# Performance Analysis of Amplify-and-Forward Cooperative Networks in Non-Identically Distributed Nakagami-m Channels with Best Relaying Selection

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Abstract --- This paper investigates the performance, analysis of amplify and forward (AF) cooperative networks with the best relay selection in independent and non-identical Nakagami-m channels and M-ary phase shift keying (MPSK) modulations. New closed form expressions are derived of the analytical upper bound, cumulative density function (CDF), probability density function (PDF), moment generating function (MGF), symbol error rate (SER) and outage probability (Pout). Simulation results reveal that the symbol error rate performance downgrades either the m parameter or the number of relays increases. Our simulations compared between identically and non-identical distributed links for a different number of relays. In addition, different modulation techniques are examined to show the effect of the modulation scheme which was used over Nakagami-m fading channels on the non-identically distributed cooperative networks. The obtained results proved the impact of m parameters, relay selection and modulation index on the SER performance.

*Index Terms*—Cooperative networks, amplify-and-forward, symbol error rate, outage probability, Nakagami-m.

## I. INTRODUCTION

The cooperative networks and collaborative strategies protocol relay has been the subject of research due to its great influence in the field of wireless communication and its applications. Many research state its role in advanced communications and many relaying strategies have been presented as well, such as amplify and forward (AF), decode and forward (DF), and compress and forward (CF) [1], [2]. In these strategies, the users serve as information sources as well as relays. In AF, the relay solely amplifies the received signal from the source, and re-transmits it to the destination without doing any further processing on the source of transmission [3], [4]. Therefore, AF relaying require low-cost hardware, implementation, which represents an ingenious solution in the field of communication systems. For this reason, performance is investigated for all the participating cooperative networks in Nakagami-m fading environments.

Nakagami-m fading model has been an important subject of study in cooperative networks because it provides the best appropriate information and realistic radio links [5].

For example, [6]-[8] analyzed the performance of cooperative diversity wireless networks using amplifyand-forward relaying over independent, non-identical (i.n.d), Nakagami-m fading channels. The error rate and outage probability are determined using the MGF of the total SNR at the destination. Furthermore, closed form expressions for the CDF, probability density function (PDF) and MGF of the total SNR [9]-[11]. The authors also show performance analysis of the best relay selection scheme which has robustness based on selecting only the best single relay. Analytical expressions for the PDF, CDF and MGF of the SNR are used to derive the expressions for outage probability over Rayleigh Fading channels.

References [12] and [13] analyzed SER's performance of a cooperative wireless network with a single relay system over independent and identical distributed links (i.i.d) of Nakagami-m fading channels. The concept of the Altamonte code is transmitted through an amplifyand-forward (AF) relay. The exact SER is determined using the MGF of the total SNR for a particular signal in the case of M-ary phase shift keying M-PSK modulation schemes, and also compared performance between independent non-identically (i.n.d) and independent identically distributed (i.i.d) links. [14] The authors used amplify and forward (AF) relaying over Nakagami-m fading channels to derive a new lower bound and asymptotic expression for the outage probability of cooperative diversity in direct sequence code-division multiple access (DS-CDMA). In their work, they studied the performance and analysis of downlink multiuser relay networks using single-relay in amplify and forward (AF) where the outage probability was evaluated at a high SNR [15].

In this paper, the performance of all relays that are participating in cooperative networks at Nakagami-m environment are investigated as well as selecting of the best relay. Then, closed-form expressions are derived for the outage probability and the average SER in M-PSK modulations as well, but BPSK is used as the basis of our analysis. Closed-form expressions for MGF, CDF and

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PDF are obtained from the end-to-end SNR at the destination. Our expressions are verified with Monte Carlo simulations. We compared the SER performance in BPSK, QPSK and 8-PSK schemes. We revealed the effect of the number of relays and the m parameter on SER. We also compared between the cumulative density function (CDF) and probability density function (PDF) for i.i.d and i.n.d distributed links in which the best relay scheme is adopted for different SNRs.

The rest of the paper is organized as follows: Section II describes the system model and the underlying assumption. Section III, derivations of MGF, CDF, and PDF expressions for the end-to-end SNR are presented Section IV, performance and analysis are done for MGF, outage probability and SER. Section V. shows the simulation results and analytical results of SER. Section VI. gives the conclusions that are drawn from this work.

#### II. SYSTEM MODEL

We consider a dual hop cooperative network model composed of a source, half duplex relay(s)  $(1 \le i \le R)$  and a destination, as shown in Fig. 1.



Fig. 1. Cooperation network system Model.

Assume that the channel state information is distinguished at the destination. We utilize time division multiplexing (TDM) to ensure the transmission from the source and the relay(s) occur in consecutive time slot.

In the first time slot, the source transmits a signal x to the relays. The received signal from the  $i^{th}$  relay can be given as:

$$y_{SR_i} = h_{SR_i} x + \eta_{SR_i} \tag{1}$$

where  $h_{SR_i}$  is the channel gain between the source and the  $i^{th}$  relay.  $\eta_{SR_i} \sim CN(0, N_0)$  is the complex additive white Gaussian noise (AWGN) between the source and the  $i^{th}$  relay while  $N_0$  is the noise variance.

In the second time slot, the  $i^{th}$  relay amplifies its received signal and forwards it to the destination. The destination receives the transmission as:

$$y_{R_iD} = G_i h_{R_iD} y_{SR_i} + \eta_{R_iD}$$
(2)

where  $h_{R_iD}$  is the channel gain between the *i*<sup>th</sup> relay and the destination and  $\eta_{R_iD} \sim CN(0, N_0)$  is the complex additive white noise. Between the  $R_i$  and D by  $G_i$  is the amplified gain expressed as:  $G_i = E_s / (E_s |h_{SR_i}|^2 + N_o)$  wit h Es as the average energy per symbol [4]. From (2), we can estimate the SNR at the destination as follows:

$$\gamma_D = \sum_{i=1}^n \frac{\gamma_{SR_i} \gamma_{R_i D}}{1 + \gamma_{SR_i} + \gamma_{R_i D}}$$
(3)

where  $\gamma_{SR_i} = |h_{SR_i}|^2 E_S / N_o$  and  $\gamma_{R_iD} = |h_{R_iD}|^2 E_S / N_o$  are the instantaneous SNR of the  $S - R_i$  and  $R_i - D$  hops, respectively.

Since, solving the exact expressions in (3) can be complicated; we derive an upper bound from (3) as:

$$\gamma_i^{up} = \min(\gamma_{SR_i}, \gamma_{R_iD}) \ge \frac{\gamma_{SR_i}\gamma_{R_iD}}{1 + \gamma_{SR_i} + \gamma_{R_iD}}$$
(4)

Therefore, the upper bound for the equivalent SNR can be written as:

$$\gamma_D^{up} \le \sum_{i=1}^n \gamma_i^{up} \tag{5}$$

The upper bound SNR given by (5) is more suitable for analysis and is shown to be quite accurate at medium and high SNR values [16].

## III. ESTIMATION OF CDF, MGF AND PDF

In order to find the expressions of the outage probability, and average SER, we need to derive the MGF, thus we first derive the CDF of  $\gamma_D^{up}$ , and then the PDF as a derivative from the CDF. Finally, we derive the MGF of  $\gamma_D^{up}$  with the help of the inverse Laplace transform.

## A. No-Relay Selection.

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Assuming that  $\gamma_{SD}$  and  $\gamma_i^{up}$  are independent, the MGF of  $\gamma_D^{up}$  can be written as.

$$M_{\gamma_{D}^{up}}(s) = \prod_{i=1}^{R} M_{\gamma_{i}}(s)$$
(6)

In order to find the MGF for the S – R – D path, it needs to derive the corresponding PDF of CDF. It is shown that when  $\gamma_{SR_i}$  and  $\gamma_{R_iD}$  are independent the CDF

of  $\gamma_i^{up} = \min(\gamma_{SR_i}, \gamma_{R_iD})$  can be expressed as [17].

$$F_{\gamma_i^{op}}(\gamma) = 1 - [1 - P_r(\gamma_{SR} \le \gamma)][1 - P_r(\gamma_{RD} \le \gamma)]$$
(7)

$$F_{\gamma_{i}^{up}}(\gamma) = 1 - [1 - F_{SR}(\gamma)][1 - F_{RD}(\gamma)]$$
(8)

$$F_{SR}(\gamma) = \Gamma(m, \frac{m}{\overline{\gamma}_{SR}^2}\gamma)$$
(9)

$$F_{RD}(\gamma) = \Gamma(m, \frac{m}{\overline{\gamma}_{RD}^2}\gamma)$$
(10)

where m is the fading parameter,  $F_{SR}(\gamma)$  and  $F_{RD}(\gamma)$  are the CDF from the source to the *i*<sup>th</sup> relay and from the *i*<sup>th</sup> relay to the destination respectively. Let :

$$\overline{\gamma}_{_{SR}} = c_{_{1}}\overline{\gamma}_{_{C}} \tag{11}$$

$$\overline{\gamma}_{RD} = c_2 \overline{\gamma}_C \tag{12}$$

where  $c_1 = \varepsilon_1 |h_{SR}|^2 \sqrt{P_s}$ ,  $c_2 = \varepsilon_2 |h_{RD}|^2 \sqrt{P_R}$ ,  $\varepsilon_1$  and  $\varepsilon_2$ are the energy consumed at the source and destination respectively. Now we consider  $c_1 = c_2 = c$ , therefore CDF  $\gamma_D^{up}$  can be expressed as:

$$F_{\gamma_{l}^{up}}(\gamma) = \left(\frac{\Gamma(m, \frac{m}{c\overline{\gamma}_{C}}\gamma)^{2}}{(\Gamma(m))^{2}} - \frac{2\Gamma(m, \frac{m}{c\overline{\gamma}_{C}}\gamma)}{\Gamma(m)}\right)$$
(13)

$$F_{\gamma_D^{up}}(\gamma) = \prod_{i=1}^{R} \left( -\frac{2\Gamma(m, \frac{m}{c\overline{\gamma_C}}\gamma)}{\Gamma(m)} + \left( \frac{\Gamma(m, \frac{m}{c\overline{\gamma_C}}\gamma)}{\Gamma(m)} \right)^2 \right) \quad (14)$$

$$f_{\gamma_D^{up}}(\gamma) = \frac{2\gamma^{m-1}e^{\frac{-\tau}{c\overline{\gamma_c}}}}{\Gamma(m)} (1 - \frac{1}{\Gamma(m)}\Gamma(m, \frac{m}{c\overline{\gamma_c}}\gamma)) \quad (15)$$

Equation (15) represents the outage probability in all relays participant (AP-AF) of Nakagami-m channels.

We derive the MGF of  $\gamma_D^{up}$ , using the PDF of  $\gamma_i^{up}$  in (15) as follows:

$$M_{g_{\gamma_D^{\rm up}}}(s) = \int_0^\infty e^{-\gamma s} p df \, d\gamma \tag{16}$$

The  $M_{g_{y_D^{up}}}$  can be given from the Laplace transform of the PDF in (15):

$$M_{g_{\gamma_{D}^{m}}}(s) = \int_{0}^{\infty} \frac{2e^{-s\gamma}}{\Gamma(m)} \left( \gamma^{m-1} e^{\frac{-m\gamma}{c\overline{\gamma}_{c}}} - \frac{1}{\Gamma(m)} \gamma^{m-1} e^{\frac{-m\gamma}{c\overline{\gamma}_{c}}} \Gamma\left(m, \frac{m}{c\overline{\gamma}_{C}}\gamma\right) \right) d\gamma$$
(17)

Solving the integral  $\int_{0}^{\infty} \gamma^{m-1} e^{\frac{-m\gamma+s\gamma}{c\overline{\gamma}_{c}}d\gamma}$  :

$$\int_{0}^{\infty} \gamma^{m-1} e^{\frac{-m\gamma + s\gamma}{c\overline{\gamma}_{c}}d\gamma} = (m-1)! \left(\frac{c\overline{\gamma}_{c}}{m} + s\right)^{m}$$
(18)

Using the identity in [18.eq (6.455.1)] we re-write (17):

$$\int_{0}^{\infty} x^{\mu-1} e^{-\beta x} \Gamma(v, \alpha x) dx = \frac{\alpha^{\nu} \Gamma(u, v)}{\mu(\alpha + \beta)^{\mu+\nu}} \times$$

$${}_{2}F_{1}\left(1, \mu + v, \mu + 1, \frac{\beta}{\beta + \alpha}\right)$$
(19)

We get the  $M_{g_{y_{D}^{w}}}$  in substituting (18) in (19) we obtain the  $M_{g_{y_{D}^{w}}}$  of as follows:

$$M_{g_{s_{j_{D}}^{m}}}(s) = \prod_{i=1}^{R} \left( (m-1)! \left( \frac{c\overline{\gamma}_{c}}{m} - S \right)^{m} - \frac{c\overline{\gamma}_{c}}{m} \right)^{m} - \frac{c\overline{\gamma}_{c}}{m} - \frac{c\overline{\gamma}_{c}}$$

$$\frac{1}{\Gamma(m)} \frac{(m/c\overline{\gamma}_C)^m \Gamma(m)}{m(2m/c\overline{\gamma}_C + s)^{2m}} {}_2F_1\left(1, 2m, m+1, \frac{m}{2m+c\overline{\gamma}_C S}\right)\right) (20)$$

## B. Best Relay Selection

Assuming R available relays, the relay selection algorithm selects the best relay (denoted relay B) such that:

$$B = \arg \max_{i \in \mathcal{W}} \{\gamma_i^{up}\}$$
(21)

where  $\psi = \{1, 2, 3, ..., R\}, \gamma_i^{\mu p}$  is the instantaneous SNR for the relay i. Therefore, the instantaneous SNR for the best relay is given by:

$$\gamma_B^{\mu\nu} = \max_{i \in \psi} \{\gamma_i^{\mu\nu}\} = \max_{i \in \psi} \{\min(\gamma_{SR_i}, \gamma_{R_iD})\}$$
(22)

The CDF of  $\gamma_B^{up}$  can be expressed as.

$$F_{\gamma_{B}^{up}}(\gamma) = \left(\frac{\Gamma(m, \frac{m}{c\overline{\gamma_{c}}}\gamma)^{2}}{(\Gamma(m))^{2}} - \frac{2\Gamma(m, \frac{m}{c\overline{\gamma_{c}}}\gamma)}{\Gamma(m)}\right)^{R}$$
(23)

Then the PDF can be expressed as follow:

$$f_{\gamma_{B}^{up}}(\gamma) = R\left(\frac{2}{\Gamma(m)}\gamma^{m-1}e^{\frac{-m\gamma}{c\overline{\gamma}_{C}}}(1-\frac{1}{\Gamma(m)}\Gamma(m,\frac{m}{c\overline{\gamma}_{C}}\gamma))\right)$$

$$\times \left(\frac{\Gamma(m,\frac{m}{c\overline{\gamma}_{C}}\gamma)^{2}}{(\Gamma(m))^{2}} - \frac{2\Gamma(m,\frac{m}{c\overline{\gamma}_{C}}\gamma)}{\Gamma(m)}\right)^{R-1}$$
(24)

$$f_{\gamma_{B}^{np}}(\gamma) = \frac{4\kappa}{\Gamma(m)} \gamma^{m-1} e^{\frac{-m\gamma}{c\overline{\gamma}_{c}}} \sum_{k=0}^{m-1} {\binom{R-1}{\kappa}} \times \left( \frac{\Gamma(m, \frac{m}{c\overline{\gamma}_{c}} \gamma)^{R+k}}{(\Gamma(m))^{2k+1}} - \frac{\Gamma(m, \frac{m}{c\overline{\gamma}_{c}} \gamma)^{R+k+1}}{(\Gamma(m))^{2k}} \right)$$
(25)

where  $\binom{R-1}{K} = (R-1)!/[K!((R-1)-K)!]$  is the binomial coefficient.

We calculate the coefficients of (25) using the multinomial theorem and the identities in [19] with the help of the generalized expansion of gamma function as in [18.eq (8.352.4)], we get :

$$\frac{\Gamma(m, \frac{m}{c\overline{\gamma}_{C}}\gamma)^{R+k}}{(\Gamma(m))^{2k+1}} = \left((m-1)!\right)^{R+k} e^{\frac{-m\gamma(R+k)}{c\overline{\gamma}_{C}}} \times \sum_{|x|} \binom{R+k}{x_{1}, x_{2}, \dots, x_{R+k}} \prod_{q=1}^{R+k} \frac{m^{q}\gamma^{q}}{c\overline{\gamma}_{C}{}^{q}q}!$$
(26)

$$\frac{\Gamma(m,\frac{m}{c\overline{\gamma}_{C}}\gamma)^{R+k-1}}{(\Gamma(m))^{2k}} = ((m-1)!)^{R+k-1}e^{\frac{-m\gamma(R+k-1)}{c\overline{\gamma}_{C}}}\times$$

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$$\sum_{|\mathbf{y}|} \binom{R+k-1}{y_1, y_2, \dots, y_{R+k-1}} \prod_{f=1}^{R+k-1} \frac{m^f \gamma^f}{c \overline{\gamma}_C^f f!}$$
(27)

Then the PDF of  $\gamma_B^{\mu\rho}$  can be expressed as in (28) and the MGF is expressed in (29) respectively.

## **IV. PERFORMANCE ANALYSIS**

In the following, we utilize the previously obtained expressions to estimate the two performance matrices: outage probability and average Symbol error Rate.

## A. Outage Probability

The outage probability is defined as the probability that the end-to-end SNR falls below a certain predefined threshold value,  $\alpha$ . The outage probability, using the CDF expression (23),  $P_{out}$  can be written as :

$$P_{out} = \left(\frac{\Gamma(m, \frac{m}{c\overline{\gamma}_c}\alpha)^2}{\left(\Gamma(m)\right)^2} - \frac{2\Gamma(m, \frac{m}{c\overline{\gamma}_c}\alpha)}{\Gamma(m)}\right)^R$$
(30)

## B. Symbol Error Rate

In this section we derive the SER using the MGF derived in the previous section with the help of partial fraction expansion. Using the (29) we estimate the SER for M-PSK signals as follows:

1) *Binary Signals:* The average SER for binary signals is given by [20]

$$P_{SER} = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} M_{g_{s_D^{e_p}}}\left(\frac{g}{\sin^2\theta}\right) d\theta \qquad (31)$$

where  $g = \sin^2(\pi/M_o)$  and g = 1 for BPSK and g = 0.5 for orthogonal BFSK. By substituting (29) into (31) and after some manipulations the average SER in this case can be expressed as (32). Using partial fraction expansions and after some manipulations, (32) can be written for the BPSK signaling as (33), where the closed-form expression for  $I_1(c)$  is given by [21, 5A.9]:

$$I_1(c) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left( \frac{\sin^2 \theta}{\sin^2 \theta + c} \right) d\theta = \frac{1}{2} \left( 1 - \sqrt{\frac{c}{1+c}} \right)$$
(34)

where  $I_1(c)$  is the SER of BPSK signals.

2) M-PSK Signals: The average SER for M-PSK signals can be written as [20]

$$P_{SER} = \frac{1}{\pi} \int_{0}^{\frac{(M-1)\pi}{M}} M_{g_{\gamma_D^{\mu_p}}} \left( \frac{g_{M-PSK}}{\sin^2 \theta} \right) d\theta$$
(35)

where  $g_M - PSK = \sin^2(\pi/M)$ . And a closed-form expression for  $I_2(c)$  is given by (36) [21, 5A.15].

$$f_{j_{g}^{m}}(\gamma) = \frac{4R}{\Gamma(m)} \sum_{k=0}^{R-1} {\binom{R-1}{\kappa}} \times \left[ \sum_{|\mathbf{x}|} {\binom{R+k}{x_{1}, x_{2}, \dots, x_{R+k}}} \right]_{q=1}^{R+k} \frac{m^{q} \gamma^{q}}{c\overline{\gamma}_{C}^{-q} q!} ((m-1)!)^{R+k} \gamma^{m-1} e^{-\frac{my(R+k+1)}{c\overline{\gamma}_{c}}} \times \frac{1}{(\Gamma(m))^{2k+1}} \\ -((m-1))!^{R+k+1} \sum_{|\mathbf{y}|} {\binom{R+k-1}{y_{1}, y_{2}, \dots, y_{R+k-1}}} \right]_{f=1}^{R+k} \frac{m^{f} \gamma^{f}}{c\overline{\gamma}_{C}^{-f} f!} \gamma^{m-1} e^{-\frac{my(R+k)}{c\overline{\gamma}_{c}}} \frac{1}{(\Gamma(m))^{2k}} \right] \\ = \frac{4R}{\Gamma(m)} \sum_{k=0}^{R-1} {\binom{R-1}{k}} \left[ \frac{((m-1)!)^{R+k}}{(\Gamma(m))^{2k+1}} \sum_{|\mathbf{x}|} {\binom{R+k}{x_{1}, x_{2}, \dots, x_{R+k}}} \right]_{q=1}^{R+k} \frac{m^{q}}{c\overline{\gamma}_{c}^{-q} q!} \gamma^{m+q-1} e^{-\frac{my(R+k+1)}{c\overline{\gamma}_{c}}} + \gamma_{s} \\ - \frac{((m-1)!)^{R+k+1}}{(\Gamma(m))^{2k}} \sum_{|\mathbf{y}|} {\binom{R+k-1}{y_{1}, y_{2}, \dots, y_{R+k-1}}} \prod_{f=1}^{R+k} \frac{m^{f}}{c\overline{\gamma}_{c}^{-f} f!} \gamma^{m+\gamma-1} e^{-\frac{my(R+k+2)}{c\overline{\gamma}_{c}}} + \gamma_{s} \\ M_{g_{j_{m}^{m}}} = \frac{4R}{\Gamma(m)} \sum_{k=0}^{R-1} {\binom{R-1}{k}} \left[ \frac{((m-1)!)^{R+k}}{(\Gamma(m))^{2k+1}} \sum_{|\mathbf{y}|} {\binom{R+k}{x_{1}, x_{2}, \dots, x_{R+k}}} \sum_{q=1}^{R+k} \prod_{q=1}^{R+k} \left( \frac{m(R+k+1)}{c\overline{\gamma}_{c}} + s \right)^{-(m-q)} \Gamma(m+q) \\ - \frac{((m-1)!)^{R+k+1}}{(\Gamma(m))^{2k}} \sum_{|\mathbf{y}|} {\binom{R+k-1}{y_{1}, y_{2}, \dots, y_{R+k-1}}} \sum_{f=1}^{R+k-1} \left( \frac{m(R+k+2)}{c\overline{\gamma}_{c}} + s \right)^{-(m-f)} \Gamma(m+f) \\ \end{bmatrix}$$

$$(29)$$

$$P_{SER} = \frac{4R}{\Gamma(m)} \sum_{k=0}^{R-1} {\binom{R-1}{K}} \frac{\left((m-1)!\right)^{R+k}}{(\Gamma(m))^{2k+1}} \sum_{|x|} {\binom{R+k}{x_1, x_2, \dots, x_{R+k}}} \Gamma(m+q) \prod_{q=1}^{R+k} \frac{1}{\pi} \int_{0}^{\overline{2}} \left(\frac{m(R+k+1)}{c\overline{\gamma_c}} + \left(\frac{\sin^2\theta}{c+\sin^2\theta}\right)^R\right)^{\binom{m-q}{2}} d\theta \\ - \frac{\left((m-1)!\right)^{R+k+1}}{(\Gamma(m))^{2k}} \sum_{|y|} {\binom{R+k-1}{y_1, y_2, \dots, y_{R+k-1}}} \Gamma(m+f) \prod_{f=1}^{R+k-1} \int_{0}^{\overline{2}} \left(\frac{m(R+k+2)}{c\overline{\gamma_c}} + \left(\frac{\sin^2\theta}{c+\sin^2\theta}\right)^{R+1}\right)^{-\binom{m-f}{2}} d\theta$$
(32)

$$P_{SER} = \frac{4R}{\Gamma(m)} \sum_{k=0}^{R-1} {\binom{R-1}{K}} \frac{((m-1)!)^{R+k}}{(\Gamma(m))^{2k+1}} \sum_{[x]} {\binom{R+k}{x_1, x_2, \dots, x_{R+k}}} \Gamma(m+q) \prod_{f=1}^{R+k} \sum_{w=0}^{W_1} {\binom{W_1}{w}} \left(\frac{m(R+k+1)}{c\overline{\gamma}_c}\right)^{W_1-w-1} \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(I_1\left(\frac{\overline{\gamma}_o}{w}\right)\right)^{R(-(m-q))} d\theta \\ -\frac{4R}{\Gamma(m)} \sum_{k=0}^{R-1} {\binom{R-1}{k}} \frac{((m-1)!)^{R+k}}{(\Gamma(m))^{2k+1}} \sum_{[y]} {\binom{R+k-1}{y_1, y_2, \dots, y_{R+k-1}}} \Gamma(m+q) \prod_{f=1}^{R+k-1} \sum_{w=0}^{W_2} {\binom{W_2}{w}} \left(\frac{m(R+k+2)}{c\overline{\gamma}_c}\right)^{W_2-w-1} \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left(I_2\left(\frac{\overline{\gamma}_o}{w}\right)\right)^{-(R+1)(m-q))} d\theta$$
(33)

$$I_2(c) = \frac{1}{\pi} \int_0^{\frac{M}{M}} \left( \frac{\sin^2 \theta}{\sin^2 \theta + c} \right) d\theta = \frac{(M-1)}{M} \left\{ 1 - \sqrt{\frac{c}{1+c}} \left( \frac{M}{(M-1)\pi} \right) \left[ \frac{\pi}{2} + \tan^{-1} \left( \sqrt{\frac{c}{1+c}} \cot(\frac{\pi}{M}) \right) \right] \right\}$$
(36)

where  $I_2(c)$  is the SER of MPSK signals.

### V. SIMULATION RESULTS

In this section, we provide numerical results for the PDF, CDF, outage probability ( $P_{out}$ ) and SER expressions. Our results are obtained from different values of M-PSK modulation index, m and R. We validate and compare our derived expressions using Monte Carlo simulations under Nakagami-m fading channels.



Fig. 2. PDF of  $\gamma_D^{up}$  in R=1,2,3 for the i.i.d and i.n.d when SNR=0 dB at Nakagami-m fading channels.



Fig. 3. CDF of  $\gamma_D^{up}$  in *R*=1,2,3 for the i.id and i.n.d when SNR=0 dB at Nakagami-m fading channels.

In Fig. 2 and Fig. 3 we compare the PDF and CDF values respectively for both i.i.d and i.n.d. The results are obtained from different values of R at SNR=0 dB. The figures reveal that if R increases both the PDF and the CDF shift to the right direction. Moreover, the peak value of the PDF becomes smaller which means that it will be more evenly distributed. The result also shows that  $(\gamma_{p}^{w})$  is enhanced and the overall SNR improves as the number relays increases.

Fig. 4 shows the outage probability as a function of the SNR and threshold, for different values of R. Assuming the SNR =5 and 10 dB we can see that by increasing the number of relays R, the outage probability decreases.



Fig. 4. Outage probability for non-identically distributed links versus  $\alpha$  with average SNR =10,5 dB and R= 1,2,3



Fig. 5. SER with versus average SNR coherent BPSK with m = 0.5, 1, 2.

Fig. 5 shows a comparison of the SER for different values of m for BPSK signals. The result reveals that while increasing the values of m the SER decreases. Fig. 6 shows the performance of SER evaluated in 8PSK signals for different values of m. The results of this Figure prove that the SER decreased the value of m increased.

Fig. 7 shows the performance of SER versus different values of R in BPSK signals with m = 1. The results of this figure reveals that the SER increases as R increases. For example when SNR= 10 dB SER for R = 1,2,3, equals 0.608188, 0.561404 and 0.5 for BPSK respectively.



Fig. 6. SER with versus average SNR coherent 8PSK with m=0.5,1,2.



Fig. 7. Comparison SER with versus average SNR of BPSK for Nakagami-m when m=1.



Fig. 8. Comparison SER with versus average SNR of 8PSK for Nakagami-m when m=1.

Fig. 8 shows the behavior of 8PSK modulated signals with SNR=10 dB. As we see from the figure, the SER achieves  $P_{SER} = 0.77193$ , 0.716375, 0.6666667 as the number of relays increases.

Fig. 9 shows the comparison of the SER performance for the BPSK, QPSK and 8PSK signals respectively with R =2 and m=1. As we see from the figure, if SNR =10 dB the average SER equals 0.716375, 0.649123, 0.561404 for 8PSK,QPSK and BPSK respectively. As it is clear from the figure, the BPSK signal gets less SER as the average SNR increases for a constant number of relays.



Fig. 9. Comparison SER with versus average SNR of different modulation when R=2 and m=1.

## VI. CONCLUSIONS

In this work, we analyzed the performance of the SER in multiple relays cooperative networks with Nakagamim fading channels. Closed-form expressions are derived for CDF, PDF and MGF for an arbitrary number of relays and m parameters. We proposed the best relay scheme to obtain higher outage probability and SER performance. Monte Carlo simulations are used to validate our expressions. Identically distributed and non-identically distributed links of cooperative networks are compared for different values of R and m parameters for low and high SNR. We also provided numerical results to show the performance of SER for different M-PSK signals as well as for different values of R and m parameters for low and high SNR. The obtained results give us important guidelines on the design of high performance cooperative networks in more a practical characteristic environment.

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