATLAB, with the simulation time of 100 slot times to calculate the average throughput on different scenes. The simulation results are shown in Fig. 8.

Fig. 7. Throughput of FA-PBCA and PBCA in adjustment processes

Fig. 8. Average throughput of four control algorithms
From the Fig. 8, it can be seen that there have been large differences between estimated node number $m$ and actual node number $n$. The higher the throughout performance of the FA-pPCA is, compared with pPCA; the higher the throughout performance of the FA-PBCA will be, compared with PBAC. Fig. 8(a), (b) is the average throughput of circumstances which estimate node is comparatively large (e.g. $m=50,100$), real node number of system changes from 2 to 150. It shows that the larger estimate nodes and small real node is, the lower throughput will be. In addition, the throughput of pPCA or PBAC with fast control algorithm is apparently higher than pPCA or PBAC without fast control algorithm. With the corresponding, Fig. 8(c), (d) is the average throughput of circumstances which estimate node is comparatively large (e.g.$m=5,10$), real node number of system changes from 2 to 150. It shows that the small estimate nodes and larger real node is, the lower throughput will be. In addition, the throughput of pPCA or PBAC with fast control algorithm is apparently higher than pPCA or PBAC without fast control algorithm. On the spot of large initial node number, the average throughout performance of pPCA is higher than that of PBAC; while on the spot of small initial node number, the average throughout performance of PBAC is higher than that of pPCA. When $\beta \in [0.5, 1]$ ($\beta = m/n$), FA-pPCA has the similar throughout performance with that of pPCA and FA-pPCA also has the similar throughout performance with that of PBAC.

V. CONCLUSIONS

When the node number in the system changes sharply, p-persistent Control Algorithm has the problem of long accommodation time, which can hardly adapt to the sudden changes of node number. Fast adaptive control algorithm (FA) divides the channel state into idle state and non-idle state. When there are seven idle slot times are detected, the actual node number will be 1/2 of estimated node number at confidence interval of 0.95. The estimated node number $M$ can be adjusted into $M/2$. In the same way, channel state can be divided into collision state and non-collision state. When there are seven collision slot times detected, the estimated node number $M$ can be adjusted into $2M$. The simulation result shows that FA has relatively higher performance in the situation of replying to the sharp changes of web node number.

REFERENCES


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