Image Reconstruction of Compressed Sensing Based on Improved Smoothed l_0 Norm Algorithm

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Abstract—This paper investigates the problem of image reconstruction of compressed sensing. First, an improved smoothed l_0 norm (ISL0) algorithm is proposed by using modified Newton method to improve the convergence speed and accuracy of classical smoothed l_0 norm (SL0) algorithm, and to increase calculation speed and efficiency. The choice of algorithm parameter is discussed and the algorithm convergence is proven. Then, the proposed ISL0 algorithm is applied to reconstruct images of compressed sensing. We preserve low-pass wavelet coefficients after single layer wavelet transform, only measure the high-pass wavelet coefficients. Then, ISL0 algorithm is utilized to recover high-pass wavelet coefficients, and inverse wavelet transform is performed to obtain the original image. Finally, simulation results are given to demonstrate the effectiveness of the proposed algorithm. It is shown that, compared with classical SL0, SP and OMP algorithms, the proposed ISL0 algorithm performs better not only in reconstruction quality, but also in calculation complexity and noise robustness.

Index Terms—Compressed sensing, image reconstruction, smoothed l_0 norm algorithm, single layer wavelet transform, modified Newton method.

I. Introduction

The recently developed Compressed Sensing (CS) [1], [2] framework is a novel technique of data acquisition. It suggests that the exact reconstruction of a sparse or compressible signal can be realized from a small number of random projections or measurements through using an optimization process from these projections. Although the encoding process is simply linear projection, the reconstruction requires some non-linear algorithms to find the sparsest signal from the measurements. Therefore, the development of fast reconstruction algorithm with reliable accuracy and optimal (nearly) theoretical performance is one of challenging questions of CS research. It is also one of key factors to put CS theory into practical applications [3].

A series of sparse signal reconstruction algorithms of CS theory have been proposed. Among existing reconstruction algorithms, the famous Basis Pursuit (BP) algorithm [4] aims at the l_1 minimization using linear programming. While it requires a minimal number of measurements, its high computational complexity may prevent it from practical large-scale applications. Another popular class of sparse recovery algorithms is based on the idea of iterative greedy pursuit. The most representative ones include the Matching Pursuit (MP) algorithm [5] and Orthogonal Matching Pursuit (OMP) algorithm [6]. The reconstruction complexity of these algorithms are significantly lower than that of BP method. However, they require more measurements for perfect reconstruction and lack provable reconstruction quality. Although greedy algorithms with a backtracking mechanism, such as Subspace Pursuit (SP) algorithm [7], have offered comparable theoretical reconstruction quality to the linear programming methods along with low reconstruction complexity, they assume that the sparsity of a signal is known for exact recovery [8]. Unfortunately, such an assumption may not be available in many practical applications [9]. More recently, Mohimani *et al* [10] proposed smoothed l_0 norm (SL0) algorithm which runs much faster than the competing algorithms while providing the same or better reconstruction accuracy without having to require the sparsity as prior knowledge. However, SL0 algorithm uses steepest descent method to approach the optimal solution, there exist “notched effect” in search direction and the step-size is usually estimated with experiences. Therefore, the computational performance of this method is still not efficient enough.

In this paper, we improve the performance of SL0 algorithm so as to obtain a more efficient algorithm named improved SL0 (ISL0) algorithm, in which modified Newton method is used to avoid the influence of “notched effect”, and variable step-size Newton method is utilized to increase calculation speed and efficiency. Then, the proposed ISL0 algorithm is employed to image reconstruction of CS based on single layer wavelet transform. The numerical simulation results show that, compared to several existing algorithms, ISL0 algorithm yields improved image reconstruction quality, running time and noise robustness.

The rest of the paper is organized as follows. Section II presents CS theory framework. In Section III, the

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proposed ISL0 algorithm is described in details. Section IV is about the image reconstruction on the base of ISL0 algorithm and single layer wavelet transform. Experimental results are presented in Section V and the paper concludes in Section VI.

II. CS Framework

Consider a length-N, real-valued signal $\boldsymbol{x}$ indexed as $x(n)$, $n \in \{1, 2, \cdots, N\}$. In CS theory, the signal $\boldsymbol{x}$ to be acquired and subsequently reconstructed is typically assumed to be sparse or compressible in an orthogonal basis $\boldsymbol{\Psi} = [\psi_1, \psi_2, \cdots, \psi_N]$ which provides a K-sparse representation of $\boldsymbol{x}$; that is

$$\boldsymbol{x} = \sum_{n=1}^{N} \theta(n)\psi_n = \sum_{q=1}^{K} \theta(n_q)\psi_{n_q} \tag{1}$$

where $\boldsymbol{x}$ is a linear combination of K vectors chosen from $\boldsymbol{\Psi}$, $\{n_q\}$ are the indices of those vectors, and $\{\theta(n)\}$ are the corresponding coefficients. Alternatively, we can write in matrix notation

$$\boldsymbol{x} = \boldsymbol{\Psi\theta} \tag{2}$$

where $\boldsymbol{x}$ is a $N\times1$ column vector, and $\boldsymbol{\theta}$ denotes a $N\times1$ column vector with K nonzero entries. Let $\|\cdot\|_p$ denote the l_p norm, then we can write $\|\boldsymbol{\theta}\|_0 = K$.

According to the CS theory, such a signal $\boldsymbol{x}$ can be acquired through the following random linear projection:

$$\boldsymbol{y} = \boldsymbol{Ax} = \boldsymbol{A\Psi\theta} = \boldsymbol{\Theta\theta} \tag{3}$$

where $\boldsymbol{y}$ is the sampled vector with $M<<N$ data points, $\boldsymbol{A}$ represents an $M\times N$ measurement/sensing matrix, and $\boldsymbol{\Theta} = \boldsymbol{A\Psi}$.

To recover signal $\boldsymbol{x}$, the approach is to seek a solution of the l_0 minimization problem

$$\min_{\boldsymbol{\theta}} \|\boldsymbol{\theta}\|_0 \quad s.t. \quad \boldsymbol{y} = \boldsymbol{A\Psi\theta} \tag{4}$$

Obviously, the above minimization problem is a NP-hard problem. The solution of (4) is not unique, we need to enumerate all the possible $\boldsymbol{\theta}$ that meet the condition. Fortunately, the above problem becomes computationally tractable if the sensing matrix $\boldsymbol{\Theta}$ satisfies a restricted isometry property (RIP) which introduced by Candès and Tao in [11], [12].

III. ISL0 Algorithm

A. SL0 Algorithm

In order to reduce the calculation complexity of (4), SL0 algorithm adopts a sum function $F_\sigma(\boldsymbol{x})$ to approximate the l_0 norm of vector $\boldsymbol{x}$. Here

$$F_\sigma(\boldsymbol{x}) = \sum_{i=1}^{N} f_\sigma(x_i) \tag{5}$$

with

$$f_\sigma(x_i) = 1 - \exp(-x_i^2 / 2\sigma^2) \tag{6}$$

and x_i (i=1,2…N) representing the components of vector $\boldsymbol{x}$. It can be easily verified that

$$\lim_{\sigma\to0} f_\sigma(x_i) = \begin{cases} 1 & if\ x_i \neq 0 \\ 0 & if\ x_i = 0 \end{cases} \tag{7}$$

which yields that

$$\|\boldsymbol{x}\|_0 = \lim_{\sigma\to0} F_\sigma(\boldsymbol{x}) \tag{8}$$

Consequently, the minimization l_0 norm problem (4) switches to the following problem

$$\min_{\boldsymbol{x}} F_\sigma(\boldsymbol{x}) = \sum_{i=1}^{N} (1 - \exp(-x_i^2 / 2\sigma^2) \quad s.t. \quad \boldsymbol{y} = \boldsymbol{Ax} \tag{9}$$

From (6), it is clear that the value of σ specifies a trade-off between accuracy and smoothness of the approximation. For small value of σ, $F_\sigma(\boldsymbol{x})$ contains lots of local extreme. Thus it will be tough to minimize this function for very small values of σ. Nevertheless, as the value of σ grows, the function becomes smoother and smoother, and hence the approximation accuracy of l_0 norm will be reduced. Therefore, SL0 algorithm solves a sequence of problems of the form

$$\min F_\sigma(\boldsymbol{x}) \quad s.t. \quad \boldsymbol{y} = \boldsymbol{Ax} \tag{10}$$

through constructing a progressively decreasing array $\sigma = [\sigma_1, \sigma_2, ..., \sigma_J]$ to optimize each corresponding objective function until σ_J is small enough, so as to eliminate the influence of the local extreme value and obtain the global optimal value of the smoothed function when parameter $\sigma = \sigma_J$.

B. Basic Idea of ISL0 Algorithm

SL0 algorithm adopts the classical first-order steepest descent method (also known as gradient descent method) to solve problem (10). Steepest descent method consists of iteration of the form $\boldsymbol{x} \leftarrow \boldsymbol{x} + l_k(-\nabla F_\sigma(\boldsymbol{x}))$, where steepest descent direction is the negative gradient of $F_\sigma(\boldsymbol{x})$. In addition, the step-size parameter l_k should be decreasing. Note that for smaller values of σ, the function $F_\sigma(\boldsymbol{x})$ is more "fluctuating", hence smaller step-sizes should be applied for the minimization.

Following [10], we set $l_k = t\sigma_k^2$ for some small positive constant t. However, this method still exists two drawbacks as follows.

a) Gradient is an extremely local pointer and does not point to the global minimum. This hill-climbing search is often in zigzag motion and may move towards a wrong direction, and we refer to this as "notched effect". As a result, the convergence speed will be strongly hindered in search direction and the accuracy of estimation for l_0 norm will be reduced correspondingly.

b) Search step-sizes cannot be estimated. Usually, it's estimated with experiences, which lacks theoretical support.

Slow convergence is typical of steepest descent direction. In convex optimization the Newton direction is often used as the search direction to overcome the drawback of steepest descent direction [13]. If the Newton direction is not necessarily a descent direction, then a modified Newton direction is preferred [14]. Motivated by these facts, we resort to the modified Newton method to solve the above problem.

The Newton direction of the objective function can be given as

$$\boldsymbol{d} = -\nabla^2 F_\sigma(\boldsymbol{x})^{-1}\nabla F_\sigma(\boldsymbol{x}) \tag{11}$$

where

$$\begin{aligned}\nabla F_\sigma(\boldsymbol{x}) &= \left[\frac{\partial f_\sigma(x_1)}{\partial x_1},\dots,\frac{\partial f_\sigma(x_N)}{\partial x_N}\right]^T \\ &= \left[\frac{x_1}{\sigma^2}e^{-\frac{x_1^2}{2\sigma^2}},\dots,\frac{x_N}{\sigma^2}e^{-\frac{x_N^2}{2\sigma^2}}\right]^T\end{aligned} \tag{12}$$

and

$$\nabla^2(F_\sigma(\boldsymbol{x})) = \begin{bmatrix} \frac{\partial^2 f_\sigma(x_1)}{\partial^2 x_1^2} & 0 & \cdots & 0 \\ 0 & \frac{\partial^2 f_\sigma(x_2)}{\partial x_2^2} & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \frac{\partial^2 f_\sigma(x_N)}{\partial x_N^2} \end{bmatrix} \tag{13}$$

with

$$\frac{\partial^2 f_\sigma(x_k)}{\partial x_k^2} = \frac{\sigma^2 - x_k^2}{\sigma^4}e^{-\frac{x_k^2}{2\sigma^2}} \tag{14}$$

To ensure that the Newton direction is descent direction, the Hessian matrix $\nabla^2(F_\sigma(\boldsymbol{x}))$ is required to be positive definite in Newton direction $\boldsymbol{d}$. In other words, all the diagonal elements in Hessian matrix should be positive. Thus the Hessian matrix shown in (13) should be modified to meet this requirement. Let

$$\boldsymbol{Q} = \nabla^2(F_\sigma(\boldsymbol{x})) + \varepsilon_k \boldsymbol{E} \tag{15}$$

where $\varepsilon_k (k = 1,\dots,N)$ is a group of appropriate positive numbers, $\boldsymbol{E}$ is identity matrix. Choosing $\varepsilon_k = \frac{2x_k^2}{\sigma^4}\exp(-\frac{x_k^2}{2\sigma^2})$ as modification coefficients, thus the diagonal elements

$$Q_{ii} = \frac{x_k^2 + \sigma^2}{\sigma^4}e^{-\frac{x_k^2}{2\sigma^2}}, \quad (i = 1,\dots,N) \tag{16}$$

of matrix $\boldsymbol{Q}$ are all positive. And the modified Newton direction can be obtained as

$$\begin{aligned}\boldsymbol{d} = -\boldsymbol{Q}^{-1}\nabla F_\sigma(\boldsymbol{x}) = \Bigg[&-\frac{\sigma^2 x_1}{x_1^2+\sigma^2},\dots, \\ &-\frac{\sigma^2 x_k}{x_k^2+\sigma^2},\dots,-\frac{\sigma^2 x_N}{x_N^2+\sigma^2}\Bigg]^T\end{aligned} \tag{17}$$

Define λ as the Newton step-size. Under normal circumstance, modified Newton method adopts full-Newton step-size in the iterative process, namely, $\lambda = 1$. However, when the selected initial value is far away from the optimal solution, the solution accuracy and the iterative convergence might not be guaranteed. To overcome these shortcomings, we would like to use variable Newton step-size rather than full-Newton step-size. This can be obtained by adjusting the step-size value of λ in accordance with some criterion that can provide an approximate measure of the adaptation process state. For solving the minimization problem (10), here, we let step-size λ meet the following inequation in the iterative process

$$F_\sigma(\boldsymbol{x} - \lambda\boldsymbol{Q}^{-1}\nabla F_\sigma(\boldsymbol{x})) < F_\sigma(\boldsymbol{x}) \tag{18}$$

It should be remarked that Newton direction $\boldsymbol{d} = -\boldsymbol{Q}^{-1}\nabla F_\sigma(\boldsymbol{x})$ is a feasible descent direction (see Theorem 1 of section 3.4) to the minimization problem (10), hence there must exist some step-size $\lambda > 0$ that satisfies (18). Thus we can attempt to start with an initial step-size $\lambda_0 = 1$, and check whether λ_0 satisfies (18). If λ_0 satisfies (18), let $\lambda = \lambda_0$. Otherwise, reduce the step-size λ_0 by backtracking along the modified Newton direction $\boldsymbol{d}$ until step-size satisfies (18).

The improved smoothed l_0 norm (ISL0) algorithm is as follows.

- Initialization: Let $\boldsymbol{x}^0$ equal to the minimum l_2 norm solution of $\boldsymbol{y}=\boldsymbol{Ax}$, namely, $\boldsymbol{x}^0 = \boldsymbol{A}^T\left(\boldsymbol{AA}^T\right)^{-1}\boldsymbol{y}$; choose a suitable decreasing sequence $[\sigma_1,\sigma_2,\dots,\sigma_J]$ for σ; initial residual $\boldsymbol{\beta}_0 = 0$.
- For $j = 1,2,\dots,J$

Step1 Let $\sigma = \sigma_j$, $\boldsymbol{x} = \boldsymbol{x}^{j-1}$.

Step2 Minimize (approximately) the function $F_\sigma(\boldsymbol{x})$ on the feasible set $\boldsymbol{\xi}=\{\boldsymbol{x}|\,\boldsymbol{y} = \boldsymbol{Ax}\}$ through modified Newton method.

1) Search direction:

$$\boldsymbol{d} = -\boldsymbol{Q}^{-1}\nabla F_{\sigma_j}(\boldsymbol{x}) = [-\frac{\sigma^2 x_1}{x_1^2+\sigma^2},\dots,-\frac{\sigma^2 x_k}{x_k^2+\sigma^2},\dots,-\frac{\sigma^2 x_N}{x_N^2+\sigma^2}]^T$$

2) Determine the search step-size λ which satisfies the following condition:

$$F_{\sigma_j}(\boldsymbol{x} - \lambda\boldsymbol{Q}^{-1}\nabla F_{\sigma_j}(\boldsymbol{x})) < F_{\sigma_j}(\boldsymbol{x})$$

3) Let $\boldsymbol{x} \leftarrow \boldsymbol{x} + \lambda\boldsymbol{d}$.

4) Project $\boldsymbol{x}$ onto the feasible set $\boldsymbol{\xi}$ with

$$x \leftarrow x - A^T(AA^T)^{-1}(Ax - y)$$

and calculate the residual $\beta = y - Ax$.

5) If the residual of the adjacent iterations meets $\|\beta - \beta_0\|_2 < \varepsilon$, then stop. Otherwise, let $\beta_0 = \beta$, and return to step2. Further decrease σ until σ is small enough.

Step3 Set $x^j = x$.

- Final answer is $\hat{x} = x^J$.

C. The Choice of Parameter σ

Usually, parameter σ is chosen as $\sigma_j = \rho\sigma_{j-1}$, $j = 2,...,J$, $\rho \in [0.5,1)$ is the decline factor of the decreasing array $[\sigma_1, \sigma_2, ..., \sigma_J]$. Generally, the sequence of σ is chosen with $\rho = 0.5$, and σ_1 and σ_J are chosen as follows. Let $\bar{x} = \max_i |x_i^0|$, for fast algorithm convergence, parameter σ should meet the following condition [15]:

$$f_\sigma(\bar{x}) = 1 - \exp(-\bar{x}^2 / 2\sigma_1^2) \le \frac{1}{2} \Rightarrow \sigma_1 \ge \frac{\bar{x}}{\sqrt{2\ln 2}} \quad (19)$$

For simplicity, we choose $\sigma_1 = \max_i |x_i^0|$. At the same time, as discussed in section 3.1, when $\sigma_J \to 0$, $F_{\sigma_J}(x)$ can better reflect the sparsity of vector x, but it is more sensitive to noise. Therefore, the minimum iteration value of parameter σ_J should not be too small. In addition, it also determines the termination process of ISL0 algorithm. Here the algorithm terminates under the condition that the value of parameter $\sigma_J < 10^{-3}$.

D. Convergence Analysis of ISL0 Algorithm

Theorem 1: When parameter $\sigma = \sigma_J$, the sequence generated by ISL0 algorithm for solving the minimization problem (10) converges.

Proof: According to the solving procedure of ISL0 algorithm, to prove the convergence of the sequence produced by the ISL0 algorithm, we only need to prove the sequence generated by modified Newton method for solving the minimization problem (10) converges when parameter $\sigma = \sigma_j$.

Since matrix Q is positive definite via modified Newton method, therefore its inverse matrix $G = Q^{-1}$ is still positive definite, namely

$$\nabla F_{\sigma_j}(x)^T G \nabla F_{\sigma_j}(x) > 0 \quad (20)$$

Let $d = -G\nabla F_{\sigma_j}(x)$, we have

$$\nabla F_{\sigma_j}(x)^T d < 0 \quad (21)$$

As a consequence, $-G\nabla F_{\sigma_j}(x)$ is a feasible descent direction for function $F_{\sigma_j}(x)$. Moreover, step-size λ satisfies the following condition

$$F_{\sigma_j}(x - \lambda G \nabla F_{\sigma_j}(x)) < F_{\sigma_j}(x) \quad (22)$$

As stated above, modified Newton method generates a decreasing sequence. Since $F_{\sigma_j}(x)$ is a continuous and differentiable function, then for a given initial value, the corresponding level set is bounded. Accordingly, the sequence generated by modified Newton method for solving the minimization problem (10) converges when parameter $\sigma = \sigma_j$. Thus Theorem 1 holds.

IV. Image Reconstruction Based on ISL0 Algorithm and Single Layer Wavelet Transform

In the traditional process of image processing based on CS, the multi-layer wavelet transform is usually adopted to make image sparse. However, as the reconstruction performance strengthens along with the increasing decomposition layers, the wavelet transform and inverse transform will bring out greatly increasing calculation correspondingly. Literature [16] has proposed an optional method based on single layer wavelet transform and OMP algorithm, which not only effectively reduces the amount of data that reconstructing image needs, but also obviously improves the reconstruction quality. Nevertheless, OMP algorithm requires a given sparsity of images and the reconstruction accuracy is still unsatisfactory.

In this section, we propose an image reconstruction method based on single layer wavelet transform and ISL0 algorithm. Inspired by literature [16], single layer wavelet transform is carried on the original image to obtain the low frequency sub-band LL_1 and the high frequency sub-bands $\{LH_1, HL_1, HH_1\}$. Note that the low frequency sub-band contains the most energy of image and non-sparse, which plays a crucial role in image reconstruction. While the high frequency sub-bands reflect the detailed information such as edges and textures, and are sparse signals. Therefore, we merely measure the high frequency sub-bands in the first level while the wavelet decomposing coefficients in low frequency sub-band should be retained.

The reconstruction method can be described as follows.

1) Implement single wavelet transform to the primitive image ($N \times N$), and then obtain the low frequency sub-band LL_1 and three high frequency sub-bands $\{LH_1, HL_1, HH_1\}$.
2) According to the sampling rate of the original image, determine M of the observation matrix, and then construct an $M \times N/2$ matrix A. Unlike literature [16], here we adopt scrambled block Hadamard matrix [17] instead of Gaussian random matrix for its simple structure, less storage space, good reconstruction performance and friendly hardware implementation. Thus matrix A can be written as

$$A = R_M W P_{N/2} \quad (23)$$

where R_M is an operator which chooses M rows of $WP_{N/2}$ uniform at random, the $N/2 \times N/2$ matrix $\boldsymbol{W}$ is a block diagonal matrix with the form

$$W = \begin{bmatrix} W_B & & & \\ & W_B & & \\ & & \ddots & \\ & & & W_B \end{bmatrix} \tag{24}$$

and $\boldsymbol{W}_B$ denoting the $B\times B$ Hadamard matrix, $P_{N/2}$ is a scrambling operator which reorders the $N/2$ columns of W randomly. Retain the low frequency sub-band without measurement, and take measurement on the high frequency sub-bands to get three measured value matrices.

3) For reconstruction, ISL0 algorithm is firstly employed to recovery the three measured value matrices. Then associates the high frequency sub-bands coefficients with the preserved low frequency sub-band coefficients, and take on wavelet inverse transform to get the reconstructed image.

V. Simulation Results

In this section, the performance of the proposed approach is experimentally verified and is compared with SL0, SP and OMP algorithms. Some classical gray images of size 256×256 are selected to conduct the simulation. All experiments are performed in MATLAB R2010a using an Intel 3.1GHZ processor with 4GB of memory and under Windows XP operating system.

A. Image Reconstruction Quality Comparison under Noiseless Condition.

In experiment, sym8 wavelet is selected as the sparsity basis, M is taken as 30, 40…, 100, and the reconstruction quality is evaluated by peak signal-to-noise ratio (PSNR). To ensure the experiment results without loss of generality, all the simulations are repeated 200 times to obtain the averaged statistical results of each algorithm. Fig. 1 is the reconstruction quality comparison of image Lena with different algorithms. It shows that ISL0 algorithm outperforms SL0, SP and OMP algorithms, and with the increase of M, the advantages become more obvious. Fig. 2 shows that the quality of the ISL0 algorithm has been improved and the details of image Lena are better reconstructed when M=50.

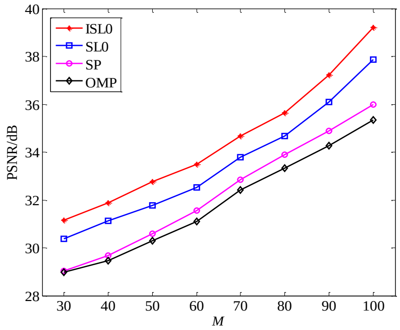


Fig. 1. Reconstruction quality comparison of four algorithms for noiseless image lena.

(a) Original image

(b) OMP
PSNR=30.28dB

(c) SP
PSNR=30.59dB

(d) SL0
PSNR=31.76dB

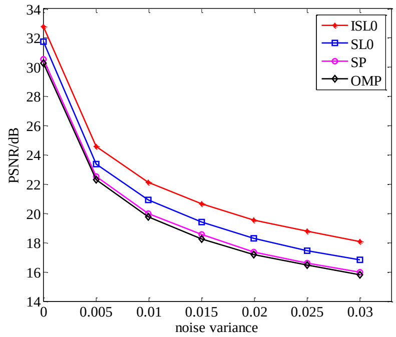
(e) ISL0
PSNR=32.75dB

Fig. 2. Reconstruction effect comparison of different algorithms for noiseless image Lena.

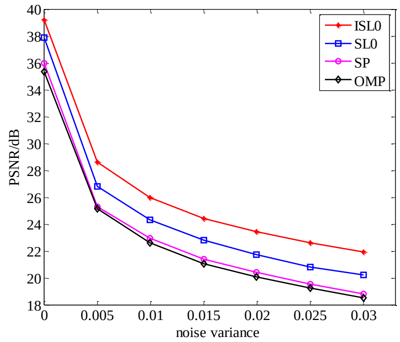
TABLE I: Reconstruction Quality Comparison with Different Images and *M*.

Image	Algorithm	PSNR(dB)			
		M=50	*M*=70	*M*=90	Avg
Peppers	ISL0	**33.49**	**35.58**	**38.20**	**35.76**
	SL0	32.70	34.73	37.13	34.85
	SP	31.75	33.61	36.13	33.83
	OMP	31.32	33.12	35.48	33.31
Boat	ISL0	**29.14**	**31.49**	**34.41**	**31.68**
	SL0	28.29	30.53	33.38	30.73
	SP	27.48	29.67	32.37	29.84
	OMP	26.97	28.94	31.45	29.12
Camera man	ISL0	**30.27**	**33.65**	**36.86**	**33.59**
	SL0	29.56	32.85	36.14	32.85
	SP	28.47	31.52	34.63	31.54
	OMP	27.99	30.85	33.83	30.89
Baboon	ISL0	**23.56**	**25.16**	**27.40**	**25.37**
	SL0	22.43	23.93	26.15	24.17
	SP	21.73	23.06	25.11	23.30
	OMP	21.31	22.58	24.49	22.79

Table I presents the PSNRs for another four natural images Peppers, Boat, Cameraman and Baboon with different M, and their average PSNR which is denoted as Avg. It can be seen that, the proposed ISL0 algorithm has the highest reconstruction quality, which increases as M increases. Besides, compared with SP and OMP algorithms, the reconstruction quality of SL0 algorithm has an average improvement of 0.87 to 1.96dB. Moreover, the reconstruction quality of ISL0 algorithm outweighs that of SL0 algorithm about 0.74 to 1.2dB in average.



(a) M=50



(b) M=100

Fig. 3. Reconstruction quality comparison of four algorithms for noisy image Lena.

(a) noisy image

(b) OMP
PSNR=22.31dB

(c) SP
PSNR=22.52dB

(d) SL0
PSNR=23.40dB

(e) ISL0
PSNR=24.57dB

Fig.4. Reconstruction effect comparison of different algorithms for noisy image Lena.

B. *Image Reconstruction Quality Comparison under Noise Condition.*

To verify the robustness of ISL0 algorithm under noise condition, imnoise function is employed to add white Gaussian noise to the tested images. Noise variance is taken as 0~0.03 in simulation. Fig. 3 gives the reconstruction quality comparison of image Lena under noise condition. Fig. 4 is the reconstruction effect comparison of noisy image Lena with different algorithms (where M=50, noise variance is 0.005), indicating that the ISL0 algorithm is better in noise robustness.

As shown in Fig. 3, the performance of four algorithms is in decline with the increase of noise variance. However, the downward trend of the curve corresponding to ISL0 algorithm is most gentle which reveals that ISL0 algorithm obtain the best reconstruction quality in cases of the same noise variances. Especially when M is comparatively large, the reconstruction performance superiority will be more apparent.

Table II and Table III show the reconstruction quality comparison for another four images under noise condition. It is clear from Table II and Table III that ISL0 algorithm performs better than other three algorithms, and the PSNR can be improved around 1.13~2.42dB in cases of M=50. On the whole, it shows that compared with other three classical algorithms, the proposed ISL0 algorithm provides better robust performance against noise.

TABLE II: RECONSTRUCTION QUALITY OF DIFFERENT ALGORITHMS WITH DIFFERENT IMAGES. (HERE NOISE VARIANCE IS 0.01, M=50)

Image	ISL0	SL0	SP	OMP
Peppers	**22.08**	20.84	20.03	19.65
Boat	**21.50**	20.32	19.49	19.14
Cameraman	**21.96**	20.77	19.95	19.58
Baboon	**19.80**	18.62	17.88	17.47

TABLE III: RECONSTRUCTION QUALITY OF DIFFERENT ALGORITHMS WITH DIFFERENT IMAGES. (HERE NOISE VARIANCE IS 0.02, M=50)

Image	ISL0	SL0	SP	OMP
Peppers	**19.47**	18.20	17.51	17.13
Boat	**19.04**	17.85	17.10	16.71
Cameraman	**19.34**	18.19	17.56	17.12
Baboon	**17.92**	16.79	16.04	15.63

C. *Average Running Time Comparison*

Fig. 5 shows the averaged reconstruction computation time of image Lena over 200 runs at different values of M.

Obtained results reveal that, for a large range of M, ISL0 algorithm and SL0 algorithm behave more efficient than the SP and OMP algorithms, and the complexity of ISL0 algorithm is slightly lower than that of the SL0 algorithm. Although ISL0 algorithm needs to calculate Hessian matrix and its inverse matrix, however, due to the fact that the objective function in (13) has a special nature that only the elements on the diagonal are non-zero, as a result, has no significant increase in calculation. In addition, SL0 algorithm adopts the classical first-order steepest descent method, which implies that the convergence speed is relatively slow. Thus the calculation speed of SL0 algorithm turns out to be slower than ISL0 algorithm for its better convergence speed by modified Newton method. Along with the increment of M, the advantage of ISL0 algorithm becomes more pronounced than SP algorithm and OMP algorithm, while the disparity between ISL0 algorithm and SL0 algorithm almost stays the same. More importantly, the proposed algorithm has improved the reconstruction performance without increasing the computation complexity or even slightly lower than that of SL0 algorithm.

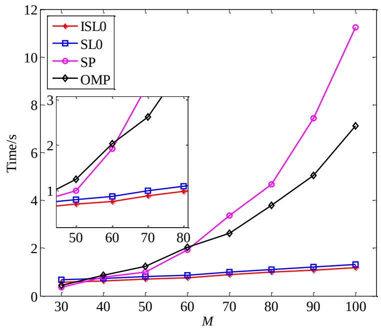


Fig. 5. Execution time comparison of different algorithms.

VI. Conclusion

In this paper, a new method is proposed to reconstruct image of CS, which is based on an improved smoothed l_0 norm algorithm and single layer wavelet transform. We have considered modified Newton method to avoid the influence of "notched effect" and combine variable step-size Newton method so as to accelerate the optimization and guarantee its convergence. Simulation results are presented which indicate that the proposed algorithm, comparing with other competing algorithms, improved both in terms of image reconstruction performance, noise robustness and execution time.

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