Design and Analysis of an OFDM-based Differential Chaos Shift Keying Communication System

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Abstract—In this paper, a new non-coherent chaos-based communication scheme, named orthogonal frequency division multiplexing based different chaos shift keying (OFDM-DCSK), is presented inspired by multi-carrier direct sequence code-division multiple access (MC-DS-CDMA) system, multi-carrier DCSK (MC-DCSK) system, and channel estimation of OFDM system. In this scheme, all of the occupied subcarriers are grouped into several groups. In each group, one chaotic reference is transmitted over the central subcarrier, while multiple modulated data streams are transmitted over the remaining subcarriers. Therefore, this transmitter structure increases the spectral efficiency, compared with the conventional DCSK system. At the receiver side, no radio frequency delay circuits are needed to demodulate the received data, which makes the system easy to implement in practice. The bit error rate (BER) performance of the OFDM-DCSK system is derived based on Gaussian approximation under additive white Gaussian noise (AWGN) channel. Simulation results are given under AWGN and two-path Rayleigh flat fading channel, which confirm the effectiveness of this new design.

Index Terms—Chaos communications, differential chaos shift keying, orthogonal frequency division multiplexing DCSK, non-coherent demodulation.

I. INTRODUCTION

Chaos-based communication has been extensively studied in recently years because chaotic signals have the following properties: non-periodic, deterministic, noise-like, wideband, and easy to generate. A number of chaotic modulation schemes were proposed, among which differential chaos shift keying (DCSK) was the most suitable one due to its good noise performance and simple transceiver requirement [1], [2].

In order to improve the spectral efficiency of the DCSK system, high-efficiency DCSK (HE-DCSK) was proposed in [3]. However, the receiver requires a radio frequency (RF) delay line, which is not easy to integrate in CMOS technology. In [4] and [5], the authors proposed a code shifted DCSK (CS-DCSK) to overcome the problem of RF delay. The reference and the information bearing signals are separated by Walsh code sequences and chaotic code sequences, respectively. H. Yang and G. Jiang proposed reference-modulated DCSK (RM-DCSK) for the purpose of low-complexity and high data rate. Mary DCSK had been studied in [6] as a multilevel version of DCSK to increase data rate, but the system complexity is fairly higher, compared with DCSK system. In [7], the authors proposed a multicarrier DCSK (MC-DCSK), which enhanced the energy-efficient, solved the RF delay problem, and improved the bit error rate (BER) performance. In [8], the authors proposed a non-coherent chaotic communication system based on orthogonal frequency division multiplexing (OFDM). In [9], the authors proposed a coherent chaotic direct-sequence spread spectrum communication system based on OFDM technique in combination with Mary phase shift keying modulation.

The feasibility of using chaotic communications in multiple-input and multiple-output (MIMO) channels had been proved in [10], in which Alamouti code was used as the space-time block code (STBC). And then the authors in [11] analyzed the BER performance of MIMO-DCSK system. However, extra hardware is needed for estimating the channel state information (CSI) to deal with the STBC decoding. In order to solve the CSI problem and reduce the implementation complexity, an analog STBC-DCSK scheme is proposed in [12], where two or three transmit antennas and single receiver antenna was considered.

Numerous contributions derived the analytical performance of DCSK system [13]-[15]. Gaussian approximation (GA) can provides good estimates of the BER for very large spreading factors [15]. However, when the spreading factor is low, GA suffers from a low precision [2]. Accurate method, which is based on chaos bit energy distribution, to predict the BER performance for DCSK system was given in [13] for additive white Gaussian noise (AWGN), Rice and Rayleigh channels. There are also lots of papers focused on the performance analyzing of DCSK system in cooperative communication, such as [16], [17].

Motivated by [7], we attempt to design a new transmit structure at the transmitter side. Recall that the chaotic signals are suitable for spread-spectrum communications, and thus we focus on the structure of multi-carrier direct sequence code-division multiple access (MC-DS-CDMA) [18]. Moreover, OFDM is a special case of multi-carrier. One of the key to the decoding of the OFDM codes is the accurate estimation of channel parameters [19]. The basic strategies for OFDM channel estimation are decision-
In Section III, we analyze the system bit error rate performance under AWGN channel with Gaussian approximation method. In Section IV, simulations are given for the proposed scheme under AWGN and two-path Rayleigh channels and followed by the concluding remarks in Section V.

II. OFDM-DCSK Scheme

Due to the paper limit, only the baseband implementation of OFDM-DCSK is described in this section.

A. Signal Format and Chaotic Generator

According to the 3GPP technical report [22], for a given transmission bandwidth \((B_w)\), the number of occupied subcarriers \((N_{sub})\) should be less than the FFT size \((N_F)\) for OFDM. Let \(N_{pilot}\) be the number of pilot subcarriers (reference signal subcarriers in this work), and \(N_{data}\) be the number of data subcarriers \((N_{pilot} + N_{data} = N_{sub})\). Inspired by the comb-type pilot pattern in OFDM, we propose a new design for DCSK communication system, named as OFDM-DCSK, in order to increase the data rate and solve the problem of RF delay.

In this paper, we first introduce a new design of OFDM-DCSK system. On the transmitter side, all of the occupied subcarriers are grouped into several groups (assume \(L\) groups). In each group, one subcarrier is assigned to transmit the reference slot, while the other subcarriers (assume \(M\) subcarriers) will carry the data slots. This design not only increases the data rate and saves the transmitted bit energy because one chaotic reference is used to transmit \(M\) bits, but also solves the RF delay problem. Then, we analyze the BER performance under AWGN channel with Gaussian approximation, which assumes that the correlate output follows the normal distribution. Moreover, numerous simulation results are given under AWGN and two-path Raleigh flat fading channels. Finally, we compare the accuracy of the BER expression with the numerical performance.

Different with [7], we propose an OFDM-DCSK scheme in this work, while in [7] the authors proposed multicarrier DCSK scheme. In fact, the idea of this work is motivated by [7]. Different with [9], no chaotic sequence production device is needed at the receiver end, which makes our design much more easier to imply in practice. Different with [8], we divided all of the occupied subcarriers into several groups. At the receiver end, the data are recovered via the reference signal (the middle subcarrier) in the current group, while in [8] all of the data are detected via the same reference signal (the first subcarrier). As well know that OFDM system can be considered as a wideband system. Each subcarrier forms a narrowband system and the adjacent subcarriers’ have similar channel gains. For AWGN channel, maybe all of the subcarriers in an OFDM symbol have similar channel gains, while for flat fading Rayleigh channel, different subcarriers have different channel gains. Thus, it is very important to divided all of the occupied subcarriers into several groups and the data are detected by interference signal in the current group, which is similar as the concept of resource block in 3GPP LTE.

Different with [7], an OFDM based structure is used in this work, which can enhance the performance in fading channel. Unlike the system structure in [7], we adopt the classical OFDM structure in 3GPP LTE standard, which makes it easier to apply in practice.

The remainder of this paper is organized as follows. In Section II, we propose the OFDM-based DCSK scheme. In Section III, we analysis the system bit error rate
0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1] are transmitted in this group. Let $N_{\text{sym}}=68$, $M=16$, $L=4$, $\beta=16$. The first group of the OFDM-DCSK is shown in Fig. 2.

![Fig. 2. An example of transmitted signal format](image)

The chaotic sequences are generated by the second-order Chebyshev polynomial function (CPF), which is given as $x_{j+1} = 1 - 2x_j^2, (-1 < x_j < 1, \neq \{0, \pm 0.5\})$. For the $l$-th group, let $\mathbf{R}_l = [x_{l,0}^j, x_{l,1}^j, \ldots, x_{l,\beta-1}^j]$ be the reference chaotic sequence. Different chaotic sequences are generating with different initial values. Moreover, all of the chaotic sequences are normalized, therefore, their mean values are all zero and mean squared values are unity, $E(x_j) = 0$ and $E(x_j^2) = 1$. For a large spreading factor, the correlation properties of the two chaotic sequences $\mathbf{R}_l$ and $\mathbf{R}_k$ can be approximated as $E[\mathbf{R}_k \cdot \mathbf{R}_l] = \beta$ for $k = l$, and $E[\mathbf{R}_k \cdot \mathbf{R}_l] = 0$ for $k \neq l$ with $\mathbf{R}_l \cdot \mathbf{R}_l$ denoting for the dot product of these two vectors.

![Fig. 3. Modulator](image)

### B. Transmitter Description

The transmitter structure of OFDM-CDMA is shown in Fig. 3. For each transmission time of a OFDM-DCSK frame, information bits $\mathbf{b} = [b_0, b_1, \ldots, b_{M\beta L}] \in \{0, 1\}$ are modulated with BPSK as $\mathbf{d}$. Then convert the series $\mathbf{d}$ into parallel data sequences. For the $l$-th subcarriers group, the parallel data sequences are given as $\mathbf{d}_l = [d_{l,0}, d_{l,1}, \ldots, d_{l,\beta-1}]^T$. Moreover, information bit $d_{m,l}$ is spread by chaotic reference sequence $x_l$ as $d_{m,0} = d_{m,1} = \ldots = d_{m,\beta-1}$ and allocated to the $m$-th subcarrier in the $l$-th group. Finally, the parallel sequences are converted into series sequences after the IFFT, and transmitted by wireless channels. Additive white Gaussian noise (AWGN) channel is considered in this paper.

### C. Receiver Description

The receiver structure is shown in Fig. 4. In our receiver, there is no RF delay circuit, compared with DCSK scheme, and no channel estimation or equalization, compared with OFDM system. Non-coherent detection method is used in this paper. The decoding process is similar for different subcarriers groups. Therefore, we focus on the first subcarriers group in the following part of this subsection. First, the output of the central subcarrier is stored in matrix $P$ and the other $M$ data signals are stored in the second matrix $S$. The matrix $P$ and $S$ are given as follows, respectively.

$$P = \begin{pmatrix} x_{0,0}^j + n_{0,0}^j & x_{1,0}^j + n_{1,0}^j & \cdots & x_{\beta-1,0}^j + n_{\beta-1,0}^j \\ x_{0,1}^j + n_{0,1}^j & x_{1,1}^j + n_{1,1}^j & \cdots & x_{\beta-1,1}^j + n_{\beta-1,1}^j \\ \vdots & \vdots & \ddots & \vdots \\ x_{0,\beta-1}^j + n_{0,\beta-1}^j & x_{1,\beta-1}^j + n_{1,\beta-1}^j & \cdots & x_{\beta-1,\beta-1}^j + n_{\beta-1,\beta-1}^j \end{pmatrix}$$

(1)

$$S = \begin{pmatrix} d_{1,0}^j & \cdots & d_{\beta-1,0}^j \\ \vdots & \ddots & \vdots \\ d_{1,M-1}^j & \cdots & d_{\beta-1,M-1}^j \\ n_{0,0}^j & \cdots & n_{\beta-1,0}^j \\ \vdots & \ddots & \vdots \\ n_{0,\beta-1}^j & \cdots & n_{\beta-1,\beta-1}^j \end{pmatrix}$$

(2)

where $n_{m,j}^j$ is the $j$-th sample of AWGN added to the reference signal, $n_{m,j}^l$ is the $j$-th sample of AWGN added to the $m$-th subcarrier of first subcarriers group.

![Fig. 4. Receiver structure](image)

After $\beta$ chip durations, all of the samples are stored for an OFDM-DCSK frame, and the decoding steps are activated. Finally, the transmitted $M$ bits are decoded in parallel via computing the sign of the resultant vector of the matrix product.
where \( \times \) is the matrix product and \( ' \) is the matrix transpose operator. In fact, Eq. (3) is equal to the following steps:

1) initial \( m = 1 \);
2) calculate the sum of the observation signal
\[
D_m' = \sum_{j=0}^{\beta-1} (d_j^1 x_j^1 + n_m^1_j) (x_j^1 + n_j^1) ;
\]
3) compare with the threshold zero, which is equal to
\[
d_m' = \text{sign}(D_m') ;
\]
4) if \( m=M \), stop; else go to step 2).

III. PERFORMANCE ANALYSIS

In this section, we first calculate the energy efficiency of the proposed OFDM-DCSK scheme. And then the bit error rate performance derivation under AWGN channel with Gaussian approximation method is given.

A. Energy Efficiency

Compared with the DCSK system, the energy efficiency of the proposed system is improved. Let \( E_{\text{data}} \) and \( E_{\text{ref}} \) be the energies to transmit the data and reference, respectively. For a conventional DCSK system, the transmitted bit energy \( E_b \), is given by
\[
E_b = E_{\text{data}} + E_{\text{ref}}.
\]
Assume the data and the reference have the same energies, which is \( E_{\text{data}} = E_{\text{ref}} = \sum_{j=0}^{\beta-1} x_j^2. \) Then, for a given bit, the transmitted energy \( E_b = 2E_{\text{data}} = 2\sum_{j=0}^{\beta-1} x_j^2. \) While in our OFDM-DCSK system, in each subcarriers group, one reference energy \( E_{\text{ref}} \) is shared with \( M \) transmitted bits, for a given bit, the transmitted energy \( E_b = E_{\text{data}} + E_{\text{ref}} / M = \frac{M+1}{M} E_{\text{data}}. \)

Define data-energy-to-bit-energy ratio (DBR) as
\[
\text{DBR} = \frac{E_{\text{data}}}{E_b}.
\]

For conventional DCSK system, the DBR is 1/2, which means that 50% of the bit energy \( E_b \) is used to transmit the reference. For OFDM-DCSK system, the DBR is \( M/(M+1) \). When \( M > 10 \), the reference energy accounts for less than 10% of the total energy \( E_b \) for each bit. Thus the energy efficiency of OFDM-DCSK system is increased, compared with DCSK system.

B. BER Derivation

We derive the BER performance of the OFDM-DCSK scheme over AWGN channels. Each subcarriers group has the same structure and each information-bearing subcarrier in the same group also has the same structure, thus we focus on the \( m \)-th, \( (m = 1,2,\ldots,M) \), information-bearing subcarrier of the \( l \)-th, \( (l = 1,2,\ldots,L) \), subcarriers group for BER performance derivation. Gaussian approximation (GA) method is employed in this work, which is a valid method for large spreading factors. Thus the distributions can be characterized via computing their mean and variance values. Due to the sensitive to initial conditions property, we can deduce that different chaotic sequences generated from different initial conditions are independent with each other. Moreover, according to [23], the Gaussian noise and the chaotic sequences are also independent with each other. Recall that the chaotic sequences are generated by CPF and normalized with zero mean and unity variance, and for a large spreading factor, the correlations between two chaotic sequences are \( E[R_k R_{k'}] = 0 \), for \( k \neq l \).

The observation signal \( D_m' \) for the \( m \)-th subcarrier of the \( l \)-th subcarriers group is given by
\[
D_m' = \sum_{j=0}^{\beta-1} (d_j^1 x_j^1 + n_m^1_j) \times (x_j^1 + n_j^1).
\]

where \( n_m^1_j \) and \( n_j^1 \) are two independent Gaussian noises coming from information-bearing bit and chaotic reference bit, respectively.

For easiness expression, (6) can be transformed as
\[
D_m' = \sum_{j=0}^{\beta-1} d_j^1 x_j^1 + \sum_{j=0}^{\beta-1} x_j^1 (d_j^1 n_j^1 + n_m^1_j) + \sum_{j=0}^{\beta-1} n_j^1 n_m^1_j.
\]

In (7), the first term is the useful signal, while the other two terms are zero-mean additive noise interferences.

Let \( E_b^i \) be the transmitted bit energy for a given data sequence in subcarriers group \( l \), which is given by
\[
E_b^i = \frac{M+1}{M} \sum_{j=0}^{\beta-1} x_j^1.
\]

Then, (7) can be written as follow
\[
D_m' = d_j^1 \frac{M+1}{M} E_b^i + W + Z.
\]

where
\[
W = \sum_{j=0}^{\beta-1} x_j^1 (d_j^1 n_j^1 + n_m^1_j),
\]
\[
Z = \sum_{j=0}^{\beta-1} n_j^1 n_m^1_j.
\]

For a given \( m \)-th subcarrier of the \( l \)-th subcarriers group, the mean and variance of the decision variable are given as follows
\[
E(D_m') = d_j^1 \frac{M+1}{M} E_b^i.
\]
\[
Var(D_m') = E(D_m'^2) - E(D_m')^2 = E\left(\left(\sum_{j=0}^{\beta-1} x_j^1 d_j^1 n_j^1 + \sum_{j=0}^{\beta-1} n_j^1 n_m^1_j\right)^2\right)\]
\[
= E\left(\sum_{j=0}^{\beta-1} x_j^1 n_m^1_j\right)^2 + E\left(\sum_{j=0}^{\beta-1} n_j^1 n_m^1_j\right)^2.
\]
Assume $n_{m,j}$ and $n_j'$ are wideband AWGN with zero mean and power spectral density of $N_0/2$. Recall that the Gaussian noise and the chaotic sequences are independent and different chaotic sequences are also independent with each other, (13) can be simplified as

$$\text{Var}(D_m') = \frac{M+1}{2M} E_n N_0 + \beta N_0^2 / 4.$$

Assume the bit energy is a deterministic variable, using (12) and (14), the bit error probability can be computed as

$$\text{BER} = \frac{1}{2} \Pr(D_m' < 0 | d_m' = +1) + \frac{1}{2} \Pr(D_m' > 0 | d_m' = -1)$$

$$= \frac{1}{2} \operatorname{erfc} \left( \frac{E(D_m') | d_m' = +1}{\sqrt{2 \text{Var}(D_m') | d_m' = +1}} \right)$$

$$= \frac{1}{2} \operatorname{erfc} \left( \frac{d_m' M+1}{M E_n} + \frac{\beta N_0^2}{4} \right)$$

$$= \frac{1}{2} \operatorname{erfc} \left( \frac{M+1 N_0}{M E_n} + \frac{(M+1)^2 \beta}{2} \left( \frac{N_0}{E_n} \right)^2 \right)^{\frac{1}{2}}$$

where $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$ is the complementary error function.

Recall that the other information bits have the same structure as the above one, thus the BER performance for OFDM-DCSK can be described as

$$\text{BER}_{\text{OFDM-DCSK}} = \frac{1}{2} \operatorname{erfc} \left( \frac{M+1 N_0}{M E_n} + \frac{(M+1)^2 \beta}{2} \left( \frac{N_0}{E_n} \right)^2 \right)^{\frac{1}{2}}$$

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, the simulation results are given for OFDM-DCSK system over AWGN channels and two-path Rayleigh channels. The parameters of simulation are set according to the standard of 3GPP LTE [22], part of which are given as Table I.

<table>
<thead>
<tr>
<th>Table I: Part of Simulation Parameters</th>
</tr>
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<tbody>
<tr>
<td>Bandwidth (MHz)</td>
</tr>
<tr>
<td>FFT size</td>
</tr>
<tr>
<td>$N_{\text{ref}}$ (3GPP)</td>
</tr>
<tr>
<td>$N_{\text{sub}}$ (this work)</td>
</tr>
<tr>
<td>$N_{\text{total}}$</td>
</tr>
<tr>
<td>$N_{\text{data}}$</td>
</tr>
</tbody>
</table>

Fig. 5 shows the simulation and BER expression for different FFT sizes with multicarrier number $M = 16$ under AWGN channels. It can be shown that the performances of OFDM-DCSK system almost the same for different FFT size. This conclusion satisfies the BER expression in (16).

In Fig. 6, the simulation results, the BER performance in theory of OFDM-DCSK system, and the BER performance in theory of DCSK system are given for different spread factor $\beta$ with multicarrier number $M = 36$ under AWGN channels. It can be seen that the OFDM-DCSK shows different with DCSK system. For a given $E_b/N_0$, as spread factor $\beta$ increase, the BER performance gets better first and then becomes worse for DCSK system, while for OFDM-DCSK system, the BER performance is becoming worse and worse.

In Fig. 7, we compare the performance of OFDM-DCSK from the simulation results and in theory.
with the DCSK system under AWGN channels. The OFDM-DCSK system has better performance, compared with DCSK for different spread factor $\beta$. For OFDM-DCSK system, the performance gets worse with the increasing of spread factor $\beta$, which is similar with DCSK system or MC-DCSK system. It can be seen that the performance gets better as the increase of the multi-carrier number $M$. It has the same trend with different $E_b/N_0$. Fig. 8 shows the simulation results for a given $E_b/N_0$ with different multicarrier number $M$.

Therefore, it should have better performance and can be used in practice.

V. CONCLUSIONS

In this paper, we have designed and analyzed an energy efficient non-coherent OFDM-DCSK system. The performance of the proposed system is studied, and the BER expression is derived for AWGN channel. Simulation results match the theoretical BER expression. Compared with DCSK system, 1) increase the energy efficiency $DBR$ from 1/2 to $M/(M+1)$, where one chaotic reference signal should be transmitted for each information bit, while $M$ bit information share one chaotic reference signal in the OFDM-DCSK system; 2) simulation results show an increase in performance as compared with the same spread factor $\beta$; 3) solve the radio frequency delay problem not only in DCSK system, but also in HE-DCSK or CS-DCSK systems. Compared with OFDM system, no channel estimation is needed at the receiver side and no CSI feedback is needed to the transmitter side. Compare with the OFDM-CSK in [8], we can obtain better BER performance under two-path Rayleigh channels. It is much easier to apply in practice.

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REFERENCES


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