# An Efficient Construction and Low Complexity Collaborative Decoding of Reed-Solomon Concatenated with Modified Polar Codes

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***Abstract*** —Polar codes are the first explicit code sequences that provably achieve capacity for discrete memory-less channels. However, their performance with successive cancellation (SC) decoding is unimpressive at practical block-lengths. In this paper, we propose a scheme of concatenating Reed-Solomon (RS) with modified polar codes to improve the performance of SC. Firstly, short repetition block codes are used as outer block codes to modify the construction of polar codes. This step can improve the performance of SC with a lower complexity compared with the other schemes. And secondly, to reduce the probability of the error propagation of SC, the modified polar codes are concatenated with RS codes. We divide the decoding process of the polar codes into several pieces. After one piece is decoded, the RS decoder is used to correct the residual errors from polar codes in this piece. By this collaborative method, the number of the error propagation can be reduced obviously. Simulation shows that the proposed scheme can obtain a good error-rate performance at finite block lengths and have a significantly better error-decay rate with a lower complexity.

***Index Terms***—Polar codes, repetition codes, successive cancellation decoding, reed-solomon codes, concatenated codes

## I. Introduction

Polar codes, proposed by Arikan [1], are proved to achieve the symmetric capacities of the binary-input discrete memory-less channels (B-DMCs) under low-complexity $O(N\log N)$ encoding and successive cancellation (SC) decoding , where N is the block length. Although polar codes have astonishing asymptotic performance, the finite-length performance of polar codes under SC decoding is not satisfying.

Due the poor minimum Hamming distance of polar codes, optimizing its decoding algorithm has been the subject of various work, e.g. [2]-[5]. Although polar codes can achieve the ML performance by the SC-List(SCL) decoding [2], the SCL decoders are not suit to be used in practical applications since their long latency and low throughput problems caused by theinherent serial nature of SC [4]. Another method of concatenating polar codes with RS codes proposed in [5] has shown the possibility of improving the bound on the error-decay rate of polar codes. Unfortunately, this work used a conventional method, which required the cardinality of the outer RS codes alphabet to be exponential in the block length of the inner polar codes. Hence this method is infeasible for implementation in practical systems. In [3], a new scheme for concatenating binary polar codes with interleaved RS codes is proposed to capture the capacity-achieving property of polar codes, while having significantly better error-decay rate. However, the complexity of Generalized minimum distance (GMD) with Maximum-Likelihood (ML) decoding for the outer codes in [3] is slight higher, which increases the complexity of the polar decoder by a constant factor of $\sum_{i=0}^{t-1} s^i / t$ , $2^t$ is the cardinality.

In this paper, we use RS codes to concatenate with modified polar codes to improve the performance of the polar codes while reserving a lower decoding complexity and latency. It is known that there exist some unpolarized channels in the construction of polar codes. But these channels may be chosen as the information channels according to a design rate. However, they can make the performance of SC decoding worsen. In the proposed scheme, firstly, we construct some short repetition block codes to protect those less reliable information bits of the polar codes. This method can improve the performance of SC with a lower complexity. Then, the modified polar codes are concatenated with RS codes. We also provide efficient collaborative decoding algorithm to further reduce the probability of the error propagation of SC. Simulation shows that the proposed scheme can significantly improve the error-rate performance at finite block lengths and have a better error-decay rate with a lower complexity.

## II. Preliminaries of the Proposed Scheme

### *A. Polar Coding and Successive Cancellation Decoding*

Let $W$ denote a binary input discrete and memory-less channel with input $u \in \{0,1\}$ , output $y$ which may be arbitrary, and transition probabilities $W(y\,|\,u)$ . The polar coding can be described as follows: a binary source block

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$u_0^{N-1}$ which consists of $K$ information bits and $N-K$ frozen bits is mapped to a code block $X_0^{N-1}$ via $X_0^{N-1} = u_0^{N-1} G_N$. The matrix $G_N = B_N F^{\otimes n}$, where $B_N$

$$\mathbf{F} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

is the bit-reversal permutation matrix.

In [1], Arikan proposed a low-complexity implementation of the SC decoder of polar codes. Let $\hat{u}_0^{N-1}$ denote the estimated value of the source block $u_0^{N\text{-}1}$. For $i = 0,1,2,...,N-1$, the decoder computes the likelihood ratio (LR) $L_N^{(i)}$ of $u_i$ given the channel output $y_0^{N-1}$ and previously decoded value $\hat{u}_0^{i-1}$. The likelihood functions $L_N^{(i)}$ can be computed recursively as following (1)

$$L_N^{(i)} = \frac{W_N^{(i)}(y_0^{N-1}, \hat{u}_0^{i-1} \mid u_i = 0)}{W_N^{(i)}(y_0^{N-1}, \hat{u}_0^{i-1} \mid u_i = 1)} \tag{1}$$

If $i$ is not an information-bit index, the decoder knows that the bit $u_i$ is set to $0$. Otherwise, it makes the hard decision based on the value of $L_N^{(i)}$. The above decoding process can be regarded as a greedy search algorithm on a code tree. But, in each level, only the one of two edges with larger probability is selected out for further processing. The process is shown in Fig.1. As a result, the SC decoder cannot find the truth path with the largest metric in many cases.

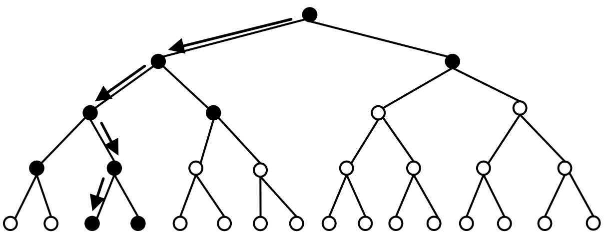

Fig. 1 Searching process of SC decoder over a code tree with $N=4$ and $K=4$.

### B. The RS-Polar Scheme with Interleaving

In [3], a scheme for concatenating polar codes with outer RS codes is proposed, which can be illustrated in Fig. 2 . But different from the conventional method of concatenation, the scheme uses the method of concatenating the polar codes with interleaved block codes to improve the error-decay rate of polar codes, which avoids making the cardinality of the outer RS code alphabet be exponential in the block length of the inner polar code [5].

In order to further improve the performance, SC-GMD-AML decoding [3] is used to decode the RS-polar concatenated codes. But, this complexity increases along with the increase of the block length of the outer RS codes.

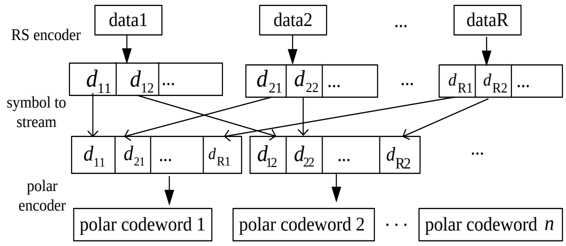


Fig. 2. The scheme of polar codes with outer interleaved RS codes

## III. AN EFFICIENT CONSTRUCTION AND LOW COMPLEXITY COLLABORATIVE DECODER

### A. The Construction Method of RS Codes Concatenated with Polar Codes

Since RS codes are maximal distance separable codes, they have the largest bounded-distance error-correction capability at a specified code rate and excellent burst error-correction capability. Due to the two advantages of the scheme in [3], we still choose RS codes as the outer code of the proposed scheme. The method of construction is as follows.

Let $n$ and $m$ denote the block lengths of the polar codes and RS code, respectively. Let $k$ denote the number of information bits of each polar codeword. The symbols of all RS code-words are drawn from the finite field $F_2^t$, with cardinality $2^t$. Here, $k$ is supposed to be divisible by $t$. The message sent to the channel is divided into a $p \times q$ matrix. Every row of the matrix is encoded by the RS encoder into an $m$-length RS codeword. So, the matrix becomes a $p \times m$ matrix. Before we use polar encoder to encode every column into an $n$-length polar code, each symbol of the RS codeword should be mapped to its binary image with $t$ bits. Therefore, the rates of RS codes and polar codes are $q/m$ and $k/n$ respectively, where $k = pt$. In this scheme, the numbers of RS and polar code-words are is $p = k/t$ and $m$, separately. Hence, the total length of the sent data is $N = nm$. The above method of construction is shown in Fig. 3.

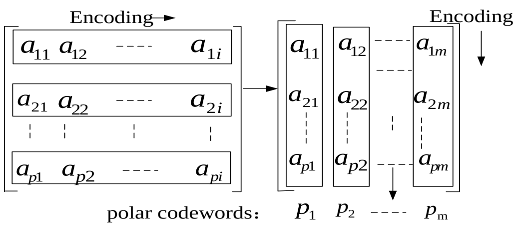


Fig. 3 The construction method of concatenating modified polar codes with RS codes.

It is known from the channel polarization theory of polar codes that not all of the selected information channels have the same performance. Fig.4 shows that some of the information channels are very strong and almost noiseless channels, while others are weaker channels. This implies that an unequal protection is needed. In this paper, we select some more reliable frozen bits as [7] to protect those less reliable information bits. Next, we need to modify the construction of the polar codes.

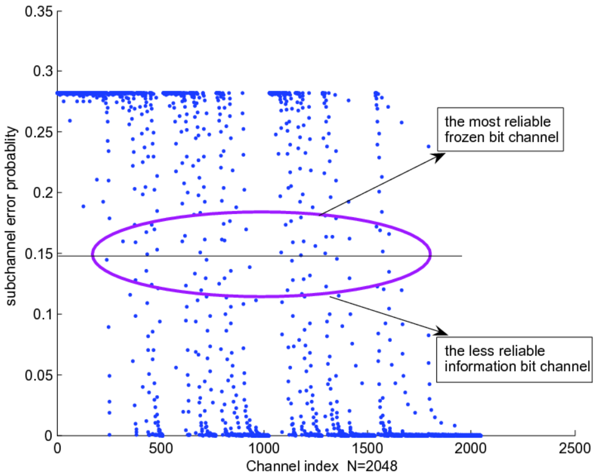


Fig. 4 Plot of $Z(W_N^i)$ versus $i=1,2,...,N=1024$ over BEC $\varepsilon=0.5$ construction.

### B. *The Construction of Modified Polar Codes*

In [1], the first important step of constructing capacity-achieving polar codes is to choose good bit-channels according to the mutual information values of channels *W*. The smaller the value of this parameter, the least reliable the channel is. So, firstly, the mutual information values of channels W should be computed. Methods for approximating the mutual information values in more general cases are described in [1], [8]-[11]. In this scheme, we use the following (2) based on Gaussian approximation [8] and [12] to compute the error probability of each channel in additive Gaussian white noise (AWGN) channel ,

$$\pi_i \approx Q(\sqrt{E[L_{n-1}^{(i)}]/2}) \tag{2}$$

where the parameter $L_{n-1}^{(i)}$ is the expected value of the LLR message from channel *W*. According to the design rate, the channels with smaller error probabilities are selected out, the remaining channels are used as frozen channels.

Since every symbol of a RS codeword is mapped to its binary image with t bits, the information bits of a polar codeword can be divided into $k/t$ groups. So, the number of information bits in each group is $t$ bits. The modified construction method of polar codes consists of the following four steps.

Step 1: Find the least reliable information-bit channel according to the error probabilities of channels from each information-bit group respectively. This step needs to compare the error probabilities of channels between the $t$ information-bit channels.

Step 2: In each group, we should find the more reliable frozen-bit channel which index should be larger than that of the least reliable information-bit channel. Otherwise, the frame error probability (FER) performance of polar codes will be deteriorated.

The above two steps can be illustrated as algorithm $1$, where $t$ is set to $4$ and so the number of information bits of a sub-block is $4$.

Step 3: In each group, the chosen frozen-bit and information-bit are used to construct a repetition block by setting the value of the chosen frozen-bit to be of the chosen information-bit, respectively. If the frozen bit and information bit satisfied the conditions are not found in a group, the stage can be ignored directly.

Step 4: The source block $u_0^{n-1}$ which includes repetition blocks is used as input to the encoder of polar codes to obtain a codeword.

In order to reserve the total rate of the modified polar codes to be unchanged, we enlarge the set of information symbols by using the chosen frozen bit channels after several repetition block codes constructed. So, from the above process, it can be seen that the rate and block length of the modified polar codes are both unchanged.

**Algorithm 1** Finding Two Bit Indexes in a Sub-Block

**Input**:
- $i_{block}$: a sub-block,
- $num$: number of bit indexes in $i_{block}$ ,
- $chosenI[3]$: 4th information-bit index in $i_{block}$ ,
- $chosenI[i_{block}\times 4+3]$ : 4th information-bit index of $i_{block}$ ,
- $chosenI[i_{(block\text{-}1)}\times 4+3]$: the first bit index of $i_{block}$ ;

**Output**:
- $p_1$ : least reliable information-bit index,
- $p_2$: most reliable frozen-bit index;

```
    Initialization
choose p1 from the sub-block ;
if (the first sub-block) then
    if( num<4 ) then
        continue ;
    else
        if( (chosenI[3]- p1 ) >1) then
            find the first frozen-bit index F1 from p1 to chosenI[3] ;
            find p2 from p1 to chosenI[3] ;
        else
            continue ;
        endif
    endif
else
    if ((chosenI[i_block ×4+3]-chosenI[i_(block-1) ×4+3]-1)<4) then
        continue ;
    else
        find the index F1 from p1 to chosenI[i_block ×4+3] ;
        find p2 from p1 to chosenI[i_block ×4+3] ;
    endif
endif
```

### C. *Collaborative Decoding of RS-Polar*

As we know, SC algorithm of the polar codes makes decisions on the bits one-by-one in a pre-chosen order. In particular, in order to decode the *i*-th bit, the algorithm uses the values of all the previous $i-1$ bit decisions. Therefore, there is no chance to correct it in the future decoding procedure once a bit is wrongly determined. In this paper, since the information block of the polar code is protected with a RS code, the RS code can be used to correct the errors in the decoded bits of polar codes by SC evolving. This can potentially mitigate the error propagation problem and consequently results in improvement at finite block lengths. Compared with [3], the decoding algorithm is modified as following five stages.

Stage 1: For each of m polar code-words, the decoder firstly divides the received block into $k/t$ groups. The length of the information bits in each group is $t$. So, the decoder begins to decode the bits from the first bit until after decoding the *t*-th information bit. After decoding the first $t$ information bits from each polar code-word, the decoder obtains mt hard-decision information bits.

Stage 2: Convert the $mt$ hard-decision output bits into $m$ symbols over $F_{2^t}$ to form a RS codeword and pass it to the bounded-distance RS decoder.

Stage 3: Encode the output of the RS decoder, then send the new RS codeword which should be converted into binary form to the SC decoder.

Stage 4: Update the just decoded $t$ information bits of each polar codeword using the new obtained RS codeword, and then continue SC decoding.

Stage 5: Increase $j$ by one and repeat, while $j < k/t$.

The above decoding process is shown as Fig. 5.

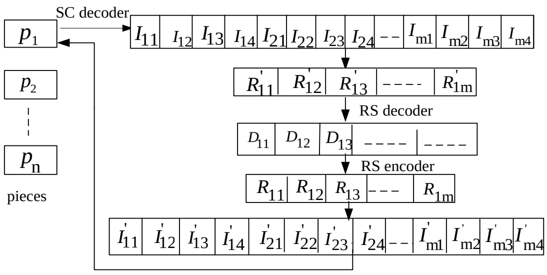


Fig. 5. Collaborative decoding of Reed-Solomon concatenated with modified polar codes, $t = 4$.

### D. Decoding Algorithm of Modified Polar Codes

For decoding of the modified polar codes, the original SC algorithm needs to be modified slightly. In each group of the received block, we assume that two source bits with indexes $g_1$ and $g_2$ are protected by a short block code, for some $g_1 > g_2$. Let $g_0$ denote the index of the first bit in a group. The decoding process is described as following four stages.

Stage 1: Decoding between the bits $u_{g_0}$ and $u_{g_1}$

When decoding the bit $u_i$ with the index $i$ smaller than $g_1$, the SC decoder is performed as described in [1] to decide the values of these bits.

Stage 2: Decoding between the bits $u_{g_1}$ and $u_{g_2}$

Let $M_{(n-1)}^{(i)}(u_i = \hat{u}_i \mid y_0^{(n-1)}, \hat{u}_0^{(i-1)})$ denote the conditional probability of the given estimated value $\hat{u}_i$. When decoding the bit $u_{g_1}$ which is protected by a short block code, instead of taking a hard decision on $u_i$, the decoder creates two decoding paths with $\hat{u}_{g_1} = 0$ and $\hat{u}_{g_1} = 1$.. For those bits which indexes are smaller than $g_2$, the decoder continues performing the original SC decoding rule along the two branches, respectively. On reaching the bit $u_{g_2}$, the estimated value of this bit in each of the branches depends on the start estimated value of the bit $u_{g_1}$. And the corresponding metric $M_{(n-1)}^{(i)}(u_i = 0 \mid y_0^{(n-1)}, \hat{u}_0^{(i-1)})$ or $M_{(n-1)}^{(i)}(u_i = 1 \mid y_0^{(n-1)}, \hat{u}_0^{(i-1)})$ of each path is recorded respectively. The bit sequences of the two branches can be denoted as the following (3) and (4)

$$s_0 = 0,\ \hat{u}_{(0,g_1+1)},\ \hat{u}_{(0,g_1+2)},\ \cdots,\ \hat{u}_{(0,g_2-1)},\ 0 \tag{3}$$

$$s_1 = 1,\ \hat{u}_{(1,g_1+1)},\ \hat{u}_{(1,g_1+2)},\ \cdots,\ \hat{u}_{(1,g_2-1)},\ 1 \tag{4}$$

Stage 3: Select the more likely of the two sequences. The probability of the $s$ th path $P_s$ is calculated as (5)

$$P_s = \prod_{r=g_1}^{i} M_n^{(r)}(1 \oplus u_r \mid y_0^{n-1}, \hat{u}_0^{r-1}) \qquad s = 1, 2 \quad and \quad g_1 \le i \le g_2 \tag{5}$$

where $u_r = 0$ or 1 is determined by the $s$-th decoding path.

Stage 4: Decoding after the bit $u_{g_2}$.

After decoding the bit $u_{g_2}$, the branch with the highest probability is selected out as the output bit sequences from the index $g_1$ to $g_2$. The other path is discarded. Finally, the bits after the bit $u_{g_2}$ are determined by the original SC algorithm until after decoding the last information bit in the group. The decoding idea is visualized in Fig. 6 for a simple example code with $u_{20} = u_{25}$

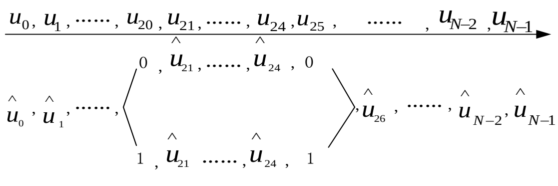

Fig. 6. The modified SC decoding scheme of polar codes.

Using the above decoding algorithm, not only the FER of the RS-Polar code will be potentially improved, but also the decoding latency can be significantly reduced because of the parallel processing in decoding those $m$ code-words of polar codes.

### E. Decoding Complexity

In the proposed scheme, the decoder of the polar code performs as conventional SC decoding, and creates a new branch only when reaching the bits protected by a short block code. Hence, the decoding complexity of modified polar using short block codes becomes at most to be doubled since we exclude some overlapping blocks. So, the complexity of decoding $m$ inner polar codes is $O(2nm\log n)$. The proposed scheme has the same complexity in decoding $n$ RS codes with SC-GMD-ML scheme. While, SC-GMD-ML decoding increases the complexity of polar decoder by a constant factor of $\sum_{i=0}^{t-1} 2^i/t$ because the decoder should compute the probabilities of all the $2^t$ symbols by traversing all the possible $2^t$ paths, for each consecutive $t$ bits. Hence, the proposed scheme has a lower complexity than SC-GMD-ML decoding [3].

## IV. Simulation Results

Transmission over AWGN channel is assumed. We firstly observe the FER performance of the modified polar coding scheme not concatenated with RS codes and compare it with the performances of original SC and SCL. Fig. 7 shows the simulation results for the three schemes of polar block length $N = 256$, $N = 1024$ and $N = 8192$. It can be seen that the decoder of modified polar coding scheme obtains a constant coding gain of approx 0.25dB over the original polar coding with SC decoding at all SNR regions. This is different from SCL decoder which achieves significant gains in the low-SNR regions, while the advantage vanishes with increasing SNR. And also, the recent results in other references show the space of FER performance improvement under SCL decoding is not significant with list size increasing. But, the decoder of the modified polar coding scheme can outperform even a list decoder with larger list size or an ML decoder in high SNR regions with lower complexity [14]. Therefore, the beneficial effects can be achieved when this scheme is combined with the other scheme.

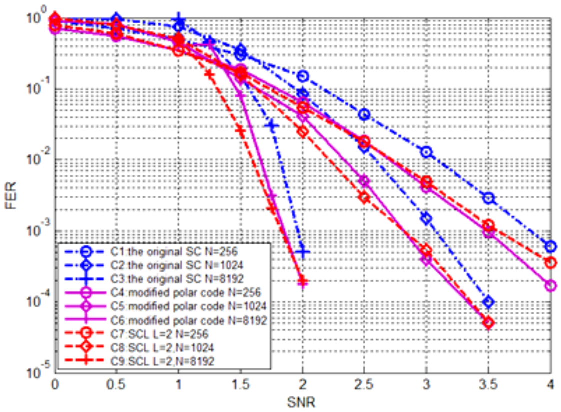


Fig. 7. The comparison performances of the polar code concatenated with repetition block codes between several block lengths.

In our simulation, the block length of inner code is a 512-length modified polar code and the outer code is 15-length RS code over $F_2^4$, where $t = 4$. The total rate of the scheme is set to 1/3. The rate of the modified polar codes is $r_p = 0.39$. So, the length of the total code block in this scheme is 7680. To construct the inner code, the 51 symbols with the same position in every RS code-words are first converted into binary image with 4 bits separately. Then the 204 bits are input as information bits to an encoder of the polar codes. Following that, the 512-bits indexes of polar codes are divided into 51 sub-blocks. And in each sub-block, there are 4 information bits. Then, the information-bit index with the highest probability and the chosen-bit index with the lower probability in each sub-block are selected out respectively. In our simulation, the chosen information-bit indexes and their corresponding chosen frozen-bit indexes are shown in the following (6).

$$\begin{aligned} I_{index} &= \{119,175,229,310,339,357,391,403,406\} \\ F_{index} &= \{122,180,232,312,340,360,396,404,408\} \end{aligned} \tag{6}$$

According to the Section 3.2, we set the values of the chosen frozen bits to be the values of the corresponding chosen information bits to construct 9 repetition short blocks. Then, these information bits are encoded by the polar encoder into a code-word. Following the same idea, we can obtain total 15 code-words of polar codes. From the above construct process, we can see that the repetition short blocks in the inner polar codes can protect those least reliable information bits by more reliable frozen bits. This scheme can improve the performance of the polar codes but with a quite smaller increase of complexity. And also, the rate and length of the polar codes remain unchanged. On the other hand, the outer RS codes used gives the protection to the inner polar codes since RS codes are maximal distance separable codes and they have excellent burst error-correction capability.

When decoding the polar code-words, the decoder does not stop performing the decoding operation until after decoding 4 information bits. In this sub-block, the decoder starts as the original SC operation. When decoding the chosen information bit, for example, the bit with index 119, two paths are created from this bit. Following the two paths respectively, the decoder continues decoding the next bits until meeting the chosen frozen bit with index 122 in this sub-block. At the end of each path, the decoder set $\hat{u}_{122}$ to be equal to $\hat{u}_{119}$, respectively. And then, from the two paths, the path with the highest probability is selected out as the estimated values from the bits $u_{119}$ to $u_{122}$. Following the same above decoding process, the decoder operates others polar code-words. Then the 4 information bits in a sub-block are converted into a symbol in the finite $F_{2^4}$, the 15 symbols together are seen as a RS code-word and sent to the RS decoder.

In the proposed scheme, since the block size of the outer RS code is only 15 and all the 15 polar codes in the concatenated code can be decoded in parallel, the decoding latency of the concatenated code is dominated by the inner polar codes. The comparison performances between the new scheme and other schemes are shown in Fig. 8 and Fig. 9.

Here, we compare the proposed scheme with the other schemes under two cases.

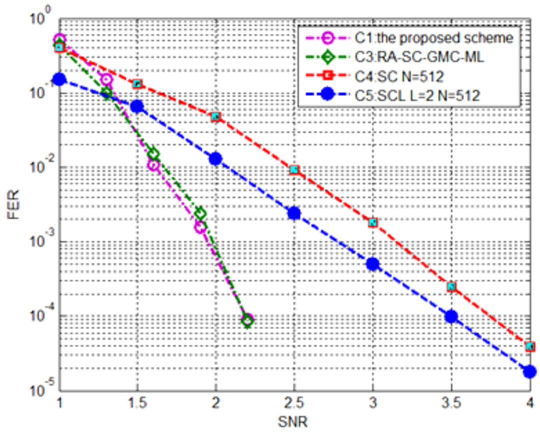


Fig. 8. The comparison of FER performance between several schemes under the same decoding latency.

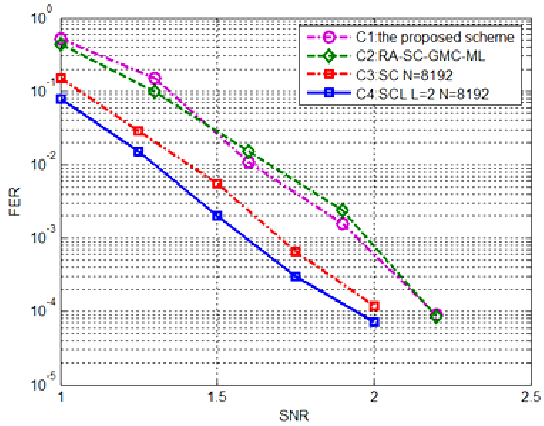


Fig. 9. The FER performance comparison of several schemes under the almost same block length.

One case is under the same decoding latency. From the above analysis, it can be seen that the proposed scheme has the same decoding latency with the original polar codes when the block length of the inner polar codes in the proposed scheme amounts to the block length of the original polar codes. Here, we choose 512-length inner polar codes and the total code 1/3-rate in our scheme to compare with the 512-length and 1/3-rate original polar codes. From the Fig. 8, we can see that the FER performance of the proposed scheme is better than SCL which list size $L$ is 2 but only with a half decoding latency. At the FER of $10^{-4}$, the proposed scheme has more than 2dB SNR gain over the 512-length and 1/3-rate original polar codes under SC decoding at the same decoding latency. And also, Fig. 8 shows the proposed scheme has almost performance with the RA-SC-GMD ML scheme [4].

The other case is under the same block length, where we consider the almost same block length between different schemes. In the simulation, the total block length of the proposed scheme is $512\times15=7680$. The 8192-length original polar codes with the same rate under SC and SCL are used to compare with the proposed scheme.

The result is the polar codes outperform the proposed scheme and RA-SC-GMD ML scheme [4] with about 0.2dB at the FER of as shown in Fig. 9. One of the reasons of the result is that block length of the proposed scheme is a little less than that of the original polar codes. Another side, the original polar codes need $13\times2^{13}=1.06\times10^{5}$ operations of LLR combinations in the decoding process. While, the LLR operations of the proposed scheme include at most $2\times9\times15\times2^{9}$ LLR operations from the inner polar decoders and $15^{2}\times33$ operations over $GF(2^{4})$ from the outer RS decoders. Hence, the decoding complexity of the original polar code is higher than the proposed scheme. Although the original of polar codes obtains almost 0.2dB performance gains lower complexity exponent to obtain the same performance with the original polar codes by using the proposed scheme. So, the propose scheme is more practical. From the Fig. 9, SCL has a better performance than the new scheme, but, we think the decoding latency and decoding complexity of SCL become unbearable with the list size increased.

From Fig. 8 and Fig. 9, we can see that the proposed scheme has almost the same performance with RA-SC-GMD ML scheme. The advantage of the new scheme is that the total complexity is lower. While SC-GMD-ML decoding increases the complexity of polar decoder by a constant factor of $\sum_{i=0}^{3}2^{i}/4$ because the decoder should compute the probabilities of all the symbols by traversing all the possible 16 paths, for each consecutive 4 bits. So, the proposed scheme has lower decoding complexity than RA-SC-GMD ML scheme.

## V. Conclusions

In this paper, we use the polar codes as inner codes and RS codes as outer codes to construct the concatenated codes. Different from other existing schemes, we modify the construct method of the inner polar codes. Since a proportion of some bit-channels are neither completely "noiseless" nor completely "noisy" in the polarized process of the polar codes, some frozen bits are chosen to transmit check information for those less reliable information bits. Therefore, in the proposed scheme, we firstly choose some less reliable information bits and more reliable frozen bits to construct some simple short blocks. By using this method, the performance of the inner polar codes is improved with a lower complexity. And, in order to further decrease the probability of error propagation of SC, we design a collaborative decoding method of RS codes and modified polar codes. Comparing the scheme in [4], the proposed scheme has the almost performance with the scheme under ML decoding but with a lower decoding complexity.

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