An Embedded Markov Chain Modeling Method for Movement-Based Location Update Scheme

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Abstract —In this paper, an embedded Markov chain model is proposed to analyze the signaling cost of the Movement-Based Location Update (MBLU) scheme under which a Location Update (LU) occurs whenever the number of cells crossed reaches a threshold, called movement threshold. Compared with existing literature, this paper has the following advantages. 1) This paper proposes an embedded Markov chain model in which the cell residence time follows Hyper-Erlang distribution. 2) This paper considers the Location Area (LA) architecture. 3) This paper emphasize the dependency between the cell and LA residence times using a fluid flow model. Close-form expressions for the signaling cost produced by LU and paging operations are derived, and their accuracy is validated by simulation. Based on the derived analytical expressions, we conduct numerical studies to investigate the impact of diverse parameters on the signaling cost.

Index Terms—Embedded markov chain, hyper-erlang distribution, Location Management (LM), Movement-Based Location Update (MBLU).

I. Introduction

A. Motivation

In a cellular network, the current location of a User Equipment (UE) is tracked by the network so that the UE can successfully receive the incoming calls. Location management is responsible for tracking the current location of the UE. LM contains two basic operations, i.e., Location Update (LU) and paging. LU is the process through which an UE periodically updates its location information in the network database. Paging is the process through which the network sends broadcasting messages in a paging area to page a called UE. In Global System for Mobile Communication (GSM), the coverage of the network is partitioned into non-overlapped Location Areas (LAs) each of which consists of a number of cells. The network adopts a two-tier architecture including Home Location Register (HLR) and Visitor Location Register (VLR) databases. The HLR and VLR databases respectively record an UE's permanent and temporary information, such as location information, identity information and so on.

LM schemes are divided into two categories, i.e., static and dynamic LM schemes. Under the static LM scheme, a LU is performed only when an UE crosses an LA boundary, and the paging area for all the UEs is the same, i.e., an LA. The static LM scheme is not cost effective, because the mobility characteristics of individual UEs are neglected. Suppose that a UE has frequent incoming calls, frequent LUs are propitious to reduce the paging cost. While for a UE with high mobility, reducing the number of LUs contributes to decrease the LU cost. Thus, an effective LM scheme should consider the mobility characteristics of individual UEs.

To overcome the defects of the static scheme, three dynamic LM schemes are proposed, i.e., Distance-Based Location Update (DBLU) scheme [1], Time-Based Location Update (TBLU) scheme [2], and Movement-Based Location Update (MBLU) scheme [3]-[13]. Under the three dynamic LU schemes, a LU is performed whenever the traveled distance, elapsed time, and crossed cells reaches the predefined threshold, called distance threshold, time threshold and movement threshold, respectively. A large number of studies showed that the MBLU scheme is the most cost-effective and computation-efficient as it does not need to consider the network topology [3]-[6]. This paper proposes a mathematical model to investigate the performance of the MBLU scheme.

B. Existing Studies

Based on whether the LA architecture, the existing studies about the MBLU scheme are mainly divided into two categories.

Studies [3]-[6] belong to this group considering LA architecture. Li *et al.* in [3] first proposed a mathematical model to analyze the performance of MBLU scheme with HLR/VLR architecture in GSM networks. However, the proposed model in [3] neglected the VLR LUs deduced by LA boundary crossings, which is proved by Wang *et al.* in [5]. On the basis of the proposed model in [3], Rodriguez-Dagnino *et al.* in [4] utilized renewal theory to analyze the signaling cost, but the defect appearing in [3] still exist. Besides, both the two models assumed that the cell boundary was independent with LA boundary, such that one cell may belong to more than one LA. Wang *et*

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al. in [5] proposed a new mathematical model to describe the VLR LU due to the movement threshold achieving or the LA boundary crossing. However, [3]-[5] neglected the dependency between the cell residence time and LA residence time, such that the derived results are not accurate. Studies [2], [7]-[13] belong to this group without considering the LA architecture. Although these studies proposed some optimal paging schemes and new analyzing approaches [8], [9], the mathematical models were imperfect. Recently, Wang *et al.* in [6] proposed a mathematical model with two call handling models to analyze the signaling cost. In addition, the size of paging area has been determined in [6]. However, the proposed model considered that the cell residence time follows exponential distribution, which does not accurately describe the real network. On the basis of [6], this paper proposes a comprehensive mathematical model to investigate the performance of the MBLU scheme through relaxing the limitation that the cell residence time follows exponential distribution. As the size of paging area has been determined in [6], this paper pays only attention to the LU.

C. Our Contributions

The main contributions of this paper are as follows. 1) An embedded Markov chain model is proposed to investigate the MBLU scheme. In the proposed model, the cell residence time follows Hyper-Erlang distribution (HERD), which can capture the mobility and traffic characteristics of each UE. In addition, this paper does not use the residual life theorem used in the literature [3]-[5], [7]-[14]. This theorem is contradictory in many cases. 2) HLR/VLR architectures are considered. 3) This paper uses fluid flow model to depict the dependency between cell residence time and LA residence time. Compared with the existing models, the proposed model in this paper is the most cost-efficient. The model developed and results derived in this paper are instrumental to the implementation of the MBLU schemes in wireless communication networks.

The rest of this paper is organized as follows. Sect. 2 introduces system model, including LA structure, HERD, and fluid flow model. Sect. 3 first tests the accuracy of the proposed model and then investigates the impact of various parameters on the signaling cost. Sect. 4 concludes this paper.

II. System Model

A. LA Structure

The LA structure employed in this paper is depicted in Fig. 1. As shown in Fig. 1, this paper considers that all the cells in the network are regular hexagons of the same size, and each cell has six neighbor cells. The innermost cell denoted by $(0,0)$ is the last registered cell, and it is surrounded by ring 1. The coordinate (i, j) denotes the $(j+1)$th cell in the ith ring. It follows that

$$j = \begin{cases} 0, & if\ \ i = 0, \\ 0,1,\dots,6i-1, & otherwise. \end{cases}$$

Denote by R the radius of an LA, e.g., $R = 5$ in Fig. 1. Denote by $N(R)$ the number of cells in a LA. It follows from Fig. 1 that

$$N(R) = 3R^2 - 3R + 1, \qquad R = 1, 2, \dots$$

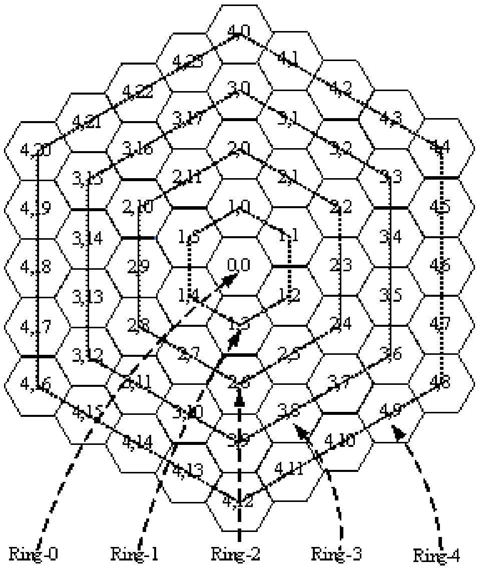

Fig. 1. LA structure with $R = 5$.

B. HERD

HERD is the sum of m independent Erlang distributions. Denote by α_i The initial probability of j stage, $0 < \alpha_j < 1$, $j = 1, 2, \dots, m$ and $\alpha_1 + \alpha_2 + \cdots + \alpha_m = 1$. Erlang distribution contains r stages [15], [16]. Denote by T_i, $i = 1, 2, \dots, r$, the time spent in i stage, each of which has probility density function (pdf) $\eta_j e^{-\eta_j t}, j = 1, 2, \dots, m$ .Suppose that at the end of the first stage, after time T_1, the second stage is started and so on, event occurring at the end of the rth stage. Thus, the event-time, denoted by X, is $T_1 + T_2 + \cdots + T_r$. The stage of each Erlang distribution of HERD is denoted by r_j. Without loss of generality, we assume that $r_1 < r_2 < \cdots < r_m$. Denote by η_j the scale parameter of the jth Erlang distribution. Denote the pdf and the Laplace transform of HERD by $h(t)$ and $h^*(s)$, respectively. It follows that

$$\begin{cases} h(t) = \sum_{j=1}^{m} \alpha_j \dfrac{(\eta_j t)^{r_j - 1}}{(r_j - 1)!} \eta_j e^{-\eta_j t}, & t \ge 0, \\ h^*(s) = \sum_{j=1}^{m} \alpha_j \left(\dfrac{\eta_j}{\eta_j + s} \right)^{r_j}. \end{cases} \tag{1}$$

Denote the mean and variance of HERD by $E(\tau)$ and δ, it follows that

$$E(\tau)=\sum_{j=1}^{m}\alpha_j\frac{r_j}{\eta_j}, \tag{2}$$

$$\delta=\sum_{j=1}^{m}\alpha_j\frac{r_j}{\eta_j^2}. \tag{3}$$

C. Fluid Flow Model

Fluid flow model has been used to depict the boundary crossing rate of an UE in a closed region in [14], [17]. To use the fluid flow model to calculate the boundary crossing rate, the following two assumptions are necessary. 1) The velocities of the UE at different locations are independent and identically distributed (i.i.d.). 2) The movement direction of an UE in a closed region is uniformly distributed over $[0, 2\pi)$, and UEs are uniformly distributed throughout the entire closed region. Denote by v, l, and a the average speed of an UE, the perimeter of the closed region, and the area of the closed region, respectively. It follows from [6], [14], [17] that

$$\eta=\frac{vl}{\pi a} \tag{4}$$

For a hexagonal shaped cell, denote by η_c the cell boundary crossing rate, it follows that

$$\eta_c=\frac{vl_c}{\pi a_c}$$

where l_c and a_c represent the perimeter and area of a cell, respectively. Similarly, denote by η_{L} the LA boundary crossing rate, it follows that

$$\eta_{\mathrm{L}}=\frac{vl_{\mathrm{L}}}{\pi a_{\mathrm{L}}}$$

where l_{L} and a_{L} represent the perimeter and area square of an LA, respectively. This paper considers the situation in which a cell boundary and a LA boundary coincide. Let β be the probability an UE crosses a cell boundary but does not cross an LA boundary. Since $\eta_{\mathrm{L}}=(1-\beta)\eta_c$, we can derive that

$$\beta=1-\frac{\eta_{\mathrm{L}}}{\eta_c}=1-\frac{l_{\mathrm{L}}}{l_c}\frac{a_c}{a_L}$$

Denote by σ the side length of a cell. Referred to the LA structure in Sect. II-A and paper [6], it follows that

$$\begin{cases}\dfrac{a_c}{a_{\mathrm{L}}}=\dfrac{a_c}{N(R)a_c}=\dfrac{1}{3R(R-1)+1},\\ \dfrac{l_{\mathrm{L}}}{l_c}=\dfrac{6(2R-1)\sigma}{6\sigma}=2R-1,\\ \beta=\dfrac{3R^2-5R+2}{3R^2-3R+1}.\end{cases}$$

This paper uses the fluid flow model to emphasize the dependency between the cell residence time and the LA residence time. [6] has proved that when the cell residence time follows an exponential distribution, the LA residence time must also be so. This paper will adopt the conclusion proved in [6].

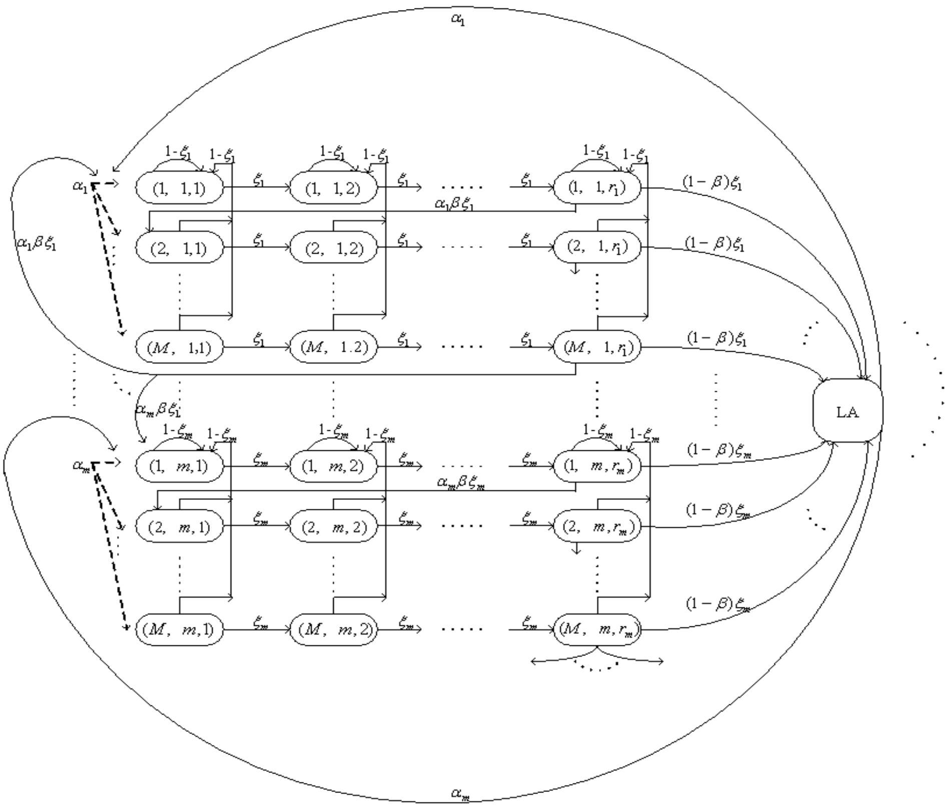

Fig. 2. An embedded markov chain for the MBLU scheme

III. Signaling Cost of the MBLU Scheme

A. Embedded Markov Chain

This paper considers that the call arrival process is Poisson process with rate λ_c and cell residence time follows HERD. Denote by t_c the call-inter arrival time. Fig. 2 shows an embedded Markov chain for the MBLU scheme. Here each stage of an Erlang distribution denotes a state in the chain. State LA represents the state an UE moves into a new LA during t_c. Denote by t_θ and t_{LA} the stage and the LA residence times, respectively. The movement counter records the number of movements starting from zero. Each state indicates the value of the counter. After the call, the chain returns to the state in which the counter resets. When one Erlang distribution residence time expires, the UE crosses a cell boundary.

Denote by (i,j,k) the state in which the movement counter reaches i in the kth stage of the jth Erlang distribution, $i=1,2,...,M$, $j=1,2,...,m$, and $k=1,2,...,r_j$. Denote by Θ the state space of the embedded Markov chain shown in Fig. 2. Denote by $\Re$ the event that an UE crosses a cell but does not cross an LA boundary, and by $\overline{\Re}$ the complementary event of $\Re$. From Sect. 2.1, β denotes the probability that $\Re$ happens. ξ_j represents the probability that the call inter arrival time large than the stage residence time, it follows that

$$\xi_j = \frac{\eta_j}{\eta_j + \lambda_c}$$

For a purpose of demonstration, take part of the HERD as an example. The following four reasons could cause the state transition. 1) If the call inter-arrival time is smaller than the stage residence time, namely, $t_c < t_\theta$, $\theta \in \Theta - \text{LA}$, after the call the chain transmits to the corresponding state in which the counter is one. 2) If the call inter-arrival time is larger than the stage residence time, namely, $t_c > t_\theta$. If $\theta = (i,j,k)$ and $k \neq r_j$, the chain transmits to the next state $(i,j,k+1)$. While $k = r_j$ and $\Re$ occurs, the chain transmits to $(i+1,1,1)$. 3) If the call inter-arrival time is larger than the cell residence time, and the event $\overline{\Re}$ occurs, the chain transmits to the state LA. An UE initiates a LU operation towards both VLR and HLR, and then the chain transmits to the initial state in which the counter is one. 4) If the call inter-arrival time is larger than the cell residence time, and at the same time the counter achieves the threshold and the event $\Re$ occurs, the chain transmits to the initial state in which the counter is one. An UE initiates a LU operation towards VLR. According to the above depiction and the proposition in paper [18], the one-step transition probabilities of the embedded Markov chain can be shown as

$$\begin{cases}
\Pr\left((1,1,1)\to(1,1,1)\right) = \Pr\left(t_c < t_{1,1,1}\right) = 1-\xi_1, \\
\Pr\left((1,1,1)\to(1,1,2)\right) = \Pr\left(t_c > t_{1,1,1}\right) = \xi_1, \\
\Pr\left((1,1,i)\to(1,1,i)\right) = \Pr\left(t_c < t_{1,1,i}\right) = 1-\xi_1, \\
\Pr\left((1,1,i)\to(1,1,i+1)\right) = \Pr\left(t_c < t_{1,1,i}\right) = \xi_1, \\
\Pr\left((1,1,r_1)\to(2,1,1)\right) = \Pr\left(t_c > t_{1,1,r_1}, \Re\right) = \alpha_1\beta\xi_1, \\
\Pr\left((1,1,r_1)\to(2,j,1)\right) = \Pr\left(t_c > t_{1,1,r_1}, \Re\right) = \alpha_j\beta\xi_1, \\
\Pr\left((1,1,r_1)\to \text{LA}\right) = \Pr\left(t_c > t_{1,1,r_1}, \overline{\Re}\right) = (1-\beta)\xi_1, \\
\Pr\left((k,1,i)\to(k,1,i+1)\right) = \Pr\left(t_c > t_{k,1,i}\right) = \xi_1, \\
\Pr\left((k,1,i)\to(1,1,i)\right) = \Pr\left(t_c < t_{k,1,i}\right) = 1-\xi_1, \\
\Pr\left((k,1,r_1)\to(k+1,1,1)\right) = \Pr\left(t_c > t_{k,1,r_1}, \Re\right) = \alpha_1\beta\xi_1, \\
\Pr\left((k,1,r_1)\to(k+1,j,1)\right) = \Pr\left(t_c > t_{k,1,r_1}, \Re\right) = \alpha_j\beta\xi_1, \\
\Pr\left((k,1,r_1)\to \text{LA}\right) = \Pr\left(t_c > t_{k,1,r_1}, \overline{\Re}\right) = (1-\beta)\xi_1, \\
\Pr\left((M,1,r_1)\to(1,1,1)\right) = \Pr\left(t_c > t_{M,1,r_1}, \Re\right) = \alpha_1\beta\xi_1, \\
\Pr\left((M,1,r_1)\to(1,j,1)\right) = \Pr\left(t_c > t_{M,1,r_1}, \Re\right) = \alpha_j\beta\xi_1, \\
\Pr\left((M,1,r_1)\to \text{LA}\right) = \Pr\left(t_c > t_{M,1,r_1}, \overline{\Re}\right) = (1-\beta)\xi_1, \\
\Pr\left(\text{LA}\to(1,1,1)\right) = \alpha_1, \\
\Pr\left(\text{LA}\to(1,j,1)\right) = \alpha_j,
\end{cases}$$

where $i=2,3,...,\text{r}_1$, $j=2,3,...,m$, and $k=2,3,...,M-1$. Denote by $\Pi_{\theta'}$ the equilibrium probability of θ', $\theta' \in \Theta$, and Π_i the sum of the states of the ith Erlang distribution, e.g.,

$$\Pi_1 = \sum_{i=1}^{\text{r}_1}\sum_{j=1}^{M}\Pi_{j,1,i.} \tag{5}$$

Assume that $A = \sum_{i=1}^{M}\Pi_{i,1,1}$. The following three equations can be derived

$$\Pi_{\text{VLR}} = \beta\sum_{i=1}^{m}\xi_i\Pi_{M,m,r_i},$$

$$E(\tau_e) = \sum_{i=1}^{m}E(\tau_i)\Pi_i,$$

$$\Pi_{\text{LA}} = (1-\beta)\xi_1\sum_{i=1}^{M}\Pi_{i,1,r_1} + \cdots + (1-\beta)\xi_m\sum_{i=1}^{M}\Pi_{i,m,r_m},$$

where Π_{LA} and Π_{VLR} respectively represent the equilibrium probability of state LA and the state in which the counter reaching the threshold, and $E(\tau_e)$ represents the average time between state transitions. Similarly, we list some examples to illustrate the relationship between the states. From Fig. 2, a part of the balance equations of the chain can be shown as

$$\Pi_{1,1,1} = (1-\xi_1)A + \alpha_1\Pi_{\text{LA}} + \alpha_1\Pi_{\text{VLR}}.$$

The relationship of each state within one Erlang distribution is

$$\begin{cases}
\Pi_{2,1,1} = \alpha_1\beta\sum_{i=1}^{m}\xi_i\Pi_{1,i,r_i}, \\
\Pi_{2,1,2} = \Pi_{2,1,1}\xi_1, \\
\vdots \\
\Pi_{2,1,r_1} = \Pi_{2,1,1}\xi_1^{r_1-1}.
\end{cases}$$

We can derive that $\Pi_{i,j,k} = \Pi_{i,j,1}\xi_j^{k-1}$, for $i = 2,3,\ldots,M, j = 1,2,\ldots,m, k = 1,2,\ldots,r_j$.

The relationship of each state among all Erlang distributions is

$$\begin{cases} \Pi_{2,1,1} = \alpha_1\beta\sum_{i=1}^{m}\xi_i\Pi_{1,i,r_i}, \\ \Pi_{2,2,1} = \alpha_2\beta\sum_{i=1}^{m}\xi_i\Pi_{1,i,r_i}, \\ \vdots \\ \Pi_{2,m,1} = \alpha_m\beta\sum_{i=1}^{m}\xi_i\Pi_{1,i,r_i}. \end{cases}$$

We can derive that

$$\begin{cases} \Pi_{2,2,1} = \frac{\alpha_2}{\alpha_1}\Pi_{2,1,1}, \\ \vdots \\ \Pi_{2,m,1} = \frac{\alpha_m}{\alpha_1}\Pi_{2,1,1}. \end{cases}$$

The general formula is

$$\Pi_{i,j,1} = \frac{\alpha_j}{\alpha_1}\Pi_{i,1,1}, \quad i = 2,3,\ldots,M \quad j = 1,2,\ldots,m$$

It follows by the same way that

$$\begin{cases} \Pi_{1,j,1} = \frac{\xi_1}{\xi_j}\frac{\alpha_j}{\alpha_1}\left(1-\xi_j\right)A + \alpha_j\Pi_{\text{LA}} + \alpha_j\Pi_{\text{VLR}}, \\ \vdots \\ \Pi_{1,j,r_j} = \frac{\xi_1}{\xi_j}\frac{\alpha_j}{\alpha_1}\left(1-\xi_j^{r_j}\right)A + \alpha_j\xi_j^{r_j-1}\Pi_{\text{LA}} + \alpha_j\xi_j^{r_j-1}\Pi_{\text{VLR}}. \end{cases}$$

We can derive that

$$\begin{aligned} \Pi_{\text{LA}} &= \left(1-\beta\right)\xi_1 A + \cdots + \left(1-\beta\right)\xi_m\frac{\xi_1}{\xi_m}\frac{\alpha_m}{\alpha_1}A \\ &= \frac{\alpha_1+\alpha_2+\cdots+\alpha_m}{\alpha_1}\left(1-\beta\right)\xi_1 A \\ &= \frac{\xi_1}{\alpha_1}\left(1-\beta\right)A. \end{aligned} \tag{6}$$

Now we deduce the relationship of each state of A.

$$\begin{cases} \Pi_{2,1,1} = \alpha_1\beta\sum_{i=1}^{m}\xi_i\Pi_{1,i,r_i}, \\ \Pi_{1,1,r_1} = \left(1-\xi_1^{r_1-1}\right)A + \alpha_1\xi_1^{r_1-1}\Pi_{\text{LA}} + \alpha_1\xi_1^{r_1-1}\Pi_{\text{VLR}}, \\ \Pi_{1,2,r_2} = \frac{\xi_1}{\xi_2}\frac{\alpha_2}{\alpha_1}\left(1-\xi_2^{r_2}\right)A + \alpha_2\xi_2^{r_2-1}\Pi_{\text{VLR}}, \\ \vdots \\ \Pi_{1,m,r_m} = \frac{\xi_1}{\xi_m}\frac{\alpha_m}{\alpha_1}\left(1-\xi_m^{r_m}\right)A + \alpha_m\xi_m^{r_m-1}\Pi_{\text{VLR}}. \end{cases}$$

Let $\text{a} = \sum_{i=1}^{m}\alpha_i\xi_i^{r_i}$. We can derive that

$$\Pi_{2,1,1} = \left(1-\text{a}\right)\beta\xi_1 A + \text{a}\alpha_1\beta\Pi_{\text{LA}} + \text{a}\alpha_1\beta\Pi_{\text{VLR}}.$$

It follows that

$$\begin{cases} \Pi_{3,1,1} = \text{a}\beta\Pi_{2,1,1}, \\ \vdots \\ \Pi_{M,1,1} = \text{a}^{M-2}\beta^{M-2}\Pi_{2,1,1}. \end{cases}$$

It follows that

$$\Pi_{\text{VLR}} = A\frac{\xi_1}{\alpha_1}\frac{\beta\left(1-\text{a}\beta\right)\left(\text{a}\beta\right)^{M-1}}{1-\left(\text{a}\beta\right)^M} \tag{7}$$

Denote by τ_θ the state residence time in state θ, $\theta \in \Theta$. It follows that

$$\begin{cases} \tau_{\text{LA}} = 0, \\ \tau_\theta = \min\left(t_c, t_\theta\right) \end{cases}$$

The mean residence time of each state can be expressed as

$$E\left(\tau_j\right) = \frac{1}{\eta_j + \lambda_c}$$

Denote by τ_e the time between state transitions. It follows that

$$\tau_e = \sum_{\theta\in\Theta}\Pi_\theta\tau_\theta,$$

$$E\left(\tau_e\right) = \sum_{j=1}^{m}E\left(\tau_j\right)\Pi_j.$$

We can derive that

$$\begin{aligned} \Pi_1 &= \Pi_{1,1,1} + \Pi_{1,1,2} + \cdots + \Pi_{1,1,r_1} \\ &\quad + \Pi_{2,1,1} + \Pi_{2,1,2} + \cdots + \Pi_{2,1,r_1} \\ &\quad + \cdots \\ &\quad + \Pi_{M,1,1} + \Pi_{M,1,2} + \cdots + \Pi_{M,1,r_1} \\ &= r_1 A - \frac{\xi_1\left(1-\xi_1^{r_1}\right)}{1-\xi_1}A + \alpha_1\frac{1-\xi_1^{r_1}}{1-\xi_1}\Pi_{\text{LA}} + \alpha_1\frac{1-\xi_1^{r_1}}{1-\xi_1}\Pi_{\text{VLA}} \\ &\quad + \frac{1-\xi_1^{r_1}}{1-\xi_1}\Pi_{2,1,1} + \cdots + \frac{1-\xi_1^{r_1}}{1-\xi_1}\Pi_{\text{M},1,1} \\ &= r_1 A \end{aligned}$$

It follows that

$$\begin{cases} \Pi_2 = \frac{\xi_1}{\xi_2}\frac{\alpha_2}{\alpha_1}r_2 A, \\ \vdots \\ \Pi_M = \frac{\xi_1}{\xi_m}\frac{\alpha_m}{\alpha_1}r_m A. \end{cases}$$

According the above formulas, we derive that

$$E\left(\tau_e\right) = A\frac{\xi_1}{\alpha_1}\sum_{i=1}^{m}\alpha_i\frac{r_i}{\lambda_i}$$

Define n_{HLR} and n_{VLR} as the expected number of HLR LUs and VLR LUs per unit time, respectively. It follows that

$$n_{\text{HLR}} = \frac{\Pi_{\text{LA}}}{E\left(\tau_e\right)} = \frac{1-\beta}{\sum_{i=1}^{m}\alpha_i\frac{r_i}{\lambda_i}}, \tag{8}$$

$$n_{\text{VLR}} = \frac{1-\left(\text{a}\beta\right)^M - \beta\left[1-\left(\text{a}\beta\right)^{M-1}\right]}{\left[1-\left(\text{a}\beta\right)^M\right]\sum_{i=1}^{m}\alpha_i\frac{r_i}{\lambda_i}}. \tag{9}$$

Define N_{HLR} and N_{VLR} as the expected number of HLR LUs and VLR LUs during call inter-arrival time, respectively. It follows that

$$N_{\mathrm{HLR}} = \frac{1}{\lambda_c} n_{\mathrm{HLR}} = \frac{1-\beta}{\lambda_c \sum_{i=1}^{m} \alpha_i \frac{r_i}{\lambda_i}} \quad (10)$$

$$N_{\mathrm{VLR}} = \frac{1-(\mathrm{a}\beta)^M - \beta\left[1-(\mathrm{a}\beta)^{M-1}\right]}{\lambda_c\left[1-(\mathrm{a}\beta)^M\right]\sum_{i=1}^{m} \alpha_i \frac{r_i}{\lambda_i}}. \quad (11)$$

Eq. (10) and (11) will be proved in Appendix.

B. Signaling Cost

Denote by δ_{HLR} / δ_{VLR} the unit cost of performing a HLR/VLR LU during t_c. This paper considers the paging scheme derived in [6]. Denote by N_{PA} and δ_{paging} the paging area size and the unit cost of paging a cell, respectively. Denote by C_{total} the total signaling cost during t_c. It follows that

$$C_{\mathrm{total}} = \delta_{\mathrm{VLR}} N_{\mathrm{VLR}} + \delta_{\mathrm{HLR}} N_{\mathrm{HLR}} + \delta_{\mathrm{paging}} N_{\mathrm{PA}}.$$

IV. Performance Evaluatoin

A. Accuracy Verification of the Analytical Formulas Through Simulation

In this subsection, the accuracy of the analytical formulas is tested by simulation. Moreover, we compare the numerical results of signaling cost obtained from [3]-[5] with the numerical results of signaling cost obtained from this paper. [3]-[6] considered the LA architecture, but papers [3]-[5] neglected the VLR LUs produced by LA boundary crossings. To embody the fairness, this paper adopts the method proposed in paper [6] in which the VLR LUs are added due to LA boundary crossings. This paper simulates a number of call arrivals, denoted by N_{call}, to obtain a performance metric. In the simulation, some parameters are assumed, $\delta_{\mathrm{HLR}} = 160$, $\delta_{\mathrm{VLR}} = 350$, $\delta_{paging} = 5$, $R = 5$, $\lambda_c = 0.001$, $\eta_c = 0.005$ and $N_{\mathrm{call}} = 1 \times 10^4$.

Fig. 3 compares the simulation results of signaling cost obtained from simulation with the numerical results of signaling cost obtained from [3]-[5] and this paper. In Fig. 3, we can get the following conclusions. 1) The analytical results are close to the simulation results, indicating that the analytical formulas derived in this paper are right. 2) The signaling cost is a downward convex function as movement threshold increases, manifesting that there is an optimal movement threshold which can minimize the signaling cost. 3) In some situations there is observable disparity between the analytical results obtained in [3–5] and the simulation result. 4) With the increase of movement threshold, the difference between the signaling costs obtained in [3]-[5] decreases.

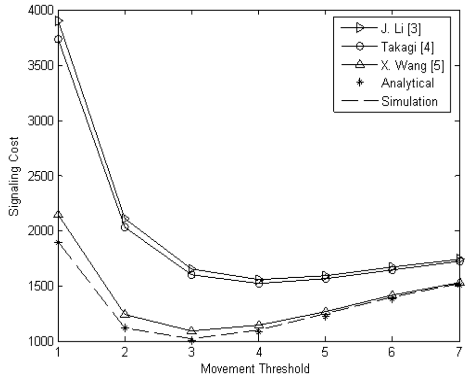

Fig. 3. Accuracy validation and comparison with existing studies.

The proposed model in [6] considered that the cell residence time follows exponential distribution, which does not accurately describe the real network. This paper relaxes the limitation that the cell residence time follows exponential distribution. The HERD can represent any distributions through setting different parameters, so that the proposed model in this paper more closes to the real network than that of [6].

B. Impact of Various Parameters on LU Cost

In this subsection, the impact of various parameters on the LU cost per call arrival under the MBLU scheme is investigated. Fig. 4-Fig. 6 show the LU cost vs. movement threshold with different parameters. In Fig. 4-Fig. 6, the following conclusions can be got.

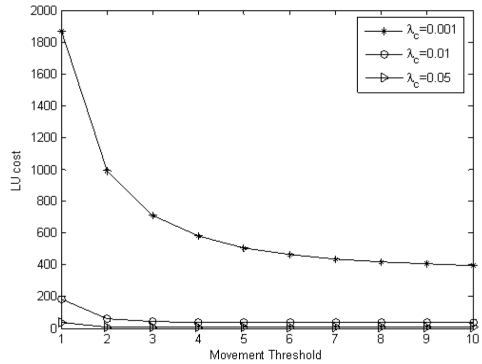

Fig. 4. LU cost vs. movement threshold with different paging rates.

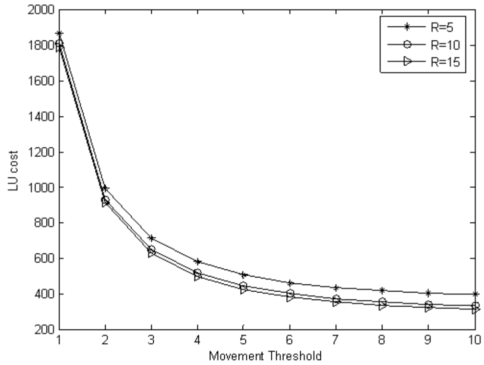

Fig. 5. LU cost vs. movement threshold with different size of LAs.

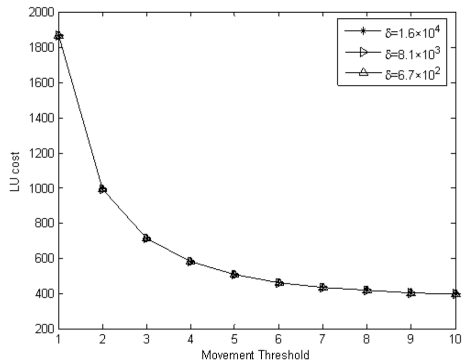

Fig. 6. LU cost vs. movement threshold with different variances of HERD.

a) Fig. 4 shows the LU cost as a function of movement threshold when $\lambda_c = 0.001$, 0.01, and 0.05. In Fig. 4, LU cost decreases as the movement threshold increases when η_c and R are fixed. In addition, when the movement threshold is fixed, the smaller the λ_c, the larger the LU cost. In the case of small λ_c, a large number of cells will be crossed during the call inter-arrival time, so that the number of LUs increases. While the movement threshold is large, the LU cost basically keeps level, because the number of LUs is very small.

b) Fig. 5 shows the LU cost as a function of movement threshold when $R = 5, 10$, and 15. In Fig. 5, when the values of λ_c and η are fixed, the larger the R, the less the LU cost. This phenomenon can be explained. As aforementioned, LUs contain two components, i.e., the VLR LUs caused by reaching the movement threshold and the VLR LUs and HLR LUs by crossing LA boundaries. Under the same movement threshold, when the LA size increases, the number of LA boundary crossings decreases during the call inter-arrival time, so that less VLR LUs and HLR LUs will be performed.

c) Fig. 6 shows the LU cost as a function of movement threshold when $\delta = 6.7\times10^2$, 8.1×10^3, and 1.6×10^4. In Fig. 6, three carves overlap, manifesting that the variance of HERD does not affect the LU cost. This conclusion indicates that the assumption that the cell residence time follows a HERD is more general.

V. CONCLUSIONS

Attributing to the developed mathematical model, analytical formulas for the signaling cost of the MBLU scheme are derived, and their accuracy is checked by simulation. With these analytical formulas, numerical studies are conducted to evaluate the impact of the LU cost. In these numerical studies, the following conclusions are observed. 1) Compared with the existing literature, the proposed model is the most accurate and is superior to existing models. 2) The signaling cost is a downward convex function of movement threshold. 3) The larger the LA radius (i.e., R), the less the LU cost. 4) The variance of Hyper-Erlang distribution (i.e., HERD) does not affect the LU cost. The model developed and results derived in this paper can guide the implementation of the MBLU scheme in wireless networks.

APPENDIX

The Proofs of (10) and (11)

Since the cell residence time follows a HERD, the number of cell boundary crossings during t_c can be expressed as

$$n = \frac{E(t_c)}{E(\tau)} = \frac{1}{\lambda_c \sum_{i=1}^{m} \alpha_i \frac{r_i}{\lambda_i}}$$

where $E(\tau)$ has been given in (2).

A. The Proof of HLR (10)

Under the fluid flow model, the number of LA boundary crossings follows a binomial distribution with $(n, 1-\beta)$. Therefore, the average number of HLR LUs during t_c can be expressed as

$$N_{\text{HLR}} = n(1-\beta) = \frac{1-\beta}{\lambda_c \sum_{i=1}^{m} \alpha_i \frac{r_i}{\lambda_i}}$$

which proves (10).

B. The Proof of VLR (11)

There are two events that can cause a VLR LU, i.e., with one event being an LA boundary crossing and the other event being the movement threshold achieving. Denote by P_{VLR} the probability that an LU occurs due to cell boundary crossings. Denote by $P(M)$ the probability that the movement counter reaches state M after which an LU occurs. It follows that

$$P_{\text{VLR}} = \beta P_M + (1-\beta) \quad . \tag{12}$$

As shown in Fig. 2, we have

$$P(M) = \frac{E(\tau_0)\sum_{i=1}^{m} \Pi_{M,i,r_i,\xi_i}}{E(\tau_e)},$$

$$E(\tau_e) = A\frac{\xi_1}{\alpha_1}\sum_{i=1}^{m}\alpha_i \frac{r_i}{\lambda_i},$$

$$\begin{cases} \Pi_{M,1,r_1} = \dfrac{\alpha_1 \xi_1^{r_1-1}}{\text{a}\beta}\Pi_{\text{VLR}}, \\ \Pi_{M,2,r_2} = \dfrac{\alpha_2 \xi_2^{r_2-1}}{\text{a}\beta}\Pi_{\text{VLR}}, \\ \vdots \\ \Pi_{M,m,r_m} = \dfrac{\alpha_m \xi_m^{r_m-1}}{\text{a}\beta}\Pi_{\text{VLR}}. \end{cases}$$

After some mathematical operations, we have

$$P(M) = \frac{\sum_{i=1}^{m}\alpha_i \frac{r_i}{\lambda_i}\Pi_{\text{VLR}}}{\beta E(\tau_e)} = \frac{\alpha_1 \Pi_{\text{VLR}}}{A\xi_1\beta}, \tag{13}$$

where

$$\Pi_{\text{VLR}} = A\frac{\xi_1}{\alpha_1}\frac{\beta\left(1-\text{a}\beta\right)\left(\text{a}\beta\right)^{M-1}}{1-\left(\text{a}\beta\right)^{M}} \qquad (14)$$

Therefore, combined (12), (13) with (14), we have

$$N_{\text{VLR}} = nP_{\text{VLR}} = \frac{1-\left(\text{a}\beta\right)^{M}-\beta\left[1-\left(\text{a}\beta\right)^{M-1}\right]}{\lambda_c\left[1-\left(\text{a}\beta\right)^{M}\right]\sum_{i=1}^{m}\alpha_i\frac{r_i}{\lambda_i}},$$

which proves N_{VLR}.

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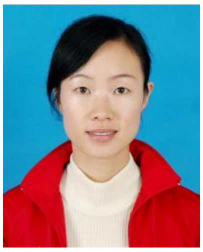

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