Outage Performance for Two-Way Relaying with Co-Channel Interference and Channel Estimation Error

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Abstract —In this paper, we consider the outage performance of a two-way decode-and-forward relaying network in the presence of multiple strong interferers at the source/destination terminals and channel estimation errors. We define the signal-to-noise ratios and signal-to-interference ratios and then use them to derive closed-form expressions for the outage probability of the system under the symmetrical and asymmetrical cases that the received powers at the relay from both terminals are the same and different. Our analysis shows that the existence of the channel estimation errors and the interferers affects the outage performance. Simulation results demonstrate that our analytical results are in excellent agreement with the Monte Carlo simulations.

Index Terms—Two-way relaying, outage probability, decode-and-forward, channel estimation error, co-channel interference

I. Introduction

Relaying technologies can improve throughout and increase communication range. They have emerged as an effective means for exploiting spatial diversity. Two-way relaying has received considerable interest over the last years due to its advantage of overcoming the half-duplex loss in one-way relay channels [1]. Until now, the performance analysis and different transmission schemes for Amplify-and-Forward (AF) and Decode and Forward (DF) two-way relaying network have been well investigated in the literature. The rate performance of two-way relaying network has been extensively studied in recent years. The achievable rate regions for half-duplex two-way relaying network were studies in [2], [3]. Later in [4], the exact outage probabilities for a three-node two-way relaying network using AF and DF schemes were presented and an adaptive AF/DF scheme was proposed to achieve the optimal outage performance. The multiple-input multiple-output (MIMO) two-way relaying network was considered in [5]-[8]. The paper [8] showed that the capacity scales linearly with the number of antennas at the source nodes and logarithmically with the number of relay nodes. Recently, the optimal diversity-multiplexing tradeoff in AF and DF MIMO two-way relaying network was tackled in [9], [10]. The two-way relaying network with cognitive spectrum sharing protocols was investigated in [11]. Other active areas spun out from the concept of relaying include relay selection [12], [13], distributed beam-forming [14], [15], coding [16], [17] and multiuser two-way relaying network [18]. The two-way relaying technology was combined with small cell [19], [20] technology.

The co-channel interference is inevitable in wireless communications, so performance analysis of one-way relay network has turned the focus to interference-limited channels [21]-[23]. One-way relaying system with noisy relay and interference-limited destination was examined and the closed-form outage probabilities were derived for both DF and AF protocols [21]. One-way relaying system with multiple interferers over Rayleigh fading channels was studied in [22]. The exact outage probability of the DF one-way relaying system with unequal-power interferers using opportunistic relay selection was provided in [23]. The DF two-way relaying network with co-channel interferences was investigated in [24], [25]. The systems' achievable rate was derived in [24]. A tight approximate expression of the average symbol error rate was derived in closed-form in [25]. The interference limits were considered in cognitive femtocells system [26].

On the other hand, for the DF relaying network, most of the work on performance analysis has been carried out under the assumption of perfect channel state information (CSI) at both relay and nodes. In practice, however, it is impossible for the relay and nodes to obtain perfect knowledge of CSI due to imperfect channel estimation algorithms, the Doppler shift or noise on the pilot signals. As such, the authors in [27]-[29] took the channel estimation error into consideration. The outage performance of DF based opportunistic cooperative communications with channel estimation errors was investigated in [27]. The symbol error rate and power allocation for DF relaying with channel estimation error was analyzed in [28]. In paper [29], the performance of DF cooperative diversity networks with imperfect channel estimation and co-channel interference was analyzed. The performance of a DF two-way relaying network with multiple interferers and channel estimation

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error is analyzed in the literature and this problem represents the main focus of this paper.

This paper is organized as follows. The system model is described in Section 2. In Section 3, we investigate the outage probability of the DF two-way relaying system with channel estimation error and multiple interferers. In Section 4, some numerical results are given to verify our analysis. Finally, Section 5 concludes the paper.

Notations-we denote P{B} as the probability of a random event B, while E{B} returns the expected value of the input random variable or event. $F_B(x)$ and $f_B(x)$ are, respectively, the cumulative density function and the probability density function of B.

II. System Model

The two-way relaying system model with co-channel interference is shown in Fig. 1, where two mobile terminals, S_1 and S_2, at the edges of two different cells exchange message with the aid of a relay node R, assuming that the direct path between them is broken. The source nodes, S_1 and S_2, are at the cell-edges and suffer from interference from other mobile/relay nodes in adjacent cells, while the relay node R is free from interference. We assume that interference is the dominating factor limiting the performance. The noises at S_1 and S_2 will be ignored, whilst R is noise-limited. The forward and backward channels are reciprocal, with α_1 denoting the channel between S_1 and R, and α_2 denoting the channel between S_2 and R. We assume that S_1, S_2 and R are equipped with one antenna only. The DF protocol is used at the relay. The DF two-way relaying strategy consists of two equal time slots.

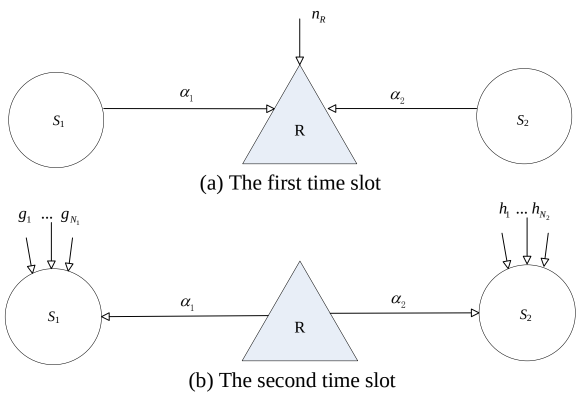

Fig. 1. The two-way relaying system model with co-channel interference

In the first time slot, S_1 and S_2 send their messages, x_1 and x_2, to R, and R has the following received signal:

$$y_R = \alpha_1 x_1 + \alpha_2 x_2 + n_R \tag{1}$$

where n_R is the complex Gaussian noise with zero mean and variance of σ_R^2. It is assumed that α_1 and α_2 are independent quasi-static Rayleigh channels with $\Omega_1 = E\{|\alpha_1|^2\}$ and $\Omega_2 = E\{|\alpha_2|^2\}$. We also denote the transmit power at S_1 and S_2 by $P_1 = E\{|x_1|^2\}$ and $P_2 = E\{|x_2|^2\}$, respectively.

In the second time slot, both x_1 and x_2 are decoded successfully, the relay R combined the decoded symbols using physical layer network coding, obtains x_R and broadcasts it to S_1 and S_2 [30]. Therefore, after two time slots, S_1 obtains

$$y_1 = \alpha_1 x_R + \sum_{i=1}^{N_1} g_i s_1^{(i)} + n_1 \tag{2}$$

where the contribution from N_1 interference signals $\{s_1^{(i)}\}_{i=1}^{N_1}$ is considered. The channels between the interferers and S_1 are $\{g_i\}$. They are assumed to be independent Rayleigh distributed with $E\{|g_i|^2\}=1$, for $i = 1,2,...,N_1$. The received signal by S_2 is

$$y_2 = \alpha_2 x_R + \sum_{j=1}^{N_2} h_j s_2^{(j)} + n_2 \tag{3}$$

where $\{s_2^{(j)}\}_{j=1}^{N_2}$ are the interference signals. The channels between the interferers and S_2 are $\{h_j\}$. They are independent Rayleigh fading with $E\{|h_j|^2\}=1$, for $j = 1,2,...,N_2$. Moreover we also assume $\{g_i\}$ and $\{h_j\}$ to be quasi-static channels. Furthermore, we assume that the influence of interference at S_1 and S_2 are dominant and therefore that the effect of n_1 and n_2 are neglected. The transmit power R is given by $P_R = E\{|x_R|^2\}$. The interference power at S_1 and S_2 is $P_{I1}^i = E\{|s_1^{(i)}|^2\}$ and $P_{I2}^j = E\{|s_2^{(j)}|^2\}$.

The channel estimation is based on training sequences (pilots) and a particular pilot power result in a certain level of channel estimation error variance [27]. The actual and the estimated channels can be modeled as

$$\alpha_1 = \hat{\alpha}_1 + e_1 \tag{4}$$

$$\alpha_2 = \hat{\alpha}_2 + e_2 \tag{5}$$

where $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are the estimated channel coefficients and e_1 and e_2 denote the channel estimation errors. Let $E\{|e_1|^2\} = E\{|e_2|^2\} = \sigma_E^2$, a parameter which reflects the accuracy of channel estimation. Since $\hat{\alpha}_1$ is assumed to be independent of e_1 and $\hat{\alpha}_2$ is assumed to be independent of e_2, $\hat{\alpha}_1$ and $\hat{\alpha}_2$ a are complex Gaussian random variables with variances $E\{|\hat{\alpha}_1|^2\} = \Omega_1 - \sigma_E^2$ and $E\{|\hat{\alpha}_2|^2\} = \Omega_2 - \sigma_E^2$.

We define the following signal-to-noise ratios (SNR):
SNR at R from S_1:

$$\gamma_{1R} = \frac{P_1 |\hat{\alpha}_1|^2}{P_1 \sigma_E^2 + \sigma_R^2} \tag{6}$$

SNR at R from S_2:

$$\gamma_{2R} = \frac{P_2 |\hat{\alpha}_2|^2}{P_2\sigma_E^2 + \sigma_R^2} \quad (7)$$

The sum-SNR at R:

$$\gamma_{MAC} = \frac{P_1 |\hat{\alpha}_1|^2}{P_1\sigma_E^2 + \sigma_R^2} + \frac{P_2 |\hat{\alpha}_2|^2}{P_2\sigma_E^2 + \sigma_R^2} \quad (8)$$

We also define the signal-to-interference ratios:
Signal-to-interference ratio at S_1:

$$\gamma_{R1} = \frac{P_R |\hat{\alpha}_1|^2}{|\bar{g}|^2 + P_R\sigma_E^2} \quad (9)$$

Signal-to-interference ratio at S_2:

$$\gamma_{R2} = \frac{P_R |\hat{\alpha}_2|^2}{|\bar{h}|^2 + P_R\sigma_E^2} \quad (10)$$

where $|\bar{g}|^2 = \sum_{i=1}^{N_1} |g_i|^2 P_{I1}^i$ and $|\bar{h}|^2 = \sum_{j=1}^{N_2} |h_j|^2 P_{I2}^j$.

III. Outage Probability Analysis

In this section, we investigate the outage probability for the two-way relaying network. Firstly we assume that all the transmit signals to be Gaussian variables and denote the transmission rates from S_1 to S_2 and from S_2 to S_1 by R_1 and R_2, respectively. According to the rate region for the two-way relaying network in [31], an outage event exactly occurred when

$$\begin{aligned} &R_1 > \min\{\frac{1}{2}\log(1+\gamma_{1R}), \frac{1}{2}\log(1+\gamma_{R2})\} \quad \text{or} \\ &R_2 > \min\{\frac{1}{2}\log(1+\gamma_{2R}), \frac{1}{2}\log(1+\gamma_{R1})\} \quad \text{or} \\ &R_1 + R_2 > \frac{1}{2}\log(1+\gamma_{MAC}) \end{aligned} \quad (11)$$

We assume that the interferers at S_1 or S_2 are of unequal power, i.e., $P_{I1}^i \neq P_{I1}^j$ and $P_{I2}^i \neq P_{I2}^j, \forall i \neq j$. Our main results is given in the following conclusion.

Conclusion: If the interferers have dissimilar power at S_1 and S_2, i.e., $P_{I1}^i \neq P_{I1}^j$ and $P_{I2}^i \neq P_{I2}^j$, $\forall i \neq j$, the outage probability P_{out} is given by (12)

$$\begin{aligned} P_{out} = {} & 1 - \psi\delta + \\ & \delta e^{-d_1}\left\{\sum_{i=1}^{N_1}\frac{\bar{\gamma}_2}{\bar{\gamma}_1(\rho_1^i\bar{\gamma}_2 - 1)}e^{-\frac{\lambda}{\beta_1 P_{I1}^i}} - \sum_{j=1}^{N_2}\frac{1}{\rho_2^j\bar{\gamma}_2(\rho_2^j\bar{\gamma}_1 - 1)}e^{-\frac{\lambda_2 - 1}{\beta_2 P_{I2}^j}} \right. \\ & \left. + \sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\frac{1}{\rho_2^j\bar{\gamma}_1\bar{\gamma}_2(\rho_2^j - \rho_1^i)}e^{-\frac{\lambda}{\beta_1 P_{I1}^i} - \frac{\lambda_2 - 1}{\beta_2 P_{I2}^j}}\right\} \\ & + \delta e^{-d_2}\left\{\sum_{j=1}^{N_2}\frac{\bar{\gamma}_1}{\bar{\gamma}_2(\rho_2^j\bar{\gamma}_1 - 1)}e^{-\frac{\lambda^{-1}}{\beta_2 P_{I2}^j}} - \sum_{i=1}^{N_1}\frac{1}{\rho_1^i\bar{\gamma}_1(\rho_1^i\bar{\gamma}_2 - 1)}e^{-\frac{\lambda_1 - 1}{\beta_1 P_{I1}^i}} \right. \\ & \left. + \sum_{i=1}^{N_1}\sum_{j=1}^{N_2}\frac{1}{\rho_1^i\bar{\gamma}_1\bar{\gamma}_2(\rho_1^i - \rho_2^j)}e^{-\frac{\lambda_1 - 1}{\beta_1 P_{I1}^i} - \frac{\lambda^{-1}}{\beta_2 P_{I2}^j}}\right\} \end{aligned} \quad (12)$$

where

$$\psi = \begin{cases} \eta_2\exp(-d_1) + \eta_1\exp(-d_2), & \bar{\gamma}_1 \neq \bar{\gamma}_2 \\ [1 + \frac{C(1) - C(\mu) - C(1-\mu)}{\bar{\gamma}_0}\exp(-\frac{C(1)}{\bar{\gamma}_0})], & \bar{\gamma}_1 = \bar{\gamma}_2 \end{cases}$$

$$\begin{aligned} & A_i^{N_1} = (P_{I1}^i)^{N_1 - 1}\prod_{\substack{k=1 \\ k\neq i}}^{N_1}(P_{I1}^i - P_{I1}^k)^{-1}, \\ & B_j^{N_2} = (P_{I2}^j)^{N_2 - 1}\prod_{\substack{l=1 \\ l\neq j}}^{N_2}(P_{I2}^j - P_{I2}^l)^{-1}, \\ & \eta_1 = \frac{\bar{\gamma}_1}{\bar{\gamma}_1 - \bar{\gamma}_2}, \quad \eta_2 = \frac{\bar{\gamma}_2}{\bar{\gamma}_2 - \bar{\gamma}_1}, \\ & \rho_1^i = \frac{1}{\bar{\gamma}_1} + \frac{1}{C(1-\mu)\beta_1 P_{I1}^i}, \quad \rho_2^j = \frac{1}{\bar{\gamma}_2} + \frac{1}{C(\mu)\beta_2 P_{I2}^j}, \\ & d_1 = \frac{C(\mu)}{\bar{\gamma}_1} + \frac{C(1) - C(\mu)}{\bar{\gamma}_2}, \\ & d_2 = \frac{C(1-\mu)}{\bar{\gamma}_2} + \frac{C(1) - C(1-\mu)}{\bar{\gamma}_1}, \\ & \delta = \sum_{i=1}^{N_1} A_i^{N_1} e^{\frac{P_R\sigma_E^2}{P_{I1}^i}} \sum_{j=1}^{N_2} B_j^{N_2} e^{\frac{P_R\sigma_E^2}{P_{I2}^j}} \end{aligned} \quad (13)$$

with

$$\beta_1 = \frac{P_1}{P_R(\sigma_R^2 + P_1\sigma_E^2)}, \beta_2 = \frac{P_2}{P_R(\sigma_R^2 + P_2\sigma_E^2)},$$

$$\bar{\gamma}_1 = \frac{P_1(\Omega_1 - \sigma_E^2)}{P_1\sigma_E^2 + \sigma_R^2}, \bar{\gamma}_2 = \frac{P_2(\Omega_2 - \sigma_E^2)}{P_2\sigma_E^2 + \sigma_R^2},$$

$$\lambda = \frac{C(\mu)}{C(1-\mu)}, \ \lambda_1 = \frac{C(1)}{C(1-\mu)}, \lambda_2 = \frac{C(1)}{C(\mu)}.$$

Proof: See Appendix.

IV. Numberical Results

Here, we present the numerical results to validate the analytical results of the outage probability. Fig. 2 shows the outage probability of DF two-way relaying network with channel estimation errors and multiple interferers for $\sigma_E^2 = 0.01, \ 0.001$. The interference power is denoted by sequences $\mathbf{P}_{I1} \overset{\Delta}{=} [P_{I1}^{(1)}, ..., P_{I1}^{(N_1)}]$ and $\mathbf{P}_{I2} \overset{\Delta}{=} [P_{I2}^{(1)}, ..., P_{I2}^{(N_2)}]$. In the simulations, we set the transmission rates by R_1=1.2, R_2=0.8 and the power of noise by $\sigma_R^2 = 1$. We set the number of the interferers by N_1=N_2=N=3, and we set the distinct interference powers as an arithmetic sequence, which is $\boldsymbol{s}_1$=$\boldsymbol{s}_2$= [2.5, 5, 7.5]. In the asymmetrical case, we assume P_1=P_2=30dB, P_R=30dB, $\Omega_1 = 0.8, \ \Omega_2 = 0.7$; In the symmetrical case, we assume $\Omega_1 = \Omega_2 = 0.7$. The comparison for the outage probability between the analytical results Eq. (12) and the simulation results is shown in Fig. 2. It is clearly observed that the analytical results Eq. (12) match the exact results perfectly. As expected, the effect of channel estimation errors also can be easily observed.

In Fig. 3, we present the outage probability performance of the DF two-way relaying network for different numbers of interferers. Here we assume $\sigma_E^2 = 0.001$. When the number of the interferers is N_1=N_2=N=3, the interference power is $\boldsymbol{s}_1$=$\boldsymbol{s}_2$= [2.5, 5, 7.5]; When the number of the interferers is N_1=N_2=N=4, the interference power is $\boldsymbol{s}_1$=$\boldsymbol{s}_2$=[2.5, 5, 7.5, 10]. The existence of different numbers of interferers affects the outage performance significantly.

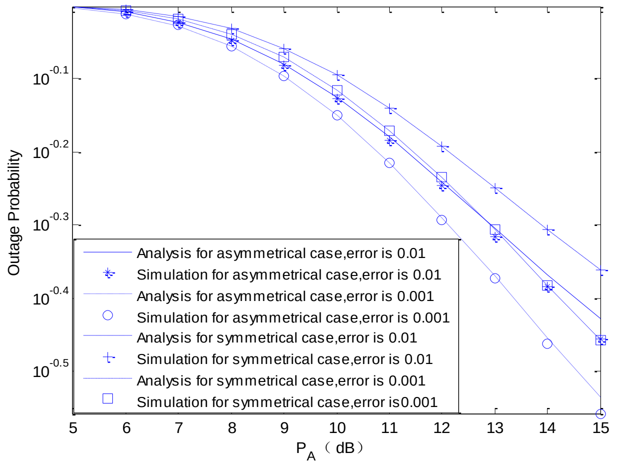

Fig. 2. Outage probability of DF two-way relaying system with different σ_E^2

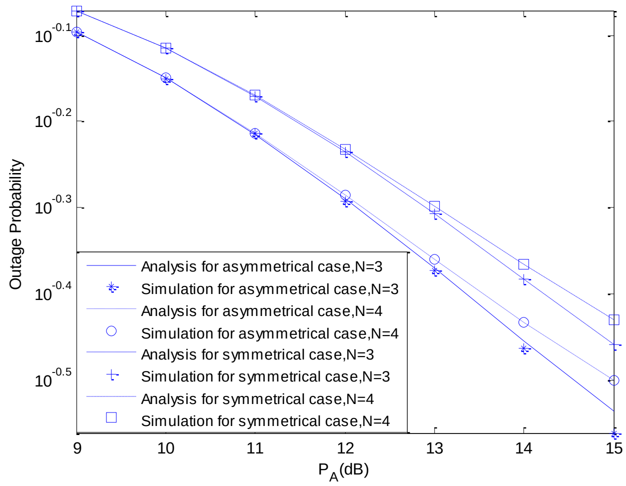

Fig. 3. Outage probability of DF two-way relaying system for different numbers of co-channel interferers

V. CONCLUSIONS

In this paper, we analyzed the outage performance of a DF two-way relaying network with channel estimation errors and co-channel interference. Firstly, the signal-to-noise ratios and signal-to-interference ratios of the system was defined. Secondly the expression of the outage probability was transformed to the function of multiple variables and the cumulative density functions and probability density functions of the variables were given. At last by employing the distribution, we derived close-form expression of the outage probability of the system under asymmetrical and symmetrical cases. Results show that our analysis is very close to the exact values.

APPENDIX PROOF

We define the function $C(x) \overset{\Delta}{=} 2^{2xR} - 1$ with $R = R_1 + R_2$. Then, (11) is recast as

$$\{\gamma_{1R} < C(\mu) \text{ or } \gamma_{R1} < C(1-\mu) \text{ or } \gamma_{2R} < C(1-\mu) \text{ or } \gamma_{R2} < C(\mu) \text{ or } \gamma_{MAC} < C(1)\} \quad (14)$$

where $\mu = R_1 / R$, for $0 \le \mu \le 1$. Therefore, the outage probability is given by

$$P_{out} = P\{Z_1 < C(\mu) \text{ or } Z_1 < \mu_1 C(1-\mu) \text{ or } Z_2 < C(1-\mu) \text{ or } Z_2 < \mu_2 C(\mu) \text{ or } Z_1 + Z_2 < C(1)\} \quad (15)$$

where

$$\mu_1 = \frac{P_1}{P_R(\sigma_R^2 + P_1\sigma_E^2)}\left(|\bar{g}|^2 + P_R\sigma_E^2\right),\ Z_1 = \frac{P_1|\hat{\alpha}_1|^2}{P_1\sigma_E^2 + \sigma_R^2}$$

$$\mu_2 = \frac{P_2}{P_R(\sigma_R^2 + P_2\sigma_E^2)}\left(|\bar{h}|^2 + P_R\sigma_E^2\right),\ Z_2 = \frac{P_2|\hat{\alpha}_2|^2}{P_2\sigma_E^2 + \sigma_R^2}$$

From (15), we see that P_{out} is affected by the cumulative density functions (CDFs) of Z_1, Z_2, μ_1 and μ_2. It is easily derived that

$$C(\mu) + C(1-\mu) \le C(1) \quad 0 \le \mu \le 1 \quad (16)$$

Therefore, we obtain the expressions of P_{out} for different cases:

$$P_{out}^{(1)} = P\{Z_1 < C(\mu) \text{ or } Z_2 < C(1-\mu) \text{ or } Z_1 + Z_2 < C(1)\}$$
$$\mu_1 \le \frac{C(\mu)}{C(1-\mu)},\ \mu_2 \le \frac{C(1-\mu)}{C(\mu)} \quad (17)$$

$$P_{out}^{(2)} = P\{Z_1 < C(\mu) \text{ or } Z_2 < \mu_2 C(\mu) \text{ or } Z_1 + Z_2 < C(1)\}$$
$$\mu_1 \le \frac{C(\mu)}{C(1-\mu)},\ \frac{C(1-\mu)}{C(\mu)} < \mu_2 \le \frac{C(1)}{C(\mu)} - 1 \quad (18)$$

$$P_{out}^{(3)} = P\{Z_1 < C(\mu) \text{ or } Z_2 < \mu_2 C(\mu)\}$$
$$\mu_1 \le \frac{C(\mu)}{C(1-\mu)},\ \mu_2 > \frac{C(1)}{C(\mu)} - 1 \quad (19)$$

$$P_{out}^{(4)} = P\{Z_1 < \mu_1 C(1-\mu) \text{ or } Z_2 < C(1-\mu) \text{ or } Z_1 + Z_2 < C(1)\} \quad (20)$$
$$\frac{C(\mu)}{C(1-\mu)} < \mu_1 \le \frac{C(1)}{C(1-\mu)} - 1,\ \mu_2 \le \frac{C(1-\mu)}{C(\mu)}$$

$$P_{out}^{(5)} = P\{Z_1 < \mu_1 C(1-\mu) \text{ or } Z_2 < C(1-\mu)\}$$
$$\mu_1 > \frac{C(1)}{C(1-\mu)} - 1,\ \mu_2 \le \frac{C(1-\mu)}{C(\mu)} \quad (21)$$

$$P_{out}^{(6)} = P\{Z_1 < \mu_1 C(1-\mu) \text{ or } Z_2 < \mu_2 C(\mu) \text{ or } Z_1 + Z_2 < C(1)\}$$
$$\mu_1 > \frac{C(1)}{C(1-\mu)} - 1,\ \mu_2 > \frac{C(1-\mu)}{C(\mu)}, \quad (22)$$
$$\mu_1 C(1-\mu) + \mu_2 C(\mu) \le C(1)$$

$$P_{out}^{(7)} = P\{Z_1 < \mu_1 C(1-\mu) \text{ or } Z_2 < \mu_2 C(\mu)\},$$
$$\mu_1 > \frac{C(\mu)}{C(1-\mu)},\ \mu_2 > \frac{C(1-\mu)}{C(\mu)}, \quad (23)$$
$$\mu_1 C(1-\mu) + \mu_2 C(\mu) \le C(1)$$

and

$$\begin{aligned} P_{out} &= P_{out}^{(1)} F_{\bar{\mu}_1}(\lambda) F_{\bar{\mu}_2}(\lambda^{-1}) \\ &+ \int_0^{\lambda}\int_{\lambda^{-1}}^{\lambda_2-1} P_{out}^{(2)} f_{\bar{\mu}_2}(\mu_2)d\mu_2 f_{\bar{\mu}_1}(\mu_1)d\mu_1 \\ &+ \int_0^{\lambda}\int_{\lambda_2-1}^{\infty} P_{out}^{(3)} f_{\bar{\mu}_2}(\mu_2)d\mu_2 f_{\bar{\mu}_1}(\mu_1)d\mu_1 \\ &+ \int_{\lambda}^{\lambda_1-1}\int_0^{\lambda^{-1}} P_{out}^{(4)} f_{\bar{\mu}_2}(\mu_2)d\mu_2 f_{\bar{\mu}_1}(\mu_1)d\mu_1 \\ &+ \int_{\lambda_1-1}^{\infty}\int_0^{\lambda-1} P_{out}^{(5)} f_{\bar{\mu}_2}(\mu_2)d\mu_2 f_{\bar{\mu}_1}(\mu_1)d\mu_1 \\ &+ \int_{\lambda}^{\lambda_1-1}\int_{\lambda^{-1}}^{\lambda_2-\frac{\alpha_1}{\lambda}} P_{out}^{(6)} f_{\bar{\mu}_2}(\mu_2)d\mu_2 f_{\bar{\mu}_1}(\mu_1)d\mu_1 \\ &+ \int_{\lambda}^{\infty}\int_{\lambda^{-1}}^{\infty} P_{out}^{(7)} f_{\bar{\mu}_2}(\mu_2)d\mu_2 f_{\bar{\mu}_1}(\mu_1)d\mu_1 \\ &- \int_{\lambda}^{\lambda_1-1}\int_{\lambda^{-1}}^{\lambda_2-\frac{\alpha_1}{\lambda}} P_{out}^{(7)} f_{\bar{\mu}_2}(\mu_2)d\mu_2 f_{\bar{\mu}_1}(\mu_1)d\mu_1 \end{aligned} \quad (24)$$

where $F_{\bar{\alpha}_1}(x)$, $F_{\bar{\alpha}_2}(x)$, $f_{\bar{\alpha}_1}(x)$, $f_{\bar{\alpha}_2}(x)$ denote the CDFs and the probability density function (PDF)s for α_1 and α_2 respectively. With $Z_1 = \frac{P_1 |\hat{\alpha}_1|^2}{P_1\sigma_E^2 + \sigma_R^2}$, $Z_2 = \frac{P_2 |\hat{\alpha}_2|^2}{P_2\sigma_E^2 + \sigma_R^2}$, the CDFs of Z_1 and Z_2 are, respectively, given by

$$F_{Z_1}(x) = 1 - e^{-\frac{x}{\bar{\gamma}_1}} \qquad x \ge 0 \quad (25)$$

where $\bar{\gamma}_1 = \frac{P_1(\Omega_1 - \sigma_E^2)}{P_1\sigma_E^2 + \sigma_R^2}$, and

$$F_{Z_2}(x) = 1 - e^{-\frac{x}{\bar{\gamma}_2}} \qquad x \ge 0 \quad (26)$$

where $\bar{\gamma}_2 = \frac{P_2(\Omega_2 - \sigma_E^2)}{P_2\sigma_E^2 + \sigma_R^2}$, and the PDFs of Z_1 and Z_2 are, respectively, given by

$$f_{Z_1}(x) = \frac{1}{\bar{\gamma}_1} e^{-\frac{x}{\bar{\gamma}_1}} \qquad x \ge 0 \quad (27)$$

$$f_{Z_2}(x) = \frac{1}{\bar{\gamma}_2} e^{-\frac{x}{\bar{\gamma}_2}} \qquad x \ge 0 \quad (28)$$

Following (17)-(23), we have

$$P_{out}^{(1)} = G_1(C(\mu), C(1-\mu)) \quad (29)$$

$$P_{out}^{(2)} = G_1(C(\mu), \mu_2 C(\mu)) \quad (30)$$

$$P_{out}^{(3)} = G_2(C(\mu), \mu_2 C(\mu)) \quad (31)$$

$$P_{out}^{(4)} = G_1(\mu_1 C(1-\mu), C(1-\mu)) \quad (32)$$

$$P_{out}^{(5)} = G_2(\mu_1 C(1-\mu), C(1-\mu)) \quad (33)$$

$$P_{out}^{(6)} = G_1(\mu_1 C(1-\mu), \mu_2 C(\mu)) \quad (34)$$

$$P_{out}^{(7)} = G_2(\mu_1 C(1-\mu), \mu_2 C(\mu)) \quad (35)$$

where

$$G_1(x,y) = 1 - \psi(x,y) \quad (36)$$

$$G_2(x,y) = 1 - \exp[-(\frac{x}{\bar{\gamma}_1} + \frac{y}{\bar{\gamma}_2})] \quad (37)$$

and $\psi(x,y)$ is

$$\psi(x,y) = \begin{cases} \eta_1 \exp[-\frac{C(1)}{\bar{\gamma}_1} + (\frac{1}{\bar{\gamma}_1} - \frac{1}{\bar{\gamma}_2})y] \\ \quad + \eta_2 \exp[-\frac{C(1)}{\bar{\gamma}_2} + (\frac{1}{\bar{\gamma}_2} - \frac{1}{\bar{\gamma}_1})x], \quad \bar{\gamma}_1 \ne \bar{\gamma}_2 \\ [1 + \frac{(C(1) - x - y)}{\bar{\gamma}_0} \exp(-\frac{C(1)}{\bar{\gamma}_0})], \quad \bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}_0 \end{cases} \quad (38)$$

where $\eta_1 = \frac{\bar{\gamma}_1}{\bar{\gamma}_1 - \bar{\gamma}_2}$, $\eta_2 = \frac{\bar{\gamma}_2}{\bar{\gamma}_2 - \bar{\gamma}_1}$.

According to the definitions

$$\alpha_1 = \frac{P_1}{P_R(P_1\sigma_E^2 + \sigma_R^2)}|\bar{g}|^2 + \frac{P_1}{P_R(P_1\sigma_E^2 + \sigma_R^2)} P_R\sigma_E^2,$$
$$\alpha_2 = \frac{P_2}{P_R(P_2\sigma_E^2 + \sigma_R^2)}|\bar{h}|^2 + \frac{P_2}{P_R(P_2\sigma_E^2 + \sigma_R^2)} P_R\sigma_E^2$$

and the CDFs of $|\bar{g}|^2$ and $|\bar{h}|^2$, the CDFs and PDFs of μ_1 and μ_2 are given by

$$F_{\bar{\mu}_1}(x) = \sum_{i=1}^{N_1} A_i^{N_1} e^{\frac{P_R\sigma_E^2}{P_{I1}^i}} \left(1 - e^{-\frac{x}{\beta_1 P_{I1}^i}}\right) \qquad x \ge 0 \quad (39)$$

$$F_{\bar{\mu}_2}(x) = \sum_{j=1}^{N_2} B_j^{N_2} e^{\frac{P_R\sigma_E^2}{P_{I2}^j}} \left(1 - e^{-\frac{x}{\beta_2 P_{I2}^j}}\right) \qquad x \ge 0 \quad (40)$$

$$f_{\bar{\mu}_1}(x) = \sum_{i=1}^{N_1} \frac{A_i^{N_1}}{\beta_1 P_{I1}^i} e^{-\frac{x}{\beta_1 P_{I1}^i} + \frac{P_R\sigma_E^2}{P_{I1}^i}} \qquad x \ge 0 \quad (41)$$

$$f_{\bar{\mu}_2}(x) = \sum_{j=1}^{N_2} \frac{B_j^{N_2}}{\beta_2 P_{I2}^j} e^{-\frac{x}{\beta_2 P_{I2}^j} + \frac{P_R\sigma_E^2}{P_{I2}^j}} \qquad x \ge 0 \quad (42)$$

where

$$A_i^{N_1} = (P_{I1}^i)^{N_1-1} \prod_{\substack{j=1 \\ j\ne i}}^{N_1} \frac{1}{P_{I1}^i - P_{I1}^j}$$

$$B_j^{N_2} = (P_{I2}^j)^{N_2-1}\prod_{\substack{l=1\\ l\neq j}}^{N_2}\frac{1}{P_{I2}^j - P_{I2}^l}$$

$$\beta_1 = \frac{P_1}{P_R(\sigma_R^2 + P_1\sigma_E^2)}\ ,\ \ \beta_2 = \frac{P_2}{P_R(\sigma_R^2 + P_2\sigma_E^2)}$$

Substituting (29)-(35), (39)-(42) into (24) and simplifying the result gives the desired result (12).

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