An Optimized Interference Alignment Algorithm Based on Max-SINR Criterion for MIMO System

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Abstract — Interference alignment (IA) is one of the promising technologies to solve the co-channel interference in wireless communication systems. It can effectively improve system capacity and reliability. Based on consideration of engineering applications, the realization of interference alignment for three-cell MIMO cooperative communication system is studied and then a precoding matrices group selection method based on the maximum of total system signal interference noise ratio (Max-SINR) criterion was proposed in this paper. Firstly, all the transformation matrices in cooperative communication system are calculated, and the precoding basis is selected according to the Max-SINR criterion. Secondly, the precoding matrices and interference suppression matrices are designed and optimized based on the singular value decomposition of channel and the maximum chord distance of matrix. Simulation results show that, the proposed optimized algorithm outperforms other interference alignment algorithms in terms of throughput and the energy efficiency. Further, the optimized algorithm also has advantage in spatial-correlated channel and has better engineering applicability than other algorithms.

Index Terms—Green communications, multiple-input multiple-output, interference alignment, precoding, massive multiple-input multiple-output

I. INTRODUCTION

Multiple-input multiple-output (MIMO) and Massive MIMO technologies [1]-[2] are considered as one of the key promising physical layer technologies for the next mobile communications, as they provide excellent spectrum utilization and system throughput. In order to meet the needs of future high-speed mobile communications, the LTE-A by 3GPP proposed a lot of new key technologies such as the enhanced multi-antenna and carrier aggregation to significantly improve the system performance. While the users in the region of the cell-edge still suffer seriously the inter-cell interference (ICI) or other cell interference (OCI) especially in the case of multi-cell network in MIMO systems with the same frequency, which seriously weakened the high spectral efficiency advantages of MIMO technology. Therefore, how to effectively suppress inter-cell interference and improve the throughput of edge users in the interference channel has been a research focus in the multi-cell MIMO system [1]-[6].

While achievable rate regions are known (see [7], [8]), characterizing the capacity region of the interference channel has remained a longstanding open problem. Recently, interference alignment (IA) technology [9]-[11] has brought a new approach to solve the above problem. It can provide better spatial multiplex gain and excellent system throughput than other interference management methods. Currently, interference alignment algorithms for multi-cell MIMO system can be divided into two types: classical algorithms based on closed-form solution [12]-[15] and typical distributed iterative algorithms [16]-[19]. All the Interference alignment algorithms aim to coordinate transmitting directions such that all received interference is confined to a predefined receiver subspace, and has been shown recently to achieve significant gains in terms of the achievable degrees of freedom (DOF).

It was shown in [12] that for a K-user interference channel with single-antenna terminals, K/2 degrees of freedom are achievable from interference alignment, which is also called the classic interference alignment algorithm. Then, an optimized precoding scheme based on eigen-channel in interference alignment for coordinated multi-point transmission systems is proposed in [13] based on [12]. Further, a novel optimized cooperative interference alignment scheme is elaborated in [9], which not only choose a group of optimized eigen-channels to transport signals, but also select a group of precoding vectors to leave the desired signal away from interference. In order to avoid the flaw of classic algorithms with relying on global ideal channel state information, K. S. Gomadam et al. [16] firstly proposed a distributed iterative interference alignment algorithm based on the channel reciprocity. Then, a joint signal and interference iterative alignment algorithm is presented for transmission over MIMO interference channel [17], which iteratively reduces both the interference power that “leaks” into the signal subspace, and the signal power that “leaks” into the interference subspace. Further, the assumption of channel reciprocity is not required for the algorithm and has better engineering applicability than other iterative algorithms.

Although the above optimized algorithms improve cooperative communication system performance...
significantly, each of the interference alignment scheme optimize its precoding matrixes based on the precoding matrix of the first base station, which dosed not analyze the impact on the system performance by preferring precoding matrix of other base stations. A precoding matrix group selection algorithm based on the maximum of total system signal interference noise ratio (Max-SINR) was proposed base on [14] in this paper.

Firstly, all the transformation matrices in cooperative communication system are calculated and the optimal precoding basis is selected according to the Max-SINR criterion. Secondly, the precoding matrices and interference mitigation matrices are designed and optimized based on the singular value decomposition of channel and the maximum chord distance of matrix. Through numerical simulations, it is proven that, compared to other known interference alignment schemes, the system throughput by the optimized algorithm are obviously improved. Further, the optimized algorithm also has advantage in spatial-correlated channel and has better engineering applicability than other algorithms.

The remainder of the paper is organized as follows. Section II introduces a general system model for the K-user (\(K=3\)) interference channel that we consider and lays down the notation that we will use. Section III reviews the classical interference alignment algorithm, and in Section IV, we present a two-stage optimization of the precoding and decoding matrices for IA method based on the maximum of total system signal interference noise ratio (Max-SINR) criterion. Section V compares the performance of our algorithm to the prior literature through simulations. Finally, the paper is terminated with conclusions in Section VI.

II. SYSTEM MODEL

The three-cell MIMO system of this paper can be modeled as a three-user interference channel model, as shown in Fig. 1. This model consists of six nodes, three of which are designated as transmitters while the other three are receivers. Each transmitter is paired with a single receiver in a 1-1 mapping. Finally, each transmitter interferes with all the receivers it is not paired with.

\[
Y_k = \sum_{j=1}^{3} H_{kj} W_j S_j + Z_k
\]

where the first term \(H_{kj} W_j S_j\) is the desired signal sent by the \(k\)-th transmitter, the second term \(\sum_{j\neq k} H_{kj} W_j S_j\) denotes the interference from other transmitters and the last one \(Z_k\) denotes the receiver thermal noise, modeled as i.i.d. zero mean, unit variance complex Gaussian random variables. In (1), \(S_j, j=1,2,3\) is \(d_j \times 1\) j-th user’s signal vector and \(d_k\) is the number of independent data streams between the \(k\)-th transmitter and receiver; \(W_j\) denotes the precoding matrix used by transmitter and \(H_{kj}\) is the channel matrix from the transmitter \(j\)-th to the receiver \(k\)-th.

At the receiver side, denoting the interference alignment suppression matrix of the \(k\)-th receiver as \(F_k\) for single-user detection which means that the interference generated by other transmitters is treated as additive noise. Thus, the decoder output vector of user \(k\) can be written as:

\[
\hat{S}_k = F_k H_{kk} W_k S_k + F_k \sum_{j=1, j\neq k} H_{kj} W_j S_j + F_k Z_k
\]

The goal of interference alignment is to choose precoder matrices \(W_j\) such that each receiver can decode its own signal by forcing interfering users to share a reduced-dimensional subspace of the user’s receive space. Thus, the interference alignment technology can reduce interferences from multiple users into single-user interference and the system can obtain the optimal degree of freedom. Design of the precoding matrix is the key to the IA algorithms. The interference from other users will be aligned to the same subspace at the receiver side by collaborative design of all precoding matrices \(\{W_j\}_{j=1}^{K}\) at transmission side for the interference alignment method. Then, the receiver can obtain the desired signal by the IA suppression matrix.

The following notations are used throughout the paper. We employ uppercase boldface letters for matrices and lowercase boldface for vectors. For any general matrix \(X\), \(X^T\) and \(X^{-1}\) denote the conjugate transpose and inverse matrix, respectively. \(tr(X)\) indicates the trace and the Frobenius norm of a matrix \(X\) and \(||X||_F = tr(XX^H)\).

III. CLASSICAL INTERFERENCE ALIGNMENT ALGORITHM

In this section, we review the IA algorithm presented in [12]. As a linear precoding method, this algorithm
enables to achieve the theoretical bound on the DOF for K-user interference channel systems with the conventional ZF suppression filter at each receiver. For the brief review, we illustrate this method for the case of $K=3$ and $M=N=2$, as shown in Fig. 1, and more general case can be found in [20] and [21].

The IA realization for each user in cooperative MIMO communication system means that the interference from other users will be aligned to the same subspace (direction of the red arrow in Fig.1). Thus, each precoding matrix has to be designed to satisfy the three interference aligning constraints described as:

\[
\text{span}(H_{12}W_2) = \text{span}(H_{13}W_3)
\]

\[
\text{span}(H_{21}W_1) = \text{span}(H_{23}W_3)
\]

\[
\text{span}(H_{31}W_1) = \text{span}(H_{32}W_2)
\]

where $\text{span}(A)$ indicates the vector space spanned by the column vectors of $A$.

The IA algorithm obtains the precoding matrices restricting the above constraints as:

\[
\text{span}(H_{12}W_2) = \text{span}(H_{13}W_3)
\]

\[
H_{21}W_1 = H_{23}W_3
\]

\[
H_{31}W_1 = H_{32}W_2
\]

The above equations, multiplying $H_{21}$ at the right side respectively, can be equivalently expressed as:

\[
\text{span}(W_1) = \text{span}( EW_1)
\]

\[
W_2 = H_{23}^{-1}H_{31}W_1
\]

\[
W_3 = H_{21}^{-1}H_{32}W_1
\]

where $E = H_{31}^{-1}H_{23}^{-1}H_{32}H_{21}^{-1}$.

Finally, we can obtained $W_i$, which can be expressed as:

\[
W_i = [e_1, e_2, ..., e_{M/2}] = v_{M/2}(E)
\]

where $e_1, e_2, ..., e_M$ are the eigenvectors of $E$. Similarly, we obtain $W_2$ and $W_3$ from (5) and (6) for the 3-user cooperative MIMO communication system. Then, the desired signal can be obtained by conventional ZF filter at each receiver.

IV. THE OPTIMIZED INTERFERENCE ALIGNMENT ALGORITHM BASED ON MAX-SINR CRITERION

In this section, we present the method of identifying the precoding and decoding matrices. We first introduce the precoding basis determination methods using the maximum of total system signal interference noise ratio (Max-SINR) criterion in [22]. Next, we optimize the precoding matrices and interference suppression matrices according to the singular value decomposition of channel and the maximum chord distance of matrix criterion.

A. Precoding Basis Determination Methods based on the Max-SINR Criterion

As shown in [12], classic interference alignment algorithm focuses on achieving of the optimal degree of freedom, which does not use the local desired channel of user. And an optimized precoding scheme based on eigen-channel in interference alignment is proposed in [7]. However the method does not consider the relationship between the precoding matrices and right singular matrix of the interfering channels. Further, in order to maximized decrease the effect of the interference from unwanted transmitters, a novel optimized cooperative interference alignment scheme is elaborated in [8], which not only choose a group of optimized eigen-channels to transport signals, but also select a group of precoding matrices to leave the desired signal away from interference.

From (5), we can easily see that the precoding matrix of the first base station has a direct impact on the design of the other two precoding matrices, thus affecting the overall performance of the MIMO cooperative communication system. However, the Reference [13] and [14] do not analyze the impact on the system performance by preferring precoding matrix of other base stations. So, a precoding basis selection algorithm based on the maximum of total system signal interference noise ratio (Max-SINR) was proposed for three-cell MIMO cooperative communication system in this paper.

Assuming any base station can first obtain the precoding matrix, (5) can be rewritten as follows:

\[
W_m = H_{ml}^{-1}W_i
\]

\[
W_n = H_{nl}^{-1}W_i
\]

where $E_i = H_{ni}^{-1}H_{mn}^{-1}H_{mi}^{-1}H_{mi}^{-1}H_{mi}^{-1}H_{mi}^{-1}$. Similarly, we can obtained $W_i$, which can be expressed as:

\[
W_i = [e_1, e_2, ..., e_{M/2}] = v_{M/2}(E_i)
\]

where $e_1, e_2, ..., e_M$ are the eigenvectors of $E_i$. Then, we obtain $W_m$ and $W_n$ from (7) and (8).

The optimized precoding basis determination method based on the Max-SINR criteria [22] can be expressed as:

\[
(E_i, E_m, E_n) = \arg \max_{W_i \in \text{arg}(E_i)} \text{SINR}
\]

\[
\text{SINR} = \frac{1}{\sum_{k=1}^{M} P_k F_k H_k W_k W_k^H H_k^H F_k} F_k^H B_k F_k
\]

\[
W_m = H_{ml}^{-1}W_i
\]

\[
W_n = H_{nl}^{-1}W_i
\]
where
\[
B_k = \sum_{j=1, j\neq k}^3 P_j H_{ij} W_j W_j^H H_{ij}^H + I_{nk}
\]

\(P_j\) is the power of \(j\)-th transmitter. In general, is a design parameter, and its choice so that the sum-rate (or any other metric) is optimized is an open problem. In this paper, we don’t consider the optimization and assume that the power of each transmitter is the same.

\(F_k\) in (9) is interference suppression matrix for the \(j\)-th receiver, which can be optimized as described in [14]. Thus, each of the receiver selects its desired signal subspace which has the minimum square of the distance to the total space of the desired signal. Hence the optimized problem can be expressed as:

\[
\begin{align*}
F_1, F_2, F_3 &= \arg \min_{F_k} \sum_{k=1}^3 \| H_{ik} W_i - F_k F_k^H H_{ik} W_i \|^2_F \\
&= \arg \min_{F_k} \sum_{k=1}^3 \| H_{ik} W_i - F_k F_k^H H_{ik} W_i \|^2_F 
\end{align*}
\]

(10)

where \(\|A\|_F^2 = tr(A^H A)\), we may simplify (10) to obtain:

\[
\begin{align*}
F_1, F_2, F_3 &= \sum_{k=1}^3 tr \left( (H_{ik} W_i - F_k F_k^H H_{ik} W_i)(H_{ik} W_i - F_k F_k^H H_{ik} W_i)^H \right) \\
&= \arg \max_{F_k} \sum_{k=1}^3 tr(F_k H_{ik} W_i W_i^H H_{ik}^H F_k)
\end{align*}
\]

(11)

where \(tr(F_k H_{ik} W_i W_i^H H_{ik}^H F_k)\) is the power of the desired signal. From (11), we can see that the larger of the desired signal power and the better effect of interference suppression.

Hence the interference suppression matrix can be obtained:

\[
F_k = V_{\text{max}(M/2)}(H_{ik} W_i W_i^H H_{ik}^H)
\]

(12)

where \(V_{\text{max}(M/2)}(A)\) represents the \(M/2\) largest eigenvectors of matrix \(A\).

According to the (9) and (12), the maximum SINR can be obtained for MIMO cooperative communication system by the selecting of precoding basis based on the Max-SINR criterion. Thus the corresponding optimal precoding matrices can be achieved at the transmitter side.

\textbf{B. Optimized Interference Alignment Algorithm}

The precoding basis determination method based on the Max-SINR criterion described in last section doesn’t use the desired channel information. Thus the system sum-rate may be further improved. Base on the criterion in [15], the proposed algorithm further optimizes the precoding matrices which are close to the desired signal channel while away from the interference channel. Thereby the effect of the interference from unwanted transmitters decreases as much as possible.

Firstly, singular value decomposition (SVD) of the channel matrix \(H_{ij}\) is expressed as follows:

\[
H_{ij} = U^{(ij)} \Sigma^{(ij)} (V^{(ij)})^H
\]

\[
= [u_1^{(ij)}, \ldots, u_M^{(ij)}] \lambda_1^{(ij)} \ldots \lambda_M^{(ij)} [v_1^{(ij)}, \ldots, v_M^{(ij)}]
\]

(13)

where \(U^{(ij)}(U^{(ij)})^H = I\) and \(V^{(ij)}(V^{(ij)})^H = I\), the columns of \(U^{(ij)}\) are orthonormal eigenvectors of \(U^{(ij)}(U^{(ij)})^H\), the columns of \(V^{(ij)}\) are orthonormal eigenvectors of \(V^{(ij)}(V^{(ij)})^H\), and \(\Sigma\) is a diagonal matrix containing the square roots of eigenvalues from \(U^{(ij)}\) or \(V^{(ij)}\) (non-zero eigenvalues of \(U^{(ij)}\) and \(V^{(ij)}\) are always the same) in descending order.

Let the base station which has the optimal \(E_j\) is the first base station, the corresponding precoding matrix \(W_j\) can be optimized as follows:

\[
W_j = \arg \max_{W_j \in \Omega(E_j)} \sum_{m=1}^{M/2} \left\| \sum_{n=1}^{M/2} \lambda_m^{(11)} [v_m^{(11)}]^H W_{lm} \right\|^2_F - \omega \sum_{j=2}^{3} \sum_{n=1}^{M/2} \left\| \lambda_m^{(11)} [v_m^{(11)}]^H W_{lm} \right\|^2_F
\]

(14)

where \(w_{lm}\) is the \(m\)-th column vector of the precoding matrix \(W_j\), \(\omega\) is a non-negative weight parameter that is determined empirically based on the overall information such as the average SNR, number of users, etc.

The first term of (14) represents the sum chord distance between the precoding matrix and the desired channel matrix and the second term represents the sum chordal distance between the precoding matrix and the interference channel. In this case, the selected optimal precoding doesn't only have a sum of the minimum chordal distance (maximum inner vector product) to the desired channel, but also have a sum of the maximum chordal distance to the interfering channel. So the useful signal can be transported as close as possible to the desired channel and away from the interference channel.

From (14), we can easily see that only the first base station of the three-cell MIMO cooperative communication system optimizes the precoding matrix at
transmitter side. However, in the case of multi-cell cooperative communication, the optimal system performance can be achieved only when the precoding matrices of the base stations contained in the system are simultaneously optimized. Thus the optimization for the cooperative system can be expressed as follows:

\[
(W_1, W_2, W_3) = \arg \max_{W_{o-cij}(E_i)} \left\{ \sum_{i=1}^{3} \sum_{m=1}^{M/2} \lambda_m^{(i)} \left\| \left( V_{m}^{(i)} \right)^H W_{m} \right\| - \omega \right\}
\]

where

\[
W_i = v_{M/2}(E_i)
\]

\[
W_m = H_{m}^{-1} W_{m}^{e} H_{m}^{-1} W_{m}^{f}
\]

\[
W_k = H_{k}^{-1} W_{k}^{e} H_{k}^{-1} W_{k}^{f}
\]

The end.

V. SIMULATION AND ANALYSIS

In order to analyze the performance and applicability of the proposed algorithm, this section presents the comparative analysis for the classic interference alignment algorithm (CJ-IA) [6], the interference alignment algorithm based on eigen-channel (SV-IA) [13], the cooperative interference alignment algorithm based on the chordal distance of matrix (CHD-IA) [14] and the optimized interference alignment algorithm based on max-SINR criterion in this paper (Max-SINR-IA).

Our simulation setup is as follows. Consider the 3 user interference channel where each node is equipped with 2 antennas. All the channels are generated according to the i.i.d. Rayleigh distribution, i.e., each entry of the channel matrices is generated independently from a complex Gaussian distribution with zero mean and unit variance. Then, we assume that each transmitter has the same transmission power constraint \( P_t \), i.e., \( P_t = P \) for \( i \in (1, 2, 3) \). Also, The weight factor \( \omega \) is a non-negative parameter that is determined empirically based on the overall information such as the average SNR, etc. and the weight vector corresponding to SNR values \( \text{SNR}=[5,10,15,20,25,30,35,40] \) is \([0.5,0.5, .2,0.2,0.1,0.05,0.05,0.01]\).

The performance of the algorithms is measured by the sum rate achieved over the interference channel by 3 users and the energy efficiency. Also, we compare the performances of the proposed algorithm with the other IA method by 1000 Monte Carlo simulation and the simulation results is shown in fig. 2 and 3, respectively.

First, Fig. 2 presents the comparison of sum rate with the SNR for different IA algorithms. As shown in Fig. 2, since the interference suppression matrix by the CJ-IA algorithm is a simple zero-forcing matrix which causes the loss of the desired signal power, the performance was significantly lower than the other schemes. Meanwhile, it is easily seen that the modified Max-SINR-IA scheme where the receive-combing vectors maximize the SINR provides a considerable performance gain than other IA methods, especially at low and intermediate SNR.
defined as the average transmitted number of bits per joule, i.e. $\eta_p = \frac{\log(1 + \text{SINR})}{E_b}$, where $E_b$ is the unit bit energy. From the plots, it is shown that the proposed methods achieve much better energy efficiency performance compared to other typical algorithms, such as CJ-IA method and SV-IA method. Especially in the case of low and intermediate SNR, the proposed Max-SINR-IA method outperforms the other typical algorithms prominently.

From the Fig. 2, Fig. 3 and Fig. 4, we confirm that the optimized IA algorithm based on Max-SINR criterion (Max-SINR-IA) increases the overall SINR at the receiver side and the distance between the desired signal space and the interference space for cooperative MIMO communication system. This increment leads to a sum rate improvement.

Further, it is revealed that the proposed IA scheme outperforms other known algorithms in terms of energy efficiency and has greater flexibility in spatial-correlated channel.

It showed be noted that the weight factor was chosen empirically in the simulations. As observed, the weight for larger SNR cases should be chosen smaller (intuitively, interference removal is more important than optimizing the direct signal at high SNRs).

VI. CONCLUSIONS

In this paper, we have proposed a non-iterative linear precoding and decoding method using a two-stage optimization process. Firstly, different from original algorithm, it takes consideration of all the transformation matrices in cooperative MIMO system and the precoding basis is selected according to the Max-SINR criterion at receiver side. Moreover, in order to make the desired signal space and the interference signal space roughly orthogonal to each other and transport signal by desired channel, we optimize the precoding matrices and interference suppression matrices based on the singular value decomposition of channel and the maximum chord distance of matrix. Through numerical simulations, we have illustrated that the proposed scheme significantly improves the sum rate and the energy efficiency compared with other known IA schemes. Also, it is the requirement of green communications which attracts much attention recently. Meanwhile, it is shown that the proposed algorithm has low implementation complexity for practical applications and has advantage in spatial-correlated channel. In the future work, we will study on the optimization of weight factor.

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