A Novel Collaborative Partner Selection algorithm Based on Nash Bargaining Game and Hungarian Method for Wireless Networks

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Abstract — In this paper, the problem sharing resource among selfish nodes and cooperative partner selection are considered in wireless networks. Each wireless node can act as not only a source, but also a potential relay in the system model. The cooperative partners are willing to jointly adjust their power levels and channel bandwidth for cooperative relaying so that an extra rate increase can be achieved. In order to bargain joint bandwidth and power allocation (JBPA) between cooperative partners, a two-user Nash bargaining solution (NBS) is proposed. Then, based on Hungarian method, multiple-user bargaining algorithm and multiple-user grouping algorithm are developed to solve partner selection in large network. By using the proposed multi-user algorithm, the optimal coalitions are formed and the rate increase of overall network is also maximized. Simulation results indicate that the total rate increment based on grouping algorithm is fully close to the optimum, and the resource allocation fairness is dependent on how much rate increase its partner can make to. All in all, based on JBPA, each user negotiates with its partner and a fair NBS is achieved. Then the optimal coalition pairs based on the Hungarian method can be determined with a limited centralized control (such as base station) for the whole network efficiency.

Index Terms — Relay selection, bandwidth allocation, power control, nash bargaining solution

I. INTRODUCTION

Cooperative diversity has been proposed for wireless network applications to enhance system coverage, link reliability and data transmission, and to decrease bit error rate (BER)\(^{[1]}\) in recent years. Generally, all nodes in a non-commercial wireless network are assumed cooperative. The cooperative strategy often benefits the network performance. For example, user cooperation is usually exploited in wireless networks with energy-limited nodes to reduce the whole network energy consumption. Hence, the fairness is not a serious problem in such scenarios. Major relevant literatures in this area are shown in the following. A strategy used for a relay to allocate power among competing users is presented in [2] while the strategies used for competing relays to gain the highest profit in terms of price from offering its power to a single user is given in [3]. The authors in [4] presented an optimum scheme for resource allocation of the relay system with a differential amplify-and-forward (AF) protocol. The research results presented in [2]-[4] were based on an asymmetric model where one user is a source and the other can be a potential relay. In [5], a \(\delta\) - improvement algorithm (DIA) using on a better response dynamic is proposed and it is proved that this algorithm can be guaranteed to converge to energy-efficient and connected topologies. The interaction among users' decisions of power level was studied as a repeated game and a reinforcement learning algorithm to schedule each user's power level based on the theory of stochastic factitious play (SFP) was proposed in [6]. In order to encouraging cooperation, a non-cooperative game theoretic framework was used to establish the critical role of altruistic nodes for small and large scale networks [7].

The studies above focus on strategies to maximize the total transmission rate or minimize the total transmission power of communication networks under some constraints. The formulated problems and their solutions focus on efficiency. The fairness issue was mostly ignored. However, in many practical scenarios, nodes' selfishness raises doubts on whether a relay node would like to spend its valuable resource in forwarding packets for other users.

For a commercial wireless network, all mobile nodes are assumed to be selfish, rational and energy-constrained. Cooperation may cause significant costs and the users bearing the greatest immediate cost may not achieve the greatest immediate benefits. In this case, a mobile user may exhaust all of its valuable resource (For example, energy and frequency spectrum) to relay other users' data, but does not obtain any immediate profits, which hurts the cooperative interests of the selfish users. Therefore, it is necessary for a network to adopt a fair strategy of distributing cooperation gains so that the individual nodes are satisfied immediately.

In our daily life, the market often serves as a central platform where buyers and sellers gather together,\[10.12720/jcm.10.2.124-135\]
negotiate transactions and exchange goods so that they can be satisfied immediately through bargaining and buying or selling. Similarly, the cooperation game theory just provides a flexible and natural tool to explore how the selfish nodes bargain with each other and mutual aid. The pioneering work can be found in the following references. In order to promote cooperation, the authors in [8] presented a price pair incentive mechanism to arbitrate resource allocation. In [9], based on the NBS, the authors proposed a novel two-tier quality of service (QoS) framework and a scheduling scheme for QoS provisioning in worldwide interoperability for microwave access networks. The authors in [10] proposed a cooperation bandwidth allocation strategy for the throughput per unit power increase. In [11], the authors considered a bandwidth exchange incentive mechanism as a means of providing incentive for forwarding data. However, only bandwidth allocation problem was considered to encourage cooperation in [10], [11]. In [12]-[14], the power allocation problem was considered to encourage cooperation. The authors in [12] considered fair power sharing between a user and its partner for an optimal signal-to-noise ratio (SNR) increase. From an energy-efficiency perspective based no NBS, the authors in [13] studied a cellular framework including two mobile users desiring to communicate with a common base station. In order to obtain both user fairness and network efficiency, a cooperative power-control game model based on Nash bargaining was formulated in [14]. Based on the Nash bargaining solution method, the authors in [15] analyzed and formulated multiple resource allocation problems including SA, PA, and simultaneous multi-resource allocation (SMRA) problems into the unified cooperative bargaining game. In order to deal with resource allocation in heterogeneous wireless networks, an algorithm based on multi-leader multi-follower Stackable games model was proposed to satisfy optimal utility of both operators and mobile users in [16].

However, the bandwidth only or power only allocation problem was studied in previous work, ignoring the JBPA in wireless network communication. Furthermore, the cooperative partner selection is a key problem [17] and also ignored when the number of mobile users is no less than three. Motivated by the aforementioned works, we constructed a symmetric wireless system model consisting of two user nodes and two destination nodes, which is shown in Fig.1.

In the model, it is assumed that each user acts as a source as well as a potential relay. Furthermore, the proposed model represents a more general scenario, comparing to previous work. By bandwidth and power exchanging, each user has the opportunity to share the other's resources (e.g., bandwidth and power) and seek other user's help to relay its data to obtain the cooperative diversity, and vice versa. The cooperation degree between partners depends on two factors: one is the bandwidth and power and the other is their channel condition, which can both benefit on the cooperative rate increment. Later, a multi-user bargaining algorithm is proposed based on optimal coalition pairs among users. With the Hungarian method and JBPA scheme obtained by the algorithm, the overall rate for the network increment can be maximized and the resource allocation possesses fairness.

The main contributions of this paper are as follows. In this paper, we study the cooperation for the node pairs based on the NBS obtained from cooperative game theory. The allocation of cooperative gain is fairness and timeliness based on NBS, which is applied to formulate the JBPA problem to guarantee fairness in this paper, i.e. the JBPA problem is formulated as a NBS game. Meanwhile, the overall network rate increase is also maximized for multiple users (the number of users $K>2$).

An optimal JBPA scheme is proposed to achieve an extra rate increase without increasing the total transmit power and the total bandwidth required. To our knowledge, this JBPA problem is still not studied in the previous references.

Since the optimal problem for NBS is no longer concave due to the consideration of the JBPA problem, the determination of the optimal JBPA values is a very difficult task. Therefore, we developed a searching algorithm, which has fast convergence to the optimum. The simulations demonstrated that the JBPA scheme achieves more rate increase.

At last, the partner selection question, the criteria for a node to select its final cooperative partner, is answered for multiple users (the number of user $K>2$). Following this question, a multi-user bargaining algorithm based on optimal coalition pairs among users is proposed to achieve the maximum overall rate increase. The optimal coalitions are formed by using the Hungarian method.

One of advantages of the proposed algorithm lies at its reduced complexity of $O_{K}^{2} K(K-1)M^2 + K^k$, where $K$ is the number of users and $M$ is the number of power levels of each user.

The rest of this paper is organized as follows. In Section 2, the system model is given. In section 3, the utility functions are presented and the JBPA problem is formulated as a $K$-person bargaining game. In Section 4, the joint resource allocation algorithm is presented. Meanwhile, a two-user algorithm and a multi-user algorithm are proposed. In Section 5, simulation result evaluation is given. Finally, this paper is concluded in Section 6.
II. SYSTEM MODEL

There are $K$ source nodes totally in the model. Any two cooperating source nodes, $N_i$ and $N_j$, and their corresponding destinations, $D_i$ and $D_j$ (in particular, $D_i=D_j$), are shown in Fig. 1. They communicate independent information over the orthogonal channels to the destinations.

The AF cooperation protocol is used in the model in two time slots. The system model is based on frequency division multiple access and each user occupies $W$ hertz bandwidth for transmission. The total power consumptions of each user in the two time slots are the same.

A. Cooperative Transmission Case

The details of cooperation between two nodes are illustrated in Fig. 2. Specifically, in time slot 1, node $N_i$ allocates $r_i$ fraction ($r_i \in (0, 1)$) of its bandwidth and 1-$s_i$ fraction ($s_i \in (0, 1)$) of its power $P_i$ to relay $r_i$ fraction of the data from node $N_j$, and it uses the $r_j$ fraction ($r_j \in (0, 1)$) of the bandwidth and $s_j$ fraction ($s_j \in (0, 1)$) of its power for its own data transmission. In time slot 2, node $N_j$ uses $r_i$ fraction ($r_i \in (0, 1)$) of the bandwidth and 1-$s_i$ fraction ($s_i \in (0, 1)$) of its power $P_j$ to forward the data originating from node $N_i$, and it uses the $r_j$ fraction of the bandwidth and $s_j$ fraction of its power for its own data transmission.

According to the cooperation details described above, a relay can forward no more than the amount of data as that originating from the source itself. There is $r_i = 1 - r_j$, which came from the result of [15]. Obviously, both $r_i$ and $r_j$ should be nonnegative for a meaningful cooperation. Then, we have

$$r_i + r_j = 1, r_i > 0, r_j > 0 \quad (1)$$

Suppose that subscript denotes source node and superscript denotes destination node. Let $G_{ij}$ $(i \neq j)$ represents the channel gain between node $N_i$ and node $N_j$, and let $G_{ji}$ denotes the channel gain between source node $N_i$ and destination node $D_j$. We assume that the noise power spectral density at different receivers is independent identical distribution with the $N_0$. The cooperative transmission consists of two stages. In time slot 1, assumed that $x_i$ is the message signal from $N_i$ to $N_j$, and destination $D_j$, then, the achieved SNR helped by $N_j$ for $N_i$ to $D_j$ is given by \[1.3\]

$$\lambda_j^i = \frac{(1-s_j)s_i P_i P_j G_{ij}^2}{\sigma_j^2 [s_i P_j G_{ij} + (1-s_j) P_j G_{ij}^2 + \sigma_j^2]} \quad (2)$$

and the effective rate of node $N_i$ at the $D_i$ is

$$r_i R_j^{\text{cooperative}} = r_i W \log (1 + \lambda_j^i + \lambda_j^i) \quad (3)$$

where $\sigma_j^2 = r_j N_0 W$ and $\lambda_j^i = s_i P_j G_{ij}^2 / \sigma_j^2$ is the SNR that results from the direct transmission (DT) from node $N_i$ to $D_i$ in the first time slot.

Similarly, the relayed SNR helped by $N_i$ for $N_j$ to $D_j$ is given by

$$\lambda_j^i = \frac{(1-s_i)s_j P_j G_{ji}^2}{\sigma_j^2 [s_j P_j G_{ji} + (1-s_i) P_j G_{ji}^2 + \sigma_j^2]} \quad (4)$$

and the effective rate of node $N_j$ at the $D_j$ is

$$r_j R_j^{\text{cooperative}} = r_j W \log (1 + \lambda_j^i + \lambda_j^i) \quad (5)$$

where $\sigma_j^2 = r_j N_0 W$ and $\lambda_j^i = s_j P_j G_{ji}^2 / \sigma_j^2$ is the SNR that results from the DT from node $N_j$ to the $D_j$ in the first time slot.

B. Direct Transmission Case

However, $N_i$ and $N_j$ may prefer transmitting its own data independently, if it could make up the opportunity cost of cooperative transmission by direct transmission, as illustrated in Fig. 3.

Then the DT rate at $D_i$ is

$$R_i^D = W \log \left(1 + \frac{P G_j^2}{\sigma_0^2} \right) \quad (6)$$

And the DT rate at the $D_j$ is

$$R_j^D = W \log \left(1 + \frac{P G_i^2}{\sigma_0^2} \right) \quad (7)$$

where $\sigma_0^2 = N_0 W$ is the AWGN received at the destination $D_i$ and $D_j$ on the condition of no partner for cooperation.

From the above introduction, it's clear that the resource allocation variables $r_i$ and $s_j$ reflect the $N_i$ 's rational decisions while $r_i$ and $s_j$ reflect the decisions of $N_j$, i.e., $N_i$ determines $r_i$ and $N_j$ determines $r_j$, and that the decisions of one user will affect the choices of its partner. Their payoff and payoff should be traded off and both users expect an optimal trade off. The following sections will focus on in particular this problem's solution that can bring about win-win results.
III. UTILITY FUNCTION AND PROBLEM FORMULATION

In this section, the utility functions of the source nodes in the system model are given and then the model is analyzed via the cooperative game theory.

A. Utility Function

For $N_i$ and $N_j$, their utility functions $U_i$ and $U_j$ can be defined as

$$U_i = r_i R_i^D$$

$$U_j = r_j R_j^D$$

(8)

(9)

For the sake of notation simplicity, we define

$$a_i = \frac{P_i G_i^j}{\sigma_0^2}, b_i = \frac{P_i G_i^j}{\sigma_0^2}, c_i = \frac{P_i G_i^j}{\sigma_0^2}$$

$$a_j = \frac{P_j G_j^i}{\sigma_0^2}, b_j = \frac{P_j G_j^i}{\sigma_0^2}, c_j = \frac{P_j G_j^i}{\sigma_0^2}$$

Then, Eq.(8) and Eq.(9) can be rewritten as

$$U_i(r_i, s_i, s_j) = r_i W \log(1 + \frac{a_i s_i}{r_i} + \frac{b_i s_i (1 - s_j)}{r_j} + \frac{c_i s_j (1 - s_j)}{r_j})$$

(10)

$$U_j(r_j, s_i, s_j) = r_j W \log(1 + \frac{a_j s_j}{r_j} + \frac{b_j s_j (1 - s_i)}{r_i} + \frac{c_j s_i (1 - s_i)}{r_i})$$

(11)

Since a node would participate in cooperative transmission only if its effective rate obtained from cooperative transmission is higher than that of DT, it is obvious that any selfish node $N_i$ will give up cooperation when $R_i^D$ is more than its payoff. Therefore, the minimal values $U_i$ and $U_j$ must be

$$U_i^\text{min} = R_i^D$$

$$U_j^\text{min} = R_j^D$$

(12)

(13)

B. Problem Formulation

The bargaining problem based on the cooperative game is described as follows. Define the set of players $\kappa = \{1,2,\ldots,K\}$ in the game. $S$ is a convex and closed subset of $S \subset \mathbb{R}^K$ to denote the set of feasible payoff allocations that the players can achieve if they cooperate. Let $U_i^\text{min}$ represents the minimal payoff that the $i$th player can accept; otherwise, it will quit cooperation. Suppose that $\{U_i \in \mathbb{S} | U_i > U_k^\text{min} \}$ is a nonempty bounded set. Define $U^\text{min} = \{U_1^\text{min}, U_2^\text{min}, \ldots, U_K^\text{min}\}$, then the pair $(S, U^\text{min})$ is called as a $K$-person bargaining problem.

Within the feasible set $S$, NBS provides a unique and fair Pareto optimal operation point. According to Nash's game theory on bargaining problem, in the analysis of the $K$-person bargaining problem, the cooperative solution should satisfy six axioms. Suppose that the cooperative solution is $U^* = (U^*_i, U^*_j)$ and the contextualized formulations of those axioms are given as follows.

1) Individual Rationality: $U_i > U_i^\text{min}$, $\forall i$.

2) Feasibility: $U^* \notin S$.

3) Symmetry: The cooperative solution is not affected when the positions of those two players are exchanged.

4) Pareto Optimality: For every $(\overline{U_i}, \overline{U_j})$, if $(\overline{U_i}, \overline{U_j}) \geq U^*, \forall i, j$, then $(\overline{U_i}, \overline{U_j}) = U^*, \forall i, j$.

5) Independence of Irrelevant Alternatives: If $U^*(S, U^\text{min}) \in S \cup S$, then $U^*(S, U^\text{min}) = U^*(S, U^\text{min})$.

6) Independence of Linear Transformations: For any monotone incremental linear function $F$, we always have $U^*(F(S), U^\text{min})) = F(U^*(S, U^\text{min}))$.

Nash proved the following theorem, showing that there is exactly one NBS satisfying the above axioms.

Theorem 1: Existence and Uniqueness of NBS: There is a unique solution function that satisfies all above six axioms, and this solution satisfies

$$U^* = \arg \max_{U_i \in \mathbb{S} \cap U^\text{min}} \prod_{i=1}^K (U_i - U_i^\text{min})$$

(14)

For the two-person bargaining problem, the NBS function is expressed as

$$U^* = \arg \max_{U_i \in \mathbb{S} \cap U^\text{min}} (U_i - U_i^\text{min})(U_j - U_j^\text{min})$$

(15)

As discussed above, each user has $U_i$ as its objective function, which is bounded above and has a convex, nonempty and closed support. The objective is to maximize all $U_i$ simultaneously and keep fair. $U_i^\text{min}$ denotes the minimal performance (direct transmit performance), and $U_i^\text{min}$ is the initial agreement point. For the problem (14), it is a JBPA problem and its objective function is not concave. Therefore, there might be an infinite number of local maximal points. The problem, then, is to find a simple way to choose the operating point in $S$ for all users, such that this point is optimal and fair. For this hard problem, we will discuss it in the next section.

IV. JOINT RESOURCE ALLOCATION ALGORITHM

Firstly, the two-user case is studied in this section, and a fast two-user bargaining algorithm is proposed. Then, a multiple-user algorithm using coalitions is developed.

A. Bargaining Algorithm for Two-User Case

Since it's impossible to reach the closed-form solution of Eq. (15), we developed a numerical search algorithm, by which the global maximum of Eq. (15) rather than a local maximum can be obtained. According to the decomposition optimization theory [19,20], the optimization problem of Eq.(15) can be equivalently decomposed into the following two problems. Firstly, the bandwidth allocation ratio can be obtained by solving

$$U^*(r_i, r_j, s_i, s_j) = \arg \max_{r_i, r_j \in [0,1]} U(r_i, r_j, s_i, s_j)$$

(16)
where \( U^*(r_i^*, r_j^*, s_i, s_j) \) is the maximal solution for given \( s_i \) and \( s_j \), not the optimal solution. \( r_i^* \) and \( r_j^* \) are the corresponding BA ratios.

Secondly, the optimal PA ratios is obtained by solving
\[
U^*(r_i^*, r_j^*, s_i, s_j) = \arg \max_{v_{s_i, s_j} \in (0,1)} U^*(r_i^*, r_j^*, s_i, s_j) \quad (17)
\]

Then, we compare all \( U^*(r_i^*, r_j^*, s_i, s_j) \) and choose the maximal one for all \( s_i \) and \( s_j \). This way, we can obtain the optimal solution, \( U^*(r_i^*, r_j^*, s_i, s_j) \).

The following gives the proof of Theorem 2 which indicate that Eq.(16) and Eq.(17) both have a unique Nash equilibrium solution.

**Theorem 2:** (Existence of Unique Nash Equilibrium)

For given \( s_i \) and \( s_j \), \( \forall s_i, s_j \in (0,1) \), the two-user bargaining game admits a unique Nash equilibrium solution \( r = (r_i, r_j) \). For given \( r_i \) and \( r_j \), \( \forall r_i, r_j \in (0,1) \), the two-user bargaining game admits a unique Nash equilibrium solution \( s = (s_i, s_j) \).

**Proof:** See Appendix A

In what follows, we firstly put forward a fast iterative algorithm for searching the maximal BA ratio.

For given \( s_i \) and \( s_j \), there exist the corresponding BA ratios \( r_i^* \) and \( r_j^* \). Substituting \( r_i^* \) and \( r_j^* \) into \( R_i^{AF} \) and \( R_j^{AF} \) respectively, we have
\[
R_i^{AF} = W \log (1 + \frac{a_i s_i}{r_i} + \frac{b_i c_i s_i (1 - s_j)}{r_i [b_i s_i + c_i (1 - s_j) + r_i]}) \quad (18)
\]
\[
R_j^{AF} = W \log (1 + \frac{a_j s_j}{r_j} + \frac{b_j c_j s_j (1 - s_i)}{r_j [b_j s_j + c_j (1 - s_i) + r_j]}) \quad (19)
\]

This way, \( R_i^{AF} \) and \( R_j^{AF} \) will not include variables \( r_i \) and \( r_j \). So we have
\[
U^*(r_i^*, r_j^*, s_i, s_j) = \arg \max_{v_{s_i, s_j} \in (0,1)} (r_i R_i^{AF} - R_i^{D})(r_j R_j^{AF} - R_j^{D}) \quad (20)
\]

For problem (20), by taking the derivative to \( r_i \) and \( r_j \) respectively, and equating them to zero, we get
\[
r_i^* = I(r_i, t), r_j(t)) = \frac{1}{2} [1 + \frac{R_i^{D}}{R_i^{P}} - \frac{R_j^{D}}{R_j^{P}}] \quad (21)
\]
\[
r_j^* = I(r_j, t), r_j(t)) = \frac{1}{2} [1 + \frac{R_j^{D}}{R_j^{P}} - \frac{R_i^{D}}{R_i^{P}}] \quad (22)
\]

It is obvious that \( r_i^* + r_j^* = 1 \), which means that there is one variable only between \( r_i^* \) and \( r_j^* \). So the iterations of the BA ratio updating can be expressed as follows
\[
\bar{r}_i(t + 1) = I(r_i(t)) \quad (23)
\]

We show next the convergence of the iterations in (23) by proving that the BA ratio updating function \( I(r_i) \) is a standard function \([21]\).

**Definition 1:** A function \( r_i > 0 \) is standard if for all \( I(r_i) \), the following properties are satisfied \([21]\):

- Positivity. \( I(r_i) > 0 \)
- Monotonic. If \( r_i > r_i' \), then \( I(r_i) > I(r_i') \)
- Scalability. For all \( \alpha > 1 \), \( a I(r_i) > I(\alpha r_i) \)

**Proposition 1:** The function \( I(r_i) \) is standard.

**Proof:** See Appendix B

In \([21]\), a proof has been given. Starting from any initial feasible BA ratios \( r_i \) and \( r_j \), the BA ratios produced after several iterations via the standard BA function always converges to a unique fixed point.

The problem (17) is a combinatorial problem involving two continuous variables, \( s_i \) and \( s_j \). However, continuous power adaptation is infeasible in practical networks because there are only several discrete power levels for each node. Suppose that the number of power levels for each node is \( M \). The computation complexity of searching the PA ratios is \( O(M^2) \). In order to explain the path to obtain the optimal allocation ratios \( r_i^* \), \( s_i^* \), and \( s_j^* \), the iteration searching algorithm is shown in Table I.

### Table I: Two-User Algorithm

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Initialization. Initialize channel state information of ( N_i ) and ( N_j ).</td>
</tr>
</tbody>
</table>
| 2    | For \( s_i \), \( (i=1, 2, \ldots, M) \)  
|      | For \( s_j \), \( (j=1, 2, \ldots, M) \)  
|      | Obtain \( \bar{r}_i \) by using the iterations in (28), and record \( r_i^* \), \( s_i \), and \( s_j \) Calculate the utility \( u_i = (U_i - U_i^{min})(U_j - U_j^{min}) \) and record it into the array \( U_i = (u_i) \). |
| 3    | Subtract the maximal element \( u_i \) and its BA ratio \( r_i^* \) and PA ratio \( s_i \) and \( s_j \). Record them as the optimums \( r_i^* \), \( s_i^* \) and \( s_j^* \). |

### B. Multiple-User Algorithm

A two-step algorithm is proposed for solving the JBPA problem for all users of a \( K \)-node network (\( K > 2 \)) in a centralized way. First, all users are grouped into pairs, in which the cooperating nodes form a coalition. Then, the Hungarian method \([22]\) is employed to solve the assignment problem. By using this algorithm, the overall rate increase can be maximized, but no more energy consumption. First, the strict coalition definition is stated as follows.

**Definition 2:** For a \( K \)-person game, any nonempty subset of the players’ set is called a coalition.

The design objective now is how to form coalitions with size two and maximize the whole network rate increase. On one hand, the power and bandwidth allocation between cooperative user pair is fair; on the other hand, the whole network rate increase can be maximized. So the partner selection question, the criteria for a node to select its final cooperative partner, is put forward.
In general, the channel gains over different users are varying. Many users may prefer a common user to form a coalition with, however only the two-user coalition is admitted. Every user greedily seeks more rate increase. Therefore, if every user is allowed to greedily seek its partner, it must choose a partner who can just help them both to obtain the maximal rate increase, but the whole network rate increase is usually ignored. This problem is illustrated with a simple example as follows.

For example, there are four users. The benefit obtained by the $i$th user from cooperating with the $j$th user is defined as $b_{ij}$. Let $b_{12} = b_{21} = 6$, $b_{34} = b_{43} = 3$, $b_{13} = b_{31} = 5$ $b_{24} = b_{42} = 5$ and other benefit values are zero, then user 1 will prefer user 2 to be his partner driven by a greed to achieve the biggest benefit for himself, and vice versa. At last, user 3 can only choose user 4 as its partner. However, if user 1 seek user 3 as its final partner and user 2 seek user 4 as its final partner, the overall benefit is obviously larger.

Therefore, deciding the coalition pairs can be treated as an assignment problem: a special structured linear programming which is concerned with optimally assigning individuals to activities, assuming that each individual has an associated value describing its rate increase. Many users may prefer a common user to form a coalition with, however only the two-user coalition is admitted. Every user greedily seeks more rate increase.

Now, the assignment problem is formulated in detail. We define the expected benefit for the $i$th user to cooperate with the $j$th user as $b_{ij}$. So, each element in the cost table $b$ can be given by

$$b_{ij} = \max(U_i + U_j - U_j^{\text{min}} - U_i^{\text{min}}, 0)$$

(24)

where $U_i$ and $U_j$ are the rates if the cooperation happens, and $U_j^{\text{min}}$ and $U_i^{\text{min}}$ are the direct transmission rates, respectively. The two-user algorithm proposed in the previous subsection can calculate each $b_{ij}$. It is reasonable that $b_{ij} = 0$. Obviously, $b_{ij} = b_{ji}$, such that $b$ is symmetric. In order to obtain $b$, $K$ users need conduct $K(K-1)/2$ two-user bargaining. The total complexity is $O(K^2 (K-1)M^3)$.

Then, a $K \times K$ assignment Table $X$ is defined, in which each component represents whether or not there is a coalition pair between two users.

$$X_{ij} = \begin{cases} 1, & \text{if user } i \text{ negotiates with user } j \\ 0, & \text{other} \end{cases}$$

(25)

Obviously, $\sum_{i=1}^{K} X_{ij} = 1, \forall i$; $\sum_{j=1}^{K} X_{ij} = 1, \forall i$ and matrix $X$ is symmetric.

So the assignment goal is to assign the nodes into pairs that ensures a maximized overall benefit, which is stated as

$$U_T = \max \frac{1}{2} \sum_{i=1}^{K} \sum_{j=1}^{K} X_{ij} b_{ij}$$

(26)

The optimal coalition pairs can be always found out by the Hungarian method. Since the Hungarian method is used for minimization optimization, The maximization problem in Eq.(26) need be changed into a minimization problem by defining $B_{ij} = \max(b_{ij}) - b_{ij}$. The Hungarian algorithm is explained briefly in Table II.

<table>
<thead>
<tr>
<th>TABLE II: HUNGARIAN METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Subtract the entries of each row by the minimum of this row. Such that: (i) Each row has at least one zero; (ii) All entries are positive or zero.</td>
</tr>
<tr>
<td>2. Subtract the entries of each column by the minimum of this column. So Each row and each column has at least one zero.</td>
</tr>
<tr>
<td>3. Select all rows and columns across which you draw lines, in such a way that all the zeros are covered and that no more lines have been drawn than necessary.</td>
</tr>
<tr>
<td>4. A test condition for optimality. (i) If the number of the lines is $K$, choose a combination from the modified cost matrix in such a way that the sum is zero. (ii) If the number of the lines is less than $K$, go to 5.</td>
</tr>
<tr>
<td>5. First find the smallest element covered by any of the lines. Then subtract it from each entry which is not covered by the lines and add it to each entry which is covered by a vertical and a horizontal line. Go back to 3.</td>
</tr>
</tbody>
</table>

Firstly, $B$ is determined by using the two-user algorithm in Table I, every user need negotiate with other $(K-1)$ users respectively. Then, the assignment Table $X$ is determined via using the Hungarian method. At last, the overall benefit can be calculated by using Eq.(26). Based on the above analysis, the multi-user JBPA algorithm is formulated in Table III.

<table>
<thead>
<tr>
<th>TABLE III: MULTI-USER ALGORITHM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Initialization. Initialize channel state information.</td>
</tr>
<tr>
<td>2. Calculate $B$: (i) Using two-user algorithm to calculate $b = {u_{ij}}$, (ii) $B = {\max(b_{ij}) - b_{ij}}$.</td>
</tr>
<tr>
<td>3. Assignment optimal coalition pairs with Hungarian method Using Hungarian method to determine $X_{ij}$.</td>
</tr>
<tr>
<td>4. Using (31) to obtain the maximal overall benefit.</td>
</tr>
</tbody>
</table>

Each user negotiates with its partner, and they adopt a fair strategy of distributing cooperation gains and the individual nodes are satisfied immediately. So the fairness guarantee can be achieved between two bargaining users. Then, a limited centralized control (such as base station) determines the optimal coalition pairs for the whole network efficiency, which is based on the Hungarian method.

Since the complexity of Hungarian method is $O(K^4)$, the overall complexity of the proposed multiple-user algorithm is $O(K^2 K(K-1)M^3 + K^4)$. However, the Hungarian method needs some limited centralized control (such as base station) to determine the optimal coalition pairs for the whole network efficiency, which is based on the Hungarian method.

The optimal coalition pairs can be always found out by the Hungarian method. Since the Hungarian method is used for minimization optimization, The maximization problem in Eq.(26) need be changed into a minimization problem by defining $B_{ij} = \max(b_{ij}) - b_{ij}$. The Hungarian algorithm is explained briefly in Table II.
pairs. Then, the decision is informed to the relevant users and the optimal power and bandwidth allocation are also informed.

C. Large Network Algorithm

For large networks with size $K$, it is impractical due to the communication overhead of the channel state information (CSI) exchange and the computational overhead of the $K(K+1)/2$ two persons Nash bargaining game.

In order to solve this problem, an iterative algorithm called multiple-user grouping algorithm for large network is proposed in Table IV.

<table>
<thead>
<tr>
<th>Table IV: Multiple Users Grouping Algorithm for Large Networks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <strong>Initialization.</strong> Initialize channel state information.</td>
</tr>
<tr>
<td>2. $K$ nodes are formed. Groups randomly. Use Algorithm 3 in each group respectively.</td>
</tr>
<tr>
<td>3. <strong>While</strong> the total rate increase is not converged <strong>do</strong></td>
</tr>
<tr>
<td>Choose two users randomly in each group which either both have no partners or are partnered.</td>
</tr>
<tr>
<td>The users chosen in group $i$ are moved into group $i + 1$. (If $i = L$, the users are moved to group 1).</td>
</tr>
<tr>
<td>Use Algorithm 3 in each group respectively. <strong>End while</strong></td>
</tr>
</tbody>
</table>

Each time through the loop, the total rate increment of each group is optimized respectively and the total rate increment is non-decreasing. So it is higher bounded by the optimal solution. Thus it remains feasible and converges.

In each subgroup, the number of user is $K/L$. In each iteration, only the users which are moved to a new group need exchange the CSI with others. Therefore, the total communication overhead of CSI exchange is reduced. Accordingly, the computational overhead is also reduced. In addition, another advantage is the algorithm can be run in parallel in each iteration. Such advantage makes the multiple-user grouping algorithm have more flexibility and agility. In Section 5, the simulation results shows that the total rate increment based on grouping algorithm is fully close to the optimum.

All nodes in commercial cooperative wireless networks are assumed selfish, rational and energy-constrained. Therefore, each node could select a partner for rate increase by solving the problem (15). The mobile users in cellular networks can choose a partner for both rate earning. In wireless sensor networks, the proposed approach helps save energy for the sensors and extend the network lifetime. Primary users may select secondary users as partners by employing the proposed scheme, which means that the primary users can obtain rate increase while the secondary users can obtain a chance to transmit its own data. Thus, the proposed scheme is Energy-efficient.

V. SIMULATION RESULTS

The simulation results using the proposed scheme are given in order to evaluate its performance. Firstly, the performance comparisons of the proposed JBPA approach with the DT scheme, the BA scheme and the PA scheme for a two-user case is provided. Then, the simulation results for the multiple-user case are shown.

The following simulations are based on following assumptions: (a) the transmission power for all source nodes is 0.1W and their channel bandwidth W=10$^3$Hz when they transmit data independently. (b) The path gain for all channels is set at $(7.75\times 10^{-3})/d^{4.6}$, where $d$ is the distance (in meters) between a transmitter and the corresponding receiver. (c) The channel between two nodes is described by the distance between them. All channels are assumed to undergo flat fading and are quasi-static. (d) The noise level of the additive white Gaussian noise (AWGN) is $5\times 10^{-14}$W.

A. Two-User Case

A two-source and two-destination network as illustrated in Fig. 4 is built for simulation continuous strategy spaces $r_1$, $r_2$, $s_1$, $s_2$. Destination nodes $D_1$ and $D_2$ are located at points (1200, 0) and (0, 0) respectively, while source nodes $N_1$ and $N_2$ at (400, 0) and (800, 0) respectively. $N_1$ is gradually moved upwards to point (400, 500). The JBPA of $N_1$ and $N_2$ is simulated for different $N_1$ positions along the line from (400, 0) to (400, 500).

Letting $Y_1$ denote the y coordinate of $N_1$, Fig. 5 shows the NBS strategies, i.e., the optimal JBPA ratios $(r_1, r_2, s_1$ and $s_2$) of both nodes when $N_1$ moves.

It can be seen that with the movement of $N_1$ from (400, 0) to (400, 500), $N_1$’s BA ratio $r_1$ is decreasing and that of $N_2$ $r_2$ is increasing correspondingly and that $r_1 < r_2$. This is because $N_2$’s channel condition is better than that of $N_1$ due to its better position in that region and because a higher increase of cooperative rate can be achieved if $N_2$ has a bandwidth higher than that of $N_1$. Therefore, $N_2$ can
contribute more to the increase of cooperative rate. Interestingly, \( N_1 \) and \( N_2 \) consume most of their own power for themselves and a small part of their power for its partner. Specifically, \( N_2 \) always allocates more power for its own data transmission and less power to relay for its partner compared to \( N_1 \). The reasons for this come from two aspects. On one hand, since \( N_2 \) has a better channel than \( N_1 \), though \( N_2 \) allocates less power than \( N_1 \) to relay data for its partner, \( N_2 \) is also able to bring about a higher rate increase for its partner than \( N_1 \). On the other hand, \( N_1 \) has to allocate much more power than \( N_2 \) to relay data for its partner so that it can to bring about the same rate increase for its partner. Therefore, it’s reasonable and fair for \( N_2 \) to allocate more power to transmit its own data and less power to relay for the sake of maximizing cooperative diversity.

Fig. 6 shows the comparison of the performance obtained by each node under different working modes, i.e. the proposed JBPA scheme, the DT scheme, BA scheme and PA scheme. Specifically, Fig. 6 (a) shows the comparison of four kinds of rates of node \( N_1 \) along the movement of \( N_1 \) from (400, 0) to (400, 500), and Fig. 6 (b) shows that of node \( N_2 \). When \( N_1 \) moves from (400, 0) to (400, 500), \( N_1 \) will cooperate with \( N_2 \) as long as the cooperation conditions are satisfied.

Fig. 6. Comparison of four kinds of rates

It can be seen in Fig. 6, the optimal rate obtained by the JBPA scheme, the BA scheme or the PA scheme is bigger than that by the DT scheme because cooperation occurs only when partner can both acquire a benefit from it. Therefore, \( N_1 \) and \( N_2 \) both has a higher transmission rate when they cooperate than when they transmit independently and directly. As shown in the Fig. 6, the optimal rate obtained by the JBPA scheme is always bigger than that by BA scheme and that by PA scheme. The BA scheme for each node is obtained via BA optimization under a fixed PA (e.g. 1:1) while the PA scheme is obtained via PA optimization based on a fixed BA (e.g. 1:1). The JBPA scheme is achieved based on optimizing JBPA. Therefore, it can be concluded that the JBPA scheme is the best strategy to maximize rate increment.

The rate increments of \( N_1 \) and \( N_2 \) are given in Fig. 7. The DT rates of \( N_1 \) and \( N_2 \) are shown in Fig. 6. As it can be seen in Fig. 7, the cooperative rate increment of \( N_1 \) is always no less than that of \( N_2 \). When \( N_1 \) is at (400,0), the channel conditions and initial resources of \( N_1 \) and \( N_2 \) are the same. Accordingly, the initial cooperative rate increment of \( N_1 \) equates that of \( N_2 \). With the movement of \( N_1 \) from (400, 0) to (400, 500), the cooperative rate increment of \( N_2 \) is always less than that of \( N_1 \). This is because \( N_2 \) pays out more power and bandwidth than \( N_1 \) to maintain their cooperation, as shown in Fig. 5. From a fairness perspective, \( N_1 \) deserves the bigger rate increase. On the other hand, since the channel condition of \( N_2 \) is better than that of \( N_1 \), \( N_2 \)'s DT rate is higher than that of \( N_1 \). Thus, the cooperative rate of \( N_2 \) should be bigger than that of \( N_1 \), which is shown in Fig. 6. In short, \( N_1 \) obtained a higher cooperative rate increment than \( N_2 \), but its rate is always no more than that of node \( N_2 \). Therefore, the NBS reflects not only the effect of the channel condition of each node on its cooperation benefit in terms of cooperative rate increment, but also the effects of the bandwidth and power contribution of each node on the cooperative rate increment to a great degree.

Fig. 7. The fairness of cooperative rate allocation

From Fig. 5 to Fig. 7, it is concluded that the proposed JBPA scheme could optimize the system performance while keeping the NBS fairness. The NBS fairness is embodied by the fact that the cooperative rate of each node is fundamentally determined by its channel condition, and that the cooperative rate increment of each node depends on its bandwidth and power contribution to maintain the cooperative transmission.

Simulations for node \( N_1 \) at (400,150), (400,300) and (400,450) are conducted respectively. It is seen from Fig. 8 that the proposed iterative algorithm has fast
convergence to the bandwidth allocation ratio $r^*_2$. In detail, it takes less than 15 iterations for $r^*_2$ to converge to the optimum.

**B. Multiple-User Case**

Multi-users simulations were conducted to test the proposed multi-user algorithm. As is shown in Fig. 9 (a), there are $K(K > 2)$ source nodes and four destination nodes. The four destination nodes are located at (800m, 800m), (800m, 0m), (0m, 800m) and (0m, 0m), respectively, and the source nodes are randomly located within the range of [0m, 800m] in the $x$-axis and [0m, 800m] in the $y$-axis.

In Fig. 9 (a), the nine users transmit data directly and randomly to their destinations with no cooperation. As is shown in Fig. 9 (b), based on Hungarian method, node 1, 7 and 4 are finally assigned to bargain with 2, 8 and 9, respectively, and node 3, 5 and 6 lost cooperative chance. It can be seen that node 9 is preferred by Node 4 and 5 to form coalitions with and that node 9 prefers node 4. Therefore, node 9 and node 4 form a coalition pair and node 5 lost cooperative chances as a result of the competition of node 4.

Define $R_A = \frac{U_r}{K} = \arg \max \frac{1}{2K} \sum_{i=1}^{K} \sum_{j=1}^{K} X_i h_j$, which represents the average rate increase per user. Monte Carlo experiments consisting of 1000 independent trials were carried out to obtain the average results. Fig. 10 shows the probabilities of a node to cooperate for different values of $K$. With the increase of $K$, the probability of a node to cooperate goes up. This is because each user is more possible to find a partner with the increasing of the number of users.

Fig. 11 shows the $R_A$ for different numbers of users. The increasing of the number of users improves the $RA$, which is the result of two reasons. Firstly, the more the users, the easier a user can be assigned a better partner to form the final coalition pair. Secondly, with the number of users increasing, more users are assigned to take part in cooperation and some even have a better partner to cooperate with. Therefore, the average benefit per user increases with the number of users increasing.
Simulations for $K=20$, $L=2$ and $K=24$, $L=3$ are conducted respectively. 1000 independent Monte Carlo trials were conducted to investigate the convergence rate. It is seen from Fig. 12 that the proposed multiple-user grouping algorithm has fast convergence. In detail, it takes about 5 iterations to converge very close to the optimum.

All in all, each user negotiates with its partner and a fair NBS is achieved, and the individual nodes are satisfied immediately. So the fairness guarantee can be achieved between two bargaining users. Based on the Hungarian method, a limited centralized control (such as base station) determines the optimal coalition pairs for the whole network efficiency. This method can solve the communication overhead of the channel state information (CSI) exchange and the computational overhead of the $K(K+1)/2$ two persons Nash bargaining game.

![Fig. 12: The average rate increase per node versus number of users.](image)

### VI. CONCLUSION

This paper analyzed the cooperative action of selfish nodes in cooperative communication networks. The JBPA problem between cooperating nodes is formulated as a cooperative game, and the NBS function is used to obtain the solution of the game. First, a two-user algorithm is developed for JBPA bargaining between two users. Then, a multi-user bargaining algorithm is presented. It utilizes the two-user algorithm to obtain the cost table $b$, and the Hungarian method to determine optimal bargaining pairs among users. Third, simulations are carried out to validate the proposed algorithms. Simulation results show that the JBPA scheme obtained by the proposed multi-user algorithm has the NBS fairness, and can achieve a more desirable performance than the DT scheme, the BA scheme and the PA scheme. Simulation results also demonstrate that the average benefit per user goes up with the increased number of users.

All in all, the novel multi-user algorithm can solve the JBPA problem while guaranteeing the NBS fairness. The overall network rate increase in the same power conditions can decrease energy consumption. The large network algorithm of multi-user algorithm can solve the communication overhead of the channel state information (CSI) exchange and the computational overhead of the $K(K+1)/2$ two persons Nash bargaining game.

Our future works will focus on other more complex cooperation modes and coalition game. The cooperation mode between node pairs in this paper will be a fundamental work for thorough research.

### APPENDIX A: PROOF OF THEOREM 2

Observe that the constraint set is convex. So if $U_i(r_i)$ is proved to be concave for $r_i$, Theorem 2 will be proved. For simplified representation, we define $A_i=1+\frac{a_i S_i}{r_i}+\frac{B_i}{r_i (C_i + r_i)}$, $B_i = b_i c_i S_i (1 - S_i)$ and $C_i = b_i S_i + c_i (1 - S_i)$. So we have

$$\frac{\partial U_i}{\partial r_i} = W \log A - \frac{WA_i - a_i S_i B_i (C_i + 2r_i)}{r_i C_i}$$

and

$$\frac{\partial^2 U_i}{\partial r_i^2} = \frac{2B_i}{\ln (C_i + r_i)} - \frac{WA_i - a_i S_i B_i}{r_i (C_i + r_i)}$$

then $\frac{\partial^2 U_i}{\partial r_i^2} < 0$. Therefore, $U_i(r_i)$ is concave for $r_i$.

For given $r_i$ and $r_j$, $\forall r_i, r_j \in (0, 1)$, there are two variables $s_i$ and $s_j$ in Eq. (22).

Observe that the constraint set is convex. So if $U_i(s_i, s_j)$ is proved to be concave for $s_i$ and $s_j$, Theorem 2 will be proved. For simplified representation, we define $f_2(s_i)=1/s_i$ and $f_2(s_i)=1/(1-s_i)$. For $f_2(s_i)$ and $f_2(s_i)$ are convex. So $f_2(s_i, s_j)=b_i f_2(s_i)+c_i f_2(s_j)+r_i f_2(s_i) f_2(s_j)$ is convex, and $f_2(s_i, s_j)$ is concave. So we have

$$U_i(s_i, s_j) = r_i W \log [1 + \frac{a_i S_i}{r_i} + \frac{b_i c_i}{r_i f_2(s_i, s_j)}]$$

Furthermore, $f(u) = \log(1+u), \forall u > 0$, is monotone increasing concave function. Since the compound function $f(u)=\log(1+u(x,y))$ is concave if $u(x, y)$ is concave and $u(x, y)>0$. Considering a positive linear combination of concave functions is concave, $U_i(s_i, s_j)$ is proved to be concave for $s_i$ and $s_j$.

### APPENDIX B: PROOF OF PROPOSITION 1

1. Positivity. It is obvious that $I(r_i) > 0$.

2. Monotonicity. If $r_i > r_i'$, then $I(r_i) < I(r_i')$.

For $R^F_i$ and $R^F_j$, by taking the derivative to $r_i$ respectively, we have

$$\frac{\partial R^F_i}{\partial r_i} = \frac{WA_i}{\ln 2} \left[ \frac{a_i S_i (1 - S_i)}{r_i} + \frac{b_i c_i S_i (1 - S_i)}{r_i^2} \right]$$

and

$$\frac{\partial R^F_j}{\partial r_i} = \frac{WA_i}{\ln 2} \left[ \frac{a_i S_i (1 - S_i)}{r_i (1 - r_i)} + \frac{b_i c_i S_i (1 - S_i)}{r_i (1 - r_i)^2} \right]$$
So $R_{ij}^{sf}$ is monotone decreasing function and $R_{ij}^{af}$ is monotone increasing function for $r_i$. Then, $R_{ij}^{af}(R_{ij}^{af})^{-1} - R_{ij}^{sf}(R_{ij}^{sf})^{-1}$ is monotone increasing function for $r_i$. Therefore, $I(\tilde{r}_i)$ is monotone increasing function for $\tilde{r}_i$.


For all $\alpha > 1$, let $\Delta I = \alpha I(\tilde{r}_i) - I(\alpha \tilde{r}_i)$.

Since $U_i(r_i)$ is monotone increasing function for $r_i$, $U_i / (1 - r_i)$ is monotone decreasing function for $r_i$. So we have

$$\alpha R_{ij}^{af}(\alpha \tilde{r}_i) = \frac{\alpha^2 \tilde{r}_i R_{ij}^{af}(\alpha \tilde{r}_i)}{\tilde{r}_i} < \frac{\alpha^2 \tilde{r}_i R_{ij}^{af}(\tilde{r}_i)}{\tilde{r}_i}$$

Then, we have

$$\Delta I = \frac{\alpha - 1}{2} + \frac{R_{ij}^{af}(\tilde{r}_i) - R_{ij}^{af}(\alpha \tilde{r}_i)}{2R_{ij}^{af}(\alpha \tilde{r}_i) R_{ij}^{af}(\tilde{r}_i)}$$

For $(1 - \alpha \tilde{r}_i) R_{ij}^{af}(\alpha \tilde{r}_i) > R_{ij}^{af}(\alpha \tilde{r}_i)$, which is the cooperation condition, we can claim that $\alpha I(\tilde{r}_i) > I(\alpha \tilde{r}_i)$.

REFERENCES


complex networks, network resource management in heterogeneous networks, wireless communications and networks, and game theory.

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