Compensation Estimation Method for Fast Fading MIMO-OFDM Channels Based on Compressed Sensing

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Abstract —In this paper, a compensation estimation algorithm is developed for Multiple-Input–Multiple-Output (MIMO) Orthogonal Frequency-Division-Multiplexing (OFDM) systems operating in a fast fading environment. In order to satisfy the constraint frequency mask, the pulse signal spectrum must be limited in the mask band. We investigate a channel estimator that exploit channel sparsity in the time and/or Doppler domain, where the channel is described by a limited number of paths, each characterized by a delay, Doppler scale, and attenuation factor, and derive the exact inter-carrier-interference pattern. The algorithm works with channel sparsity by jointly estimating the sparse coefficients vector and by reconstructing dynamic mathematical model of pulse wave functions. The proposed method exploits the intrinsic relationship between the sparse channels and the mathematical model. A dynamic mathematical model of pulse wave functions is used to construct the sparse channels. The dynamic mathematical model reconstruction is used to update the sensing matrix. The pulse signal which is band and time concentrated distribution, is conducive to optimization design of sparse MIMO-OFDM channel. The simulation results show that the proposed channel estimator can provide a considerable performance improvement in estimating doubly selective channels with few pilots and computational complexity.

Index Terms—Compressed sensing, sparse channel, channel estimation, fast fading

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a popular technique widely used for broadband wireless communication systems due to its high spectral efficiency [1]. However, the theoretical performance of Multiple-Input Multiple-Output (MIMO) [2] and OFDM systems may be degraded severely in broadband mobile applications due to the doubly selective channels. Thus, channel estimation which is employed to eliminate the distortion of the signals at the receiver is a critical component that affects the performance of MIMO-OFDM systems.

The conventional channel estimation methods for MIMO-OFDM systems have focused on pilot-assisted approaches [1]-[3] based on a quasi-static fading model that allows the channel to be invariant during an OFDM block. However, a loss of subchannel orthogonality due to time-variant multipath channels in orthogonal OFDM leads to inter-carrier interference (ICI) which increases the error of system [4]-[5]. Therefore, the channel time variation during a symbol block must be considered to support high-speed mobile channels.

Many pilot-aided channel estimation methods [3]-[5] usually estimate the channel response for a few selected subcarriers first. Those observations are used to interpolate the rest subcarriers. In such schemes, the required number of pilots depends on the coherence bandwidth of the channel, since the spacing of the pilot sequence has to satisfy the Nyquist sampling theorem to properly sample the fast fading channels. However, these schemes just consider the rich scattering environment, with the sparsity of the MIMO-OFDM channels being ignored. A number of sparse channel estimation schemes [6]-[17] have been proposed for time-frequency fading channels. The time-frequency joint sparse channel estimation scheme [17] first relies on a pseudorandom time-domain preamble, which is identical for all transmit antennas. The sparse common support property of the MIMO channels is utilized to acquire the partial common support. Then, frequency-domain orthogonal pilots are used for the channel recovery. However, the common support and the required number of pilots depend on the coherence bandwidth of channels and time of transmit antennas. The overhead of the required pilots or preamble will significantly increase as the number of transmit antennas. The blocks of transmitted OFDM symbols become large, which decreases the spectral efficiency. It may lead to the unacceptable high computational consumption, inter symbol interference and low frequency mask [18]. According to the Heisenberg uncertainty principle [19], the pulse signal is concentrated distribution in frequency domain and must disperse in the time domain. The sensing matrix of is updated by high-speed mobile channels [20]. It is not easy to obtain sparse channels that meet the requirements of band and time limited [21]. The solution of problem to construct the sparse channels requires the use of a dynamic mathematical model of pulse wave functions. However, such models often involve errors due to a variety of causes, including fast fading environments, atmospheric effects, and hardware limitations.

In this paper, we focus on the compensation estimation of mathematical model in compressed sensing (CS) based fast fading MIMO-OFDM channels. In order to achieve

Manuscript received February 25, 2015; revised July 18, 2015.
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doi:10.12720/jcm.10.7.466-473

few pilots, few computational complexity and low inter symbol interference, the pulse signal duration should be as short as possible, and be limited in the time. At the same time, in order to satisfy the constraint frequency mask, the pulse signal spectrum must is limited in the mask band. When the frequency mask changes, pulse waveform can be flexibly adjusted and corrected. The pulse signal which is band and time concentrated distribution, is conducive to optimization design of sparse MIMO-OFDM channel. The proposed method exploits the intrinsic relationship between the sparse channels and the dynamic mathematical model of pulse wave functions. The proposed method works by jointly estimating the sparse coefficients vector and by reconstructing the dynamic mathematical model. Then, the dynamic mathematical model of pulse wave functions reconstruction is used to update the sensing matrix, and the algorithm passes to the next iteration. The mathematical model is exploited to track rapidly MIMO-OFDM channels by avoiding depending on the coherence bandwidth and time of channels. We propose a CS-based fast fading channels estimator for MIMO-OFDM systems with as few measurements as possible. The computational complexity is significantly reduced by avoiding the inverse computation of the large pilot matrices.

The organization of this letter is as follows. The MIMO-OFDM system model is introduced in Section II. The CS-based channel compensation estimation is presented in Section III. Section IV depicts the experiments based on channel compensation estimation and the letter is summarized in Section V.

Notation: $(\cdot)^*$, $(\cdot)^{\bar{T}}$, $(\cdot)^H$, $(\cdot)^\dagger$, $(\cdot)^{-1}$ and $\|\cdot\|_F$ denote the complex conjugation, transpose, conjugate-transpose, inverse, pseudo-inverse operations and Forbenius norm, respectively. $diag\{\mathrm{V}\}$ is diagonal matrix whose diagonal is the vector V; $\min(\cdot)$ is to get minimum elements of an array; I_p denotes a $p\times p$ identity matrix; $I_{N\times 1}$ is an $N\times 1$ vector.

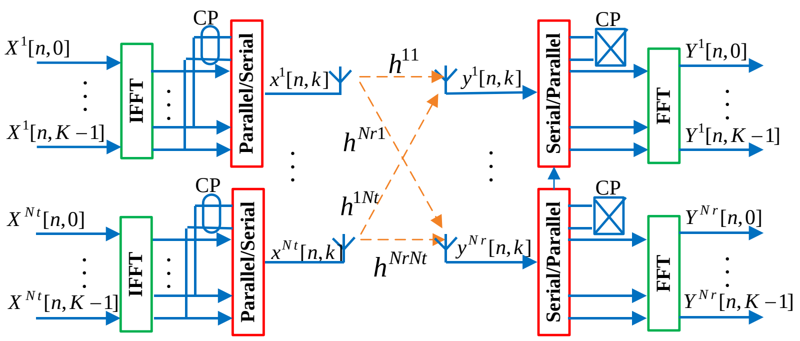

Fig. 1. MIMO-OFDM system models

II. System Model

Consider a MIMO-OFDM system with N_T transmit antennas, N_R receive antennas in Fig. 1. The OFDM system contains K subcarriers. The sample interval is T_s. The $K\times 1$ vector $\bar{\mathrm{X}}^t(n)$ denotes the OFDM symbols transmitted from the ith antenna corresponding to the nth duration time.

$$\bar{\mathrm{X}}^t(n)=\left[\bar{X}^t(1),\bar{X}^t(2),\cdots,\bar{X}^t(K)\right]^{\bar{T}} \quad (1)$$

Before these symbols are transmitted, this vector is first modulated by the inverse complex Fourier transform (IDFT), and a cyclic prefix is appended at the head of each OFDM symbol. After removing the cyclic prefix at the rth receive antenna, we obtain the $K\times 1$ vector $\mathrm{y}^r(n)$, which can be written as

$$\mathrm{y}^r(n)=\sqrt{\frac{\rho}{N_T}}\sum_{t=1}^{N_T}H^{r,t}F^H\bar{\mathrm{X}}^t(n)+\eta^t(n) \quad (2)$$

where

$$\mathrm{y}^r(n)=\left[y^r(1),y^r(2),\cdots,y^r(K)\right]^{\bar{T}} \quad (3)$$

$H^{r,t}$ is a circulant matrix [4,5] with first column given by $\left[h^{r,t^{\bar{T}}},0_{1\times(K-L)}\right]^{\bar{T}}$ in slow fading environments. The $L\times 1$ vector $h^{r,t}$ denotes the impulse response of a channel with L paths between the tth transmit antenna and the rth receive antenna, and ρ is the power allocated to the subcarriers. L is chosen to exceed the maximum delay spread. F denotes $K\times K$ the unitary complex Fourier transform (DFT) matrix. $\eta^r(n)$ is vector of the noise. It is easy to show

$$H^{r,t}=F^H diag\left\{\sqrt{K}F\left[h^{r,t^{\bar{T}}},0_{1\times(K-L)}\right]^{\bar{T}}F\right\}$$

Taking the DFT of $\mathrm{y}^r(n)$, we obtain

$$\mathrm{Y}^r(n)=\sqrt{\frac{\rho}{N_t}}\sum_{t=1}^{N_t}diag\left\{\sqrt{K}F\left[h^{r,t^{\bar{T}}},0_{1\times(K-L)}\right]^{\bar{T}}\right\}\bar{\mathrm{X}}^t(n)+\Xi^r(n) \quad (4)$$

where $\Xi^r(n)=F\eta^r(n)$. For fast fading environments, the channel matrix model defined in (4) is no longer valid.

$$\mathrm{Y}^r(n)=\sqrt{\frac{\rho}{N_t}}\sum_{t=1}^{N_T}\bar{\mathrm{X}}^t(n)\tilde{F}h^{r,t}(n)+\Xi^r(n) \quad (5)$$

where $\tilde{F}$ is $\sqrt{K}$ times the first L columns of F.

Due to the rapid time variation of the channel, the $KL(Q+1)$ channel matrix $H^{r,t}$ cannot be considered as a circulant matrix. Where Q represents the basis expansion order. Therefore, the channel matrix $H^{r,t}$ cannot be directly applied to fast fading channels without paying a performance penalty.

III. CS-BASED CHANNEL COMPENSATION ESTIMATION

A. Channel Model

For the *n*th symbol, the *t*th transmit antenna sends pilots in $M^t(n)$ subcarriers and the remaining $K-M^t(n)$ subcarriers are used for data transmission. The set of pilots chosen for different transmit antennas need not be the same or disjoint. The *i*th antenna transmit demodulated baseband data matrix is $\mathrm{X}^t(n)$ .The transmit symbols matrix can be denoted as

$$\bar{\mathrm{X}}^t(n) = \mathrm{X}^t(n)\mathrm{G}^t(n) \tag{6}$$

where the $K\times 1$ vector $G^t(n)$ frequency pulse waveform is

$$\mathrm{G}^t(n) = \left[G^t(1), G^t(2), \cdots, G^t(K)\right]^{\bar{T}} \tag{7}$$

Then, the received signal of the rth receive antenna in the nth observation symbol can be expressed as

$$\mathrm{Y}^r(n) = \sqrt{\frac{\rho}{N_t}}\sum_{t=1}^{N_T} diag\left\{\mathrm{X}^t(n)\right\}\mathrm{G}^t(n)\tilde{F}h^{r,t}(n) + \Xi^r(n) \tag{8}$$

In many cases such as the nonline-of-sight scenario, the mean angle of departure and angle of arrival of the clusters of multipath components tend to be uniformly distributed over all subcarriers. Thus, when the number of clusters is relatively large, the channel estimation technique [3]-[5] may hardly improve a considerable performance in estimating fast fading channels, because nearly all the elements of spatial domain channel matrices may not approach zero. To consider wider applications, we will focus on techniques in which the sparse channel coefficients with mathematical model.

One of the most salient characteristics of multipath wireless channels is signal propagation over multiple spatially distributed paths. The channels between different transmit-receive antenna pairs are time frequency sparse. The time domain channel vector for the rth receive antenna is

$$h^r(n) = \left[h^{r,1}(n), h^{r,2}(n), \ldots, h^{r,N_T}(n)\right]^{\bar{T}} \tag{9}$$

where the multipath channels $h^{r,t}(n)$ can be represented by

$$h^{r,t}(n) = \sum_{l=0}^{L-1} h^{r,t}(n,l)\delta(\tau-\tau_l) \tag{10}$$

where $h^{r,t}(n,l)$ and τ_l represent the gain, the delay of the path l , respectively, and $\delta(t)$ is the Dirac delta function. L is impulse responses of equal maximum resolvable paths. The path l channel $h^{r,t}(n,l)$ is represented by as

$$h^{r,t}(n,l) = \sum_{q=0}^{Q} h^{r,t}(n,l,q)e^{\frac{j2\pi(q-Q/2)k}{WK}} \tag{11}$$

where W is a positive integer, and the variation of W is associated with the maximum Doppler frequency. Q can be set to $2\left[f_{\max}KT_s\right]$, representing the basis expansion order, where $f_{\max}$ is the maximum Doppler frequency. The carrier frequency offset matrix is

$$\varPsi_q^{r,t} = \left[e^{\frac{-j2\pi(q-Q/2)}{WK}}, \cdots, e^{\frac{-j2\pi(q-Q/2)(K-1)}{WK}}\right] \tag{12}$$

The multipath channels $h^{r,t}(n)$ is the nth symbol complex impulse response of the all path, and can is be written in matrix form as

$$h^{r,t}(n) = \Psi^{r,t}\beta^{r,t}(n) \tag{13}$$

where the carrier frequency offset matrix is

$$\Psi^{r,t} = \begin{bmatrix} \varPsi_{0,0}^{r,t} & \varPsi_{0,1}^{r,t} & \cdots & \varPsi_{0,Q}^{r,t} \\ \varPsi_{1,0}^{r,t} & \varPsi_{1,1}^{r,t} & & \varPsi_{1,Q}^{r,t} \\ \cdots & & \cdots & \\ \varPsi_{L,0}^{r,t} & \varPsi_{L,1}^{r,t} & & \varPsi_{L,Q}^{r,t} \end{bmatrix}^{\bar{T}} \tag{14}$$

The sparse coefficients vector of complex impulse response is

$$\beta^{r,t}(n) = \left[h^{r,t}(n,\tau_0), h^{r,t}(n,\tau_1), \cdots, h^{r,t}(n,\tau_L)\right]^{\bar{T}} \tag{15}$$

Due to the unknown of $\beta^{r,t}(n)$, we use the complex basis including all time frequencies to sparsely represent the received channel in (15). Equation (15) means that the received signals are sparse in the time and frequency domain. For estimation of such channels, the performance can be improved through exploitation of sparsity. Exited MIMO-OFDM channel estimation techniques treat channels as spatial rich multipath. However, in many situations, MIMO-OFDM channels may tend to be time frequency sparsity due to limited scattering. Using the time frequency expansion and a measurement matrix, the sampling scheme based on CS can be designed to reduce the sampling rate in theory.

B. Subsampled at the Pilot Position

The following is presented to design the measurement matrix with finite bases. The sparse coefficients vector of channel responses are estimated at the pilot locations.

Defining $\mathrm{X}_{diag}^t = diag\left\{X^t(1), X^t(2), \cdots, X^t(K)\right\}$, the set of receive vectors at the pilot positions using the known pilot matrix are chosen by

$$\mathrm{Y}_p^r(n) = \sqrt{\frac{\rho}{N_T}}\sum_{t=1}^{N_T} S^t(n)\mathrm{X}_{diag}^t S^{t^{\bar{T}}}(n)S^t(n)\mathrm{G}^t(n)\tilde{F}h^{r,t}(n) + \Xi_p^r(n) \tag{16}$$

where $S^t(n)$ is the $M^t(n)\times K$ pilot selection matrix that chooses the $M^t(n)$ rows of the X^t_{diag} matrix according to the *k*th subcarriers pilots chosen in the *n*th symbol, $M^t(n) << N_t KL(Q+1)$. It can be noticed that the training stage entails a total of $M^t(n)$ pilots to estimate $N_t KL(Q+1)$ channels. For traditional channel estimation schemes, the spacing of the pilot sequence has to satisfy the Nyquist sampling theorem to properly sample the fast fading channels. It is necessary to establish a theoretical limit on the number of pilots required for perfect channel recovery in a noise system.

According to the theory of compressed sensing, a sparse signal can be exactly reconstructed from its measurement matrix. Based on the CS theory, (16) can be sampled by a measurement matrix. The measurement matrix satisfies the so-called restricted isometry property (RIP). According to the RIP theorems, the goal then is to design the training matrix using minimum number of pilot vectors and process the received signal matrix $Y(n)$ to obtain an estimate $\hat{h}^r(n)$ that is close to $h^r(n)$ in terms of the mean squared error (MSE). The measurement matrices satisfy the RIP [8]. The *t*th transmit antenna selection matrix

$$S^t(n) = \left[S^t(n,1), S^t(n,2), \ldots, S^t(n, M^t(n)) \right]^{\bar{T}} \qquad (17)$$

The selection matrix selects randomly the pilot symbol locations. The *n*th column $S^t(n)$ is constructed by selecting randomly $M^t(n)$ subcarriers of K subcarriers in the *n*th symbol. The $M^t(n)$ subcarriers are used for pilot transmission while the rest $K - M^t(n)$ ones are data and virtual carriers. The entries of $S^t(n)$ which determine the location of the $M^t(n)$ pilot subcarriers are prescribed by a random sequence $\{x_p\}$ that is generated at the transmitter for probing a channel. At the transmitter, the random sequence $\{x_p\}$ is generally used to probe a channel. The numbers of pilot are

$$M = \sum_{t=1}^{N_T} \sum_{k=1}^{K} M^t(n,k) \qquad (18)$$

$M \ge C\xi \log(N_t N_r KL(Q+1)/\xi)$ satisfies RIP of order ξ and parameter δ_s with probability at least $1-e^{-CM}$, where, $C \ge 1$ is the oversampling factor. We have different observations for different transmit-receive antenna pairs. Further the length $M^t(n)$ of the observation is not same for all antennas.

C. Estimation and Compensation

Using the statistical characteristic and the dominate scatter, a compensation estimation detector is proposed in this section. The feature used to detect the channel is firstly presented in the following. After the CS receiver, the maximal element is assumed to be the dominate scatter, which is corresponding to the basis. At the receiver, the dominate scatter is depicted as

$$\mathrm{Y}^r_p(n) = \sqrt{\frac{\rho}{N_t}} \sum_{t=1}^{N_T} S^t(n) \mathrm{X}^t_{diag} S^{t^{\bar{T}}}(n) S^t(n) \mathrm{G}^t(n) \tilde{F} \times \Psi^{r,t} \beta^{r,t}(n) + \Xi^r_p(n) \qquad (19)$$

In order to avoid the interference of the wireless system and improve channels sparsity, an effective method is to design appropriate pulse waveform $G^t(n)$. The channels function is limited by angular frequency and time domain. The time pulse wave functions can be the most of concentration in a given time interval $\left(-T_s/2,\ T_s/2\right)$ and maximum normalized Doppler bandwidth $v_{D\max}$, where $v_{D\max} = f_{\max} T_s$. The time pulse wave functions meet the product differential equation

$$g^t(t) = \int_{-\frac{T_s}{2}}^{\frac{T_s}{2}} \gamma^t \frac{\sin(2\pi(t-\alpha)v_{D\max})}{(\pi(t-\alpha))} d\alpha \qquad (20)$$

where γ^t is the eigenvalues of pulse wave functions. Based on the definition of equation (20), $g^t(t)$ can be shown as follows:

$$g^t(t) = \gamma^t \frac{\sin\left[\frac{v_{D\max}}{2} \sqrt{\left(\frac{2t}{T_s}\right)^2 - 1} \right]}{\sinh(\frac{v_{D\max}}{2}) \sqrt{\left(\frac{2t}{T_s}\right)^2 - 1}} \qquad (21)$$

where

$$\sinh(\frac{v_{D\max}}{2}) = \frac{1}{2}\left(e^{\frac{v_{D\max}}{2}} - e^{-\frac{v_{D\max}}{2}} \right) \qquad (22)$$

The time pulse wave functions can be expressed as

$$g^t(t) = \gamma^t u(t) \qquad (23)$$

where

$$u(t) = \frac{\sin\left[\frac{v_{D\max}}{2} \sqrt{\left(\frac{2t}{T}\right)^2 - 1} \right]}{\sinh(\frac{v_{D\max}}{2}) \sqrt{\left(\frac{2t}{T}\right)^2 - 1}} \qquad (24)$$

The frequency pulse wave functions can be written as

$$\mathrm{G}^t(n) = \gamma^t \mathrm{U}(n) \qquad (25)$$

where $U(n)$ is the Fourier transform of $\mathrm{U}(n) = Fu(t)$. It can be observed that the energy of time concentrates along (20). Doppler lines $v_{D\max}$ shares the same frequency as the dominate scatter. Equation (25) shows that channels function is limited by angular frequency and time domain.

According to the time domain waveform and amplitude spectrum, the set of receive vectors are

$$\mathrm{Y}_p^r(n)=\sqrt{\frac{\rho}{N_t}}\sum_{t=1}^{N_t}S^t(n)\mathrm{X}_{diag}^t S^{t^T}(n)S^t(n)\gamma^t\mathrm{U}(n)\tilde{F}\Psi^{r,t}\beta^{r,t}(n)$$
$$+\Xi_p^r(n) \tag{26}$$

Equation (26) can be expressed in matrix form as

$$\mathrm{Y}_p^r(n)=\Phi^t(n)\Psi^r(n)\bar{\beta}^r(n)+\Xi_p^r(n) \tag{27}$$

where

$$\mathrm{Y}_p^r(n)=\left[Y_p^r(n,0),\cdots,Y_p^r(n,M)\right]^{\bar{T}} \tag{28}$$

$$\Phi(n)=\sqrt{\frac{\rho}{N_t}}\left[S^1(n)\mathrm{X}_{diag}^1 S^{1^T}(n),\cdots,S^{N_t}(n)\mathrm{X}_{diag}^{N_t}S^{N_t^T}(n)\right]^{\bar{T}} \tag{29}$$

$$\Psi^r(n)=diag(S^1(n)\mathrm{U}(n)\tilde{F}\Psi^{r,1},\cdots,S^{N_T}(n)\mathrm{U}(n)\tilde{F}\Psi^{r,N_T}) \tag{30}$$

$$\bar{\beta}^r(n)=\left[\gamma^t\beta^{r,1}(n),\gamma^t\beta^{r,2}(n),\cdots,\gamma^t\beta^{r,N_t}(n)\right]^{\mathrm{T}} \tag{31}$$

Conventional methods assume that the channels don't include time and frequency constraints. The following optimization problem is used to channel reconstruction [13]-[17]:

$$\bar{\beta}^r(n)=\underset{\bar{\beta}^r(n)}{\arg\min}\left\{\left\|\mathrm{Y}_p^r(n)-\Phi^t(n)\Psi^r(n)\bar{\beta}^r(n)\right\|_2^2+\lambda\left\|\bar{\beta}^r(n)\right\|_1\right\} \tag{32}$$

where λ is the regularization parameter. In consideration of the time and frequency sparsity, we also need to reconstruction the eigenvalues of appropriate pulse waveform. Then, the sparse channels are estimated by using appropriate pulse wave functions. We pose the problem of joint channels estimation and eigenvalues reconstruction as the minimum cost solution of the following cost function:

$$J\left(\beta^{r,1}(n),\gamma^t\right)=\left\{\left\|\mathrm{Y}_p^r(n)-\Phi^t(n)\Psi^r(n)\bar{\beta}^r(n)\right\|_2^2+\lambda\left\|\bar{\beta}^r(n)\right\|_1\right\} \tag{33}$$

The channels estimation and eigenvalues reconstruction can be obtained as

$$\left[\beta^r(n),\gamma^t\right]=\underset{\beta^r(n),\gamma^t}{\arg\min}\,J\left(\beta^r(n),\gamma^t\right) \tag{34}$$

By solving (34), both the channels estimation and eigenvalues reconstruction can be jointly obtained. The remaining problem is how to solve (34). Using a similar joint method proposed in [17], (34) can be solved by an iterative algorithm with channels estimation, eigenvalues reconstruction and compensation of pulse wave functions. The cost function is minimized with respect to the sparse coefficients estimation. The eigenvalues are reconstructed by the channels estimation. The pulse wave functions are updated using the reconstructed eigenvalues. The sensing matrix of is updated by the pulse wave functions. The sparsity and accuracy of the channel estimation are compensated to improve by the new sensing matrix in next iterative process. The algorithm flow is outlined as follows:

Firstly, the problem of estimating the sparse complex basis expansion channel $h^r(n)$ coefficients is transformed into the problem of estimating $\beta^r(n)$.

$$\left(\beta^r(n)\right)^{i+1}=\underset{\beta^r(n)}{\arg\min}\,J\left(\beta^r(n),\left(\gamma^t\right)^i\right) \tag{35}$$

$\left(\beta^r(n)\right)^{i+1}$ is the sparse coefficients of the complex time frequency basis parameter with pulse waveform. Furthermore, when some spatial domain bins contain few physical paths due to limited scattering, the corresponding channel coefficients should approach zero. Equation (35) is actually composed of complex pulses, which means that the received signals in (35) is sparse in the time frequency domain. The cost function is minimized with respect to the sparse coefficients estimation.

Secondly, the eigenvalues are reconstructed by the channels estimation. Reconstruction of eigenvalues for pulse wave functions is:

$$\left(\gamma^t\right)^{i+1}=\underset{\gamma^t}{\arg\min}\,J\left(\left(\beta^r(n)\right)^{i+1},\gamma^t\right) \tag{36}$$

Thirdly, the pulse wave functions are updated using the reconstructed eigenvalues $\left(\gamma^t\right)^{i+1}$. The pulse wave functions $v_{D\max}$, $\mathrm{U}(n)$ are updated by using $\left(\gamma^t\right)^{i+1}$.

Fourthly, the sensing matrix $\mathrm{G}^t(n)$ of is updated by the pulse wave functions $\mathrm{U}(n)$.

Finally, the sparsity and accuracy of the channel estimation are compensated to improve by the new sensing matrix in next iterative process. The compensation sparse complex basis expansion represents band and time limited sequences with the pulse wave functions. Let $i=i+1$ and return to estimation of sparse coefficients. Terminate when

$$\frac{\left\|\left(\beta^r(n)\right)^{i+1}-\left(\beta^r(n)\right)^i\right\|_2^2}{\left\|\left(\beta^r(n)\right)^i\right\|_2^2} \tag{37}$$

Equation (37) is less than a preset threshold. This allows us to choose a suitable threshold for ignoring the small-valued channel taps and retaining only the most significant taps to reduce the effect of noise in the estimation. The energy mainly concentrates in the spatial time frequency, thereby improving the performance of the channel estimation technique. It can be observed that the energy of $\left(\beta^r(n)\right)^{i+1}$ spreads over the entire ambiguity domain, so the energy in an adjacency of the dominant time frequency, corresponding to the matched signal. When the frequency mask changes, pulse waveform can be flexibly adjusted and corrected. The pulse signal which is band and time concentrated distribution, is conducive to optimization design of sparse MIMO-OFDM channel. The proposed method exploits the intrinsic relationship between the sparse channels and the dynamic mathematical model. The proposed method works by jointly estimating the sparse coefficients vector and by reconstructing the dynamic mathematical model. Then, the dynamic mathematical model reconstruction is used to update the sensing matrix, and the algorithm passes to the next iteration. The mathematical model is exploited to track rapidly MIMO-OFDM channels by avoiding depending on the coherence bandwidth and time of channels.

D. Performance Evaluation

We define dominant non-zero channel coefficients $\beta^r(n)$'s as those which contribute significant channel power, that is, the coefficients for which $\sum E\left|h^{r,t}(n,\tau_1)\right|^2 > \varepsilon$ for some prescribed threshold $\varepsilon > 0$. Thus, the channels are sparse

$$\sum_{t=1}^{N_t}\left\|E\left[\left|\beta^r(n)\right|^2\right]\right\|_{\ell_0} << N_t KL(Q+1) \qquad (38)$$

It is easily seen that the fast fading channels encountered in practice are sparse MIMO-OFDM channels with most of the multipath energy localized to relatively small regions. The channels impulse responses are dominated by a relatively small number of dominant resolvable paths within the sparse complex basis expansion. Thus, the goal is to reconstruct the sparse channel coefficients $\beta^r(n)$ from few pilot measurements. Using the sparse complex basis expansion and a measurement matrix, the sampling scheme based on CS can be designed to reduce the sampling rate in theory. When traditional channel estimation methods are used for the doubly selective MIMO OFDM channels, the $M \times N_t KL(Q+1)$ matrix will be designed to have full column rank $N_t KL(Q+1)$, which requires $M \ge N_t KL(Q+1)$, where $N_t KL(Q+1)$ is the number of path.

The main computational cost is from of sparse coefficients estimation and eigenvalues reconstruction. In each iteration, the first step reconstruct the targets, whose complexity is order $O(MN_tK)$. The computational complexity of eigenvalues reconstruction is order $O(N_tK)$.

IV. SIMULATIONS

In order to demonstrate the performance of the proposed channel estimation, the simulations were presented in this section. The simulation parameters are chosen to be depictive of a communication system with carrier frequency to be 2.3GHz, symbol duration and guard interval to be 102.4 μs and 12.8 μs , subcarriers to be1024, and FFT size to be 1024. The performance of the MIMO-OFDM system is measured in terms of the MSE of the channel estimate, and the bit error rate (BER) versus SNR.

Fig. 2 presents the correct channel impulse response recovery probability when different numbers of pilots $[M^t(n)/(N_tK)]\%$ are used under the fast fading channel with the fixed SNR of 60 dB in a 4×4 MIMO system. Here, the correct recovery is defined as the estimation Mean Square Error (MSE) is lower than 10^{-2}. It can be seen from Fig. 2 that by utilizing the obtained intrinsic relationship between the sparse channels and the dynamic mathematical model, the required number of pilots in the proposed method is less than that in the MIMO TFT-OFDM [17] algorithm and far less than that in SOMP algorithm.

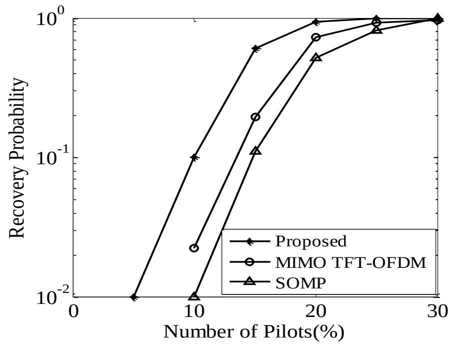

Fig. 2. Comparison of the recovery probabilities at the SNR = 60 dB.

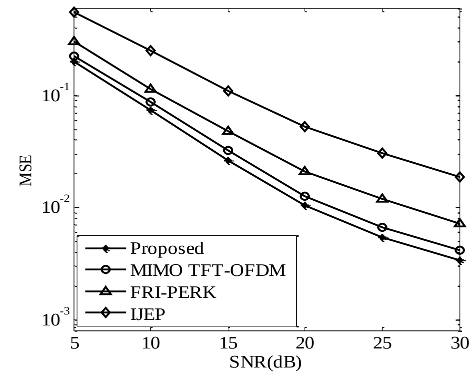

Fig. 3. MSE against SNR

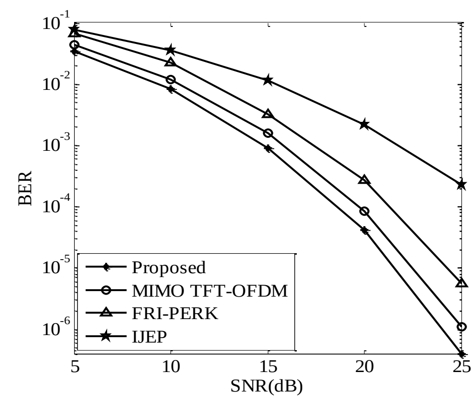

Fig. 4. BER against SNR.

Fig. 3 shows MSE of channel estimation error vs. Signal-to-Noise Ratio (SNR). The BER of each data stream is shown in Fig. 4. Fig. 3 and Fig. 4 illustrate the MSE and BER performance of different schemes under the fast fading channel with the mobile speed of 200 km/h in a 6×6 MIMO system, respectively. The joint estimation methods of IJEP [5], FRI-PERK [13] and MIMO TFT-OFDM are also evaluated for comparison. It could be seen that for the mobile channel, the proposed method yields better performance compared with IJEP, FRI-PERK and MIMO TFT-OFDM. Since the spacing of the pilot sequence doesn't satisfy the Nyquist sampling theorem to properly sample the fast fading channels. The joint channel estimation schemes of IJEP, FRI-PERK and MIMO TFT-OFDM rely on a pseudorandom time-domain preamble, which is identical for all transmit antennas. However, the required number of pilots depend on the coherence bandwidth of channels and time of transmit antennas. The overhead of the required pilots or preamble will significantly increase as the number of transmit antennas and the blocks of transmitted OFDM symbols becomes large, which decreases the spectral efficiency. The proposed method exploits the intrinsic relationship between the sparse channels and the dynamic mathematical model. The proposed method works by jointly estimating the sparse coefficients vector and by reconstructing the dynamic mathematical model. When the frequency mask changes, pulse waveform can be flexibly adjusted and corrected. The pulse signal which is band and time concentrated distribution, is conducive to optimization design of sparse MIMO-OFDM channel. The mathematical model is exploited to track rapidly MIMO-OFDM channels by avoiding depending on the coherence bandwidth and time of channels.

V. Conclusions

We proposed the CS-based channel compensation estimation for MIMO-OFDM with fast fading channels. The proposed estimation method has much better performance than traditional channel estimation method. We investigate a channel estimator that exploit channel sparsity in the time and Doppler domain, where the channel is described by a limited number of paths, each characterized by a delay, Doppler scale, and attenuation factor, and derive the exact inter-carrier-interference pattern. The algorithm works with channel sparsity by jointly estimating the sparse coefficients vector and by reconstructing the mathematical model. The proposed method exploits the intrinsic relationship between the sparse channels and the mathematical model.

Acknowledgment

This work was supported by the National Natural Science Foundation of China (61132004, 6110120), the key disciplines of Shanghai Normal University (DZL126), the Shanghai Municipal Education Commission funding scheme for training young teachers of Colleges and universities in 2012, and the general project of Shanghai Normal University (SK20131, A-7001-15-001005).

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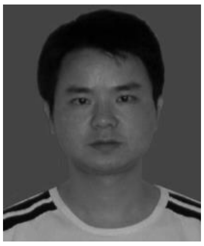

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