A Simple Joint Clock Synchronization and Localization Algorithm with Low Communication Overhead

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Abstract —A novel simple joint clock synchronization and localization algorithm based on Pairwise Broadcast Synchronization (PBS) protocol in wireless sensor networks (WSNs) is proposed in this paper. In this scheme, anchor nodes (with unknown clock bias and known position) overhear the timing message exchanges of reference node (with known clock bias and known position) and unknown node (with unknown clock bias and unknown position) without sending extra messages. Comparing this proposed algorithm with those algorithms based on two-way message exchanges mechanism, we find that this new algorithm can reduce a lot of communication overhead and requires less reference nodes. The simulation results indicate that this algorithm can ensure high accuracy and its estimation performance meets the Cramer-Rao Lower Bounds (CRLB) simultaneously. Considering the tradeoff between the estimation performance and communication overhead, we can see that the proposed algorithm performs better than the available joint clock synchronization and localization algorithms.

Index Terms—Wireless sensor networks, clock synchronization, localization, communication overhead, energy-efficient

I. Introduction

With the rapid development of Micro-Electro-Mechanical Systems (MEMS) and wireless communications, Wireless Sensor Networks (WSNs) attract a lot of attention. WSNs, which comprise a large number of wireless sensing devices, can be widely deployed for energy-agile, data-gathering, and target-tracking applications [1], [2], such as rare species localization and tracking, battlefield detecting and rescuing, industrial machinery controlling, etc. Many applications of WSNs require clock synchronization for transmission scheduling, data fusion, sleeping and activating normally. At the same time, node localization is a prerequisite to the accurate tracking ability of WSNs. WSNs generally consist of many small sensors whose energy resource are quite limited and cannot be charged in workplace, but a large proportion of its total energy consumption is spent on clock synchronization and node localization [3], [4], energy efficient synchronization protocols are highly desirable.

Clock synchronization is a crucial requirement to ensure optimal data fusion and coordinated wake and sleep periods [5], [6]. Several clock synchronization protocols have been proposed for WSNs, such as Reference-Broadcast Synchronization (RBS) [7], Time-sync Proto-col for Sensor Networks (TPSN) [8], Flooding Time Synchronization Protocol (FTSP) [9] and so on. The data exchange mechanism can be roughly divided into three categories, i.e. two-way data exchange mechanism for exchanging timing message between two adjacent nodes [10], one-way data exchange mechanism in which a set of nodes receive timing message of master node without sending message [11], hybrid data exchange mechanism which combines both one-way and two-way mechanisms [12]. Because the two-way data exchange mechanism needs a lot of communication overhead and the estimation performance of one-way data exchange mechanism is low, Pairwise Broadcast Synchronization (PBS) protocol [12] with hybrid data exchange mechanism has attracted more attention. Joint Maximum-Likelihood Estimator (JMLE) algorithm based on PBS protocol is proposed in [13], but the assumption with constant propagation delay between any two nodes in the algorithm is impractical.

On the other hand, node localization ensures WSNs accurately locate the moving target. Range-based localization algorithms which localize the node by measuring the actual distance between adjacent nodes and range-free localization algorithms [14] which determine position of the node by network connectivity without measuring the distance between adjacent nodes are two types of localization algorithm in WSNs, and the range-based localization algorithms can achieve higher estimation performance. In general, the range-based algorithms utilize the metrics of Time of Arrival (TOA) [15] and Time Difference of Arrival (TDOA) [16], e.g., Best Linear Unbiased Estimator-Least Squares Calibration (BLUE-LSC) and Best Linear Unbiased Estimator-Linear

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Least Squares (BLUE-LLS) based on TOA algorithm is obtained in [17], Ref. [18] proposed an efficient localization algorithm based on TDOA. However, it is important to note that the prerequisite of the range-based localization algorithms is time synchronization between sending and receiving node.

Due to the time synchronization and node localization have a close connection, researchers proposed a unified framework to jointly solve time synchronization and node localization at the same time. This joint algorithm can be roughly divided into two categories, i.e., joint clock synchronization and range-based localization algorithm and joint clock synchronization and range-free localization algorithm. Joint clock synchronization and range-based localization algorithm proposed in [19] uses two-way message exchange between any two nodes to estimate clock parameters of unknown nodes and distance between every two nodes. Based on the algorithm in [19], ref. [20] proposed a novel Asymmetrical Time-Stamping and Passive Listening (ATPL) protocol for joint clock synchronization and ranging, and it can reduce lots of communication overhead. Joint clock synchronization and range-free localization algorithm proposed in [2] uses Least Squares (LS) estimator to jointly estimate parameters of location and clock of unknown node based on two-way data exchange mechanism. To reduce communication overhead and computational complexity, the algorithm in [21] uses the linearized equations from TOA measure-ments and applies a Weighted Least Square (WLS) criterion to closely approach estimation performance of the LS solution.

Because of the low-power limitation of the WSNs nodes, we use PBS scheme to jointly solve the clock synchronization and localization of unknown node. The proposed algorithm requires less communication overhead and fewer reference nodes, so it can prolong the service life of nodes and improve algorithm's flexibility. We analyze and simulate estimation model and performance of the proposed algorithm, and the analytical and simulation results show that estimation performance of this algorithm can meet corresponding Cramer-Rao Lower Bound (CRLB) and very closely approach the estimation performance of algorithms in [2] and [21].

The remainder of this paper is organized as follows: the system model of the proposed algorithm is outlined in Section II; Section III detailed analyzes the process of this algorithm; the CRLB is derived in Section IV and numerical simulation results are discussed in Section V; Section VI concludes this paper.

II. System Model

By analyzing the clock synchronization and localization algorithms in WSNs, we can find that fewer reference nodes can improve flexibility of algorithm and less communication overhead can prolong the service life of sensor nodes. We use the message exchange mode of PBS protocol to jointly solve the clock synchronization and localization of unknown node. The proposed algorithm requires fewer reference nodes and less communication overhead, so the network has longer service life and the algorithm has better flexibility.

Consider a simple joint synchronization and localization system as shown in Fig. 1 which composed of $M+2$ wireless nodes: one unknown node S to be estimated with unknown clock bias θ_s and unknown position $\mathbf{p}_s = [x_s \quad y_s]^T$, one reference node R with known clock bias $\theta_r = 0$ and known position $\mathbf{p}_r = [x_r \quad y_r]^T$, and M anchor nodes N_i with unknown clock bias θ_i and known position $\mathbf{p}_i = [x_i \quad y_i]^T$, $(i = 1, \cdots, M)$. We assume perfect communication (no data loss and no failure) at the physical layer. Node R and node S exchange timing message with each other, and nodes N_i $(i = 1, \cdots, M)$ passively listen to the timing message between node R and node S without sending any extra messages.

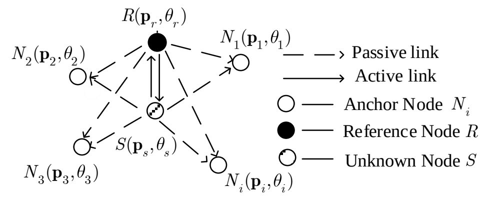


Fig. 1. The signaling mechanism for the proposed algorithm

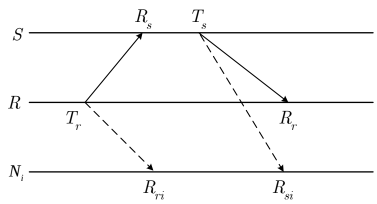


Fig. 2. Timestamp list for the proposed algorithm

As shown in Fig. 2, reference node R sends a synchronization message to unknown node S at time T_r, node S records its reception time R_s for the message and replies to node R at time T_s, then node R records the reception time of node S 's reply as R_r. At the same time, the anchor node N_i, $(i = 1, \cdots, M)$ records the reception time of the message from node R as R_{ri} and the reception time of the message from node S as R_{si}. Then, we can obtain a set of timestamps $T_r, R_s, R_{ri}, T_s, R_r, R_{si}\ {}_{i=1}^{M}$. The aforementioned procedure can be modeled as

$$\begin{aligned} R_s &= T_r + t_{sr} + e_{i,1} + \theta_s \\ R_{ri} &= T_r + t_{ri} + e_{i,2} + \theta_i \\ R_r &= T_s + t_{sr} + e_{i,3} - \theta_s \\ R_{si} &= T_s + t_{si} + e_{i,4} + \theta_i - \theta_s \end{aligned} \tag{1}$$

where t_{sr}, t_{si}, t_{ri} represents the propagation delay between unknown node S and reference node R, the propagation delay between node S and anchor node N_i, the propagation delay between node R and anchor node N_i, ($i=1,\cdots,M$), respectively. For simplicity, we normalize the propagation velocity as $v=1$, then $t_{sr}=\|\mathbf{p}_s-\mathbf{p}_r\|$, $t_{si}=\|\mathbf{p}_s-\mathbf{p}_i\|$, and $t_{ri}=\|\mathbf{p}_r-\mathbf{p}_i\|$. The symbols $e_{i,1}\ e_{i,2}\ e_{i,3}\ e_{i,4}$, ($i=1,\cdots,M$), are measurement errors, which are assumed as independent, and identically distributed (*i.i.d*) Gaussian variables with zero-mean and common variance σ^2.

III. Joint Timing and Location Estimation Algorithm

A. *Joint Clock Synchronization and Localization of Unknown Node*

Rewriting (1), we can obtain a modified equation,

$$\begin{aligned} t_{si}=\|\mathbf{p}_s-\mathbf{p}_i\| &= R_{si}-T_s-R_{ri}+T_r+t_{ri} \\ &+\theta_s-(e_{i,4}-e_{i,2}) \end{aligned} \tag{2}$$

After adjusting and squaring the two sides of (2), we can obtain

$$\begin{aligned} & T_i^2-\|\mathbf{p}_i\|^2+\theta_s^2-\|\mathbf{p}_s\|^2+e_i^2-2T_i\theta_i-2\theta_s e_i \\ & =-2T_i\theta_s-2\mathbf{p}_i^T\mathbf{p}_s \end{aligned} \tag{3}$$

where $e_i\equiv e_{i,4}-e_{i,2}$ ($i=1,\cdots,M$) are modeled as *i.i.d* Gaussian with zero-mean and common variance $2\sigma^2$, and $T_i=R_{si}-R_r-R_{ri}+R_s+t_{ri}$.

By stacking data from all M anchors, the system model in (3) can be compactly expressed as in vector-matrix form

$$\mathbf{A}\mathbf{q}=\mathbf{b}+\mathbf{1}\eta+\mathbf{w} \tag{4}$$

where the unknown parameter vector to be estimated is $\mathbf{q}=\left[\theta_s\ \mathbf{p}_s^T\right]^T$, $\mathbf{1}$ denotes the $M\times 1$ vector of ones, $\eta=\theta_s^2-\|\mathbf{p}_s\|^2$, measurement vector $\mathbf{b}$ and measurement matrix $\mathbf{A}$ are known from the message exchange,

$$\mathbf{b}=\begin{bmatrix} T_1^2-\|\mathbf{p}_1\|^2 \\ \vdots \\ T_M^2-\|\mathbf{p}_M\|^2 \end{bmatrix},\quad \mathbf{A}=\begin{bmatrix} -2T_1 & -2\mathbf{p}_1^T \\ \vdots & \vdots \\ -2T_M & -2\mathbf{p}_M^T \end{bmatrix} \tag{5}$$

For sufficiently small noise conditions, measurement error vector is

$$\mathbf{w}=\begin{bmatrix} e_1^2-2T_1e_1-2\theta_s e_1 \\ \vdots \\ e_M^2-2T_Me_M-2\theta_s e_M \end{bmatrix}\approx\begin{bmatrix} -2(\theta_s+T_1)e_1 \\ \vdots \\ -2(\theta_s+T_M)e_M \end{bmatrix} \tag{6}$$

Suppose $C_{\mathbf{w}}=E\ \mathbf{w}\mathbf{w}^T\ /2\sigma^2$, i.e.

$$C_{\mathbf{w}}=\begin{bmatrix} (2T_1+2\theta_s)^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & (2T_M+2\theta_s)^2 \end{bmatrix} \tag{7}$$

where $E\ \mathbf{A}$ is expected value of random matrix $\mathbf{A}$. So the estimator vector can be denoted as [21]

$$\hat{\mathbf{q}}=\mathbf{h}+\mathbf{g}\eta \tag{8}$$

where

$$\begin{aligned} \mathbf{h} &= \ \mathbf{A}^TC_{\mathbf{w}}^{-1}\mathbf{A}\ ^{-1}\mathbf{A}^TC_{\mathbf{w}}^{-1}\mathbf{b} \\ \mathbf{g} &= \ \mathbf{A}^TC_{\mathbf{w}}^{-1}\mathbf{A}\ ^{-1}\mathbf{A}^TC_{\mathbf{w}}^{-1}\mathbf{1} \end{aligned} \tag{9}$$

If η can be estimated, say $\hat{\eta}$, then the solution of $\mathbf{q}$ is

$$\hat{\mathbf{q}}=\mathbf{h}+\mathbf{g}\hat{\eta}=\begin{bmatrix} \hat{\theta}_s \\ \hat{\mathbf{p}}_s \end{bmatrix} \tag{10}$$

So we can yield the following relationship

$$\begin{aligned} \hat{\mathbf{q}}^T\mathbf{V}\hat{\mathbf{q}} &= \mathbf{g}^T\mathbf{V}\mathbf{g}\hat{\eta}^2+2\mathbf{g}^T\mathbf{V}\mathbf{h}\hat{\eta}+\mathbf{h}^T\mathbf{V}\mathbf{h} \\ &= \hat{\theta}_s^2-\|\hat{\mathbf{p}}_s\|^2=\hat{\eta} \end{aligned} \tag{11}$$

where

$$\mathbf{V}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{12}$$

We can obtain a new quadratic equation of $\hat{\eta}$ as follow by rewriting (11),

$$\mathbf{g}^T\mathbf{V}\mathbf{g}\hat{\eta}^2+\ 2\mathbf{g}^T\mathbf{V}\mathbf{h}-1\ \hat{\eta}+\mathbf{h}^T\mathbf{V}\mathbf{h}=0 \tag{13}$$

By defining $\Delta\equiv\ 2\mathbf{g}^T\mathbf{V}\mathbf{h}-1\ ^2-4\mathbf{g}^T\mathbf{V}\mathbf{g}\mathbf{h}^T\mathbf{V}\mathbf{h}$, we can obtain $\hat{\eta}$ by solving quadratic equation (13) in four cases as follow [21].

(1) $\mathbf{g}^T\mathbf{V}\mathbf{g}=0$: the solution of $\hat{\eta}$ is simple, i.e.

$$\hat{\eta}=\mathbf{h}^T\mathbf{V}\mathbf{h}/(1-2\mathbf{g}^T\mathbf{V}\mathbf{h}) \tag{14}$$

(2) $\mathbf{g}^T\mathbf{V}\mathbf{g}\neq 0$ and $\Delta=0$: there exists only one solution for $\hat{\eta}$, i.e.

$$\hat{\eta}=\ 1-2\mathbf{g}^T\mathbf{V}\mathbf{h}\ \big/\ 2\mathbf{g}^T\mathbf{V}\mathbf{g} \tag{15}$$

(3) $\mathbf{g}^T\mathbf{V}\mathbf{g}\neq 0$ and $\Delta>0$: there exists two solutions for $\hat{\eta}$, i.e.

$$\begin{pmatrix} \hat{\eta}_1 \\ \hat{\eta}_2 \end{pmatrix}=\frac{1-2\mathbf{g}^T\mathbf{V}\mathbf{h}\pm\sqrt{\Delta}}{2\mathbf{g}^T\mathbf{V}\mathbf{g}} \tag{16}$$

(Note: From simulation results we find $\hat{\eta}_2$ can achieve higher performance than $\hat{\eta}_1$, so we choose $\hat{\eta}_2$ as solution in case (2).)

(4) $\mathbf{g}^T\mathbf{V}\mathbf{g}\neq 0$ and $\Delta<0$: there is no valid solution exists for $\hat{\eta}$. This situation may occur when the measurement errors are too large. If this happens, indicator $\Delta<0$ can be used to lead to an alternative algorithm also with closed solution, for example separately treated clock synchronization and localization.

Initializing the $C_{\mathbf{W}}=\mathbf{I}$, i.e. the identity matrix, we can obtain $\hat{\eta}$ according to the algorithm process, then update

$C_{\mathbf{W}}$ by (7). Repeating the above process, we can get more accurate estimators.

B. Clock Synchronization of Anchor Nodes

From (1) we have

$$e_{i,2} = R_{ri} - T_r - \theta_i - t_{ri} \tag{17}$$

If there are N rounds message exchange, the likelihood function for θ_i is given by

$$\begin{aligned} \mathrm{L}(\theta_i) = \frac{1}{(2\pi\sigma^2)^N} \exp\Bigg\{ -\frac{1}{2\sigma^2} \times \\ \left[\sum_{n=1}^{N} \; R_{ri,n} - T_{r,n} - t_{ri} - \theta_i \;\; ^2 \right] \Bigg\} \end{aligned} \tag{18}$$

where $R_{ri,n}, T_{r,n}$ are the time parameters of R_{ri} and T_r at n -th message exchange, respectively. Differentiating the log-likelihood function leads to

$$\frac{\partial \ln \mathrm{L}(\theta_i)}{\partial \theta_i} = \frac{1}{\sigma^2} \left[\sum_{n=1}^{N} \; R_{ri,n} - T_{r,n} - t_{ri} - \theta_i \;\; \right] \tag{19}$$

Hence, the maximum likelihood estimation (MLE) of clock offset is given by

$$\hat{\theta}_i = \frac{1}{N} \sum_{n=1}^{N} \; R_{ri,n} - T_{r,n} - t_{ri} - \theta_i \tag{20}$$

C. Performance Bound Derivation

The CRLB of clock and location parameters of unknown node: Below we give the bias and error variance of the proposed algorithm in three cases having solution according to the method of [21].

(1) $\mathbf{g}^T \mathbf{V} \mathbf{g} = 0$:

$$S = 2\sigma^2 \operatorname{tr} \; \mathbf{Q}^T \mathbf{Q} \;\; , \; D = E \;\; \mathbf{u}^T \mathbf{V} \mathbf{u} \tag{21}$$

(2) $\mathbf{g}^T \mathbf{V} \mathbf{g} \neq 0$ and $\Delta = 0$:

$$S = E \;\; \|\mathbf{u}\|^2 \;\; , \; D = -2\sigma^2 \tag{22}$$

(3) $\mathbf{g}^T \mathbf{V} \mathbf{g} \neq 0$ and $\Delta > 0$:

$$S = 2\sigma^2 \operatorname{tr} \; \mathbf{Q}^T \mathbf{Q} \;\; , \; D = E \;\; \mathbf{u}^T \mathbf{V} \mathbf{u} \tag{23}$$

where

$$\mathbf{u} = \;\; \mathbf{A}^T C_{\mathbf{W}}^{-1} \mathbf{A} \;\; ^{-1} \mathbf{A}^T C_{\mathbf{W}}^{-1} \;\; \mathbf{w} - \sigma^2 \mathbf{1} \tag{24}$$

$$\mathbf{Q} = \left(\frac{2\mathbf{g}\mathbf{q}^T \mathbf{V}}{1 - 2\mathbf{g}^T \mathbf{V} \mathbf{q}} + \mathbf{I} \right) \;\; \mathbf{A}^T C_{\mathbf{W}}^{-1} \mathbf{A} \;\; ^{-1} \mathbf{A}^T C_{\mathbf{W}}^{-1/2} \tag{25}$$

$$\begin{aligned} & E \;\; \mathbf{u}^T \mathbf{V} \mathbf{u} \;\; = 2\sigma^2 \times \\ & \operatorname{tr} \;\; C_{\mathbf{W}}^{-1/2} \mathbf{A} \;\; \mathbf{A}^T C_{\mathbf{W}}^{-1} \mathbf{A} \;\; ^{-1} \mathbf{V} \;\; \mathbf{A}^T C_{\mathbf{W}}^{-1} \mathbf{A} \;\; ^{-1} \mathbf{A}^T C_{\mathbf{W}}^{-1/2} \end{aligned} \tag{26}$$

$$E \;\; \|\mathbf{u}\|^2 \;\; = 2\sigma^2 \operatorname{tr} \;\; C_{\mathbf{W}}^{-1/2} \mathbf{A} \;\; \mathbf{A}^T C_{\mathbf{W}}^{-1} \mathbf{A} \;\; ^{-2} \mathbf{A}^T C_{\mathbf{W}}^{-1/2} \;\; . \tag{27}$$

Here we use $\operatorname{tr} \; \mathbf{A}$ to denote the trace of matrix $\mathbf{A}$.

Based on their sum and difference, the individual timing and positioning error variances can be obtained, i.e.

$$E \;\; (\hat{\theta}_s - \theta_s)^2 \;\; = \frac{S + D}{2} \tag{28}$$

$$E \;\; \|\hat{\mathbf{p}}_s - \mathbf{p}_s\|^2 \;\; = \frac{S - D}{2} \tag{29}$$

The CRLB of clock offsets of anchors: Differentiating (19) leads to

$$\frac{\partial^2 \ln \mathrm{L}(\theta_i)}{\partial \theta_i^2} = -\frac{N}{\sigma^2} \tag{30}$$

Hence, the CRLB for the clock offsets are given by

$$\operatorname{var}(\hat{\theta}_i) \geq -E \left[\frac{\partial^2 \ln \mathrm{L}(\theta_i)}{\partial \theta_i^2} \right]^{-1} = \frac{\sigma^2}{N} \quad i = 1, \cdots, M \tag{31}$$

From the simulation later we can see that the derived estimation performances all are consistent to the simulated ones.

IV. Performance Analysis

A. Communnication Overhead

Now we consider the communication overhead of proposed algorithm. The joint clock synchronization and node localization algorithm based on efficient LS estimator used in Ref. [2] is two-way data exchange mechanism for exchanging timing message between reference nodes and unknown nodes. So the total numbers of sending and receiving messages of [2] are given by

$$N_{\mathrm{LS}t} = N_{\mathrm{LS}r} = 2NEL \tag{32}$$

where $E(E \geq 3)$ is the number of reference nodes, $N(N \geq 3)$ is the total number of exchanging timing information of nodes, L (generally is 1) is the number of unknown node.

Joint Synchronization and Localization Using TOAs proposed in [21] use the TOAs of reference nodes and unknown nodes, so the total numbers of sending and receiving messages of [21] are

$$N_{\mathrm{WLS}t} = N_{\mathrm{WLS}r} = NEL \tag{33}$$

where the number of reference nodes must meet $E \geq 4$.

The proposed algorithm in this paper is based on the PBS algorithm in [12], so the total number of sending and receiving messages are same as [12]

$$\begin{aligned} N_{\mathrm{PBS}t} &= 2NL \\ N_{\mathrm{PBS}r} &= 2N(M+1)L \end{aligned} \tag{34}$$

where $M(M \geq 4)$ is the number of anchor nodes, the number of reference node is 1.

Energy consumption of wireless sensor node when sending 1 bit message is much larger than that when receiving 1 bit message (if the distance is 100 meters, the energy consumption for sending is 21 times larger than receiving) [22]. From the analysis above, it is observed that the energy consumption of the proposed algorithm is the least in all three algorithms. And the proposed algorithm requires only one reference node, so it is more flexibility.

B. Computational Comlexity

Energy consumption and computational capacity are limited for most wireless sensor nodes. High complexity algorithms for sensor nodes may led to great burden for CPU (such as, long processing time, accelerated aging and so on), and accelerate energy consumption. It is very important to consider a lower complexity joint synchronization and localization algorithm.

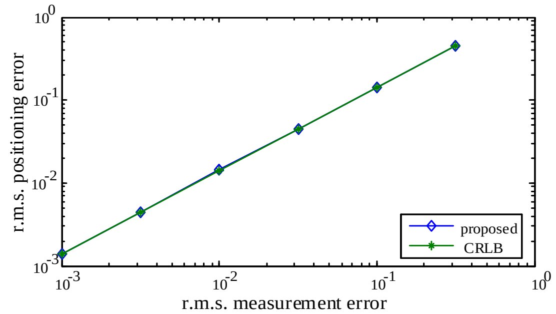
Suppose that only one round of data exchange and one iteration are performed, the computational complexity analysis of the proposed algorithm for joint 1-D timing and 2-D positioning using M anchor nodes is as follows.

- $C_{\mathbf{W}}^{-1}$: Considering $C_{\mathbf{W}}$ is a M-dimensional diagonal matrix, totally M multiplications required for $C_{\mathbf{W}}^{-1}$;
- $\mathbf{A}^T C_{\mathbf{W}}^{-1}$: Considering $\mathbf{A}^T$ is a $3\times M$ matrix, and $C_{\mathbf{W}}^{-1}$ is a M-dimensional diagonal matrix, totally $3M$ multiplications required;
- $\mathbf{A}^T C_{\mathbf{W}}^{-1}\mathbf{A}$: Considering $\mathbf{A}^T C_{\mathbf{W}}^{-1}$ is a $3\times M$ matrix, and $\mathbf{A}$ is a $M\times 3$ matrix, totally $9M$ multiplications and $9(M-1)$ additions required;
- $\left(\mathbf{A}^T C_{\mathbf{W}}^{-1}\mathbf{A}\right)^{-1}$: Considering $\mathbf{A}^T C_{\mathbf{W}}^{-1}\mathbf{A}$ is a 3-dimensional square matrix, totally 6 multiplications and 12 additions required;
- $\left(\mathbf{A}^T C_{\mathbf{W}}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T C_{\mathbf{W}}^{-1}$: totally $9M$ multiplications and $9(M-1)$ additions required;
- $\mathbf{b}$: totally $3M$ multiplications and $2M$ additions required;
- $\mathbf{g}$ and $\mathbf{h}$ from (9): totally $3M$ multiplications and $6M-4$ additions required;
- Δ from $\mathbf{g}$ and $\mathbf{h}$: 3 multiplications and 2 additions required for $\mathbf{g}^T\mathbf{V}\mathbf{g}$, $\mathbf{g}^T\mathbf{V}\mathbf{h}$ and $\mathbf{h}^T\mathbf{V}\mathbf{h}$ respectively, so totally 13 multiplications and 8 additions required for Δ from $\mathbf{g}$ and $\mathbf{h}$;
- $\mathbf{h}+\mathbf{g}\hat{\eta}$: totally 3 multiplications and 3 additions required.

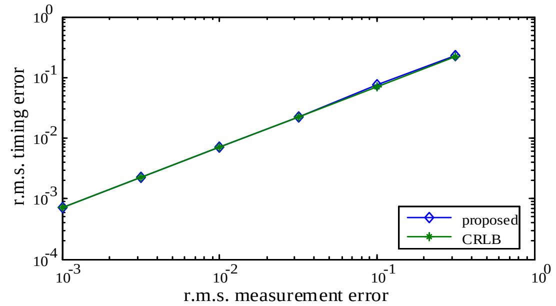
We can find that the proposed algorithm has the highest computational complexity when $\mathbf{g}^T\mathbf{V}\mathbf{g}\neq 0$ and $\Delta>0$. Suppose that the total number of exchanging timing information of nodes is N and Z rounds of iteration for update $C_{\mathbf{W}}$ is performed, the summarized computational complexity of the proposed algorithm is around $28MZN$ multiplications and $26MZN$ additions. More than half of the computations are used to calculate $\left(\mathbf{A}^T C_{\mathbf{W}}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T C_{\mathbf{W}}^{-1}$. So the proposed algorithm have comparable computational complexity $\mathcal{O}(N)$ as compare to the algorithm in [22], but the computational complexity of the algorithm in [2] is $\mathcal{O}(N^3)$.

V. Simulation Results

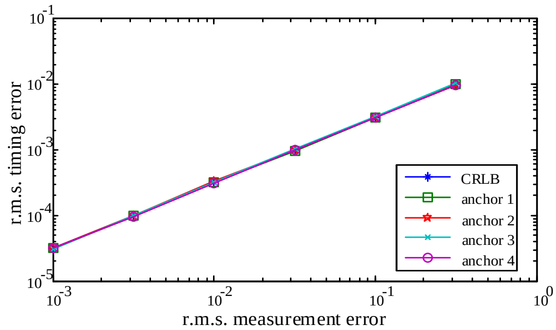
Simulation studies on the estimation performance are presented to verify the effectiveness of the proposed algorithm. We compare our proposed algorithm with the LS solution [2] and WLS [21] algorithm, as well as CRLB. In this simulation, there are six wireless sensors, one reference node, one unknown node and four anchor nodes, randomly placed within a circle with radius 500 distance units. The clock bias is randomly generated within 0.5-1.0 time units. All simulation results are average of 1000 independent runs. Fig. 3 shows the estimation performance with the noise variance increasing. From the figure we can obviously find the proposed algorithm has an estimation performance that is very close the corresponding CRLB, no matter positioning or timing performance.



(a) Positioning performance of unknown node.



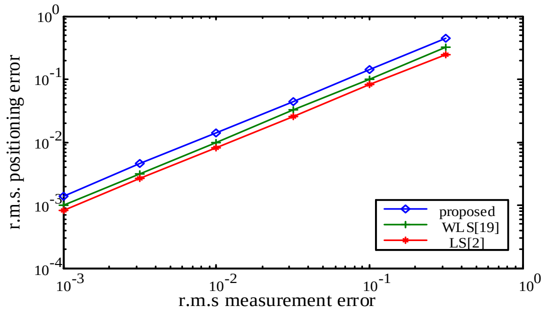
(b) Timing performance of unknown node.



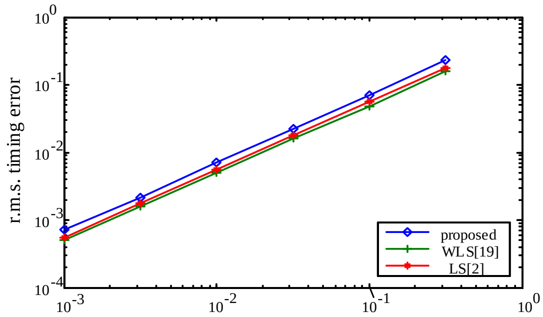
(c) Timing performance of anchors.

Fig. 3. Estimation performance of the proposed algorithm

We compare the estimation performance of the proposed algorithm with the LS algorithm in [2] and WLS algorithm in [21]. Fig. 4 shows the positioning and timing estimation performance of these three algorithms with the noise variance increasing, respectively. And we can see that the proposed algorithm is close to the estimation performance of the other two algorithms.

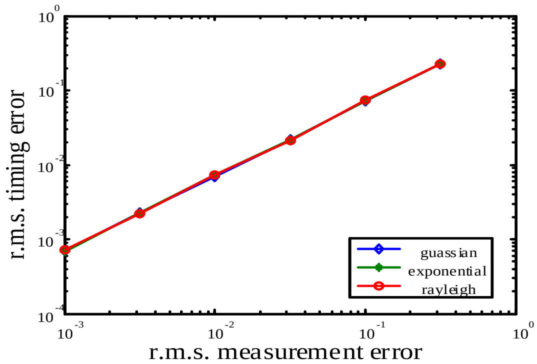


(a) Positioning performance.

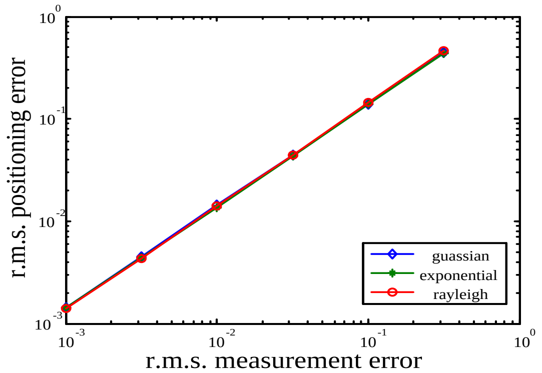


(b) Timing performance.

Fig. 4. Estimation performance comparison



(a) Positioning performance.



(b) Timing performance.

Fig. 5. Estimation performance with different noise distribution models

Random delays of information exchange between the nodes have different distribution models in different application conditions and the environments, such as Gaussian distribution, Exponential distribution, and Rayleigh distribution. In the proposed algorithm we supposed the random delay is Gaussian distribution. In order to study the robustness of the proposed algorithm in different noise environments, we simulate this algorithm in three above different noise distribution models without any modification of the algorithm. The simulation results are shown in Fig. 5.

From these simulation results, the robustness of the proposed algorithm for different noise environment is verified. The estimation performance under different noise environment is close to the CRLB, and can be very close to the estimation performance of algorithms in [2] and [21]. Hence the proposed algorithm is effective and robust with high estimation performance.

VI. Conclusions

In this paper we propose a simple joint clock synchronization and localization algorithm based on Pairwise Broadcast Synchronization (PBS) protocol in WSNs, i.e. anchor nodes (with unknown clock bias and known position) overhear the timing message exchanges of reference node (with known clock bias and position) and unknown node (with unknown clock bias and position) without sending extra messages. The proposed algorithm requires less reference node and communication overhead than others. By analyzing and simulating the estimation performance of the proposed algorithm, we see that the estimation performance of this proposed algorithm can meet CRLB and be close to the estimation performance of algorithms in [2] and [21]. So we can find the proposed algorithm has good trade-off between estimation performance and communication overhead.

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