Tradeoff Analysis between Spectrum Efficiency and Energy Efficiency in Heterogeneous Networks (HetNets) Using Bias Factor

Mengting Liu, Yinglei Teng, and Mei Song
School of Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing, 100876, China
Email: {liumengting, songm}@bupt.edu.cn; lilytengtt@gmail.com

Abstract —Recently, Small Base Stations (SBSs) are added to macro-cell layout to extend network coverage and increase system capacity. Whereas, most of the users incline to connect to Macro Base Stations (MBSs) due to their overwhelmingly higher transmit power. As a result, the imbalanced distribution of load leads to the underutilization of the resources of SBSs, which may deteriorate the network performance. The small Cell Range Expansion (CRE) aims at solving the problem by setting a positive bias factor. Although this technology turns out to be effective, it's not easy to understand the corresponding effect on network performance. Therefore, the average area weighted load per cell is derived to address this issue. Meanwhile, we utilize Area Weighted Spectrum Efficiency (AWSE) and Area Weighted Energy Efficiency (AWEE) to observe the effect of bias factor from the perspective of cells per tier. Further, the efficiency (SE) and efficiency (EE) of different tiers are calculated on tier level. Simulation results show that there is a tradeoff between the overall SE and EE with varying bias factors, which provides a new insight into the analysis of network performance.

Index Terms—Heterogeneous networks, bias, area weighted, spectrum efficiency, energy efficiency

I. INTRODUCTION

Driven by the explosive demands for mobile data traffic, the evolution from the conventional cellular networks into dense, irregular Heterogeneous Networks (HetNets) is ongoing and accelerating. The low powered Small Base Stations (SBSs) show its advantage in extending network coverage and increasing system capacity. However, since the transmit power of Macro Base Stations (MBSs) is overwhelmingly larger than that of SBSs, a majority of users incline to connect to MBSs which can provide the highest reference received power. Consequently, the imbalanced distribution of load leads to the underutilization of SBSs as well as large power consumption, which may have an impact on the network performance including Spectrum Efficiency (SE) and Energy Efficiency (EE).

Recently, Cell Range Expansion (CRE) has been proved as a promising approach to solve the aforementioned problem [1], [2]. However, most of the related works emphasize the improvement of network performance brought by CRE including rate coverage, SINR coverage, and fifth percentile rate [3], [4] but lack explicit quantitative analysis of load per cell and the impact of bias factor on system performance. Once CRE is employed in the network, the value of bias factor and the corresponding variation of load have remarkably effect on network performance including SE, EE and the relation between them. In fact, there is a large body of literature focusing on the tradeoff between SE and EE with respect to different factors [5]-[8] or in different scenarios [9]-[12]. Ref. [5] claims that there is a tradeoff between SE and EE for single cell as well as multi-cell networks and explores the impact of bandwidth on EE. Also, tradeoff of SE and EE in multi-cell network with different interfering scenarios is studied in [6], and the simulation part shows that the relation between SE and EE varies according to varying transmit powers. More recently, the variations of SE and EE with a series of factors including the number of antennas, the number of multiplexing users, system bandwidth and cell radius are investigated in [7]. Meanwhile, [8] studies the effect of estimation error, transmission power budget and channel-to-noise ratio on the SE-EE tradeoff relationship and provides a way to select the trade-off parameters with different design requirements. However, none of them involves load or bias factor in the network models. Then followed in [9], the evaluation of EE with respect to load is explored in cognitive small cell networks and the numerical results illustrate that the increase of load helps to improve EE in case of perfect sensing. In addition, the authors of [10]-[12] emphasize that load conditions should be taken into account to better balance the SE-EE tradeoff. Therefore, a dynamic traffic-aware reconfiguration scheme is proposed to maximize the average EE while guaranteeing the system performance in [10]. Meanwhile, [11] and [12] study the SE-EE tradeoff relation in different load conditions, i.e., high load condition and low load condition and show that both of SE and EE can be significantly increased under low load condition. However, few works explicitly investigate the effect of bias factor on SE and EE. So our paper aims

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to provide a new perspective for the tradeoff between SE and EE using bias factor.

In our paper, a general two-tier HetNet is presented. Different tiers are recognized by the transmit power, Base Station (BS) density and bias factor. Similar to [13], we assume that each user connects to the BS offering the highest Bias-Based Received Power (BRP) but the key difference being the interference only comes from co-tier cells except its serving BS. Specifically, we provide the following theoretical contributions. Firstly, we get the average area weighted load per cell to capture the load condition per cell in different tiers, which is distinguished from the load per cell in [13] because of the consideration of area discrepancy in different cells. Next, the average ergodic rate per user is derived in form of multiple integral. Corresponding to area weighted load per cell, the Area Weighted Spectrum Efficiency (AWSE) and Area Weighted Energy Efficiency (AWEE) per cell are proposed on cell level. Then based on the average ergodic rate, the SE and EE of different tiers as well as the whole network are deduced. Finally, the tradeoff between SE and EE is studied using the bias factor. However, rather than giving the optimum bias factor, the main contribution of this paper is to provide a new insight into the tradeoff between SE and EE using bias factor.

The remaining sections of the paper are organized as follows. In Section II, a two-tier HetNet model is presented. Section III describes the network performance by means of several metrics including area weighted load, AWSE, AWEE per cell, SE and EE of different tiers as well as the whole network. In Section IV, we provide numerical results of the developed metrics and discuss the relation between SE and EE. Finally, Section V concludes the paper.

II. System Model

Modeling the locations of BSs and users by means of Homogeneous Poisson Process Point (HPPP) is frequently mentioned in earlier works and has been proved to be effective by numerical results when it's compared with actual scene [14]. For simplicity, a two-tier HetNet is considered in this paper, which consists of a tier of M_1 MBSs distributed according to an HPPP Φ_1 with density λ_1, overlaid with a tier of M_2 SBSs also modeled as an HPPP Φ_2 with density λ_2. Apart from the density, BSs belonging to different tiers also differ in transmit power (P_1 and P_2 respectively) and the bias factor (B_1 and B_2 respectively). In our model, the users are scattering over $\mathbb{R}^2$ according to another HPPP Φ_u with intensity λ_u and the mobility of users is out of consideration. To illustrate specifically, the network topology is shown in Fig. 1 where macro cells are overlaid with denser small cells. The blue points and black points represent MBSs and SBSs respectively while the borders of macro cells are shown by blue lines. The small cells with $B_2 = 20$ (black solid lines) broaden the coverage of SBSs with $B_2 = 1$ (no bias, grey fill parts).

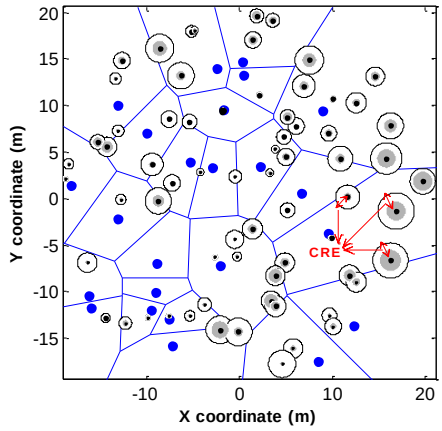

Fig. 1. Illustration of a two-tier HetNet with varying bias factors

The following analysis uses the notion of typicality of HPPP, which is made precise using Palm theory and Slivnyak-Mecke theorem [15, Chapter4]. Therefore, without loss of generality, we utilize a typical user and a typical BS lying at the origin as the objects of analysis.

In this paper, the power loss propagation model is considered with path loss exponent α , and for the random channel effects, we assume that the users experience Rayleigh fading from the *k*th BS in *i*th tier following an exponential distribution with unit mean, which is denoted by $h_{i,k}$, i.e. $h_{i,k} \sim \exp(1)$. Therefore, the power received at the typical user from the *k*th BS at a distance $x_{i,k}$ in *i*th tier is $P_i h_{i,k} x_{i,k}^{-\alpha}$. In addition, the noise is assumed additive with power σ^2 in the network. In this paper, we assume that the BSs belonging to the same tier operate in the same spectrum but the working frequency is different for the BSs in different tiers, i.e., all of the MBSs operate at the same spectrum while all the SBSs work at another higher spectrum. Intra-cell interference is not considered since Orthogonal Frequency Division Multiple Access (OFDMA) is employed within each cell. Moreover, the bandwidth allocated to each user is assumed as 1Hz for convenience. Thus, for each user, the interference only comes from all other co-tier BSs except its serving BS. In this case, the SINR of the typical user from the serving BS (the *l*th BS) in *i*th tier is

$$SINR_i(x) = \frac{P_i h_{i,l} x_{i,l}^{-\alpha}}{\sum_{k=1,k\neq l}^{M_i} P_i h_{i,k} x_{i,k}^{-\alpha} + \sigma^2} . \quad (1)$$

III. SE-EE Tradeoff Analysis

This section first formulates Association Probability (AP) and the average ergodic rate of a typical user, which are crucial to analyze the network performance. Then the average area weighted load per cell is further derived. Finally, we give AWSE and AWEE per cell as well as the SE and EE of different tiers and the whole network.

A. Related Metrics

1) Association probability (AP):

According to the assumption of open access of SBSs, each user is allowed to access one BS in either of the two tiers. For users, an association with *i*th tier is considered in terms of maximum BRP, which is defined as (the macro cell layer and small cell layer are same with tier 1 and tier 2 for convenience)

$$P_{r,i} = B_i P_i \min_k \{x_{i,k}\}^{-\alpha} = B_i P_i D_i^{-\alpha}, i \in \{1,2\} \tag{2}$$

where $B_1 = 1$ and $B_2 > 1$.

The association principle described above leads to the formulation of AP as expatiated below.

Lemma 1. The probability that a typical user is associated with the *i*th tier is given by

$$p_{Ai} = \frac{1}{1+\bar{\lambda}_{ji}(\bar{B}_{ji}\bar{P}_{ji})^{\frac{2}{\alpha}}}, i, j \in \{1,2\}, j \neq i \tag{3}$$

where $\bar{\lambda}_{ji} \triangleq \frac{\lambda_j}{\lambda_i}, \bar{P}_{ji} \triangleq \frac{P_j}{P_i}, \bar{B}_{ji} \triangleq \frac{B_j}{B_i}$.

Proof: See Appendix A.

2) Average ergodic rate:

Theorem 1. The average ergodic rate per bandwidth with the unit of bit/s/Hz of a typical user in *i*th tier is

$$\begin{aligned}\mathcal{R}_i = &\frac{2\pi\lambda_i}{p_{Ai}} \int_0^\infty \int_0^\infty \frac{x}{1+y} \exp\{-ys(x) \\ &-\pi x^2 \left(\lambda_i y^{\frac{2}{\alpha}} \int_{\left(\frac{1}{y}\right)^{\frac{2}{\alpha}}}^\infty \frac{1}{1+u^{\frac{\alpha}{2}}} du + \sum_{j=1}^2 \lambda_j \left(\bar{B}_{ji}\bar{P}_{ji}\right)^{\frac{2}{\alpha}} \right)\} dydx\end{aligned} \tag{4}$$

where $s(x) = x^\alpha P_i^{-1} \sigma^2$.

Proof: See Appendix B.

Based on the idea of the law of total probability, the average ergodic rate per bandwidth in the whole network of a typical user is given as

$$\begin{aligned}\mathcal{R} = \sum_{i=1}^2 p_{Ai}\mathcal{R}_i = &\sum_{i=1}^2 2\pi\lambda_i \int_0^\infty \int_0^\infty \frac{x}{1+y} \exp\{-ys(x) \\ &-\pi x^2 \left(\lambda_i y^{\frac{2}{\alpha}} \int_{\left(\frac{1}{y}\right)^{\frac{2}{\alpha}}}^\infty \frac{1}{1+u^{\frac{\alpha}{2}}} du + \sum_{j=1}^2 \lambda_j \left(\bar{B}_{ji}\bar{P}_{ji}\right)^{\frac{2}{\alpha}} \right)\} dydx\end{aligned} \tag{5}$$

Although (4) and (5) are not closed-forms, the expressions can be easily computed numerically.

B. Load Measurement

To characterize the load condition per cell, the expectation of load per cell needs to be calculated.

Theorem 2. The average area weighted load per cell in *i*th tier is

$$E_i = \frac{4.5}{3.5}\left(\frac{\lambda_u p_{Ai}}{\lambda_i}\right) \tag{6}$$

Proof: See Appendix C.

In particular, as B_2 increases to infinity, p_{A2} infinitely gets close to 1 according to (3) and E_2 limits to $4.5/3.5(\lambda_u/\lambda_2)$. Specifically, the expectation is deduced in the sense of per unit area by the area-weighted average. And apparently, the load is presented as the function of $\lambda_u p_{Ai}/\lambda_i$ (noted as L_i), which can be interpreted as the ratio of the densities of access users and their associated BSs in the *i*th tier. In fact, regardless of the area discrepancy of the co-tier cells, L_i can also measure the average load per cell in line with the statement in [13] and the coefficient in front is caused by the area approximation.

C. SE and EE Analysis

1) AWSE and AWEE per cell:

Different from area spectrum efficiency and area energy efficiency described in [16], this paper adopts AWSE and AWEE which take the size of association area into consideration and provides more objective evaluation and more plausible insights.

For the network performance evaluation, we employ the notion of AWSE per cell defined as the mean of the achievable rates in the network per unit bandwidth per cell by area weighted averaging. According to the definition, we express the AWSE per cell in *i*th tier using the average ergodic rate from (4) as follows with the unit of bit/s/Hz,

$$\begin{aligned}AWSE_i = E_i\mathcal{R}_i &= \frac{4.5}{3.5}\left(\frac{\lambda_u p_{Ai}}{\lambda_i}\right)\mathcal{R}_i \\ &= \frac{9\pi\lambda_u}{3.5} \int_0^\infty \int_0^\infty \frac{x}{1+y} \exp\{-ys(x) \\ &-\pi x^2 \left(\lambda_i y^{\frac{2}{\alpha}} \int_{\left(\frac{1}{y}\right)^{\frac{2}{\alpha}}}^\infty \frac{1}{1+u^{\frac{\alpha}{2}}} du + \sum_{j=1}^2 \lambda_j \left(\bar{B}_{ji}\bar{P}_{ji}\right)^{\frac{2}{\alpha}} \right)\} dydx\end{aligned} \tag{7}$$

Likewise, we define the AWEE per cell as the area weighted mean of the achievable rates in the network per unit bandwidth per watt per cell. Thus we can get AWEE per cell in *i*th tier with the unit of bit/J/Hz as

$$\begin{aligned}AWEE_i = \frac{AWSE_i}{E_i P_i} = \frac{E_i\mathcal{R}_i}{E_i P_i} = &\frac{2\pi\lambda_i}{p_{Ai}P_i} \int_0^\infty \int_0^\infty \frac{x}{1+y} \exp\{-ys(x) \\ &-\pi x^2 \left(\lambda_i y^{\frac{2}{\alpha}} \int_{\left(\frac{1}{y}\right)^{\frac{2}{\alpha}}}^\infty \frac{1}{1+u^{\frac{\alpha}{2}}} du + \sum_{j=1}^2 \lambda_j \left(\bar{B}_{ji}\bar{P}_{ji}\right)^{\frac{2}{\alpha}} \right)\} dydx\end{aligned} \tag{8}$$

where the power consumption per cell ($E_i P_i$) is determined by the load and BS transmit power and it's justified with the assumption that the BS transmit power for each user is fixed at P_i and virtually independent of the load of different cells.

2) Network SE and EE:

Similar to the definition of SE and EE, the SE of different tiers and the whole network can be derived as

$$\begin{aligned}SE_i = M_i E_i \mathcal{R}_i = &\frac{9\pi\lambda_u M_i}{3.5} \int_0^\infty \int_0^\infty \frac{x}{1+y} \exp\{-ys(x) \\ &-\pi x^2 \left(\lambda_i y^{\frac{2}{\alpha}} \int_{\left(\frac{1}{y}\right)^{\frac{2}{\alpha}}}^\infty \frac{1}{1+u^{\frac{\alpha}{2}}} du + \sum_{j=1}^2 \lambda_j \left(\bar{B}_{ji}\bar{P}_{ji}\right)^{\frac{2}{\alpha}} \right)\} dydx\end{aligned} \tag{9}$$

and

$$SE=\sum_{i=1}^{2}M_iE_i\mathcal{R}_i=\sum_{i=1}^{2}\frac{9\pi\lambda_u M_i}{3.5}\int_0^\infty\int_0^\infty\frac{x}{1+y}\times$$
$$\exp\{-ys(x)-\pi x^2(\lambda_i y^{\frac{2}{\alpha}}\int_{\left(\frac{1}{y}\right)^{\frac{2}{\alpha}}}^{\infty}\frac{1}{1+u^{\frac{\alpha}{2}}}du+\sum_{j=1}^{2}\lambda_j\left(\bar{B}_{ji}\bar{P}_{ji}\right)^{\frac{2}{\alpha}})\}dydx \tag{10}$$

In the special case with $\sigma^2=0$ (no noise), (10) can be simplified to

$$SE\big|_{\sigma^2=0}=\sum_{i=1}^{2}M_iE_i\mathcal{R}_i=\sum_{i=1}^{2}\frac{9\pi\lambda_u M_i}{3.5}\int_0^\infty\int_0^\infty\frac{x}{1+y}\times$$
$$\exp\{-\pi x^2(\lambda_i y^{\frac{2}{\alpha}}\int_{\left(\frac{1}{y}\right)^{\frac{2}{\alpha}}}^{\infty}\frac{1}{1+u^{\frac{\alpha}{2}}}du+\sum_{j=1}^{2}\lambda_j\left(\bar{B}_{ji}\bar{P}_{ji}\right)^{\frac{2}{\alpha}})\}dydx \tag{11}$$

According to (9), it can be clearly seen that SE_i largely depends on the ratio of bias factors in different tiers (SE_i is a decreasing function of $\bar{B}_{ij}$). Specifically, when B_1 stays constant, SE of macro cell tier (SE_1) declines with increasing B_2 while SE of small cell tier (SE_2) shows an increase. Additionally, it's the increase of SE_2 that matters at first and then the decrease of SE_1 becomes dominant as B_2 grows. Hence, it can be inferred that network SE (the sum of SE_1 and SE_2) increases first and then drops down based on (10) and (11).

Further, the EE of different tiers and the whole network can be given by

$$EE_i=\frac{M_iE_i\mathcal{R}_i}{M_iE_iP_i}=\frac{2\pi\lambda_i}{p_{Ai}P_i}\int_0^\infty\int_0^\infty\frac{x}{1+y}\exp\{-ys(x)$$
$$-\pi x^2\left(\lambda_i y^{\frac{2}{\alpha}}\int_{\left(\frac{1}{y}\right)^{\frac{2}{\alpha}}}^{\infty}\frac{1}{1+u^{\frac{\alpha}{2}}}du+\sum_{j=1}^{2}\lambda_j\left(\bar{B}_{ji}\bar{P}_{ji}\right)^{\frac{2}{\alpha}}\right)\}dydx \tag{12}$$

and

$$EE=\frac{SE}{\sum_{i=1}^{2}M_iE_iP_i}=\frac{1}{\sum_{i=1}^{2}M_iE_iP_i}\{\sum_{i=1}^{2}\frac{9\pi\lambda_u M_i}{3.5}\int_0^\infty\int_0^\infty\frac{x}{1+y}\times$$
$$\exp[-ys(x)-\pi x^2(\lambda_i y^{\frac{2}{\alpha}}\int_{\left(\frac{1}{y}\right)^{\frac{2}{\alpha}}}^{\infty}\frac{1}{1+u^{\frac{\alpha}{2}}}du+\sum_{j=1}^{2}\lambda_j\left(\bar{B}_{ji}\bar{P}_{ji}\right)^{\frac{2}{\alpha}})]dydx\} \tag{13}$$

Similarly, in the special case with $\sigma^2=0$ (no noise), (13) can be further simplified as

$$EE\big|_{\sigma^2=0}=\frac{SE}{\sum_{i=1}^{2}M_iE_iP_i}=\frac{1}{\sum_{i=1}^{2}M_iE_iP_i}\{\sum_{i=1}^{2}\frac{9\pi\lambda_u M_i}{3.5}\int_0^\infty\int_0^\infty\frac{x}{1+y}$$
$$\times\exp[-\pi x^2(\lambda_i y^{\frac{2}{\alpha}}\int_{\left(\frac{1}{y}\right)^{\frac{2}{\alpha}}}^{\infty}\frac{1}{1+u^{\frac{\alpha}{2}}}du+\sum_{j=1}^{2}\lambda_j\left(\bar{B}_{ji}\bar{P}_{ji}\right)^{\frac{2}{\alpha}})]dydx\} \tag{14}$$

According to (9) and (12), we notice that EE_i and SE_i share the same forms but differ in the denominator, i.e., power consumption per cell. As $\bar{B}_{ij}$ increases, both the load per cell and SE per tier declines as mentioned before. And note that the load variation is dominant in case of small $\bar{B}_{ij}$ while SE per tier prevails with large $\bar{B}_{ij}$. Therefore, EE per tier expressed by the ratio of SE and load per tier increases first and then drops down. Similar conclusion related to the EE of the whole network can be drawn based on (10), (13) and (14).

IV. Simulation Results

In this part, we present several numerical simulations and give relative analysis in this section. In this paper, MBSs and SBSs are modeled in an HPPP way and mutually independent. Also, the users are distributed to another HPPP and the mobility of users isn't taken into consideration. For the sake of clarity, the simulation parameters are listed in Table I.

TABLE I: SIMULATION PARAMETERS

Symbol	Definition	Value
P_1	The transmit power of MBSs	20 W
P_2	The transmit power of SBSs	0.02W
σ^2	The power of noise	-174dBm/Hz
W	The physical bandwidth for each user	1 Hz
α	The path loss exponent	4
λ_1	The density of MBSs	0.001/m^2
λ_2	The density of SBSs	0.005/m^2
λ_u	The density of users	0.02-0.1/ m^2 0.05/ m^2 (default)
R	The radius of network	200m
B_1	The bias factor of tier1	1
B_2	The bias factor of tier2	1-2000

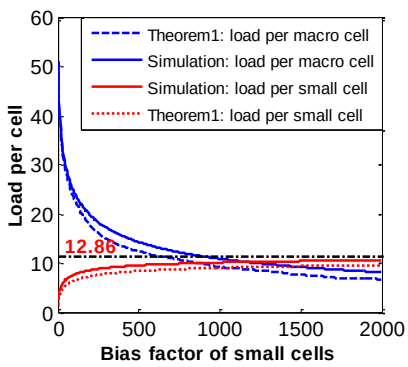

Fig. 2. Average area weighted load per cell in a two-tier HetNet.

A. The Effect of Bias Factor on Load per Cell

The variation of area weighted load per cell with the bias factor is shown in Fig. 2. It can be seen that the simulated and theoretical results match reasonably well as expected. As depicted in Fig. 2, the increase of B_2 decreases the area weighted load per macro cell while conducing to the increase of the load per small cell. Furthermore, with the increase of bias factor, the load per small cell increases less rapidly and approaches a fixed value infinitely, which can be calculated by

4.5/3.5(λ_u/λ_2) (12.86 in Fig. 2). The reason is that more users incline to connect to SBSs with increasing bias factor but the average load per small cell can't increase infinitely due to the limitation of AP in (6).

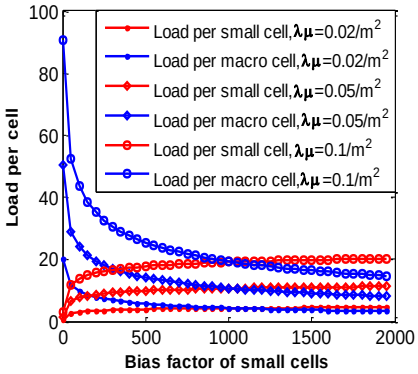

Fig. 3. Average area weighted load per cell with varying user densities

Similar trend of the load variation per cell with bias factor can be observed in Fig. 3 where different user density is used. As is shown, the bias where the load per cell in different tiers shares the same value is invariant to the user density. The reason lies in that the ratio of the average load per cell in different tiers is irrelevant to the density of users according to (6). In fact, it's the ratio of BS densities, the ratio of BS transmit powers in different tiers and the bias factors that have a great effect on the association of users.

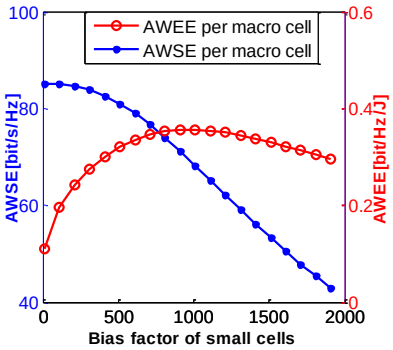

Fig. 4. AWSE and AWEE per macro cell with varying bias factors

B. The AWSE AWEE per Cell

Fig. 4 and Fig. 5 illustrate the AWSE and AWEE per macro cell as well as per small cell respectively. From Fig.4, we can see the AWSE curve keeps dropping with the increase of B_2. Meanwhile, the AWEE curve flares up at low bias factor, but shows a decline when the bias factor over a certain value. The same metrics of small cells can be observed from Fig. 5. It shows that the AWSE per small cell increases more and more slowly and stays stable at high bias factor. Moreover, the AWEE per small cell curve has the same tend with that of macro cell but decreases much slower at high bias factor. This is because load is the key factor for AWSE, high value of B_2 decreases the AWSE per macro cell and increases the AWSE per small cell by pushing more users to SBSs. For macro cells, fewer users mean less power consumption. Hence, the AWEE per macro cell expressed by the ratio of AWSE and load increases first and then drops down. For small cells, increasing load brings more power consumption but also higher AWSE, resulting in the increase of AWEE at first but a following slight decline.

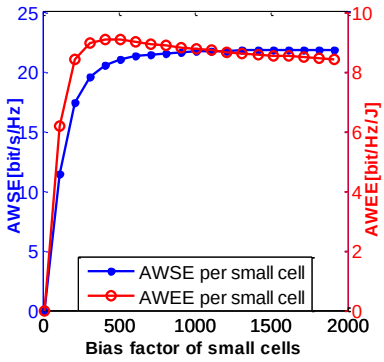

Fig. 5. AWSE and AWEE per small cell with varying bias factors

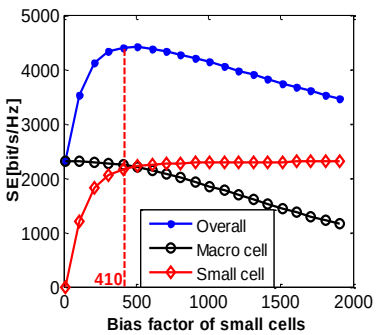

Fig. 6. SE comparison with varying bias factors

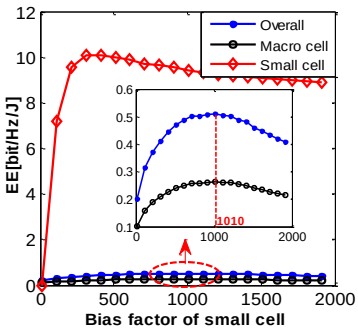

Fig. 7. EE comparison with varying bias factors

C. The Tradeoff between SE and EE with Varying Bias Factor

It can be observed from Fig. 6 and Fig. 7 that the SE and EE of macro cells and small cells exhibit the same trend with the corresponding AWSE and AWEE per cell, which are in line with (9) and (12). Furthermore, the overall SE curve and EE curve present to be similar with varying B_2 (from rising to decline). However, the optimum bias factor for SE isn't consistent with that of EE ($B_2 = 410$ for maximum SE and $B_2 = 1010$ for maximum EE) and remains agnostic depending on the network metrics. Specifically, the EE curve exhibits a rising trend when SE reaches its optimum and bias optimization to achieve the highest EE will reduce SE

due to the load offloading from MBSs to SBSs. According to the results, there is a tradeoff between SE and EE and SE acts more sensitive to the variation of bias factors than EE. Hence, the set of bias factor should depend on the metrics of network performance being concerned about. All of these results are able to provide practical guidelines for the set of bias factor in the HetNets.

V. Conclusions

In this paper, a two-tier HetNet model is presented and stochastic method is utilized to measure the network performance. The area weighted load as well as AWSE and AWEE per cell is employed to capture the load condition from the perspective of cells while SE and EE of different tiers aim to judge the network on tier level. Simulation results show that our proposed performance metrics turn out to be efficient in describing network performance. Both analysis and simulations indicate that there is a tradeoff between SE and EE of the whole network performed as the corresponding optimum bias factors aren't consistent due to the different effects of the bias factors on them. All of these results can provide a new insight into the tradeoff between SE and EE.

Appendix A

According to the access principle of maximum BRP, the AP of a typical user with *i*th tier can be given by

$$\begin{aligned} P_{Ai} &= P\left[\bigcap_{j\neq i}\left(P_iB_iD_i^{-\alpha} > P_jB_jD_j^{-\alpha}\right)\right] \\ &\overset{(a)}{=} \prod_{j\neq i} P\left(P_iB_iD_i^{-\alpha} > P_jB_jD_j^{-\alpha}\right) \\ &= \prod_{j\neq i} P\left[D_j > (\bar{P}_{ji}\bar{B}_{ji})^{\frac{1}{\alpha}} D_i\right] \\ &= \int_0^\infty \prod_{j\neq i} P\left[D_j > (\bar{P}_{ji}\bar{B}_{ji})^{\frac{1}{\alpha}} r\right] f_i(r)dr \end{aligned} . \tag{15}$$

where (a) follows from the independence of Φ_1 and Φ_2.

Then $P\left[D_j > (\bar{P}_{ji}\bar{B}_{ji})^{1/\alpha} r\right]$ and $f_i(r)$ can be derived using the property of HPPP [15, Chapter2]. Meanwhile, $P\left[D_j > (\bar{P}_{ji}\bar{B}_{ji})^{1/\alpha} r\right]$ means there is no BS closer than $(\bar{P}_{ji}\bar{B}_{ji})^{1/\alpha} r$ in the *j*th tier. Therefore,

$$P\left\{D_j > (\bar{P}_{ji}\bar{B}_{ji})^{\frac{1}{\alpha}} r\right\} = e^{-\pi\lambda_j\left((\bar{P}_{ji}\bar{B}_{ji})^{\frac{1}{\alpha}} r\right)^2} . \tag{16}$$

and the PDF of the distance between the typical user and its serving BS is

$$f_i(r) = \frac{dF_i(r)}{dr} = \frac{d\{P(r_i \le r)\}}{dr} = \frac{d\left(1-e^{-\pi\lambda_i r^2}\right)}{dr} = 2\pi\lambda_i r e^{-\pi\lambda_i r^2} \tag{17}$$

Then the desired result can be derived by plugging (16) and (17) into (15).

Appendix B

Based on the definition of average ergodic rate, we can derive

$$\begin{aligned} \mathcal{R}_i &= \int_0^\infty \mathbb{E}_{SINR_i}\left[\ln\left(1+SINR_i(x)\right)\right] f_{X_i}(x)dx \\ &= \int_0^\infty\int_0^\infty \ln\left[1+\varphi_i(x)\right] f_{\Phi_i}\left[\varphi_i(x)\right] d\varphi f_{X_i}(x)dx \\ &= \int_0^\infty\int_{\varphi=0}^\infty\int_{y=0}^{\varphi} \frac{1}{1+y} f_{\Phi_i}\left[\varphi_i(x)\right] dy d\varphi f_{X_i}(x)dx \\ &\overset{(a)}{=} \int_0^\infty\int_{y=0}^\infty \left[\int_{\varphi=y}^\infty f_\Phi(\varphi)d\varphi\right]\frac{1}{1+y} dy f_{X_i}(x)dx \\ &= \int_0^\infty\int_0^\infty \frac{\mathbb{P}\left[\varphi_i(x) > y\right]}{1+y} dy f_{X_i}(x)dx \end{aligned} \tag{18}$$

where (a) follows from changing the order of integration and the SINR of the typical user in (18) can be rewritten as $\varphi_i(x) = \frac{Pg_{i,0}x^{-\alpha}}{T}$, where $T = \sum_{j=1}^2 I_j + \sigma^2$. From [4], the distribution of the distance between the typical user and its serving BS is simplified as

$$f_{X_i}(x) = \frac{2\pi\lambda_i x}{p_{Ai}} \exp\left[-\pi\sum_{j=1}^2 \lambda_j (\bar{P}_{ji}\bar{B}_{ji})^{2/\alpha} x^2\right] \tag{19}$$

Therefore, the probability $\mathbb{P}\left[\varphi_i(x) > y\right]$ is

$$\begin{aligned} &\mathbb{P}\left[\varphi_i(x) > y\right] = \mathbb{P}\left(g_{i,0} > x^\alpha P_i^{-1} yT\right) \\ &= \int_0^\infty \exp\left(-x^\alpha P_i^{-1} yt\right) f_T(t)dt \\ &= \exp\left[-ys(x)\right]\int_0^\infty \exp\left(-x^\alpha P_i^{-1} yI_i\right) f_T(t)dt \\ &= \exp\left[-ys(x)\right] L_{I_i}\left(x^\alpha P_i^{-1} y\right) \\ &\overset{(b)}{=} \exp\left[-ys(x)\right]\exp\left\{-\pi\lambda_i y^{2/\alpha} x^2 \int_{(1/y)^{2/\alpha}}^\infty \frac{1}{1+u^{\alpha/2}} du\right\} \\ &= \exp\left\{-ys(x) - \pi\lambda_i y^{2/\alpha} x^2 \int_{\left(\frac{1}{y}\right)^{2/\alpha}}^\infty \frac{1}{1+u^{\alpha/2}} du\right\} \end{aligned} \tag{20}$$

with $s(x) = x^\alpha P_i^{-1}\sigma^2$ and (b) follows the conclusion in [13].

Applying (19) and (20) to (18), we get the result in (4).

Appendix C

From [4], the probability generating function (PGF) of the number of users associated with the typical BS is

$$G_{N_i}(z) = \mathbb{E}\left[\exp(\lambda_u S_i(z-1))\right] = 3.5^{4.5}(3.5 + \frac{\lambda_u P_{Ai}}{\lambda_i}(1-z))^{-4.5} \tag{21}$$

Resort to the central moment functions of PGF [15, Chapter4], the expectation of the load per cell can be given by

$$E(N_i) = G_{N_i}{}'(1^-) . \tag{22}$$

where the *n* order central moment function is

$$\begin{aligned} G_{N_i}{}^{(n)}(z) = 3.5^{3.5}\frac{\Gamma(n+4.5)}{\Gamma(3.5)}\left(\frac{\lambda_u P_{Ai}}{\lambda_i}\right)^n \times \\ \left(3.5 + \frac{\lambda_u P_{Ai}}{\lambda_i}(1-z)\right)^{-(4.5+n)} \end{aligned} . \tag{23}$$

Setting n with 1 gives $G_{N_i}{}'(z)$. Then by replacing z with 1, we can obtain E_i

$$E_i = E(N_i) = \frac{4.5}{3.5}(\frac{\lambda_u p_{Ai}}{\lambda_i}) \tag{24}$$

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Meng-Ting Liu was born in Zhejiang Province, China, in 1992. She received the B.S degree in communication engineering from Minzu University of China, in 2013. She is now a postgraduate in the School of Electronic Engineering at Beijing University of Posts and Telecommunications (BUPT). Her research interests include stochastic geometry theory, performance analysis and CoMP in the ultra-dense network.

Ying-Lei Teng received the B.S degree from Shandong University, China, in 2005, and received the PhD degree in electrical engineering from BUPT in 2011. She is now working as an Associate Professor in School of Electronic Engineering at BUPT. Her current research interests include performance analysis and optimization theory in the HetNet, cross-layer design technology and UDN.

Mei Song received the B.E. and M.E. degrees from Tianjin University, China. She is currently a Professor with BUPT, China. Her current research interests include resource allocation and mobility management in heterogeneous and cognitive network, cooperative communications, SDN and other advanced technology in future communications.