A PAPR Reduction Method Based on Differential Evolution

Min Wang[1] and Bin Xiao[2]
[1]School of Electrical Engineering and Information, Southwest Petroleum University
[2]School of Computer Science, Southwest Petroleum University, Chengdu 610500, China
Email: wangmin80616@163.com

Abstract —High peak-to-average ratio power (PAPR) is a critical practical problem for an OFDM system. Many techniques, such as clipping and filtering (ICF), cognitive clipping, have been proposed to deal with the issue. However, it needs to determine the relationship between error vector magnitude (EVM) and clipping ratio (CR). In this paper, we formulate a universal PAPR optimization model with EVM constraint, and propose a differential evolution (DE) algorithm with a time complexity of $O(N\log_2 N)$. The optimization parameter of the new approach is clipping noise. The EVM constraint guarantees proper receiver operation that is specified by most modern communication standards. The proposed DE algorithm employs the noise vector as the population. It adjusts three crucial control parameters to minimize cost function which is the amount of PAPR reduction. Simulation results show that our proposed method can offer good performance in PAPR and bit error rate (BER).

Index Terms—Peak-to-average power ratio, clipping and filtering, differential evolution, error vector magnitude, adaptive step size

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a very attractive multicarrier modulation technique [1]. It can offer high spectral efficiency, immunity to frequency selective fading and low inter-symbol interference. However, OFDM often suffer from high (peak-to-average ratio) PAPR which results in high scope power amplifier. On one hand, it decreases the signal-to-quantization noise ratio and leads to in-band distortion and out-of-band radiation. On the other hand, it degrades the efficiency of the power amplifier in the transmitter.

Many PAPR reduction methods have been proposed to address this issue. These include adaptive step size cognitive clipping [2], tone injection [3]-[5], iterative clipping and filtering (ICF) [6]-[9], coding [10]-[13], selected mapping [14]-[17] and partial transmit sequences [18]-[22]. Among these techniques, ICF technique is quite successful that requires no modification to the receiver structure. It is the simplest approach to obtain a reduction of the PAPR. However, the filtering operation causes in-band distortion and out-of-band spectral re-growth. The in-band distortion can degrade bit error performance and the frequency-domain filtering results in peak re-growth. The ICF method requires much iteration to approach a desired PAPR reduction that makes computational complexity increase exponentially.

Ma and Shao [2] proposed an adaptive step size cognitive clipping method. It reduces the number of iterations by using the dynamic error vector magnitude (EVM) step size. However, the major drawback is that it needs to determine the relationship between EVM and clipping ratio (CR) in advance. For different modulation type, it needs a lot of simulations to configure of the optimal CR look-up-table for different EVM threshold. Min et. al addressed a number of feature selection problems from the viewpoint of constraint satisfaction problems [23]. Min *et al* considered the reduct problem of numeric data with both error ranges and test costs[24].

In this paper, we formulate a universal PAPR optimization model with EVM constraint, and propose a differential evolution (DE) algorithm with a time complexity of $O(N\log_2 N)$. It is a universal PAPR optimization model, which can be applied to various communication systems. It does not need to configure the optimal CR look-up-table in advance. We only need to input modulated signal without considering the CR. The optimization objective is the desirable PAPR reduction. We adopt the EVM constraint in the new problem. It can guarantee accurately decoding the data in the receiver that is specified by most modern communications standards.

We propose a DE algorithm to solve the optimal PAPR problem. There are three major aspects of our DE algorithm. Firstly, we employ the noise vector as the population to parallel direct search the optimal solution. Secondly, we define the amount of PAPR reduction as cost function. Finally, three parameters including population size, scaling factor and crossover rate are adjusted to achieve the best performance. Complexity analyses demonstrate that the time complexity of our approach is $O(N\log_2 N)$.

Simulations are done with MATLAB 2010. It has been carried out to evaluate the performance of the proposed algorithm. The simulations assumed that the data were 16-QAM modulated and the system contained $N=1024$ subcarriers. It shows that our proposed method can

Manuscript received November 28, 2014; revised June 20, 2015.
This work was supported by National Science Foundation of China under Grant Nos. 61379089
Corresponding author email: xb-xiaobin@163.com
doi:10.12720/jcm.10.6.435-441

achieve better bit error rate (BER) performance without degradation of PAPR reduction.

The paper is organized as follows. In Section 2, we briefly review previous studies about adaptive step size clipping method and the definition of PAPR and EVM. In section 3 we define a universal PAPR optimization problem. In sections 4 and 5 we adopt a DE algorithm for solving the optimization problem and discuss its complexity. Simulation results are provided in Section 6. Finally, the conclusion is given in Section 7.

II. Related Works

This section briefly reviews the related work. We start from two metrics to quantify OFDM system performance, PAPR and EVM. Next, we describe previous studies work, adaptive step size cognitive clipping method.

A. Peak-To-Average Power Ratio

For digital implementation, we need to express the signal in discrete time. The discrete-time OFDM symbol with N subcarriers can be defined as

$$x(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X(k)e^{j\frac{2\pi nk}{LN}},n=0,\cdots,LN-1 \tag{1}$$

In equation (1), $X(\mathrm{k})\in C^n$ is an OFDM constellation.

To approximate the true PAPR of (1), the L times oversampling technique is usually used, where L oversampling factor is.

The PAPR is a measure commonly used to quantify the envelope fluctuations of multicarrier signals. The PAPR of the transmitted OFDM symbol is defined as

$$\mathrm{PAPR}=\frac{\max\limits_{0\le n\le N-1}\left|x(n)\right|^2}{E\left[\left|x(n)\right|^2\right]} \tag{2}$$

In equation (2), $E\left[\bullet\right]$ denotes the expected value.

B. Error Vector Magnitude

EVM is extensively applied to describe the distortion of the processed OFDM symbol, which directly affects the BER performance at the receiver.

For an OFDM block, each data symbol is mapped to a constellation point. However, various transmitter impairments or some deliberate modifications will cause the actual constellation point to deviate from the ideal location. The difference between the ideal constellation point and the deviated point is called the error vector.

For a single OFDM symbol, its EVM is defined as

$$\mathrm{EVM}=\frac{1}{S_{\max}}\sqrt{\frac{1}{N}\sum_{0}^{N-1}\left|E(k)\right|^2} \tag{3}$$

In equation (3), $S_{\max}$ is the average power of the carrier modulation scheme and $E(k)$ is the error vector at the kth subcarrier in-band.

$$E(k)=X'(k)-X(k) \tag{4}$$

In equation (4), $X(k)$ is an ideal constellation and $X'(k)$ is the constellation that is actually transmitted.

C. Adaptive step size method

In paper [2], an adaptive step method is proposed to reduce PAPR of OFDM signals. It uses dynamic EVM step size to adjust clipping level for PAR reduction. Fig. 1 includes two major components: ∇EVM selecting block and EVM_i selecting block, which select appropriate step size ∇EVM and EVM_i for iteration processing.

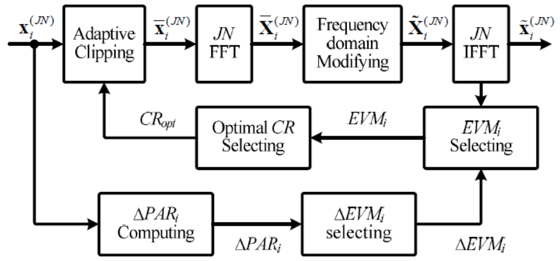


Fig. 1. Structure of adaptive step size method

However, it needs to determine the relationship between EVM and CR. For different modulation type and number of subcarriers, the inhibition ability of reducing PAPR is different. It needs a lot of simulations to configure of the optimal CR look-up-table for various communication systems in advance.

III. Problem Definition

In this section, we formulate a universal PAPR optimization model with EVM constraint. It is an EVM constraint PAPR optimization problem. It is no use for configuring the CR look-up-table in advance, and can be applied to any communication systems.

We focus on the clipping noise which is generated by various ways such as ICF, adaptive step size cognitive clipping. We will reduce the amplitude of the original signal by looking for the optimal noise vector.

We define an N-dimensional noise vector $e(n)$ as

$$e(n)=x(n)-x'(n) \tag{5}$$

In equation (5), $e(n)$ is the distortion introducing by clipping with various methods. Noise achieves the same purpose, namely to reduce high amplitude of original signal. We denote $E(k)$ is the frequency-domain vector of $e(n)$, which is the error vector between the original signal and the signal after clipping. It directly determines the size of the EVM. It will not bring any out-band radiation because of using the frequency domain clipping. Among all constellations $E(k)\in C^n$ that satisfy EVM constraint, we seek one with the minimum possible PAPR. We call $E(k)$ the PAPR-reduction vector.

The EVM constraint PAPR minimizing problem can be defined as follows:

Problem 1: the minimal PAPR problem

$$Input : X(k);$$
$$Output : e(k), x'(n)$$
$$Constraint : EVM_{\max}$$
$$Optimization \quad objective : \min|PAPR|$$

The input OFDM symbol is 16-QAM modulated signal. The output OFDM symbol in the time-domain can be written as

$$x'(n) = \mathrm{IFFT}(X'(k)) = x(n) + \sum_{k=0}^{N-1} E(k)\omega_{n,k} \tag{6}$$

$$\omega_{n,k} = \frac{1}{\sqrt{N}} \exp(j\frac{2\pi nk}{M}) \qquad n = 0, \ldots, N-1 \tag{7}$$

It represents that the time-domain clipped signal obtained by the frequency domain signal after M point IFFT operations.

Problem 1 has an optimization objective, which is to minimize the cost function.

If we want to guarantee that the signal OFDM symbol is not distorted too much, we define the constraint function as follows

$$\frac{\sqrt{\frac{1}{N}\sum_{1}^{N}|E(k)|^2}}{S_{\max}} \le EVM_{\max} \tag{8}$$

Namely, all $E(k)$ satisfies the EVM constraint, where $EVM_{\max}$ is permitted worst distortion on a single symbol specified by most modern communications.

IV. Algorithm Design

This section presents differential evolution (DE) algorithm. The DE algorithm is applied to parallel search the noise vector achieving the desirable PAPR reduction [25]-[28].

Our approach tries to construct a general model, only considers the modulated signal as input, and gets a desirable output signal. It can meet with most modern communication standards. We provide a proposed DE algorithm to search the optimal $E(k)$ and apply it to solve the PAPR reduction problem. It could be implemented in parallel processing frame to reduce the computational burden and run time.

The DE method usually has four main procedures: initialization, mutation operation, crossover operation and selection operation. Like the evolutionary algorithm, DE is an efficient and powerful population-based stochastic search method for solving optimization problems over continuous space.

Algorithm 1 shows the DE-based optimization scheme for PAPR reduction. The input of the algorithm includes frequency-domain error range, control parameters N_p, P_m, P_c. The output includes fitness function $f(E_{best,G})$, $E(k)$ and $x'(n)$.

TABLE I: DE-BASED OPTIMIZATION ALGORITHM

Algorithm 1 DE-based optimization algorithm

Inputs:
The frequency-domain error range, $X(k)$, Np, Pm, Pc
Outputs: $f(E_{best,G}), E(k), \quad x'(n)$
Method: Differential evolution.
Begin;
G=0;
Create a random initial population $E_{i,G} \quad i = 1, \cdots, Np$;
for G=1 to $G\max$ do
  for i=1 to Np do
    Select randomly $r_1 \ne r_2 \ne r_3 \ne i$;
    $j_{rand} = \lfloor rand[0,1) * D \rfloor$;
    $M_{i,G} = E_{r1} + Pm(E_{r2} - E_{r3})$;
  end for;
  for j=1 to D do
    $w_{i,G}^{j} = \begin{cases} m_{i,G}^{j}, & if\,(rand[0,1) \le P_m)\,or\,(j = j_{rand}), \\ e_{i,G}^{j}, & otherwise. \end{cases}$;
  end for
  for j=1 to Np do
    if $f(W_{i,G}) \le f(E_{i,G})$ then
      $E_{i,G+1} = W_{i,G}$;
      $f(E_{i,G+1}) = f(W_{i,G})$;
      if $f(W_{i,G}) < f(E_{best,G})$ then
        $E_{best,G} = W_{i,G}$;
      end if;
    end if;
  end for;
end for;
End.

The algorithm terminates after the gap is small enough, or after the maximum number of iterations has been reached.

1) Parameter setup

Choosing suitable control parameter values is a problem-dependent task. In our scheme, there are three real control parameters in the algorithm, which are: the scale factor P_m, crossover probability P_c and population size N_p.

The rest of the parameters are maximal number of generations $G_{\max}$ which may serve as a stopping condition and low and high boundary constraints of noise vector.

2) Initialization

It begins with a randomly initiated population of noise vector $E(k)$. DE is a parallel direct search method which should cover the entire search space as much as possible with the search space constrained by the prescribed minimum and maximum bounds.

$$E_{\min} = \{e_{\min}^{1}, \ldots, e_{\min}^{D}\} \tag{9}$$

$$E_{\max} = \{e_{\max}^{1}, \cdots, e_{\max}^{D}\} \tag{10}$$

Equations (9), (10) are the minimum and maximum bounds of noise vector $E(k)$. The initial value of the jth parameter in the ith individual is generated by

$$e_{i,0}^{j} = e_{\min}^{j} + rand\left[0,1\right] \bullet (e_{\max}^{j} - e_{\min}^{j}),\ j = 1,2,\cdots,D \quad (11)$$

3) Mutation

After initialization, DE employs the mutation operation to produce a mutant vector $M_{i,\mathrm{G}}$ corresponding to target vector $E_{i,G}$ in the current generation.

For each target vector $E_{i,G}$ is generated by

$$M_{i,\mathrm{G}} = E_{r1} + P_m(E_{r2} - E_{r3}) \quad (12)$$

In equation (13), $r1$ $r2$ and $r3$ are selected randomly and are mutually different. The scaling factor Pm is a positive control parameter for scaling the difference vector.

4) Crossover

To increase the diversity of the population, crossover operator is carried out. In this paper, DE employs the binomial crossover defined as follows

$$w_{i,G}^{j} = \begin{cases} m_{i,G}^{j}, & \text{if } (rand\left[0,1\right) \le P_m) \text{ or } \\ & (j = j_{rand}),\ j = 1,2,\cdots,D \\ e_{i,G}^{j}, & \text{otherwise} \end{cases} \quad (13)$$

In equation (13), $rand_{i,j}\left[0,1\right)$ is a uniformly distributed random number, $j_{rand} \in \left[1,2,\cdots,D\right]$ is a randomly chosen index, which ensures that $W_{i,G}$ gets at least one component from the mutant vector $M_{i,\mathrm{G}}$.

5) Selection

During selection, the offspring W_i competes with the initial solution candidate E_i .If it is fittest with respect to the cost function, it replaces E_i in the next generation $(G+1)$, i.e.

$$E_{i,G+1} = \begin{cases} W_{i,G}, & if\,(W_{i,G}) \le f(E_{i,G}) \\ E_{i,G}, & otherwise \end{cases} \quad (14)$$

V. Complexity Analysis

The computational complexity of our approach method is mainly reflected in the mutation and selection. While the number of samples for DE is $G*Np$. We can denote G is the maximum number of iterations and Np is the population size. In this paper, we set $Np = 50$ to achieve a good optimization result. It is far less than the number of subcarriers $N = 1024$. Each sample is calculated by using the N -point FFT. The number of the multiplications with our approach algorithm to find a sub-optimal solution is the $O(N \log_2 N)$.

VI. Simulations and AnalySIS

Simulations in the 64-bit MATLAB platform have been carried out to evaluate the performance of the proposed systems including comparison of control parameters, the complementary cumulative distribution function (CCDF) of the PAPR, BER. The simulations assumed that the data were 16-QAM modulated and the system contained $N = 1024$ subcarriers, 20MHz bandwidth.

A. Comparison of Control Parameters

The performance of the conventional DE algorithm highly depends on the chosen trial vector generation strategy and associated parameter values used. Inappropriate choice of strategies and parameters may lead to premature convergence or stagnation. The efficiency of the search for the global minimum is very sensitive to the setting of values P_m, P_c and N_P. A series of simulations were conducted to select the optimal control parameters.

1) The scale factor P_m

Fig. 2 shows the obtained optimal objective function curve using different Pm values. The scale factor P_m is a value in the range $\left[0,2\right]$ that controls the amount of perturbation in the mutation process. When the scaling factor P_m value is between $\left[0.5,1\right]$, the results obtained by the algorithm is better. If the $P_m < 0.5$ or $P_m > 1$, we cannot get high quality solutions. When $P_m = 0.5$, we can get the desirable objective function value of 5.892.

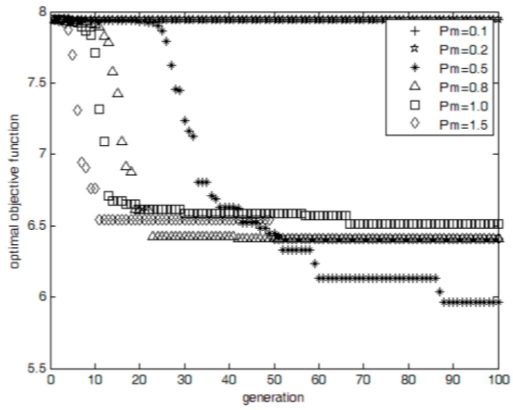


Fig. 2. Finding optimal scale factor P_m

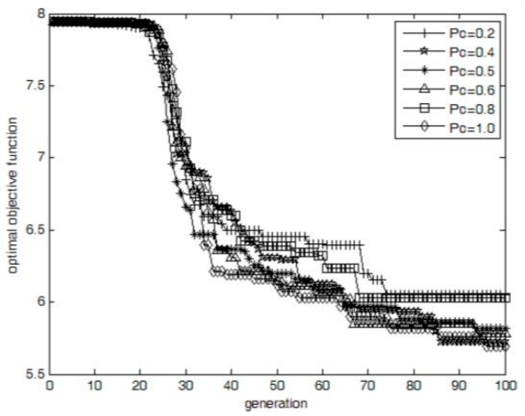


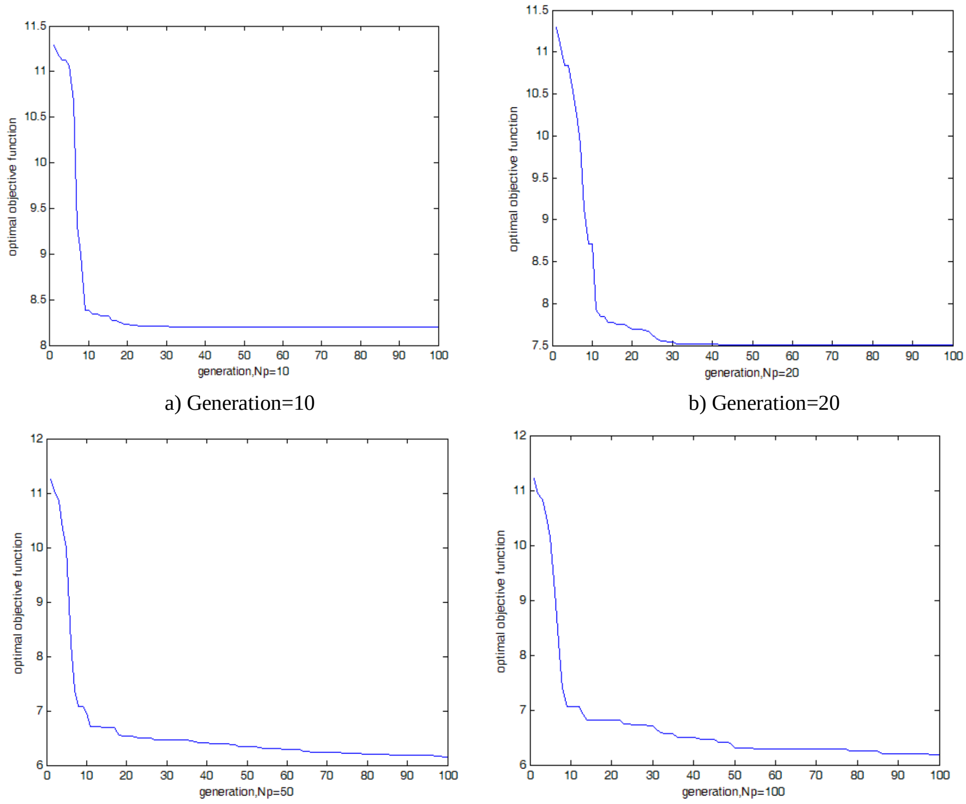
Fig. 3. Finding optimal crossover probability P_c

2) A crossover probability P_c

Fig. 3 shows the obtained optimal objective function curve using different P_c values. From Fig. 3, we can see a value for P_c from the range $[0.6, 0.8]$ is best because each trial vector frequently competes with a target vector. Using low values of P_c can cause the search to take longer than a simple random search .When $P_c = 1$ the population may stagnate because the size of the pool of potential trial vectors is limited.

3) Population size Np

Fig. 4 shows the obtained optimal objective function curve using different N_p values. In order to test the influence of population size to our proposed DE algorithm, we set the scaling factor $P_m = 0.5$, crossover factor $P_c = 0.8$, the population size from 10 to 100. When the population size increases to a certain number, the accuracy of solutions will no longer increase. The reason is the larger population can keep the diversity of population, but will reduce the speed of convergence. Diversity and convergence speed must maintain a certain balance. Therefore, accuracy will be decreased, when the population size is too large, if not to increase the maximum generation. It can achieve the good optimization result when the population size is between $[30, 50]$.



a) Generation=10 b) Generation=20

c) Generation=50 d) Generation=100

Fig. 4. Finding optimal population size N_p

B. Complementary Cumulative Distribution Function

The PAPR reduction performance is evaluated by the complementary cumulative distribution function (CCDF) of the PAPR. We will generate 100, 00 OFDM symbols to obtain the CCDF of PAPR. It denotes the probability that a PAPR exceeds a certain threshold. Fig. 5 shows the CCDFs of the PAPR for the original, adaptive step size method and our proposed method, respectively. As we can see, an impressive PAPR reduction of close to 6.1dB at the 10^2 CCDF level is possible while still meeting the constraints of EVM. It is obvious that our proposed method by using DE algorithm can offer the same PAPR reduction performance as that of the adaptive step size cognitive clipping method with much iteration.

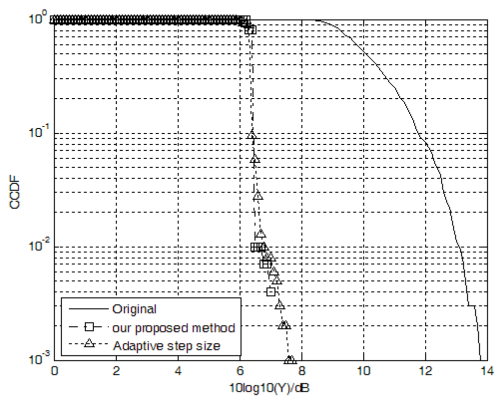


Fig. 5. PAPR-reduction performance of original method, our method, adaptive step size method (1024 subcarriers, 16QAM, L=4).

C. BER

Fig. 6 plots the bit error rate curves of the original signal, the adaptive step size method, and our proposed method in an additive white Gaussian noise (AWGN) channel. From the figures, it is observed that the DE algorithm achieve almost the same BER performance as that the original unprocessed method. There is less 0.2dB Eb/N0 degradation at 10^{-6} BER level. In most communication system, this degradation is negligible because of the channel coding. The signal to noise ratio (SNR) of our proposed method is better than adaptive step size method for a given BER. The reason is the adaptive step size method only considers the average EVM, while the BER performance of a clipped OFDM system depends on the in-band distortion. For example, at a BER level of 10^{-4}, our proposed method has an SNR about 1dB better than that of the adaptive step size method.

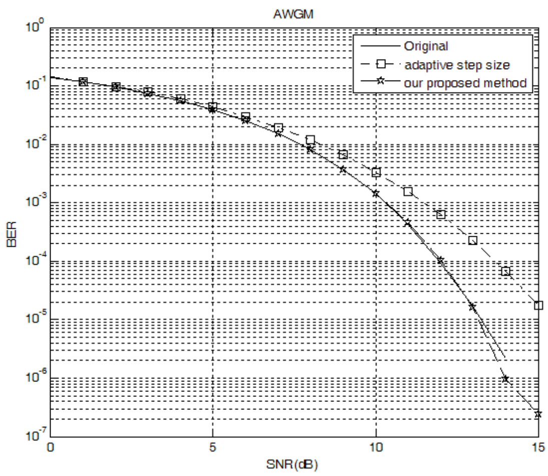


Fig. 6. BER comparison for original OFDM symbols, clipped OFDM symbols using adaptive step size method and our proposed DE method in an additive white Gaussian noise (AWGN) channel.

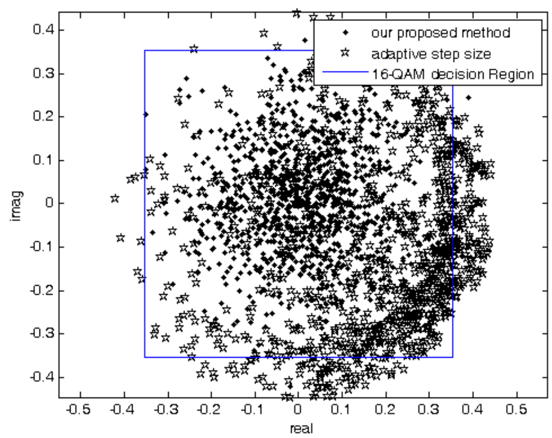


Fig. 7. Constellation of individual optimal 16-QAM OFDM symbols with our method and adaptive step size method.

D. Spectral Characteristics

Fig. 7 plots individual error vectors for our proposed method with EVM constraint and the adaptive step size method. A constellation point is said to be feasible if it is located within the associated feasible region. From Fig. 7, we can see that the original ICF signals exceeds the EVM specification and outside the decision region boundary. The reason is that the basic clipping violates the EVM constraint. The clipped signal must be corrected in the frequency domain or it will be incorrectly decoded by the receiver. At the same time, we can see that the signal using DE algorithm satisfies the EVM constraint and almost all inside the decision region boundary. It can guarantee achieve the better BER performance.

VII. Conclusion

In this paper, a desirable solution can be obtained with the EVM constraint by constructing the PAPR optimization universal model. The satisfactory solution will be solved by DE algorithm efficiently. Complexity analysis show the time complexity of our approach is $O(N\log_2 N)$. Our proposed method can avoid solving the CR with a lot of simulations. Simulations results compare our proposed method to that of the adaptive step size method. It shows that our proposed method can achieve better BER performance without degradation of PAPR reduction. We believe that a real-time implementation is feasible given the capabilities of current day CMOS technology.

Acknowledgment

The authors would like to thank the anonymous reviewers and the editors for their valuable comments and suggestions that helped improve the quality of this paper.

References

[1] J. Jang and K. B. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE Journal on Selected Areas in Communications*, pp. 171-178, Feb. 2003.

[2] W. Ma, S. Shao, and Y. Tang, "Adaptive Step Size for OFDM CFR with cognitive clipping," presented at the Wireless Communications Networking and Mobile Computing Meeting, Sep. 2010.

[3] S. H. Han, J. M. Cioffi, and J. H. Lee, "Tone injection with hexagonal constellation for peak-to-average power ratio reduction in OFDM," *IEEE Communications Letters*, vol. 9, pp. 646-648, Oct. 2006.

[4] M. G. Damavandi, A. Abbasfar, and D. G. Michelson, "Peak power reduction of OFDM systems through tone injection via parametric minimum cross-entropy method," *IEEE Transactions on Vehicular Technology*, vol. 62, pp. 1838-1843, May 2013.

[5] T. Habruseva, D. Arsenijević, and M. Kleinert, "Optimum phase noise reduction and repetition rate tuning in quantum-dot mode-locked lasers," *Applied Physics Letters*, vol. 2, pp. 21-22, Feb. 2014

[6] Y. Wang, W. Chen, and C. Tellambura, "Genetic algorithm based nearly optimal peak reduction tone set selection for adaptive amplitude clipping PAPR reduction," *IEEE Transactions on Broadcasting*, vol. 58, pp. 462-471, Aug. 2012.

[7] X. Zhu, W. Pan, and H. Li, "Simplified approach to optimized iterative clipping and filtering for PAPR reduction of OFDM communications," *IEEE Transactions on Signals*, vol. 61, pp. 1891-1901, May 2013.

[8] L. Nadal, M. M. Svaluto, and J. M. Fàbrega, "Low complexity PAPR reduction techniques for clipping and quantization noise mitigation in direct-detection O-OFDM systems," *Optical Fiber Technology*, vol. 20, pp. 208-216, Jan. 2014.

[9] C. Sharma, P. K. Sharma, and S. K. Tomar, "A modified Iterative Amplitude clipping and filtering technique for PAPR reduction in OFDM systems," presented at the Networks and Computer Communications meeting, Oct. 365-368, 2011.

[10] S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," *IEEE Wireless Communications*, vol. 12, pp. 56-65, April 2005.

[11] L. Li and D. Qu, "Joint decoding of LDPC code and phase factors for OFDM systems with PTS PAPR reduction," *IEEE Transactions on Vehicular Technology*, vol. 62, pp. 444-449, Jan 2013.

[12] S. Ahmed and M. Kawai, "Adaptive selection of spreading code subsets from orthogonal binary code sets for reduced PAPR in MC-CDMA systems," *Journal of Communications*, vol. 5, pp. 537-544, July 2010.

[13] Y. Sha, J. Chen, and G. Wang, "PAPR reduction method for SFBC MIMO-OFDM systems," *Journal on Communications*, vol. 18, pp. 25-28, Oct. 2009.

[14] M. F. Naeiny and F. Marvasti, "Selected mapping algorithm for PAPR reduction of space-frequency coded OFDM systems without side information," *IEEE Transactions on Vehicular Technology*, vol. 60, pp. 1211-1216, March 2011.

[15] Y. Shao, N. Chi, and J. Fan, "Generation of 16-QAM-OFDM signals using selected mapping method and its application in optical millimeter-wave access system," *IEEE Photonics Technology Letters*, vol. 24, pp. 1301-1303, Aug. 2014.

[16] H. B. Jeon, J. S. No, and D. J. Shin, "A low-complexity SLM scheme using additive mapping sequences for PAPR reduction of OFDM signals," *IEEE Transactions on Broadcasting*, vol. 57, pp. 866-875, Dec. 2011.

[17] L. Yang, K. K. Soo, and Y. M. Siu, "A low complexity selected mapping scheme by use of time domain sequence superposition technique for PAPR reduction in OFDM system," *IEEE Transactions on Broadcasting*, vol. 54, pp. 821-824, Dec. 2008.

[18] S. H. Han and J. H. Lee, "PAPR reduction of OFDM signals using a reduced complexity PTS technique," *IEEE Signal Processing Letters*, vol.11, pp. 887-890, Nov. 2004.

[19] N. Taspinar, A. Yildirim, and M. Yildirim, "Partial transmit sequences for PAPR reduction using parallel tabu search algorithm in OFDM systems," *IEEE Communications Letters*, vol. 15, pp. 974-976, Sep. 2011.

[20] P. Varahram and W. F. Al-Azzo, "A low complexity partial transmit sequence scheme by use of dummy signals for PAPR reduction in OFDM systems," *IEEE Transactions on Consumer Electronics*,vol. 56, pp. 2416-2420, Nov. 2010.

[21] J. Hou, J. Ge, and J. Li, "Peak-to-average power ratio reduction of OFDM signals using PTS scheme with low computational complexity," *IEEE Transactions on Broadcasting*, vol. 57, pp. 143-148, March 2011.

[22] L. Wang and Y. Cao, "Low computational complexity PTS scheme for PAPR reduction in OFDM system," *Journal on Communications*, vol. 9, pp. 245-252, Oct. 2009.

[23] F. Min, Q. Hu, and W. Zhu, "Feature selection with test cost constraint," *International Journal of Approximate Reasoning*, , vol. 55, no. 1, pp. 167-179, 2014.

[24] F. Min and W. Zhu, "Attribute reduction of data with error ranges and test costs," *Information Sciences*, vol. 211, pp. 48-67, 2012.

[25] H. Singh and L. Srivastava, "Modified differential evolution algorithm for multi-objective VAR management," *International Journal of Electrical Power Energy Systems*, vol. 55, pp. 731-740, Feb. 2014.

[26] Y. Wang and Z. Cai, "Combining multi-objective optimization with differential evolution to solve constrained optimization problems," *IEEE Transactions on Evolutionary Computation*, vol. 16, pp. 117-134, July 2012.

[27] X. Li and M. Yin, "An opposition-based differential evolution algorithm for permutation flow shop scheduling based on diversity measure," *Advances in Engineering Software*, vol. 55, pp. 10-31, Jan. 2013.

[28] S. M. Elsayed, R. A. Sarker, and D. L. Essam, "An improved self-adaptive differential evolution algorithm for optimization problems," *IEEE Transactions on Industrial Informatics*, vol. 9, pp. 89-99, Feb. 2013.

Min Wang was born in Hunan Province, China, in 1980. She received the B.S. degree from the Southwest University of China, Chongqing, in 2002. She received the M.S. degree from the University of Southwest Petroleum of China, Chengdu, in School of Electrical Engineering and Information. Her research interests include peak-to-average power ratio, signal processing, and information theory.